



## 1 Objective:

Determining the  $(x, y, z)$  coordinates and arrival times  $(t)$  of the 30 most prominent clicks (highest signal-to-noise ratio) detected in a given recorded audio file.

Two Solution Approaches are taken into consideration:

1. Fixed Sound Speed (1490 m/s)
2. Estimated Sound Speed

## 2 Introduction:

In marine research, it's crucial to locate underwater objects using sound, and this process is known as underwater acoustic positioning. This helps us pinpoint where things are underwater by analyzing when sound signals reach different hydrophones. The report here introduces a method for figuring out the positions  $(x, y, z)$  of underwater sources using the time delays  $(t)$  recorded from underwater clicks. In this report, we suggest two solutions: one assumes a constant speed of sound  $(c)$ , and the other considers an estimated speed of sound  $(c')$ . The main goal is to precisely find the locations of the sources of acoustic signals. The fixed speed solution assumes a steady speed, while the estimated speed solution allows for variations. Both methods aim to reduce errors in position calculations, using optimization techniques to improve accuracy. The report highlights the importance of achieving precise and reliable underwater acoustic source localization for effective marine research applications.

## 3 Methodology:

### 3.1 Hydrophone Configuration:

The study employed a setup consisting of five underwater hydrophones labelled as H1, H2, H3, H4, and H5. These hydrophones were strategically placed to detect acoustic signals emitted by marine organisms. The arrangement of these hydrophones was thoughtfully planned to improve the precision of identifying the sources of sound through triangulation. Figure 1 shows the actual physical configuration of the hydrophones.

The array's configuration was established based on precise coordinates and certain assumptions to ensure computational efficiency:

- H1 and H2 were positioned in alignment with the x-axis at coordinates  $(x_1, y_1, -z_1)$  and  $(x_2, y_2, -z_1)$  respectively, with the assumption that  $z_1$  equals  $z_2$ .
- H3 and H4 were situated along the y-axis at coordinates  $(x_3, y_3, 0)$  and  $(x_4, y_4, 0)$  respectively, under the assumption that  $z_3 = z_4 = 0$ .

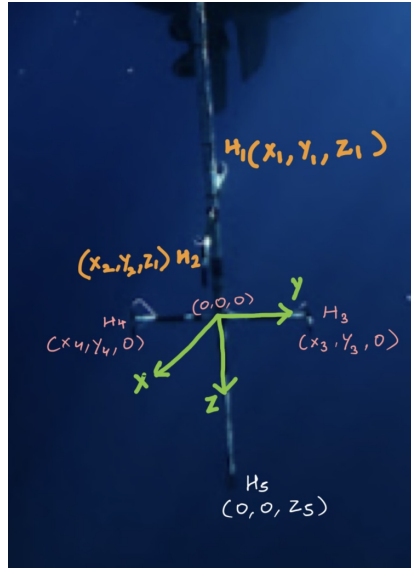


Figure 1: Considered reference positions of the Hydrophones

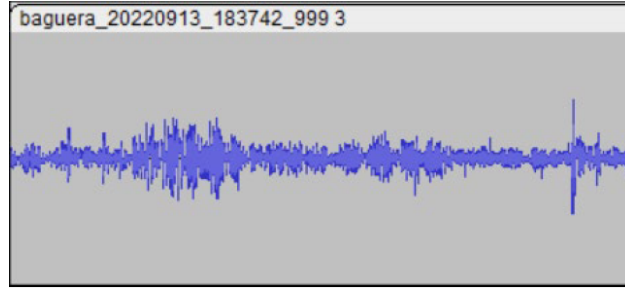


Figure 2: Raw audio signal

- H5 was placed under the origin, and its coordinates were  $(0, 0, z_5)$ .

The precise distances between the Hydrophones are discussed in the table 1.

