General DP Remarks

Optimal Substructure

- Create optimal solution to problem using optimal solutions to subproblems.
- Can't use DP if optimal solution to a problem does not require subproblem solutions to be optimal.
  → Often happens when subproblems are *not independent* of each other.

Overlapping Subproblems

- For DP to be useful, recursive algorithm should require us to compute optimal solutions to the *same subproblems* over and over again.
- In total, there should be a small number of distinct subproblems (i.e. polynomial in input size).

LCS

$$LCS[i,j] = \begin{cases} 1 + LCS[i-1, j-1] & \text{if } x_i = y_i \\ \max(LCS[i-1,j], LCS[i,j-1]) & \text{otherwise} \end{cases}$$

		j=0	j=1	j=2	j=3	j=4	j=5	j=6	j=7	j=8
			p	r	i	n	t	i	n	g
i=0		0	0	0	0	0	0	0	0	0
i=1	s	0	0	0	0	0	0	0	0	0
i=2	p	0	1↖	1←	1←	1←	1←	1←	1←	1←
i=3	r	0	1↑	2↖	2←	2←	2←	2←	2←	2←
i=4	i	0	1↑	2↑	3↖	3←	3←	3←	3←	3←
i=5	n	0	1↑	2↑	3↑	4↖	4←	4←	4←	4←
i=6	g	0	1↑	2↑	3↑	4↑	4↑	4↑	4↑	5↖
i=7	t	0	1↑	2↑	3↑	4↑	5↖	5←	5←	5←
i=8	i	0	1↑	2↑	3↖	4↑	5↑	6↖	6←	6←
i=9	m	0	1↑	2↑	3↑	4↑	5↑	6↑	6↑	6↑
i=10	e	0	1↑	2↑	3↑	4↑	5↑	6↑	6↑	6↑

OBST

$$e[i,j] = \begin{cases} 0 & \text{if } i = j-1 \\ \min_{i \le r \le j}\{e[i, r-1] + e[r+1, j] + w(i,j)\} & \text{if } i \le j \end{cases}$$

$root[i,j]$ = root of subtree with keys $k_i, \ldots, k_j$ for $1 \le i \le j \le n$

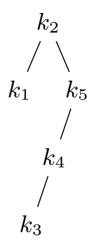
$$\begin{aligned} w[1,\ldots,n+1, 0, \ldots, n] = & \text{ sum of probabilities} \\ w[i, i-1] &= 0 \text{ for } 1 \le i \le n \\ w[i,j] &= w[i, j-1] + p_j \text{ for } 1 \le i \le j \le n \end{aligned}$$

Consider 5 keys with search probabilities $p_1 = 0.25, p_2 = 0.2, p_3 = 0.05, p_4 = 0.2, p_5 = 0.3$

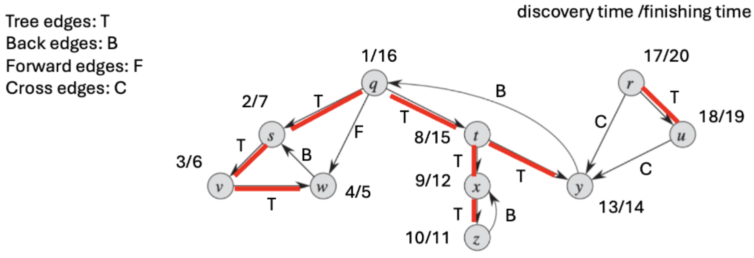
w	0	1	2	3	4	5
1	0	0.25	0.45	0.5	0.7	1.0
2		0	0.2	0.25	0.45	0.55
3			0	0.05	0.25	0.55
4				0	0.2	0.5
5					0	0.3
6						0

e	0	1	2	3	4	5
1	0	0.25	0.65	0.8	1.25	2.1
2		0	0.2	0.3	0.75	1.35
3			0	0.05	0.3	0.85
4				0	0.2	0.7
5					0	0.3
6						0

r	0	1	2	3	4	5
1		1	1	1	2	2
2			2	2	2	4
3				3	4	5
4					4	5
5						5
6						



DFS



$$(q\,[s\{v(ww)v\}s]\,[t\{x(zz)x\}yyt]q)\,(r[uu]r)$$

Tree Edges Are edges in depth-first forest G_π. Edge (u,v) is a tree edge if v was first discovered by exploring edge (u,v)
Back Edges Are edges (u,v) connecting a vertex u to an ancestor in a depth-first tree. We consider self-loops, which may occur in directed graphs, to be back edges.
Forward Edges Are nontree edges (u,v) connecting a vertex u to a descendant v in a depth-first tree.
Cross Edges Are all other edges. They can go between vertices in the same depth-first tree, as long as one vertex is not an ancestor of the other, or they can go between verticies in different depth-first trees.

