Solving Equations using Lineare Elimination Method: (2 equations)

Making @x3 -

Now,
$$4a-3b=6$$

$$\Rightarrow 12-3b=6$$

$$\Rightarrow 3b=6$$

$$\therefore b=2$$

$$\therefore (a,b)=(3,2)$$

②
$$2a+5b \cdot 46 \rightarrow 0$$
 $8a+b=32 \rightarrow 0$
 $= 160$
 $0 \times 5 \rightarrow 404+5b \rightarrow 0$

$$8a + b = 32$$

 $24 + b = 32$
 $\vdots (a, b) = (3,8)$

$$3a-3b-c=3 \rightarrow \bigcirc$$

$$-3b+2c=0 \longrightarrow \emptyset$$

times officer it colors of keeps and and

~ (V) - e. (v)

$$-66-7c=-33$$

From, (1) ->

SI 25 + 6+ X (-)

Matrix Row Reduction -> (Gravesian Elemination)

Another method to solve 3 variable eqn:

$$a+b+2c=12 \rightarrow 0$$

 $3a-3b-c=3 \rightarrow 0$
 $2a-b+6c=24 \rightarrow 0$

Rules:

Donly first eqn will contain a vorciable. Remove a from others.

-> Devide each now with the coefficient a

$$a+b+2c=12 \longrightarrow \emptyset$$

$$a-b-\frac{c}{3}=1 \longrightarrow \emptyset$$

$$a-\frac{b}{2}+3c=12 \longrightarrow \mathbb{N}$$

Now use (1) egn to remove a from the other egutions

$$(-) = \frac{1}{3} = 1$$

$$(-) = \frac{1}{3} + \frac{1}{2} + \frac{1}{2} = \frac{12}{2}$$

$$-2b - (\frac{2}{3} + 2c) = -11$$

$$= \frac{1}{3} - 2b - \frac{75c}{3} = -11$$

$$= \frac{1}{3} + \frac{1}{3} = \frac$$

$$\begin{array}{c|c}
\hline
O & -O & \rightarrow \\
\hline
O & -\frac{b}{2} + 3c = 12 \\
\hline
-\frac{a}{b} + \frac{b}{2} + \frac{2c}{b} = 12 \\
\hline
-\frac{b}{2} + \frac{b}{b} + \frac{2c}{b} = 0 \\
\Rightarrow -\frac{3b}{2} + \frac{c}{b} = 0
\end{array}$$

So, the equations
$$\rightarrow$$
 $a+b+2c=12 \rightarrow 0$
 $2b+\frac{7c}{3}=11 \rightarrow 0$
 $-\frac{3b}{2}+c=0 \rightarrow 0$

Now again from (VII) and (VIII) remove everficient of b.

Miles I. z ? - d

$$a+b+2c=12 \rightarrow \textcircled{1} \qquad b+\frac{7c}{6}=\frac{11}{2} \rightarrow \textcircled{2}$$

$$b+\frac{7c}{6}=\frac{11}{2} \rightarrow \textcircled{2}$$

$$b+\frac{2c}{3}=0 \rightarrow \textcircled{2}$$

Now, again ean remaining.

$$a + b + 2c = 12 \rightarrow \text{(W)}$$

$$b + \frac{7c}{6} = \frac{11}{2} \rightarrow \text{(X)}$$

$$c = 3$$

From,
$$(ix) \rightarrow \text{Replace value Q with it's value}$$

$$b + \frac{7 \times 3}{6} = \frac{11}{2}$$

$$\Rightarrow b + \frac{21}{6} = \frac{11}{2}$$

$$\Rightarrow b = \frac{11}{2} - \frac{21}{6}$$

$$\Rightarrow b = \frac{33 - 21}{6}$$

Now, Forom (1) replace bond e with values

$$a + 2 + 6 = 12$$
 $\Rightarrow a = 4$

The solm
$$\rightarrow$$
 (a,b,c) = (4,2,43)

0 - 52 - 4 -

N - 21 124. C.

