# Calculus for Machine Learning

Slope = $\frac{\text{Vertical change}}{\text{Horizontal change}}$

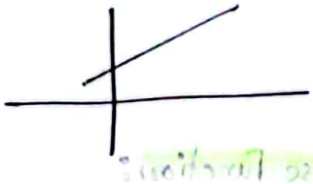
$= \frac{\Delta y}{\Delta x}$

Slope at a point = $\frac{dy}{dx}$

$y = f(x)$, $\therefore \frac{dy}{dx} = \frac{d}{dx} f(x)$ (Leibniz's notation)

Slope of a horizontal straight line is always 0. $f'(x) = 0$

For a straight line like this →

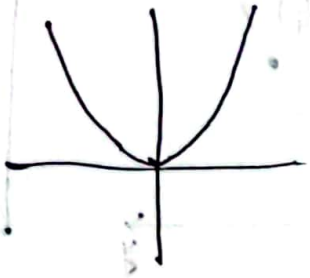


$f(x) = ax + b$

$f'(x) = a$ (slope)

Some Derivative for Quadratic Functions:

① $y = f(x) = x^2$



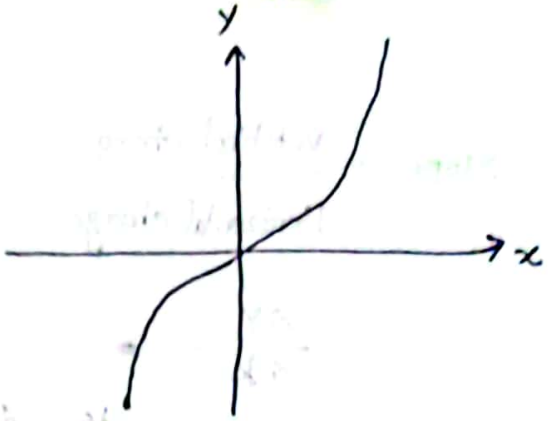
$f'(x) = \frac{d}{dx} f(x) = \frac{d}{dx}(x^2) = 2x$

Derivative of Cubic function:

$y = f(x) = x^3$

$f'(x) = \frac{d}{dx} f(x) = \frac{d}{dx}(x^3) = 3x^2$

$[a x^n \to n x^{n-1}]$



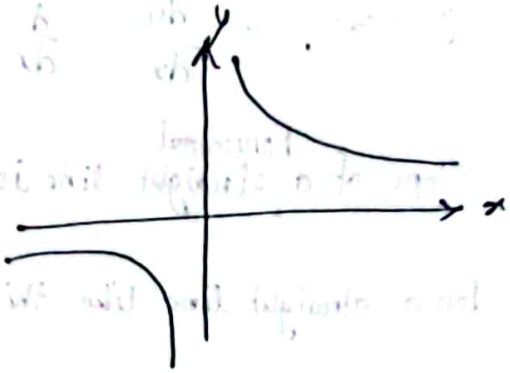

Derivative of other power function:

$y = f(x) = \frac{1}{x}$

$f'(x) = \frac{d}{dx} f(x) = \frac{d}{dx}\left(\frac{1}{x}\right) = \frac{d}{dx}(x^{-1})$