Table 1: Distance between Hydrophones

Hydrophones	H1	H2	H3	H4	H5
H1	0	108.5	103.5	102.3	191.2
H2	108.5	0	103.2	105.6	192.5
H3	103.5	103.2	0	107.5	125.5
H4	102.3	105.6	107.5	0	125.0

### 3.2 Data Processing:

In my exploration of audio processing, I selected a specific audio file namely baguera\_20220913\_183742\_999.wav as the primary input. Upon importing this file into Audacity, a notable presence of noise became apparent, as illustrated in Figure 2, which depicts the characteristic noisy data encountered. To address this, I took proactive measures to enhance the audio quality by consecutively applying a

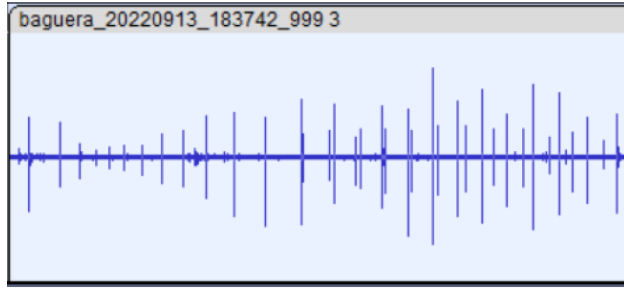


Figure 3: Sampled Audio Signal

high-pass filter of 10000Hz and an amplifier. The positive outcome of these adjustments is evident in the improved audio representation displayed below. In the course of this process, I carefully singled out clicks that, in my estimation, would generate optimal waveforms for subsequent sample calculations.

When measuring samples from the waves, it is essential to consider a few key aspects:

- To determine samples from Hydrophones 1 – 2 and 3 – 4, peak and crest analysis is imperative. This is particularly crucial as these Hydrophones are oriented in opposite directions to each other.
- When measuring in the reverse direction from the reference wave, it is essential to consistently apply a negative sign to the readings. This ensures accurate consideration of the directional shift.

### 3.3 Time Delay Calculations:

We computed time delays ( $t$ ) by calculating the difference in timestamps between click events detected by various hydrophones. This valuable information was extracted using Audacity’s label feature. The detailed data of all the 30 clicks are given in table 11. The obtained results serve as a crucial dataset, providing insights into the temporal variations between these acoustic signals.

After making an average over the data that was collected, the results for the time delays are as following:

Table 2: Average time delay:

dt12	32.03333
dt34	-5.2
dt51	-5.03333
dt52	-36.1667
dt53	-14.2333
dt54	-10.2

Positioning of the hydrophones, as determined in the earlier analysis, is described as follows, with coordinates specified in meters:

- H1: 54.78, 0.31, -70.07
- H2: -53.71, 2.06, -70.07

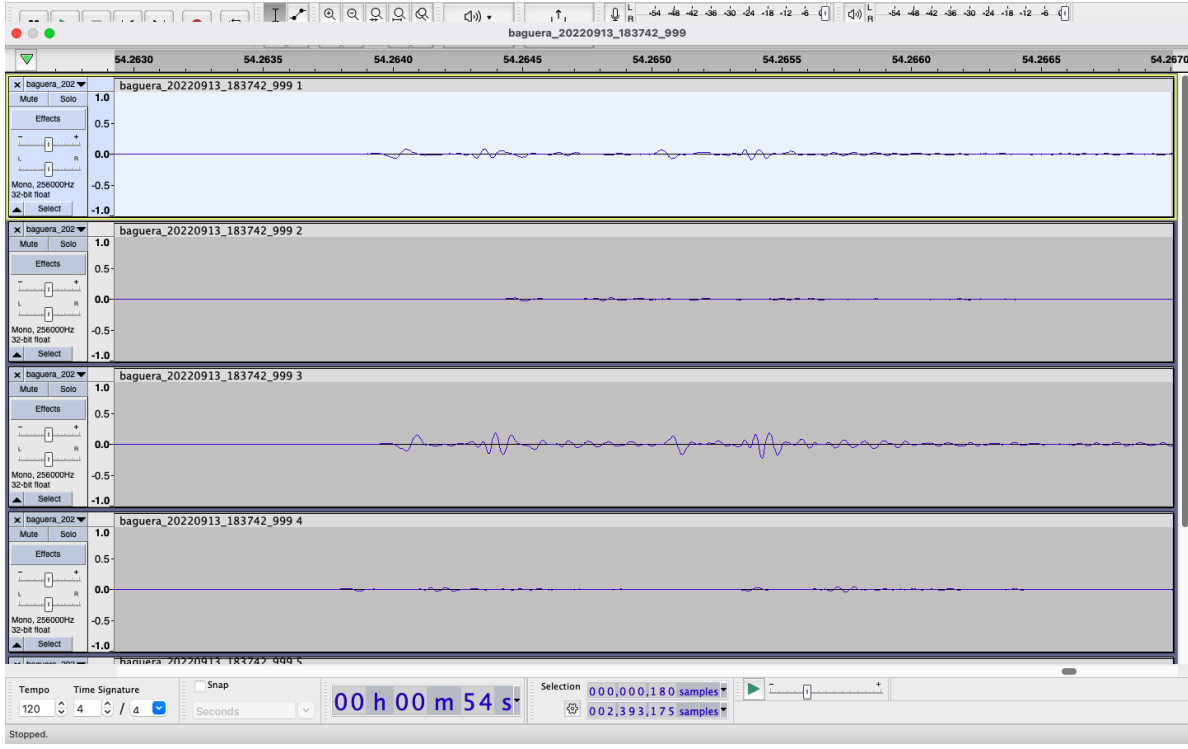


Figure 4: Audio labelling in Audacity

- H3: 1.10, 54.36, 0.00
- H4: 2.82, -53.13, 0.00
- H5: 0.00, 0.00, 113.11

This data is used for further calculations for positioning the whale based on the clicks recorded by the hydrophones.

### 3.4 Positioning of Whale

The position of the whale is defined by two coordinates: the azimuth angle and the elevation angle as in Figure 5.

- Azimuth angle( $\phi$ ): This angle is measured from the 0X axis to the 0Y axis. It provides information about the horizontal direction of the whale's position. The range of the azimuth angle spans  $[0,360]$ , allowing us to pinpoint the whale's location in relation to the X and Y axes.
- Elevation angle( $\theta$ ): This angle is measured from the 0X axis to the 0Z axis. It indicates the vertical direction of the whale's position. The range of the elevation angle extends from  $[0,90]$ , enabling us to determine how high or low the whale is in relation to the X and Z axes.

In simple words, the azimuth angle,  $\phi$ , is the angle between the whale's direction of motion and a reference direction, such as north. The elevation angle,  $\theta$ , is the angle between the whale's direction of motion and a horizontal plane. Together, these coordinates and angles offer

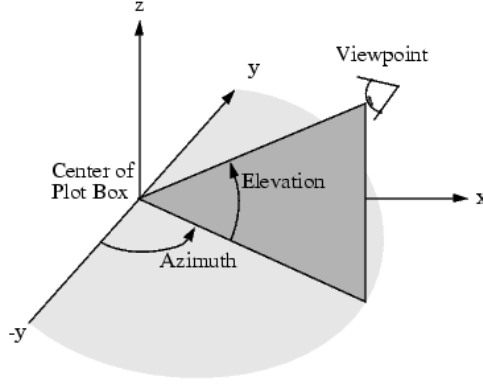


Figure 5: Azimuth angle( $\phi$ ) and Elevation angle( $\theta$ )

a comprehensive description of the whale's spatial orientation, aiding in precise localization and understanding of its position in the underwater environment. As the whale moves in a straight line, its azimuth angle remains constant. However, as the whale rises or falls in the water, its elevation angle changes.

#### 3.4.1 Assumptions:

1. Small deviations of hydrophone coordinates from the axes were neglected. For example: Z1 and Z2 are considered the same.
2. Whereas the distance between the whale and the antenna generally is much bigger than Fraunhofer's distance for the antenna, the wavefront of the signals can be considered to be plane and parallel to each other.

