Validate Power: Westfall-Young

December 31, 2021

Explanation

Westfall-Young procedures

The Westfall-Young procedure was validated separately due to its unique complications and computational burden.

First, we wrote a series of careful unit tests ensuring that our code worked as expected for each step of the WY procedure. One of these tests compared results from our procedure to the WY procedure in the multtest package, and found it matched quite well.

Second, we also tested the WY procedures using the same simulation procedure described above. For constant effect and fixed effect models, the power estimation matches well between PUM and the simulations. However, for random effects models, the two methods diverge in some cases. We find that the simulated power matches the `PUMP`-calculated power only when (1) there is a large number of blocks/clusters, and (2) the user has a large number of WY permutations. Below, we discuss the single-step procedure because it is simpler, although the same concepts hold for the step-down procedure. Although it is difficult to verify why this discrepancy occurs, our hypothesis is that this behavior occurs due to the combination of the sensitivity of the WY procedure, and the instability of the random effects model.

The goal of the WY procedure is to estimate the adjusted p-value for outcome i

$$\tilde{p}_i = Pr\left(\min_{1\leq j\leq k} P_j \leq p_i \mid H_0^C\right).$$

where P_j is a random variable representing a p-value for outcome j, and lowercase p_i is the observed realization for outcome i. We assume a set of k tests with corresponding null hypotheses H_{0i} for $i = 1, \cdots, k$. The complete null hypothesis is the setting where all the null hypotheses are true: $H_0^C = \cap_{i=1}^k H_{0i} = \{\text{all } H_i \text{are true}\}$. In order to estimate this p-value, the p-value is calculated across B permutations

$$\tilde{p}_i = \frac{1}{B}\sum_{b=1}^{B} \mathbb{1}(\min_{1\leq j\leq k} p^{\star}_{b,j} \leq p_i).$$

