

Shortest path has optimal substructure.

$$\delta(a, b) = \min(\delta(a, c) + w(c, b), \delta(a, d) + w(d, b))$$

Given a weighted, directed graph $G = (V, E)$ and a source vertex s in V , find the min cost path from s to every vertex in V .

- Bellman-Ford
 - DP
 - General case, edge weights may be negative
- Dijkstra
 - Greedy
 - Edge weights must be non-negative

Relaxation: Given a vertex v , a vertex u , and an edge (u, v) , we can relax v by

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Relax(u, v)
  if v.d > u.d + w(u, v)
    v.d = u.d + w(u, v)
    v.pi = u
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Proof by induction on relaxing the i th edge (v_{i-1}, v_i) on p

Let $w_I = \sum_1^i w(v_{i-1}, v_i)$. W_i is the shortest path weights $\delta(s, v_i)$ because of optimal substructure