Mathematical Model for Vision Corporation Allocation Problem

Sets

P : Set of plants, indexed by p
R : Set of products, indexed by r
C : Set of customers, indexed by c

Parameters

L_{pr} : Labor hours required to produce one unit of product r at plant p
M_{pr} : Machine hours required to produce one unit of product r at plant p
T_{pr} : Material required to produce one unit of product r at plant p
LC_p : Total labor hours available at plant p
MC_p : Total machine hours available at plant p
TM : Total material available across all plants
PC_{pr} : Production cost of one unit of product r at plant p
SP_{rc} : Sales price of one unit of product r to customer c
SC_{pc} : Shipping cost of one unit of product from plant p to customer c
D_{rc} : Demand for product r by customer c
IC : Inspection capacity (total units that can be inspected)

Decision Variables

x_{pr} : Number of units of product r produced at plant p
y_{prc} : Number of units of product r shipped from plant p to customer c

Objective Function

$$\text{Maximize } Z = \sum_{p\in P}\sum_{r\in R}\sum_{c\in C} SP_{rc}\cdot y_{prc} - \sum_{p\in P}\sum_{r\in R} PC_{pr}\cdot x_{pr} - \sum_{p\in P}\sum_{r\in R}\sum_{c\in C} SC_{pc}\cdot y_{prc}$$

Constraints

1. **Labor Capacity Constraints:**

$$\sum_{r\in R} L_{pr}\cdot x_{pr} \le LC_p \quad \forall p \in P$$

2. **Machine Capacity Constraints:**

$$\sum_{r \in R} M_{pr} \cdot x_{pr} \leq MC_p \quad \forall p \in P$$

3. **Material Constraints:**

$$\sum_{p \in P} \sum_{r \in R} T_{pr} \cdot x_{pr} \leq TM$$

4. **Demand Satisfaction:**

$$\sum_{p \in P} y_{prc} \leq D_{rc} \quad \forall r \in R, \forall c \in C$$

5. **Production Balance:**

$$\sum_{c \in C} y_{prc} = x_{pr} \quad \forall p \in P, \forall r \in R$$

6. **Inspection Capacity:**

$$\sum_{p \in \{1,2\}} \sum_{r \in R} \sum_{c \in \{RAYco, HONco\}} y_{prc} \leq IC$$

Non-negative Integer Constraints

$$x_{pr} \in Z_{\geq 0} \quad \forall p \in P, \forall r \in R$$

$$y_{prc} \in Z_{\geq 0} \quad \forall p \in P, \forall r \in R, \forall c \in C$$