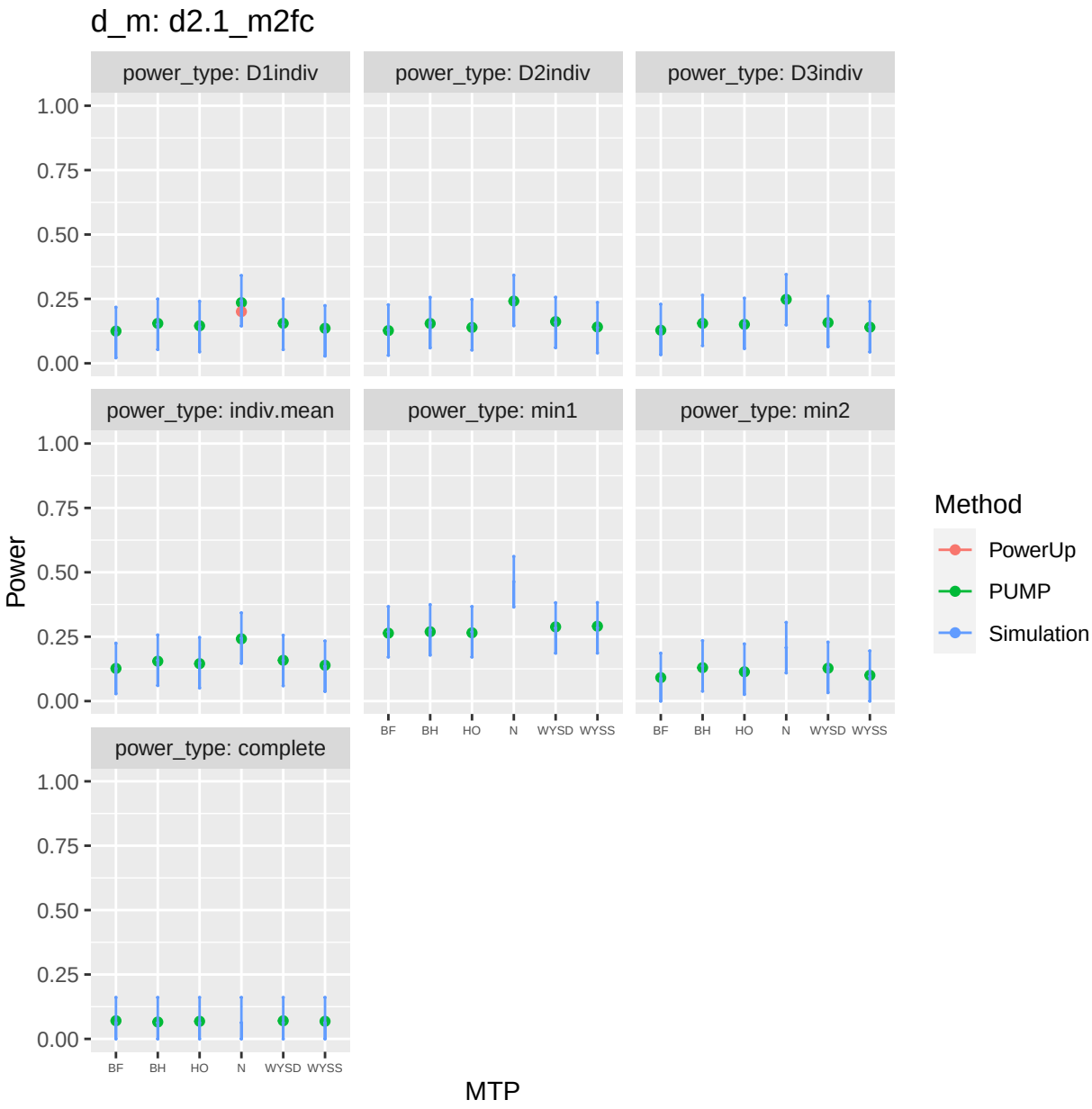
In order for the procedure to be correct, we should be generating the p-values P_j from the the true distribution of p-values under the null hypothesis. We generally are testing outcomes that are truly significant, so the observed p-values are small. Thus, looking at these expressions, we are concerned about tail behavior; we are testing whether a small observed p-value is less than the minimum of of the generated p-values of our multiple outcomes. This combination of factors means that a small discrepancy in the tails between the generated null distribution and the true distribution could have a large impact on the adjusted p-value.

Given that we are relying on tail behavior, this explains why we need both a large number of permutations, and a large number of blocks/clusters. The large number of permutations is required because we are trying to estimate a rare event. With a significant outcome, it is rare that the observed p-value will be less than the minimum of p-values generated from the null distribution. The large number of blocks/clusters is required to accurately estimate the tail behavior for random effects models. With a small number of blocks/clusters, we may not properly estimate the spread of the distribution, resulting in a poor estimate of the tail behavior.

