Some problems of Probabilities:

Question 01: A committiee of 7 members is to be formed from a group comprising 8 gentlemen and 5 ladies. What is the probability that the committee would comprise of 2 ladies?

$\rightarrow {}^{8}C_5 \times {}^{5}C_2$

Answer would be from 5 ladies select 2 $\rightarrow {}^{5}C_2$

from 8 man select 5 $\rightarrow {}^{8}C_5$ $(5+2 = 7$ people total$)$

$\therefore P(5 \text{ man}, 2 \text{ ladies}) = {}^{5}C_2 \times {}^{8}C_5$

Question 2: A committie of 7 members is to be formed from a group comprising 8 gentlemen and 5 ladies. What is the probability that the committee would comprise of at least 2 ladies?

Here, Event B = "at least 2 ladies" $\rightarrow \{2,3,4,5\}$

~~So probability of 7 member committe =~~

Probability of 7 members committee where there is at least 2 women,

$= ({}^{8}C_5 \times {}^{5}C_2) + ({}^{8}C_4 \times {}^{5}C_3) + ({}^{8}C_3 \times {}^{5}C_4) + ({}^{8}C_2 \times {}^{5}C_5)$

$= 1568$

Probabilities of picking 7 members out of 13 people $= {}^{13}C_7 = {}^{13}C_6 = 1716$ → I didn't get the transition

Favourable outcome $= \frac{1568}{1716}$

Question 03:

There are 3 boxes with the following compositions. One ball drawn from each box. What is the probability that they would be the same color?

Box	Blue	Red	White	Total
	5	8	10	23
	4	9	8	21
	3	6	7	16

Total number of outcomes of picking a ball $= {}^{23}C_1 \times {}^{21}C_1 \times {}^{16}C_1$

$= 7728$

Total number of outcome of picking a blue ball from the 3 boxes $= {}^{5}C_1 \times {}^{4}C_1 \times {}^{3}C_1$

$= 60$

Total number of outcome of picking a Red ball:

from the 3 boxes $= {}^{8}C_1 \times {}^{9}C_1 \times {}^{6}C_1$

$= 432$

Similarly for white ball $= {}^{10}C_1 \times {}^{8}C_1 \times {}^{7}C_1$

$= 560$

Number of finding the same color ball from the 3 boxes $= 60 + 432 + 560$

$= 1052$

$$\text{prob}(\text{favourable outcomes}) = \frac{1052}{7728} = 0.136$$

$= 13.6\%$

Question 04: If a coin is tossed twice, probability of obtaining one tail?

Sample space = { HH, HT, TH, TT }

∴ Event A = "One tail"

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{4} = \frac{1}{2}$$

Question 05: If a coin is tossed twice, probability of obtaining at least one tail?

S = { HH, HT, TH, TT }

Event A = " At least one tail"

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{4} = 0.75$$

Question 06: A bag contains 15 one-rupee coins, 25 two rupee coins, and 10 five rupee coins. If a coin is selected at random from the bag, then probability of not selecting a one rupee coin is?

$$P(\text{selecting a one rupee coin}) = \frac{15}{15+25+10} = \frac{15}{50}$$

$$\therefore P(\text{Not selecting a one rupee coin}) = 1 - \frac{15}{50} \quad \left[P(A') = 1 - P(A) \right]$$

$$= \frac{50-15}{50}$$

$$= \frac{35^{7}}{50_{10}} = \frac{7}{10}$$

Question 07: If two letters are taken from the word → Home, What is the probability that none of them would be vowels?

n(vowels) : 2
n(consonents) : 2

total outcome = $^4C_2 = 6$.

favourable outcome = To select 2 consonents from the consonents available.

$= {}^2C_2 = 1$

$\therefore$ Probability (favourable outcome) = $\frac{n(\text{favourable outcome})}{n(\text{total outcome})}$

$= \frac{1}{6}$

Ans

Question 08:

Two balls are drawn from a bag containing 5 white and 7 black balls at random. What is the probability that they would be of different color?

Total outcome = $^{12}C_2 = 66$

Total Number of outcome of picking a whiteball = $^5C_1$

$= 5$

Total Number of outcome of picking a black ball = $^7C_1$

$= 7$

Number of favourable outcomes (1 white and 1 black ball)

$= 7 \times 5$

$= 35$

$\therefore P(\text{favourable outcome}) = \frac{35}{66}$

Question 09: What is the chance of getting at least one defective item if three items are drawn randomly from a lot containing 6 items of which 2 are defective items?

There are at most 2 defective items from 6 items. So, 2 are defective and 4 normal

~~So, picking up at least 1 defective item from the six item~~

So 3 items can be picked like this → 1 defective, 2 item

→ 2 defective, 1 item

Total outcome : ${}^6C_3$

$= 20$

Total outcome of get picking 3 items where at least 1 item will be defective →

$= ({}^2C_1 \times {}^4C_2) + ({}^2C_2 + {}^4C_1)$

$= 2 \times 6 + 1 \times 4$

$= 16$

$\therefore P(\text{favourable outcome}) = \frac{16}{20} = 0.80$ Ans

Question 10: Two dice rolled together. What is the probability of getting no difference in points?