3 3 3 5 11 5 11 5 6 11 5 8 6

0-0+(4+1)-

Mc = = = + 4

7 = 3+ +A

11 - 20 - 21:

A. 20-15.

Mende each row with their cofficient of a

$$\alpha = \frac{3b}{4} = \frac{3}{2} \rightarrow 0$$

<u>«</u>»

$$\frac{9 + \frac{b}{5} = \frac{317}{5}}{(-)^{(+)} \frac{3b}{4} + \frac{b}{5}} = \frac{3}{2} - \frac{17}{5}$$

$$= \frac{3b}{4} + \frac{b}{5} = \frac{3}{2} - \frac{17}{5}$$

$$-\left(\frac{3b}{4}+\frac{b}{5}\right)=\frac{3}{2}-\frac{17}{5}$$

$$\Rightarrow -\frac{19b}{20} = \frac{15-34}{10}$$

0 - do + 00

$$(1) - (1) \rightarrow (1)$$

$$(2) \frac{3b}{4} = \frac{3}{2} \rightarrow (0)$$

$$(1) \frac{3b}{4} = \frac{3}{2} \rightarrow (0)$$

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$$(2) \frac{3b}{4} = \frac{3}{2} \rightarrow (0)$$

$$(3) \frac{3b}{4} = \frac{3}{2} \rightarrow (0)$$

$$\Rightarrow a + 0.2b = 34$$

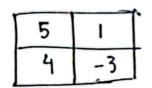
if we put value of b in (11)

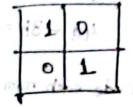
Now, if we potrag the three system into matrix

Original system

Intermediale	System_
--------------	---------

Sowed system





Reduced Row ethelon John

This above a egn on matrix was for non singulare

Now let us check for singular system:

Original System

$$a+b=10 \rightarrow \bigcirc$$

$$0 + b = 10 \longrightarrow 0$$

$$0 + 0 + 0 = 0$$

(1)×2 - (11) →

original Matrix

	•
2	2

Uppen diagonal matrix (Intermediate) Another singular system:

Original System

$$10a + 2b = 22 \rightarrow \bigcirc$$

$$2 \times (1) - (1) \rightarrow$$

$$100 + 2b = 22$$

$$-100 + 2b^{2} = 22$$

Intermediate system

$$5a + b = 11$$
 $\Rightarrow a + b = 15$
 $\Rightarrow a + b = 15$
 $\Rightarrow a + b = 0$

Original Matrix

Upper diagonal Mootrix (Intermediate system)

1	1	0'2
. †	0	0
	H. h	21000

Row echelon form

Greneral Row dechelon form in Matrix:

		1			-
200	Ly til)*	*	*	*
	0	1	*	*	*
	0	0	0	0	0
	0	0	0	0	0
αĒ	011	0	10	0	0
					1.00

Non zeno values Lican be put night after 1 only

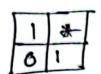
Is Everything left of \$1

Ls Everything left of o

is o Ly Everyfly is right of o

is 0 .

2x2 Matrices these echelon forms can happen



1	*
0	0_

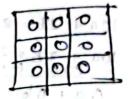
0	0
0	0

Matrices →

1	*	*	
0	1	*	
0	0	1	

1	1	*	*
۲	0	1	*
0 0	0	0	0

1	*	*
0	0	0
0	0	0



water also bearing . P) Row operations in matrix proserve singularity:

elatore lampath mayo

Means, if we perston different now operations like

- Switching rows

- Multiplying nows with scelars

The singularity will remain unchanged

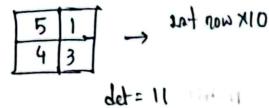
Suppose,

o to light o pulling of o

,	sabboz	و,	70	15	墨
nollo	delete	Ług.	4	3	-
		-	NA.	5 L	-

19 1 Switch TOWS -

15-4=11 (singular) det = 4 215 = -11 (singular) Then,



0-1+0

" rules My May all

B

Anothe row operation example ->

Sum now 1 + now 2, then calculate det with 1 nt now

equations

End provide no intermeture

En Pank = 0

The reank of a matrix:

System 1

$$\begin{array}{ccc} (a+b=0) & \longrightarrow & \bigcirc \\ a+2b=0 & \longrightarrow & \bigcirc \\ \end{array}$$

· 2 equations and preside 2 informations



System 2

$$a + b = 0$$

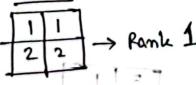
2a+2b2 0

2 equations and the the sharper next.