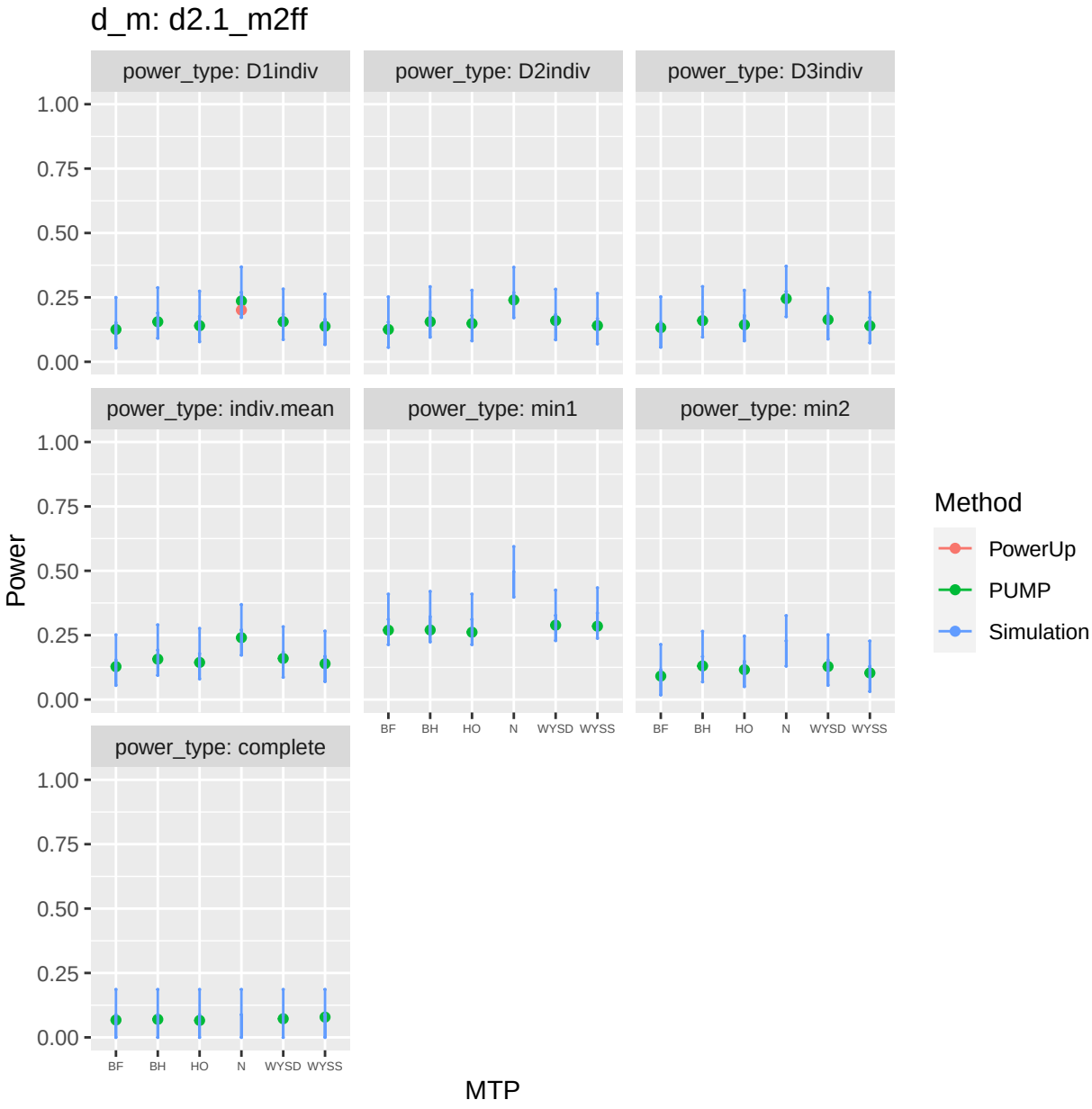
Remark due to computational burden, for certain models we only tested WY-SS and not WY-SD. In some cases, we had to reduce the number of simulations, number of permutations, or the number of units in order to run a validation that was not computationally intractable.