#### 3.4.2 Equations:

According to the assumptions, for the planar wavefront, the time difference of sound hitting hydrophones H1 and H2 can be calculated as given in Figure 7. This is exactly the same for hydrophones H3 and H4.

$$h/a = h'/a' \Rightarrow a = h * a'/h'$$

$$\Rightarrow a = h * \sin\theta$$

$$b'/a' = \cot\theta \Rightarrow a' \cot\theta = b'$$

$$\Rightarrow H_{1x} \cot\theta = b'$$

$$h = H_{1z} - H_{1x} \cot\theta$$

$$a = (H_{1z} - H_{1x} \cot\theta) \sin\theta$$

$$d = a + c = H_{1z} \sin\theta - H_{1x} \cot\theta \sin\theta + H_{5z} \sin\theta$$

$$d_{azm} = d \cos\phi = (H_{5z} - H_{1x} \cot\theta - H_{1z}) \sin\theta \cos\phi$$

$$H_{5z} \text{ and } H_{1z} \text{ has opposite sign}$$

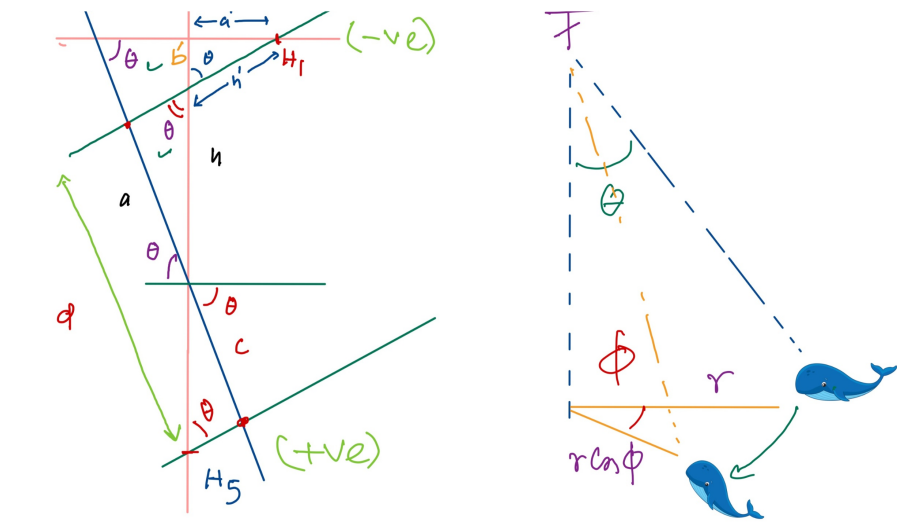


Figure 6: Equations 3

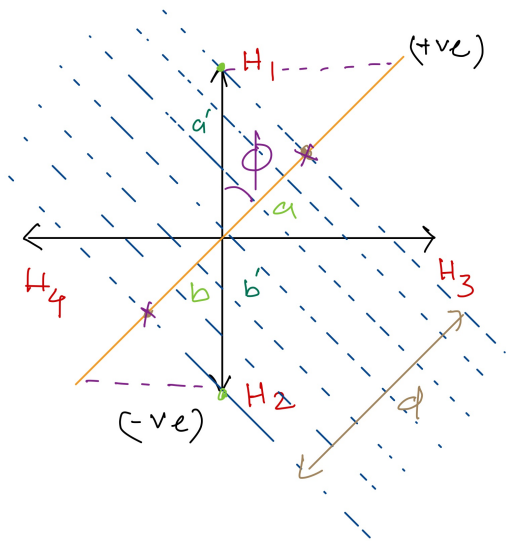


Figure 7: Equations 1 and 2

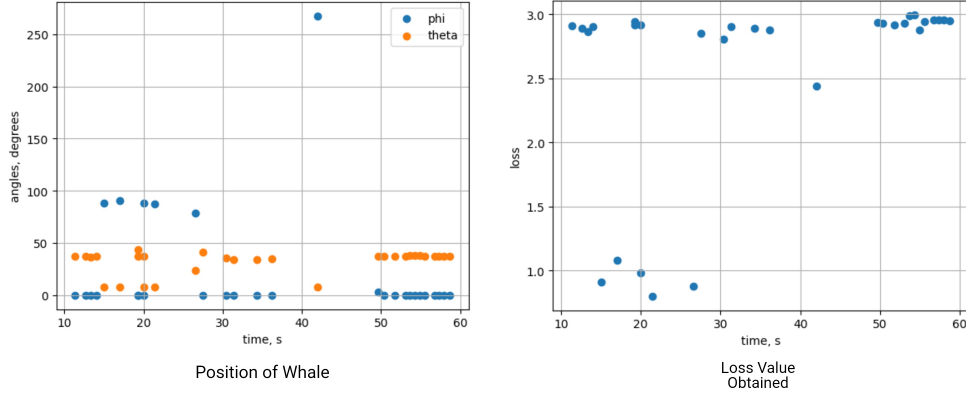


Figure 8: Results with fixed speed of sound

From here, we get the 2 equations necessary to calculate the Azimuth and Elevation angles. However, for calculating the speed of sound simultaneously with the position of the whale, we add another equation which is the time delay between the H1 and H5 hydrophone.

$$\begin{aligned}
 a &= a' \cos \phi \\
 b &= b' \cos \phi \\
 d &= a + b = a' \cos \phi + b' \cos \phi = (a' + b') \cos \phi \\
 &= (H_{1X} - H_{2X}) \cos \phi
 \end{aligned}$$

## 4 Analysis

### 4.1 Fixed Velocity

The first result is in fixed velocity of sound in water at  $1500 \text{ ms}^{-1}$ . In this result, it can be seen that the  $\phi$  (azimuth) is the same as time elapsed except in the first 30 seconds. After that, the