$n(S) \rightarrow 36$

$n(A) \rightarrow \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\} \rightarrow 6$

$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$ Ans

Question 11: There are 10 balls numbered from 1-10. If one of them is selected at random, What is the probability that the number printed on the ball would be an odd number > 4

$n(S) = 10$

$n(A) = \{5, 7, 9\}$ $\therefore P(A) = \frac{n(A)}{n(S)} = \frac{3}{10}$

Question 12: A lot of 10 electronic components is known to include 3 defective parts. If a sample of 4 components is selected at random from the lot, What is the probability that this sample does not contain more than one defective?

Means we have to find sample of 4 components contains only 1 defective or contains no defective

Total outcome (Total selectable components) : $^{10}C_4 = 210$

No of favourable outcome = $^{7}C_4 + {}^{3}C_1 \times {}^{7}C_3$ → (case 2)

→ (case 1)

$= 35 + 105$

$= 140$

$\therefore P(\text{favourable outcome}) = \frac{140}{210} = \frac{2}{3}$ A

Question 13: What is the probability that, a leap year selected at random would contain 53 saturdays?

There are 52 weeks in a 365 days year.

Each week contains 1 sat. So we have 52 sat in a year. $(52 \times 7) = 364$

In a leap year, there are 2 days extra (365, 366 No of days)

They can be (Sat, Sun) (Tue, Wed) (Fri, Sat)
(Sun, Mon) (Wed, Thu)
(Mon, Tue) (Thu, Fri)

Probability of getting sat in any of the 2 day : $\frac{2}{7}$

Ans

$\therefore P(\text{favourable outcome}) = \frac{52}{364} \times \frac{2}{7}$

Question 14: What is the probability of picking a spade or an ace not of spade from a pack of 52 cards?

$$P(\text{spade}) = \frac{13}{52}$$

$$P(\text{ace but not of spade}) = \frac{3}{52}$$

$$\therefore P(\text{favourable outcome}) = \frac{13}{52} + \frac{3}{52}$$

$$= \frac{16}{52}$$

$$= 0.30769$$

Question 15: Find the probability that a 4 digit number comprising the digits 2, 5, 6, and 7 would be divisible by 4.

Total no of outcomes = $4!$ (2, 5, 6, 7) 4 members

$= 24$

Condition of a number getting divisible by 4.

→ The ~~last sum of~~ last 2 digits have to be divisible by 4, then it can be said whole number will be divisible by 4.

First we will take two digits from (2, 5, 6, 7) and check which numbers are divisible by 4.

→ 25 26 27

52 56 57

62 65 67

72 75 76

These circled numbers are divisible by 4.

Let's make them 4 digit number with the remaining 2 digit

52 → 6752, 7652

56 → 2756, 7256

72 → 5672, 6572

76 → 2576, 5276

No of favorable outcome = 8

$$\therefore P(\text{favourable outcome}) = \frac{8}{24} = \frac{1}{3}$$

Question 16: What is the probability that 4 children selected at random have different Birthdays?

Total Days in a year = 365 (We won't take leap year)

$P(\text{Child 1 birthday}) = \frac{365}{365}$

$P(\text{Child 2 birthday}) = \frac{364}{365}$

$P(\text{Child 3 birthday}) = \frac{363}{365}$

$P(\text{Child 4 birthday}) = \frac{362}{365}$

$$P(\text{Favourable outcome}) = \frac{365}{365} \times \frac{364}{365} \times \frac{363}{365} \times \frac{362}{365}$$

$$= \frac{364 \times 363 \times 362}{(365)^3}$$

Question 17: A pair of dice is thrown together and the sum of the dice is noted to be 10. What is the probability that one of the dice has shown the point 4?

No. Total outcome = ~~36 (6²)~~ $\{(6,4), (5,5), (4,6)\} = 3$

no of favourable outcomes = $\{(6,4), (5,5), (4,6)\} = 2$

$\therefore P(\text{favourable outcome}) = \frac{2}{3}$

Question 18: In a group of 20 males and 15 females, 12 males and 8 females are service holders. What is the probability that a person selected at random from the group is a service holder given that the selected person is a male?

Total no. of outcome = 20 (Given that selected person is a male)

favourable outcome = 12

$\therefore P(\text{favourable outcome}) = \frac{12}{20}$

Ans

Question 19: If is given that a family of two children has a girl, what is the probability that the other child is also a girl?

~~1/2~~ (Because there can be either a boy or girl).

Sample space = {GB, BG, GG} = 3

No. favourable outcome = {GG} = 1

$\therefore P(\text{favourable outcome}) = \frac{1}{3}$

Question 20: If $P(A) = 5/9$, then the odds against Event A?