d2.1 models

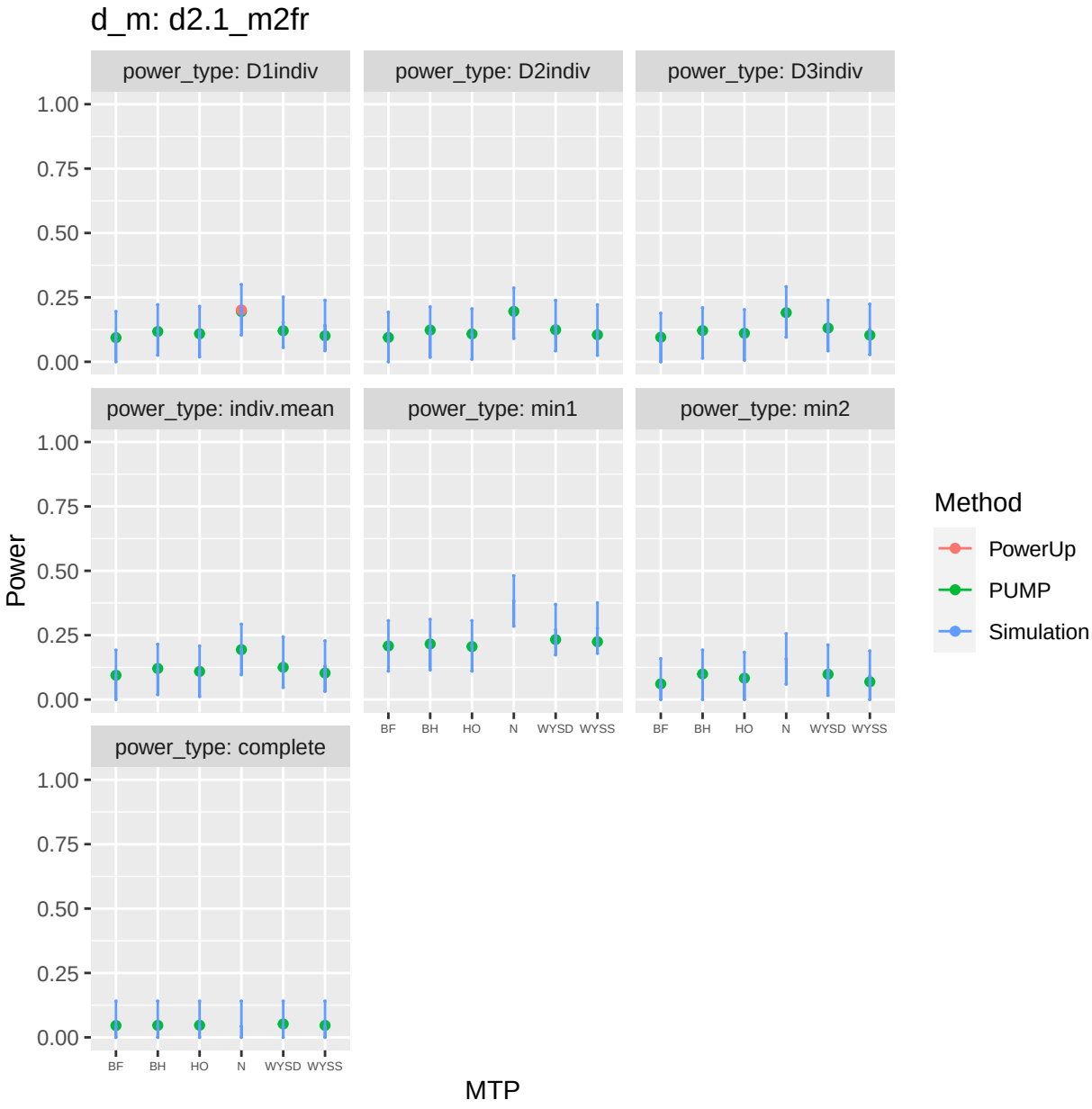
Constant effects



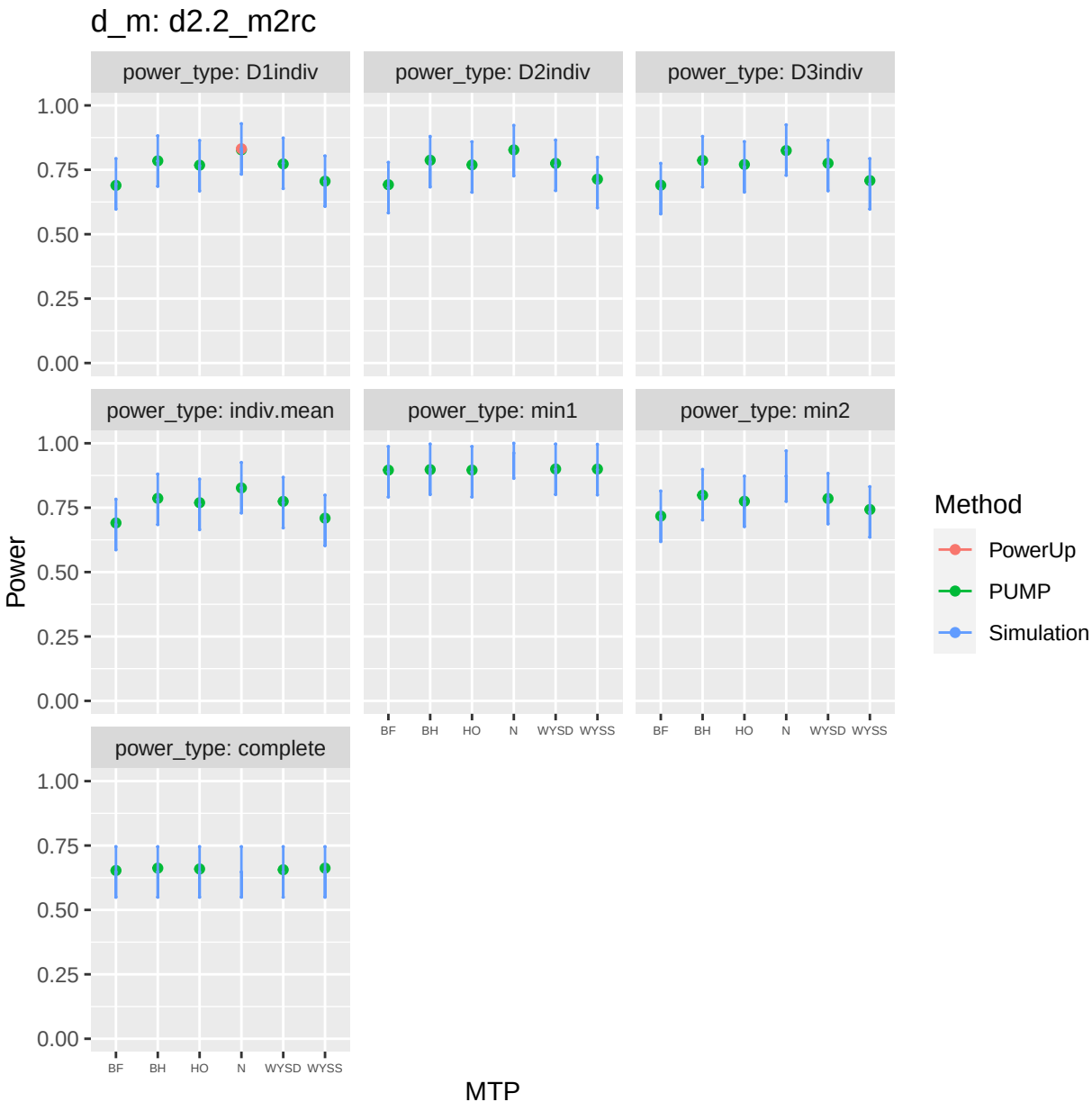
Fixed effects