Graphs

Handshaking Lemma

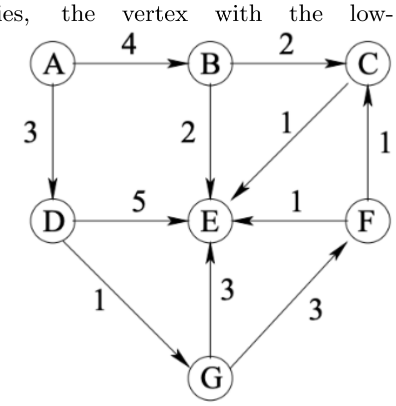
$$\sum_{v \in V} \deg(v) = 2|E|$$

BFS vs DFS

- DFS is usually for finding relationship among vertices.
- BFS Is usually for finding shortest path from a given source.

Djikstra

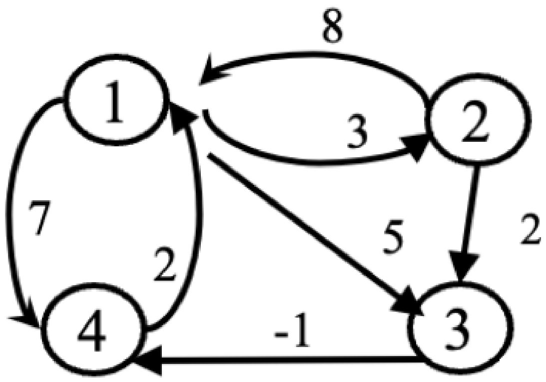
Execute Djikstra's algorithm on the graph below starting at A. If there are ties, the vertex with the lowest letter comes first.



A: 0	D: 3	B: 4	G: 4	C: 6	E: 6	F: 7
B: ∞	B: 4	G: 4	C: 6	E: 6	F: 7	
C: ∞	C: ∞	E: 8	E:6	F: 7		
D: ∞	E: ∞	C: ∞	F: ∞			
E: ∞	F: ∞	F: ∞				
F: ∞	G: ∞					
G: ∞						

Floyd-Warshall

Execute the Floyd-Warshall algorithm on the graph below, provide the D^k and P matrixes on each step.



$D^0 =$

	1	2	3	4
1	0	3	5	7
2	8	0	2	∞
3	∞	∞	0	-1
4	2	∞	∞	0

$P =$

	1	2	3	4
1	0	0	0	0
2	0	0	0	0
3	0	0	0	0
4	0	0	0	0

$D^1 =$

	1	2	3	4
1	0	3	5	7
2	8	0	2	15
3	∞	∞	0	-1
4	2	5	7	0

$P =$

	1	2	3	4
1	0	0	0	0
2	0	0	0	1
3	0	0	0	0
4	0	1	1	0

$D^2 =$

	1	2	3	4
1	0	3	5	7
2	8	0	2	15
3	∞	∞	0	-1
4	2	5	7	0

$P =$

	1	2	3	4
1	0	0	0	0
2	0	0	0	1
3	0	0	0	0
4	0	1	1	0

$D^3 =$

	1	2	3	4
1	0	3	5	4
2	8	0	2	1
3	∞	∞	0	-1
4	2	5	7	0

$P =$

	1	2	3	4
1	0	0	0	3
2	0	0	0	3
3	0	0	0	0
4	0	1	1	0

$D^4 =$

	1	2	3	4
1	0	3	5	4
2	3	0	2	1
3	1	4	0	-1
4	2	5	7	0

$P =$

	1	2	3	4
1	0	0	0	3
2	4	0	0	3
3	4	4	0	0
4	0	1	1	0

Knapsack

$$c[i,w] = \begin{cases} 0 & \text{if } i = 0 \text{ or } w = 0 \\ c[i-1,w] & \text{if } w_i > w \\ \max(c[i-1,w], c[i-1,w-w_i] + v_i) & \text{otherwise} \end{cases}$$