Problem-1:

(A) Let the expected numbers of points be Y

Dice Face	Value	Probability
1	0	1/6
2	Y−A	1/6
3	Y	1/6
4	Y	1/6
5	Y	1/6
6	Y+B	1/6

$$E(x) = \sum_{\forall i} x_i \, P(x_i) = Y$$

$$\Rightarrow Y = 0 + \frac{1}{6} * (Y-A) + 3 * Y * \frac{1}{6} + \frac{1}{6} * (Y+B)$$

$$\Rightarrow Y = \frac{1}{6}Y - \frac{1}{6}A + \frac{1}{2}Y + \frac{1}{6}Y + \frac{1}{6}B$$

$$\Rightarrow Y = \frac{5}{6}Y - \frac{1}{6}A + \frac{1}{6}B$$

$$\Rightarrow \frac{1}{6}Y = \frac{1}{6}B - \frac{1}{6}A$$

$$\Rightarrow \boxed{Y = B - A}$$ (Ans)

(B)

Bayes' rule:

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

let events C be that Chipotle is selected
" F " " Five Guys " "
" W be that review is positive

$$P(C) = P(F) = \frac{1}{2} \quad \text{(prior)}$$

$$P(W|C) = \frac{120}{200} = \frac{3}{5}, \quad P(W|F) = \frac{40}{100} = \frac{2}{5}$$

$$P(W) = P(C) * P(W|C) + P(F) * P(W|F)$$

$$= \left(\frac{1}{2} * \frac{3}{5}\right) + \left(\frac{1}{2} * \frac{2}{5}\right)$$

$$= \frac{3}{10} + \frac{1}{5} = \frac{5}{10} = \frac{1}{2}$$

$$P(C|W) = \frac{P(W|C) \cdot P(C)}{P(W)}$$

$$= \frac{3/5 * 1/2}{1/2}$$

$$= 3/5$$

$$\therefore \boxed{P(C|W) = \frac{3}{5}} \text{ (Ans.)}$$

(c) Given,

$$L(p) = p^4 (1-p)^3$$

(take log)

$$\Rightarrow \log L(p) = \log \left(p^4 * (1-p)^3\right)$$

$$\Rightarrow \log L(p) = \log \left(p^4\right) + \log \left((1-p)^3\right)$$

$$\Rightarrow \log L(p) = 4 \log (p) + 3 \log (1-p)$$

take derivative

$$\Rightarrow \frac{d}{dp} \log L(p) = \frac{d}{dp} 4 \cdot \log (p) + \frac{d}{dp} 3 \cdot \log (1-p)$$

$$\Rightarrow \frac{d}{dp} \log L(p) = 4 \cdot \frac{1}{p} + 3 \cdot \frac{1}{1-p} (-1)$$

$$= \frac{4}{p} - \frac{3}{(1-p)}$$

$L(p)$ is maximum when

$$\frac{d}{dp} \log L(p) = 0$$

$$\Rightarrow \frac{4}{p} - \frac{3}{(1-p)} = 0$$

$$\Rightarrow 4 \cdot (1-p) = 3 \cdot p$$

$$\Rightarrow 4 = 7p$$

$$\Rightarrow \boxed{p = \frac{4}{7}}$$

Here, p represents the probability of the coin turning up heads in a single flip that would best explain the observed sequence of coin flips in the question. It is also known as the maximum likelihood estimate (MLE) of the coin's bias, based on the observed data