Random effects



d2.2 models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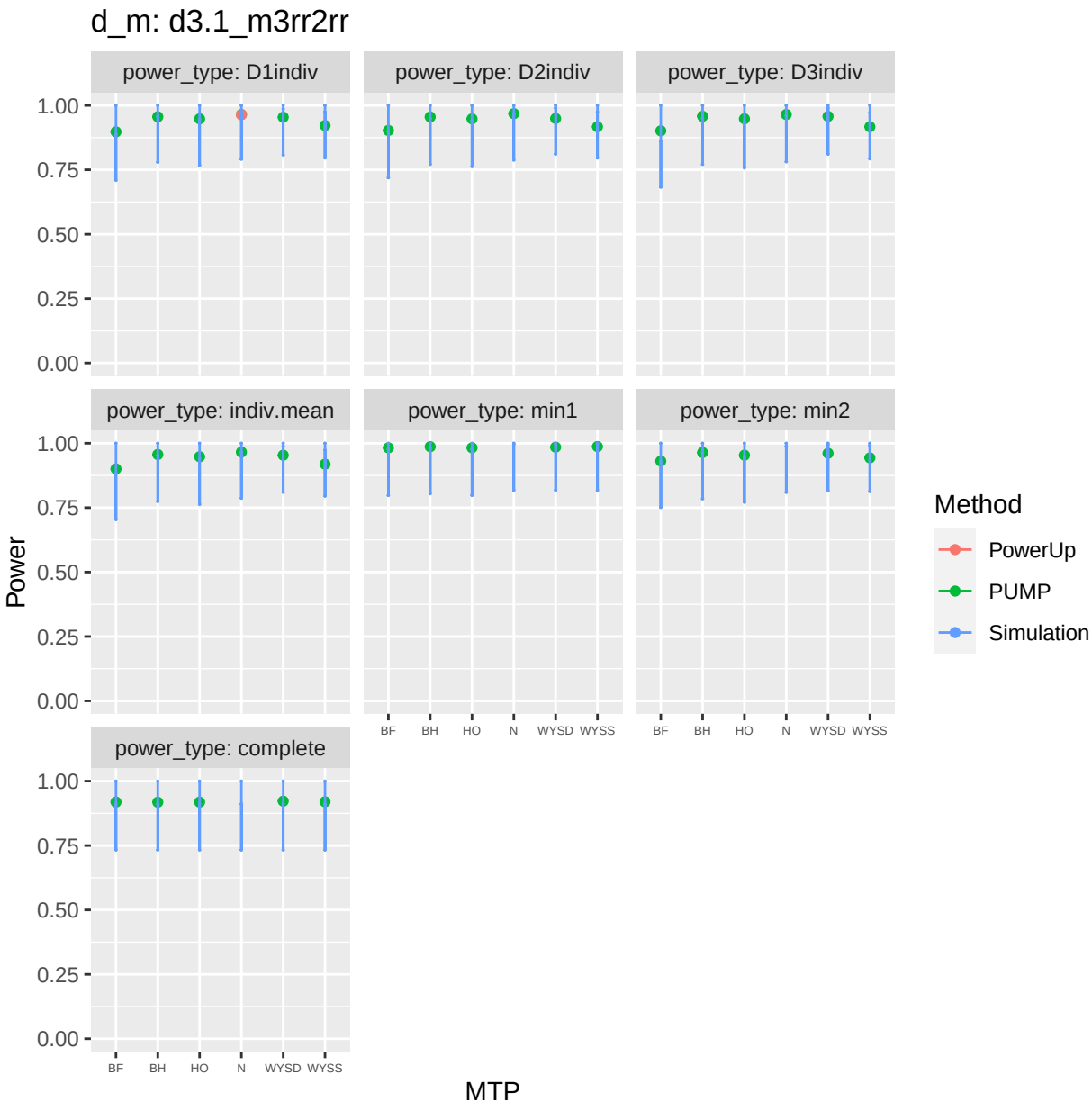
power_type: min2
power_type: complete
Power
1.00
0.75
0.50
0.25
0.00
BF
BH
HO
N
WYSD
WYSS
MTP
Method
PowerUp
PUMP
Simulation

