# Bayes Theorem Questions:

**Question 01:** Bag 01 contains 3 red and 4 black balls.

Bag 02 " 5 red " 6 black balls.

One ball randomely picked from any of the bag and found to be red. Find the probability that It was found from Bag 2.

Let, Event $E_1$ = "ball drawn from Bag 1"

$E_2$ = "ball drawn from Bag 2"

$A$ = "The drawn ball is Red"

As we have 2 bags, the probability of drawing ball from any of the bag $= \frac{1}{2}$

Means, $P(E_1) = P(E_2) = \frac{1}{2}$

$P(A|E_1)$ = Probability of getting red ball from bag 1

$= \frac{3}{7}$

Similarly $P(A|E_2) = \frac{5}{11}$ [Probability of getting red ball from bag 2]

From Bayes Theorem, $P(E_2|A) = \dfrac{P(E_2) \cdot P(A|E_2)}{P(E_2) \cdot P(A|E_2) + P(E_1) \cdot P(A|E_1)}$

$$= \frac{\frac{1}{2} \times \frac{5}{11}}{\frac{1}{2} \times \frac{5}{11} + \frac{1}{2} \times \frac{3}{7}}$$

$$= \frac{\frac{5}{22}}{\frac{5}{22} + \frac{3}{14}}$$

**Question 02:** A doctor knows that 10% of her patients have cancer. She also knows that a particular test for cancer is 90% accurate in detecting cancer in patients who actually have it, But 10% of the time it gives false positive (indicating cancer when there is none). If a patient tests positive for cancer, what's the probability that the patient actually have cancer?

Let, Event A = "the paitient have cancer"

B = "Patients test positive for cancer"

$P(A) = 10\% = 0.1 \rightarrow P(A') = 0.9$

$P(B) = 90\% = 0.9 \rightarrow P(B') = 0.1$

They asked for $\rightarrow P(\text{Patient actually have cancer} \mid \text{Patient tested positive})$

$= P(A|B)$

$$\therefore P(A|B) = \frac{P(A) \cdot P(B|A)}{P(A) \cdot P(B|A) + P(A') \cdot P(B|A')} \rightarrow \text{Bayes Theorem}$$

~~Or Bayes theorem can be written as~~

$$\cancel{P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}}$$

$$\cancel{= \frac{0.9 \times 0.1}{0.9}}$$

$\cancel{= 0.}$

$P(B|A)$ = P(Patient tested positive | Patient have cancer)

As A particular test is 90% accurate means 90% of patient will tested positive because ~~and also~~ they have cancer.

So, $P(B|A) = 90\% = 0.9$

$P(B|A')$ → Means P( Patient tested positive | Patient don't have cancer)

$= 10\% = 0.1$

$$\therefore \quad P(A|B) = \frac{P(A) \cdot P(B|A)}{P(A) \cdot P(B|A) + P(A') \cdot P(B|A')}$$

$$= \frac{0.1 \cdot 0.9}{0.1 \cdot 0.9 + 0.9 \cdot 0.1}$$

$$= 0.5$$

Ans

## Question 03:

A company knows that 2% of its products are defective. A customer buys a product and returns it, claiming that It is defective. The company tests the product and find that it is indeed defective. What is the probability that the customer is telling the truth.

Event A = "Defective products" $\therefore P(A) = 2\% = 0.02$

$\therefore P(A') = 0.98$

B = 'Customer telling the truth'

$P(A|B) = 1$, means, P(product found defective | Customer tells the truth) = 1

$$\cancel{P(B|A) = \frac{P(B) \cdot P(A|B)}{P(B) \cdot P(B|A) + P(B') \cdot P(B|A')}} = P(B) \cdot$$

~~But we are not provided with P(B) value.~~

$$\therefore P(B|A) = \frac{P(B) \cdot P(A|B)}{P(B) \cdot P(A|B) + P(B') \cdot P(A|B')}$$

But we are not provided with value P(B). So solution can't be made.

However, we can make some inferences. If

if $P(B) = 0.9$, Then $P(B|A) = 45\%$.

if $P(B) = 0.1$, Then $P(B|A) = 2\%$

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Question 5: In a legal trial, there are 12 jurors, 7 of whom are women and 5 are men. If the jury seqlection process ~~of rant~~ is random. what is the probability a random selected 3 juror panel will ~~cost~~ consist of 2 women and 1 man?

Event A: Selecting a 3-Juror panel with 2 women and 1 men.

Event B: Selecting a women on the first pick and a men on the second pick

$P(A) = P(1\text{ woman}) \times P(1\text{ woman}) \times P(1\text{ Man})$

$\cancel{= P(7/12) \times P(6/11) \times P(5/}$

$= \left(7/12 \times 6/11 \times 5/10\right)$

$= 0.159$

$P(B|A) = 1$ (Because we are certain about the selection at this moment)

$P(A') = 1 - P(A)$

$= 1 - \cancel{0.15}\, 0.159$

$= 0.841$

$P(B|A') = P($selecting a woman on the first pick and a man on the second pick $|$ ~~we~~ 2 women and 1 Man have not been selected$)$

This is the same as probability of selecting 1 women and 2 men in any order.

So, $P(B|A') = \left[\left(\frac{7}{12} \cdot \frac{5}{11} \cdot \frac{10}{10}\right) + \left(\frac{5}{12} \cdot \frac{7}{11} \cdot \frac{10}{10}\right)\right]$

$= 0.265 + 0.265 = 0.53$

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Bayes Formula, $P(A|B) = \dfrac{P(A) \cdot P(B|A)}{P(A) \cdot P(B|A) + P(A') \cdot P(B|A')}$

$$= \frac{0.159 \times 1}{0.159 \times 1 + 0.841 \cdot 0.53}$$

$$= 0.263.$$

Ans

**Question 6:** There is a diseases that affect 1% of the population. Researchers developed a diagnostic test for this disease which is 95% accurate, and a specificity of ~~&~~ 90% (meaning it correctly identify 90% of people without the diseases). If a person tests positive for the diseases. What is the probability they actually have that diseases?

Event A = "Having the diseases"

Event B = "Tested positive for the diseases"

$P(A) = 1\% = 0.01$ (Probability of having the disease)

$P(B|A) = 95\% = 0.95$ (Probability of testing positive given that have disease)

$P(A') = 100 - 1 = 99\% = 0.99$ (Probability of not having the diseases)

$P(B|A') = 100 - 90 = 10\% = 0.10$ (Probability of testing positive | Not having the diseases)

$$\therefore P(A|B) = \frac{P(A) \cdot P(B|A)}{P(A) \cdot P(B|A) + P(A') \cdot P(B|A')}$$

$$= \frac{0.01 \times 0.95}{0.01 \times 0.95 + 0.99 \cdot 0.10}$$

$$= 0.0876$$

$$= 8.76\%$$

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