$= -1\, x^{-1-1}$

$= -x^{-2}$



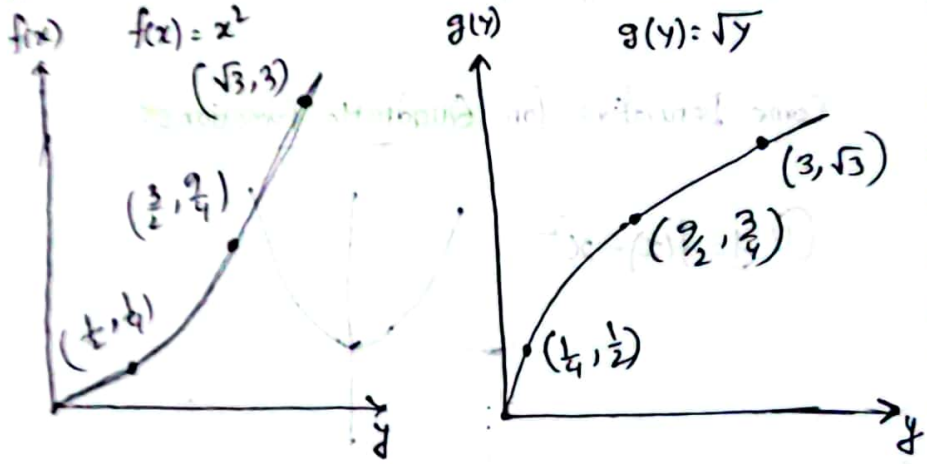

Derivative of Inverse Function:

$g(x)$ and $f(x)$ are inverses.

$g(x) = f^{-1}(x)$

$g(f(x)) = x$

$g'(y) = \frac{1}{f'(x)}$



## Derivative of trigonometric functions:

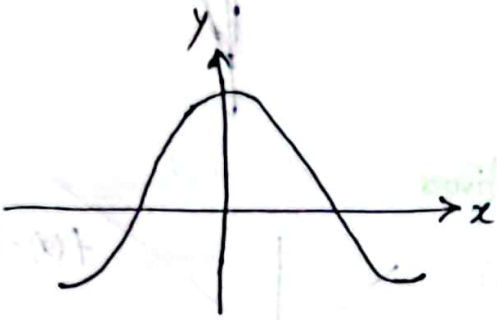

### sinx:

$f(x) = \sin x$

$\frac{d}{dx} f(x) = \frac{d}{dx}(\sin x) = \cos x$

| x | $\pi/2$ | $-\pi/2$ | 0 | $-2\pi$ |
|---|---|---|---|---|
| slope | 0 | 0 | 1 | -1 |
| cos(x) | 0 | 0 | 1 | -1 |

### cosx:




| x | 0 | $-\pi$ | $\pi/2$ | $-\pi/2$ |
|---|---|---|---|---|
| slope | 0 | 0 | -1 | 1 |
| sinx | 0 | 0 | 1 | -1 |

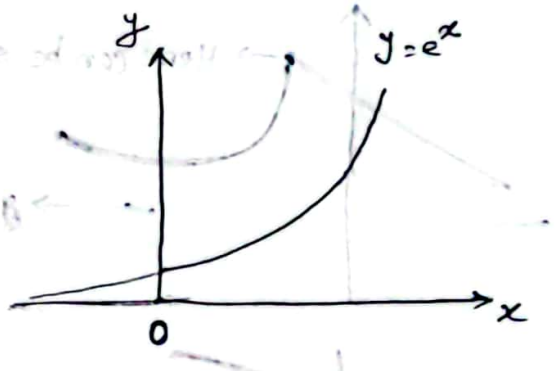
$f(x) = \cos x$

$f'(x) = \frac{d}{dx} f(x) = \frac{d}{dx} \cos x = -\sin x$

## Derivative of exponential function:

$f(x) = e^x$

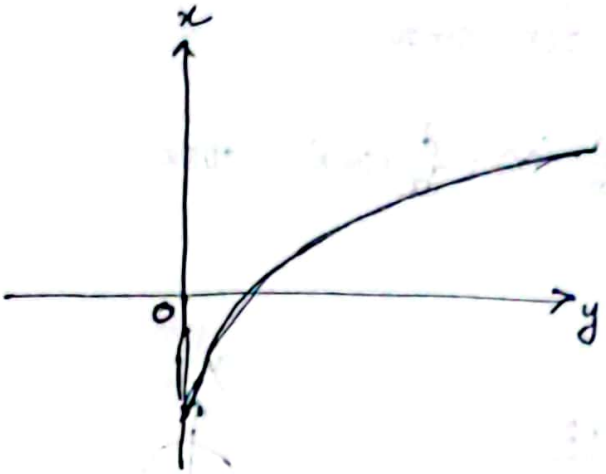
$f'(x) = \frac{d}{dx} e^x = e^x$



Derivative of logarithm:

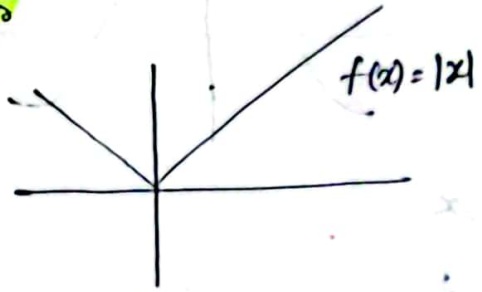
$f(x) = \log x$

$f'(x) = \frac{d}{dx} f(x) = \frac{d}{dx} \log x = \frac{1}{x}$

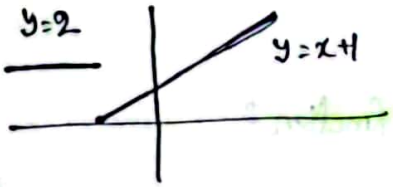



Some Functions don't have derivatives

① $f(x) = |x|$ (absolute Function)

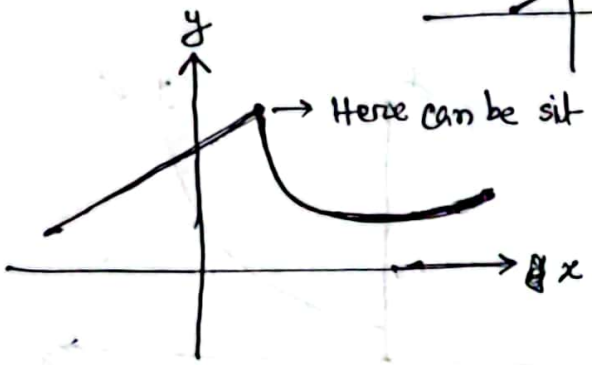



② Not continuous Functions



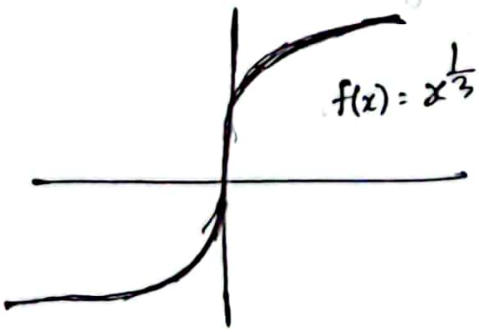

(Discontinuous)

③




→ Here can be sit multiple tangent (Note a single one)

④




(A slope can be made here but that would be parallel to y axis and 0)

## Properties of the derivative:

① Multiplication by Scalars: $\frac{d}{dx} k f(x) = k \frac{d}{dx} f(x)$

② The sum rule: $\frac{d}{dx}(u+v) = \frac{d}{dx}(u) + \frac{d}{dx}(v)$

③ The product rule: $\frac{d}{dx}(u \cdot v) = u \frac{d}{dx} v + v \frac{d}{dx} u$

④ The chain Rule:

If you have a function $y(u)$ and another function $u(x)$, where $y$ depends on $u$ and $u$ depends on $x$, then the chain rule allows you to find the derivative of the composite function $y(x)$, which depends directly on $x$. It can be expressed as follows.

If $y(u)$ is a function of $u$ and $u(x)$ is a function of $(x)$, then the derivative of $y$ with respect to $x$ $(dy/dx)$ is:

$$\frac{dy}{dx} = \left(\frac{dy}{du}\right) \times \left(\frac{du}{dx}\right)$$

The chain rule tells us that, to find the derivative of $y$ with respect to $x$, you first find the derivative of $y$ with respect to $u$ $\left(\frac{dy}{du}\right)$ and then multiply it by the derivative of $u$ with respect to $x$ $\left(\frac{du}{dx}\right)$