d3.1 models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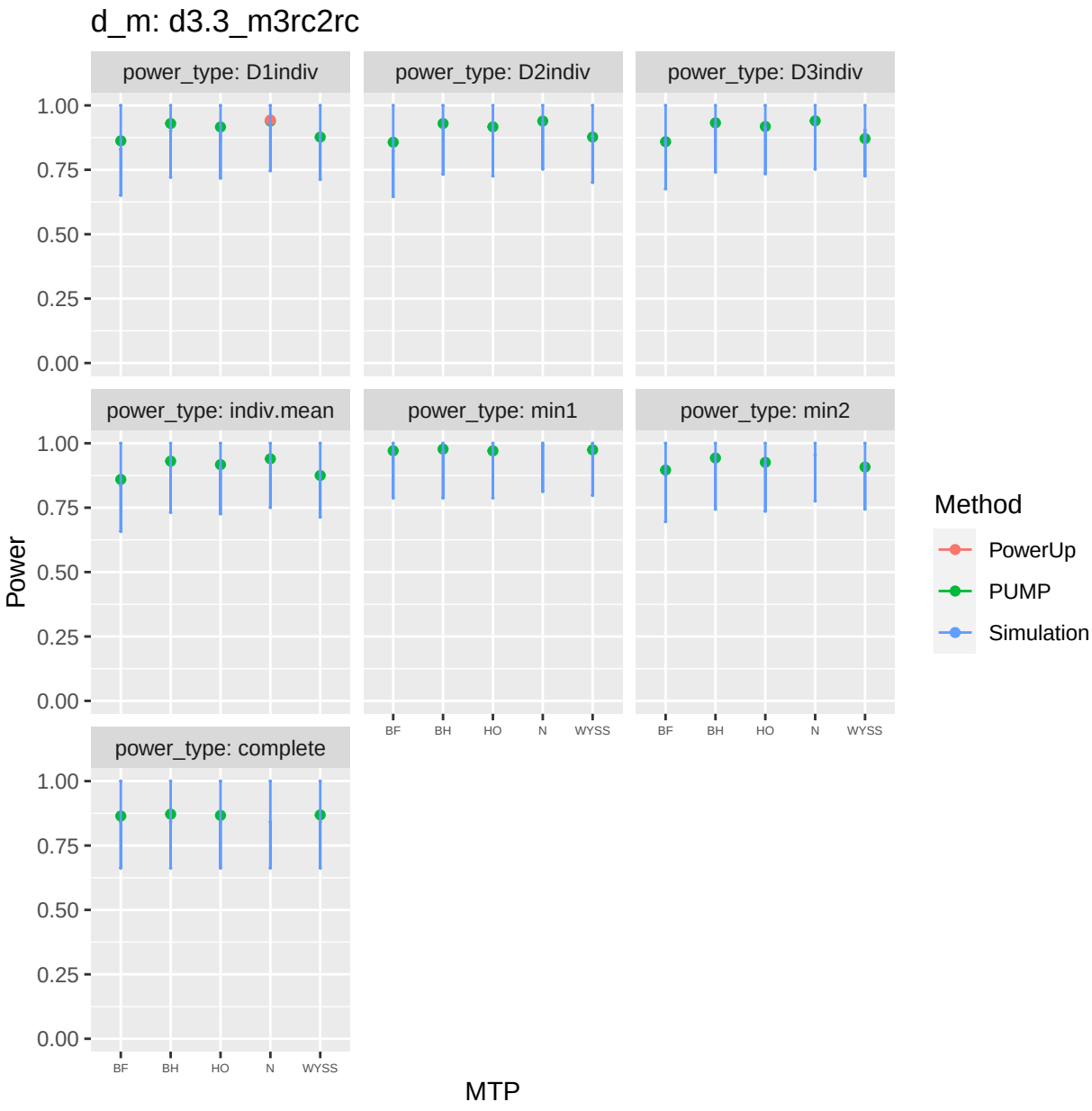
power_type: indiv.mean
power_type: min1
power_type: min2
power_type: complete
Power
1.00
0.75
0.50
0.25
0.00
BF
BH
HO
N
WYSD
WYSS
MTP
Method
PowerUp
PUMP
Simulation