azimuth angle changes and becomes stable. At around 40 second mark, the azimuth of the whale changed abruptly as there might have been some human errors while taking the measurements from the recordings. The same can be deduced for the  $\theta$  (elevation) as well. As time elapsed, the elevation stayed the same. Unlike azimuth angle, there were no spike in the estimation of theta which means that the whale remained in the same elevation throughout the time period.

However, the loss is almost stable at 3.0 except for those regions where  $\theta$  and  $\phi$  have irregular values. This suggests the optimiser may have found some local minima at that point.

### 4.2 Estimated Velocity (varying velocities)

This set of results suggests the same for elevation, azimuth and as well as loss. The estimated velocity graph suggests that various velocities were implemented to find the azimuth and elevation in this section. The velocities range mainly in the region  $1500 \text{ ms}^{-1}$  except at 40 seconds time step when it dropped around  $1480 \text{ ms}^{-1}$  which clearly indicates that some noisy measurement has been taken because acoustics wave velocity can't change within such short time frame.

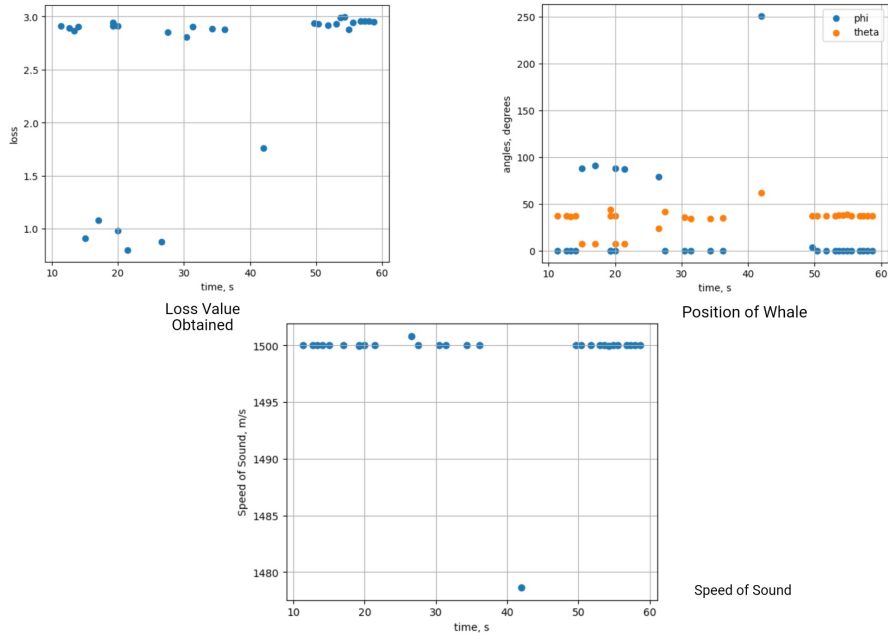


Figure 9: Results with estimated speed of sound

## 5 Conclusion

The results of the experiment are given in figure ,8, 9,10 and 11. It is indeed uncertain to determine the precise position of a whale based on the elevation and azimuth angles alone. These angles only provide information about the whale's orientation with respect to the hydrophones, but not its exact location.