But provide only one information because we can get equation 2 from equation 1.

So, Rank = L

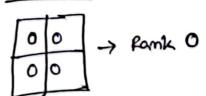
Matrix



System 3

But provide no in formation,

Matrix



Anothe formula for Rank is -

Rank = 2 - (Dimension of solution space) For 20 Metrix

How do we define singularity with Rank?

A matrix is singular it it is fully reanked.

Like, For 2D Motrix Full reanh value = 2

Forc. 3D 4 4 7 4 = 3

So, if the rearranh of a 2D instrix is 2, then the matrix is non singular. Else it is singular.

Find the Rank of below Matrix:

[5 1] [-1 3]

As the above matrix is non singular because no row and dependent on each other. So, Ramh would be 2 (as it is a 2d matrix) Also, in anothe way we can say that, the largest number of linearly independent row/column is the Ramh.

Here, largest number of independent nows = 2

50, Rank = 2

1 th (2) = [21 0 - 10]

CS CamScanner

xitted larifus

$$\begin{bmatrix} \frac{2}{6} & -1 \\ -6 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{6} & \frac{1}$$

Here Ist now can be converted to and now by multiplying

(-3) with first row. So, second now is dependent on 30, largest independent row number = 1

Arribo transly for Kowa to allow

First the Rome of bolow Modern

1 3

I worker all ma Raple = 1: virgini 10 1 to women at 11 .08

How to find Rank of a matrix morce quickly?

→ Using row echelon matrix

Original Matrix

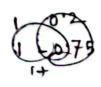
- tooker ask tool a			1	0 1	_1
(xinform 64 -3)	nolypria	non	el virtor	avedn	alt aA

-> Devide each roa by the most of newto do no

destimost co efficient (non zero)

1	0.2	$\rightarrow 0$
1	-0.75	→(1)

