Computational Photography
Prof. Manuel Menezes de Oliveira Neto
Programming Assignment
Discrete Fourier Transform

Total of Points of the Assignment: 100

The goal of this assignment is to allow you to **explore and verify several of the properties of the Discrete Fourier Transform (DFT) and its inverse** that have been presented in class. The following tasks can be easily implemented in MATLAB, but you can use any library of your preference to compute the DFT and its inverse.

1. (10 points). Given an input image f, **compute its DFT and then reconstruct two separate images** obtained using: (**1) only the real components of the DFT**; and **(2) only the imaginary components**. The images below illustrate the results obtained for the image of the cameraman. **Explain why the images seem to contain mirrored copies and why one looks much darker**.

Reconstruction using the real components only.

Reconstruction using the imaginary components only.

2. (40 points). Given an input image f, perform the following tasks:

(a) Compute the DFT using MATLAB and **reconstruct the original image manually from its cosine and sine DFT coefficients**;

(b) **Compute the DFT manually by projecting it onto cosine and sine basis functions**, and use MATLAB to reconstruct the original image using the inverse DFT (IDFT);

(c) **Compute the DFT manually** as done for (b) **and use its coefficients to reconstruct the original image manually**, as done for (a);

(d) **Compare the original image with the ones reconstructed in parts (a), (b), and (c)**, by displaying them side by side. They should be equal.

Performing the DFT and IDFT manually involve computationally expensive operations. Thus, for items (a) to (d), **use the reduced-size version of the cameraman image provided with the assignment**. After completing these tasks, you will have a good appreciation for the value of the Fast Fourier Transform (FFT) algorithm.

Remember that **MATLAB computes the DFT and IDFT** (through the commands fft2 and iffft2, respectively) **asymmetrically**, as:

$$F(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) e^{-i2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

$$f(x,y) = \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u,v) e^{i2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

3. (10 points). Explore the role of the DC term (*i.e.*, F(0,0)) by:

(a) Compute the DFT of an image f. Then set the DC term to zero. Finally, reconstruct a new image (IDFT) from the resulting spectrum;

(b) Compute the average intensity level of image f. Subtract such average intensity from all pixel values;

(c) Compare the results from (a) and (b). What do you get?

4. (10 points). Verify the effect of the **fftshift** command:

(a) Compute the DFT of an image f. Apply the fftshift command to the spectrum of f. Reconstruct an image from the resulting fftshifted spectrum;

(b) Compute a new image $g(x,y) = (-1)^{(x+y)} f(x,y)$ and compare it with the result obtained in (a);

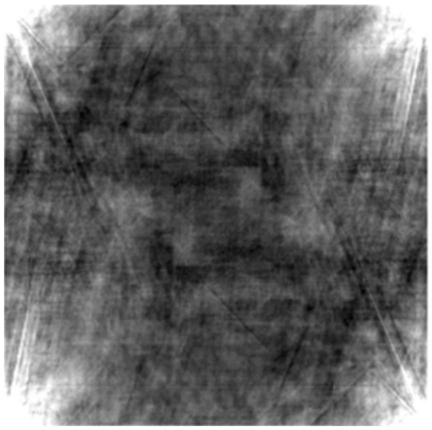
(c) What will happen in the frequency domain if you apply the fftshift command to image f (in the space domain)?

5. (30 points). Amplitude versus Phase Spectra. Given an input image f, **compute its DFT. F(u,v) can be expressed as $F(u,v) = |F(u,v)|\ e^{i\theta}$. Then reconstruct two separate images** (using the IDFT) obtained using:

(a) **Only the amplitude information**, i.e, assuming that $\theta=0$: $F(u,v) = |F(u,v)|\ e^{i0} = |F(u,v)|$;

(b) **Only the phase information**, i.e., assuming that $|F(u,v)|=1$: $F(u,v) = e^{i\theta}$. The images below illustrate the results obtained for the image of the cameraman. As you can see, **the phase information is responsible for the structure of the image**.

Tip: Check the meaning of the MATLAB commands **abs** and **angle**.



Reconstruction using only the amplitude information.

Reconstruction using only the phase information.

Handing in your Assignment

Submit a compressed file (e.g., .ZIP) containing an illustrated report (PDF) describing your results, observations, and conclusions, and the source code (e.g., MATLAB scripts) written to perform these operations.