Integrating additional coordinate-finding solvers with the azimuth and elevation solvers could potentially enhance the accuracy of whale localization. Combining the information from both sets could also lead to a reduction in the loss value, further improving the accuracy of the positioning system.

Additionally, the resulting optimiser is very sensitive towards initial conditions. If we change the initial  $\theta$ ,  $\phi$  or speed of sound guesses, the resulting values change significantly. Moreover, the position of the whale is mostly constant with some discrepancies. This might be due to the following human factors:

1. Error in calculating time intervals in Audacity
2. misinterpretation of a noise as a signal from a whale.

Also, the initial guessed speed of sound remains more or less constant even after the optimisation.



	time	dt12	dt34	dt51	dt52	dt53	dt54	phi	theta	loss
0	11.351	22	-13	-7	-29	-11	1	0.0	37.348	2.913
1	12.658	23	-10	-6	-29	-9	-1	0.0	37.07	2.889
2	13.34	21	1	-8	-30	-12	1	0.249	36.544	2.863
3	14.021	23	-11	-8	-30	-12	2	0.0	37.138	2.905
4	15.0	19	42	24	-29	24	-36	88.139	7.691	0.906
5	17.0	56	58	-9	-61	-60	22	90.698	7.695	1.077
6	19.26	19	-63	28	11	-27	74	0.0	43.712	2.946
7	19.261	21	-11	-8	-29	-11	1	0.0	37.277	2.914
8	20.002	53	62	27	64	-26	-36	87.906	7.692	0.983
9	20.002	21	-11	-8	-30	-11	0	0.0	37.277	2.914
10	21.4	21	60	34	-29	28	41	87.361	7.698	0.799
11	26.58	72	84	50	22	43	-92	78.795	23.524	0.875
12	27.546	21	-53	35	7	-26	-37	0.0	41.456	2.854
13	30.418	145	-108	-11	-153	-48	-62	0.0	35.626	2.805
14	31.36	125	-61	-50	-170	-61	5	0.0	34.324	2.904
15	34.294	122	-56	-49	-171	-57	3	0.0	34.238	2.888
16	36.16	121	-65	-39	-171	-64	2	0.0	34.682	2.878
17	42.024	-131	115	-34	108	65	-182	267.331	7.797	2.442
18	49.667	5	14	-9	-30	-11	-1	3.449	37.485	2.938
19	50.34	20	-11	-10	-30	-12	1	0.0	37.348	2.931
20	51.729	21	-11	-9	-30	-12	-1	0.0	37.277	2.92
21	53.065	20	-11	-10	-30	-12	-1	0.0	37.348	2.931
22	53.665	8	-11	-11	-18	-12	-1	0.0	38.265	2.992
23	54.264	7	-11	-11	-18	-12	-1	0.0	38.348	2.996
24	54.891	6	-11	10	-16	-12	-1	0.0	38.432	2.878
25	55.533	20	-12	-11	-32	-13	-1	0.0	37.419	2.941
26	56.758	20	-13	-13	-33	-14	-2	0.0	37.491	2.957
27	57.328	20	-13	-13	-33	-14	-1	0.0	37.491	2.957
28	57.938	20	-13	-13	-33	-14	-1	0.0	37.491	2.957
29	58.681	20	-13	-12	-33 <sup>9</sup>	-14	-2	0.0	37.491	2.951