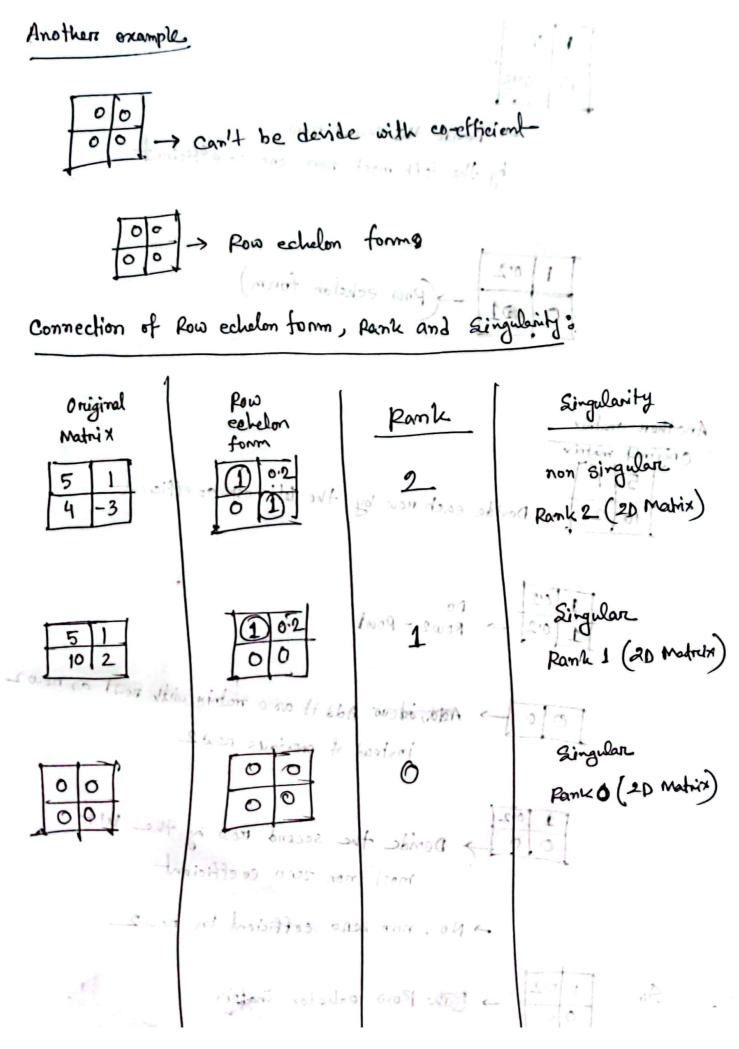
Now subtract row 1 from row 2



independent now/coturn is the Pank.

1 0'2 O -0.95 Devide the second rips by the left most non zero co efficient
Pow echelon form)
Another matrix Original matrix 5 1 Devide each now by the lettmost co efficient
1 0.2 > Rows - Rowl Rows - Rowl with nows as richo 2
instead of previous row 2 1 012 Devide the second row by the left most non zero eo efficient
No, non zeno cofficient In now2_ 50 102 → Ear Row cohelon matrix

Another



For 3D Matrix

		1							1	1	٠,	
3	-3	-1						7		1		1
2	-1	6	->	Devide	each	KOW	with.U	Lftmost	co.	elfici	ont-	-

1	1	1	1	1			
	1	-1	-0.3	1			
Į	1	-0.2	3	→	Subtreac	mous -mal	1
		•		→	u	row3 - row1	2-

$$\pi_2 - \pi_1 \rightarrow 0 - 2 - 1.3$$

$$\pi_3 - \pi_4 \rightarrow 0 - 1.5 2$$
Add this nows with existing now 1

1	1	1	
1	1	1.3	
0	-2	1-1	-

Now, K2 x 1'5 and K3 X2

,				r 3
	1	1	1	
I	0	-2	-1.3	ain is notwell to
Ī	0	0	5.95	> Deride lettmost
•		-	1	O-efficient