d3.3 models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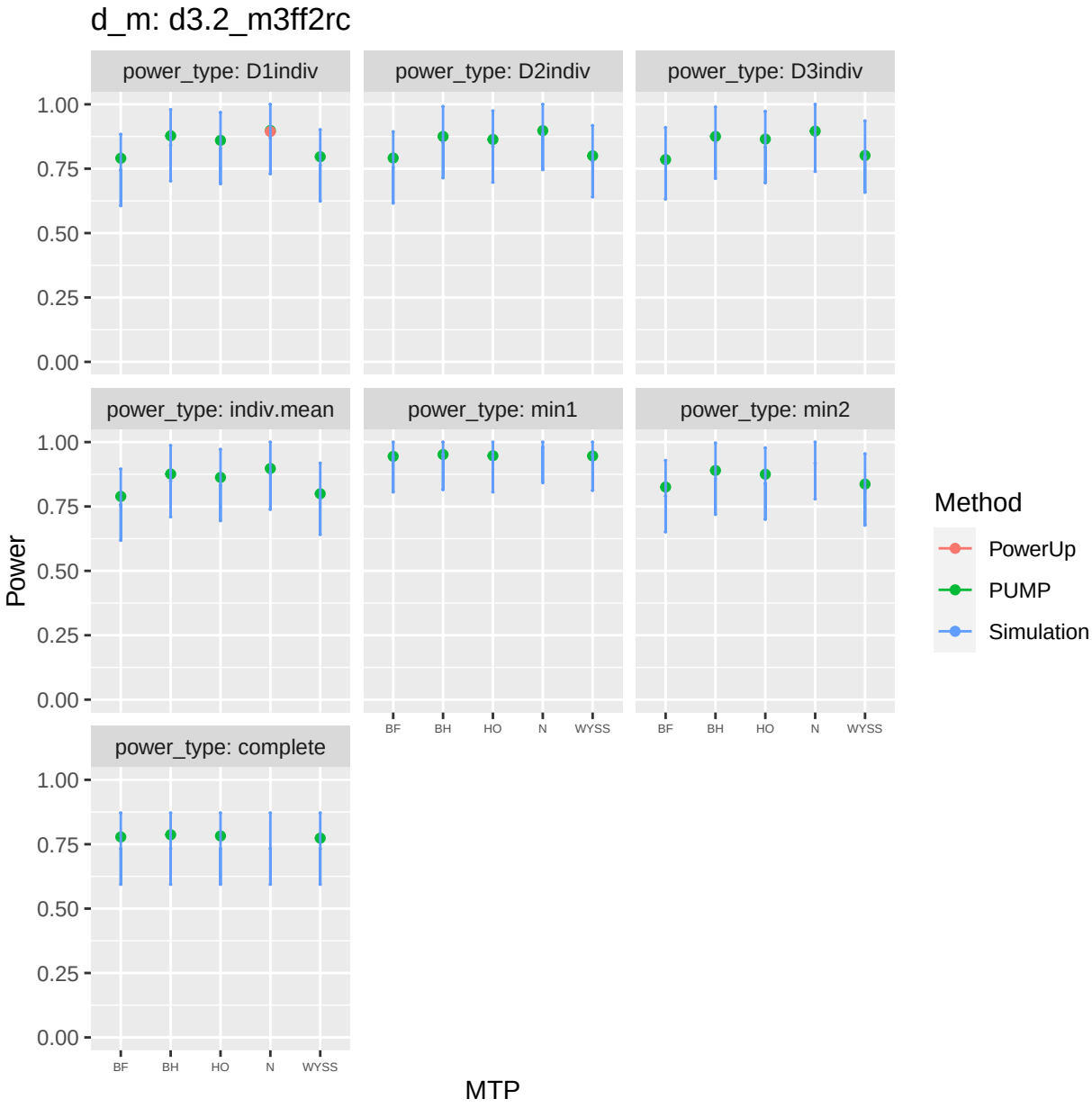
power_type: D3indiv
power_type: indiv.mean
power_type: min1
power_type: min2
power_type: complete
1.00
0.75
0.50
0.25
0.00
Power
BF
BH
HO
N
WYSS
MTP
Method
PowerUp
PUMP
Simulation

d3.2 models

Constant effects



Random effects