Figure 10: Results with fixed speed of sound

	time	dt12	dt34	dt51	dt52	dt53	dt54	phi	theta	loss	speed
0	11.351	22	-13	-7	-29	-11	1	269.998	0.024	1.014	1500.01
1	12.658	23	-10	-6	-29	-9	-1	269.998	0.026	1.051	1499.999
2	13.34	21	1	-8	-30	-12	1	268.543	90.0	1.155	1499.998
3	14.021	23	-11	-8	-30	-12	2	269.998	0.02	1.039	1499.986
4	15.0	19	42	24	-29	24	-36	274.372	90.0	1.156	1499.998
5	17.0	56	58	-9	-61	-60	22	90.001	0.011	0.648	1500.081
6	19.26	19	-63	28	11	-27	74	272.27	6.97	0.506	1500.005
7	19.261	21	-11	-8	-29	-11	1	269.998	0.021	1.036	1500.005
8	20.002	53	62	27	64	-26	-36	274.922	90.0	1.156	1499.996
9	20.002	21	-11	-8	-30	-11	0	269.998	0.021	1.036	1500.005
10	21.4	21	60	34	-29	28	41	276.198	90.0	1.156	1499.997
11	26.58	72	84	50	22	43	-92	279.142	90.0	1.156	1499.996
12	27.546	21	-53	35	7	-26	-37	273.68	9.245	0.591	1500.004
13	30.418	145	-108	-11	-153	-48	-62	278.134	43.46	0.686	1500.0
14	31.36	125	-61	-50	-170	-61	5	268.044	57.699	1.016	1499.998
15	34.294	122	-56	-49	-171	-57	3	267.873	58.659	1.033	1499.998
16	36.16	121	-65	-39	-171	-64	2	271.309	48.598	0.948	1499.999
17	42.024	-131	115	-34	108	65	-182	105.285	33.004	0.432	1499.999
18	49.667	5	14	-9	-30	-11	-1	268.378	90.0	1.155	1499.999
19	50.34	20	-11	-10	-30	-12	1	269.998	0.018	1.035	1499.997
20	51.729	21	-11	-9	-30	-12	-1	269.998	0.019	1.036	1500.008
21	53.065	20	-11	-10	-30	-12	-1	269.998	0.018	1.035	1499.997
22	53.665	8	-11	-11	-18	-12	-1	269.995	0.039	1.023	1500.003
23	54.264	7	-11	-11	-18	-12	-1	269.995	0.046	1.023	1500.006
24	54.891	6	-11	10	-16	-12	-1	270.412	3.667	1.022	1500.003
25	55.533	20	-12	-11	-32	-13	-1	269.998	0.016	1.023	1500.007
26	56.758	20	-13	-13	-33	-14	-2	269.998	0.014	1.011	1500.007
27	57.328	20	-13	-13	-33	-14	-1	269.998	0.014	1.011	1500.007
28	57.938	20	-13	-13	-33	-14	-1	269.998	0.014	1.011	1500.007
29	58.681	20	-13	-12	-33	-14	-2	269.998	0.014	1.011	1500.004