No of favourable outcome = 5

No. of total outcome = 9

No. of non favourable outcome = 9 − 5 = 4

$$\text{odds against Event A} = \frac{\text{No of non favourable outcome}}{\text{No of favourable outcome}}$$

$$= \frac{4}{5}$$ Ans

Equally likely Event: If probability is same for two or more ~~certain~~ events, then they are equally likely events.

→ Example tossing a coin → $P(H)=\frac{1}{2}$, $P(T)=\frac{1}{2}$

~~Rolling a dice → $P(1)=\frac{1}{6}$, $P(2)=\frac{1}{6}$ ..... $P(6)=\frac{1}{6}$~~

$P(\text{getting even on dice}) = \frac{3}{6} = \frac{1}{2}$
$P(\text{getting odd on dice}) = \frac{3}{6} = \frac{1}{2}$
} Equally likely events

Rolling a dice → $P(1)=\frac{1}{6}$, $P(2)=\frac{1}{6}$, ... $P(6)=\frac{1}{6}$

Impossible events: Whose probability = 0

Sure Event: Whose probability = 1

Simple event: A dice thrown and dice [2] appears (Can't broken down more)

Compound event: A dice thrown and ~~num~~ even number (2, 4, 6) appears (can be broken down)

Exhausted events: All possible events of an experiment.

Question 21: A number is selected at random from the first 1000 natural numbers. What is the probability that the number would be a multiple of 7 or 11?

No. of total outcome = 1000

Lcm of 7, 11 = 77

$n(7) = \frac{1000}{7} = 142$, $P(7) = \frac{142}{1000}$; $n(7 \cap 11) = 12$ $\frac{1000}{77} = 12$

$n(11) = \frac{1000}{11} = 90$ $P(11) = \frac{90}{1000}$ $\therefore P(7 \cap 11) = \frac{12}{1000}$

$\therefore P(7 \cup 11) = P(7) + P(11) - P(7 \cap 11)$

$= \frac{142}{1000} + \frac{90}{1000} - \frac{12}{1000}$

$= \frac{120}{1000} = 0.22$

Ans

Question 22: The probability that an accountant's job applicant has a B.com Degree is 0.85, that is a CA is 0.25 and that of both B.com and CA is 0.25 out of 500, applicants, how many would be B.com or CA?

A = B.com

B = CA

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.85 + 0.25 - 0.25$

$= 0.85$

Now, $P(A \cup B) = \frac{n(A \cup B)}{n(S)}$

$\Rightarrow n(A \cup B) = P(A \cup B) \times n(S)$

$= 0.85 \times 500$

$= 425$

Question 23: If $P(A-B) = 1/5$, $P(A) = 1/3$ and $P(B) = 1/2$, what is the probability that out of the two events A and B, only B would occur?

Means what is $P(B-A)$?

We know,

$$P(A-B) = P(A) - P(A \cap B)$$

$$\Rightarrow P(A \cap B) = P(A) - P(A-B)$$

$$= 1/3 - 1/5$$

$$= \frac{5-3}{15} = \frac{2}{15}$$

Now,

$$P(B-A) = P(B) - P(A \cap B)$$

$$= 1/2 - \frac{2}{15}$$

$$= \frac{15-4}{30} = \frac{11}{30}$$ Ans

Question 24: There are 3 person A, B and C having different ages. The probability that A survives another 5 years is 0·80, B survives another 5 years is 0·60 and C survives another 5 years is 0·50. The probability that A and B survive another 5 years is 0·46, B and C survive another 5 years 0·32 and A and C survive another 5 years 0·48 The probability that all these three persons survive another 5 years is 0·626 Find the probability that at least one of them survives another 5 years?

Formula, $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) - P(A \cap B \cap C)$

Question 25: If a card drawn at random from a pack of 52 cards, what is the chance getting a spade or ace?

A = "Spade" , B = "Ace"

$P(A) = \frac{13}{52} = \frac{1}{4}$ $P(B) = \frac{4}{52} = \frac{1}{13}$

$\therefore P(A \cap B) = \frac{1}{52}$

4 | 4, 13, 52
1, 13, 4

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{4} + \frac{1}{13} - \frac{1}{52}$

$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52}$

$= \frac{16}{52} = \frac{4}{13}$ Ans

Question 26: If $P(\bar{A} \cup \bar{B}) = 5/6$, $P(A) = 1/2$, $P(\bar{B}) = \frac{2}{3}$ $\therefore P(A \cup B) = ?$

$P(B) = 1 - P(\bar{B})$

$= 1 - \frac{2}{3}$

$= \frac{1}{3}$

$P(\bar{A} \cup \bar{B}) = P(A \cap B)'$ [De morgans law]

$\Rightarrow P(\bar{A} \cup \bar{B}) = 1 - P(A \cap B)$

$\Rightarrow P(A \cap B) = 1 - P(A' \cup B')$

$= 1 - \frac{5}{6}$

$= \frac{1}{6}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{2} + \frac{1}{3} - \frac{1}{6}$

$= \frac{3+2-1}{6} = \frac{4}{6} = \frac{2}{3}$ Ans