V				
ad	di Alu	' '	àth	
	previ			1
21			5	

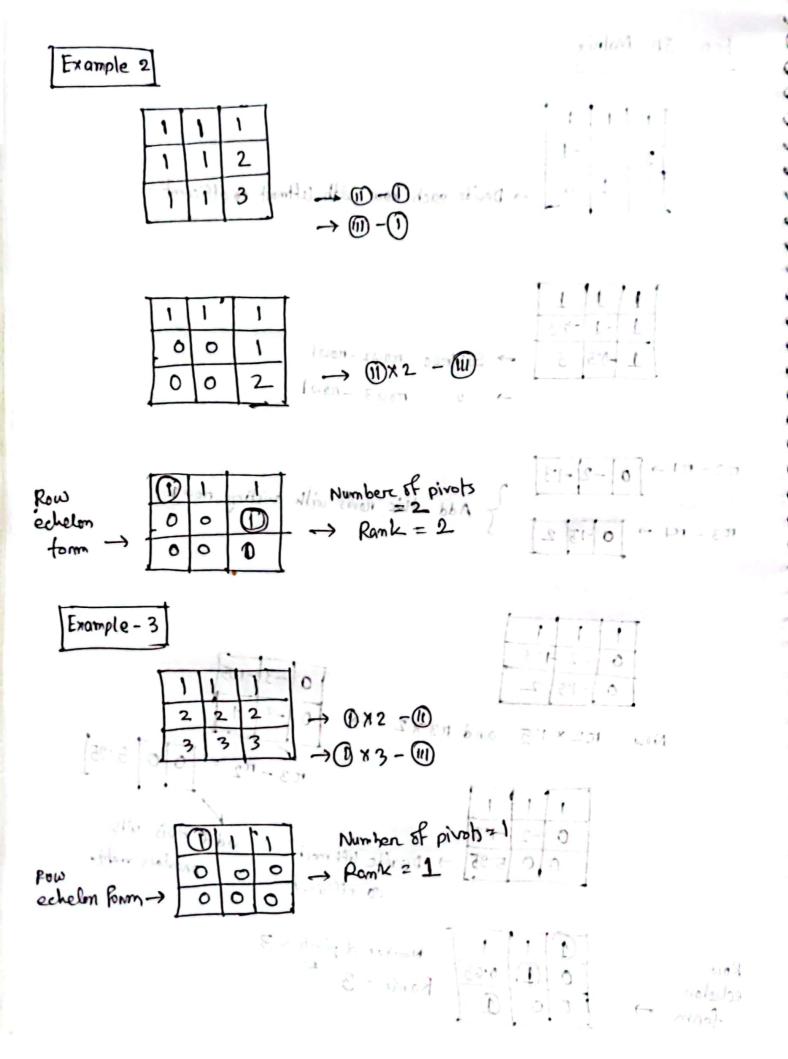
ROW	
echelon	
form	-

PPPPPPPP

	1	1	1
1	0	1	0.62
Ĭ	0	0	0

Number of pivots = 3

Rank = 3



1	0	0	0
-	0	0	0
4	0	0	0

-> Can't do any additional thing-

humber of pivot = 0

Rank = 0

Reduced Row Echelon form: (General)

١	2	*	*	*	*
Ì	0	1	*	*	*
	0	0	3	*	*
	0	0	0	-	5 *
	0	0	10	0	1
		R	20	k2	5

3	*	*	*	*
0	0	1	*	*
0	0	0	-4	*
0	0	0	0	0
0	0	0	0	0

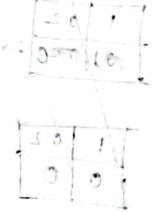
Rank = 3

-> Zero rows at the bottom

toward model out by it

- -> Each rows has a pivot (Leftmost non-Zoro entry)
- -> Every-pivot is right of the pivots on the nows above
- -> Rank of the matrix

 is the number of pivots



Reduced Row Echelon form:

Original system

Intermediale system

Solved System

Oraginal Matrix

5	1
4	-3

Uppere diagonal Natrier

1	0.2
0	1

to some Row echelon form

Diagonal Matrix

1	1	0
t	0	1

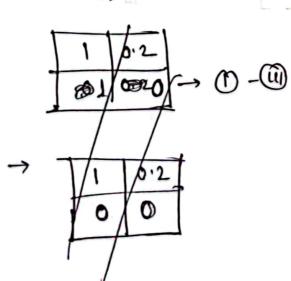
row edielon form

Row echelon torm -

1	Est.	6.2	961)	
,	:0	1	→ @×0.5	-(1)

is real seems that a

O and O ->



Now, O-00 - I'o - exchang with O

	1	0		roo	///
_	0	1	Ţ	-> Reduced rechelon	form

Reduced now echelon form in general.

	1	0	0	0	0
	9	1	0	0	Ō
	0	0	1	0	0
	0	0	0	1 0	p
1	0	0	0	0	1

1	*	0	0	*
C	0	1	0	*
0	0	0	T	*
0	0	0	0	0
0	0	0	0	0

→ is in row echelon form

→ Each phot is 1

→ Any number above the pirot

is 0

Supplied to the state of the state of

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ment adalos son bucken

Make reduced now echelon tonm from a roow echelon tonn

row eahelon form

•	eche	m	TON	VIV					mi	. 1
1				- 4	9.0	•				L.
	11	2	3	1000	moladon	Poduced	(-		1	07
	0	1	4					•		
	0	0	1							

as above con 1, only 0 can stay in reduced now echelon form

,-	anof	rela	ubo e	N ON	mi zi	-
	1	0	-5	ſ		
	0	1)		d-sel	
toun .	0	0	o In	down	Br A	<-
		-	-	l	D	

0	9)	0	0	1
0	9	^	1	7
0	0	1	0	0
9	1	0		2)
1	Ø	0	0	3

•	
0	0
1	4
0	
	0 - 0

11	0	0
0	1	0
0	0	1

neduced now echelon form