Figure 11: Results with estimated speed of sound

## 6 Code:

```
import numpy as np
import math
import pandas as pd
from scipy.optimize import minimize
from scipy.spatial import distance
import matplotlib.pyplot as plt

df = pd.read_csv("Clicks.csv")
index = 0

# Convert centimeters to meters
H1 = np.array([54.78, -0.31, -70.07]) / 100
H2 = np.array([-53.71, -2.06, -70.07]) / 100
H3 = np.array([1.10, 54.36, 0]) / 100
H4 = np.array([2.82, -53.13, 0]) / 100
H5 = np.array([0, 0, 113.11]) / 100

sampling_rate = 256000 # samples/s, kHz, magnitude resolution 32 bits

# Functions for fixed speed of sound
def spherical_fixed_sound(vars):
    # pfi - azimuth angle, theta - elevation angle
    pfi, theta = vars
    c = 1490 # Meters per second

    dt12 = int(df.loc[index]['dt12']) / sampling_rate
    dt34 = int(df.loc[index]['dt34']) / sampling_rate
    dt51 = int(df.loc[index]['dt51']) / sampling_rate
    dt52 = int(df.loc[index]['dt52']) / sampling_rate
    dt53 = int(df.loc[index]['dt53']) / sampling_rate
    dt54 = int(df.loc[index]['dt54']) / sampling_rate

    # Equation of movement of the wave in H1H2 plane
    eq1 = c*dt12*math.cos(theta) - (H1[0] - H2[0])*math.cos(pfi)
    # Equation of movement of the wave in H3H4 plane
    eq2 = c*dt34*math.cos(theta) - (H3[1] - H4[1])*math.sin(pfi)
    # Equation of movement of the wave in vertical plane for H1 and H5
    eq3 = c*dt51 - ((H5[2] - H1[2])*math.sin(theta) + H1[0]/math.tan(theta))*math.cos(pfi)

    return [eq1, eq2, eq3]

# Functions for estimated speed of sound
def spherical_variable_sound(vars):
    # pfi - azimuth angle, theta - elevation angle, c - speed of sound
```

```

pfi, theta, c = vars

dt12 = df.loc[index]['dt12'] / sampling_rate
dt34 = df.loc[index]['dt34'] / sampling_rate
dt51 = df.loc[index]['dt51'] / sampling_rate
dt52 = df.loc[index]['dt52'] / sampling_rate
dt53 = df.loc[index]['dt53'] / sampling_rate
dt54 = df.loc[index]['dt54'] / sampling_rate

eq1 = c*dt12*math.cos(theta) - (H1[0] - H2[0])*math.cos(pfi)
eq2 = c*dt34*math.cos(theta) - (H3[1] - H4[1])*math.sin(pfi)
eq3 = c*dt51 - ((H5[2] - H1[2])*math.sin(theta) + H1[0]/math.tan(theta))*math.cos(pfi)

return [eq1, eq2, eq3]

def loss_fun(vars,choice:int):
    if choice == 1:
        eqs = spherical_fixed_sound(vars)
    else:
        eqs = spherical_variable_sound(vars)

    return abs(eqs[0])+abs(eqs[1])+abs(eqs[2])

initial_guess = [math.radians(math.pi/3), math.radians(math.pi/3)]
# Bounds for angles
bnds = ((0, 2*math.pi), (0, (math.pi)/2))

phi = []
theta = []
loss = []
speed = []

# Calculating angles for fixed speed
for i in range(len(df)):
    index = i;
    result = minimize(loss_fun, initial_guess, bounds=bnds, method='SLSQP',args=(1,))
    df.at[index, 'phi'] = math.degrees(result.x[0])
    df.at[index, 'theta'] = math.degrees(result.x[1])
    df.at[index, 'loss'] = result.fun

# Printing plots for fixed speed
fig, ax = plt.subplots()
ax.scatter(df['time'], df['phi'], label="phi")
ax.scatter(df['time'], df['theta'], label="theta")
ax.legend()
ax.set_xlabel("time, s")
ax.set_ylabel("angles, degrees")

```

```

ax.grid()
plt.show()

fig, ax = plt.subplots()
ax.scatter(df['time'], df['loss'])
ax.set_xlabel("time, s")
ax.set_ylabel("loss")
ax.grid()
plt.show()

df_fixed_speed = df.round(3)

# Calculating angles and speed
phi = []
theta = []
loss = []
speed = []
bnds = ((0, 2*math.pi), (0, (math.pi)/2), (1450, 1670))
initial_guess = [math.radians(math.pi/3), math.radians(math.pi/3), 1500]
for i in range(len(df)):
    index = i;
    result = minimize(loss_fun, initial_guess, bounds=bnds, method='SLSQP', args=(2,))
    df.at[index, 'phi'] = math.degrees(result.x[0])
    df.at[index, 'theta'] = math.degrees(result.x[1])
    df.at[index, 'loss'] = result.fun
    df.at[index, 'sound speed'] = result.x[2]

# Printing plots for estimated speed
fig, ax = plt.subplots()
ax.scatter(df['time'], df['phi'], label="phi")
ax.scatter(df['time'], df['theta'], label="theta")
ax.legend()
ax.set_xlabel("time, s")
ax.set_ylabel("angles, degrees")
ax.grid()
plt.show()

fig, ax = plt.subplots()
ax.scatter(df['time'], df['loss'])
ax.set_xlabel("time, s")
ax.set_ylabel("loss")
ax.grid()
plt.show()

fig, ax = plt.subplots()
ax.scatter(df['time'], df['sound speed'])
ax.set_xlabel("time, s")

```

```
ax.set_ylabel("Speed of Sound, m/s")
ax.grid()
plt.show()

df_variable_speed = df.round(3)
```