Event: A subset of sample space called event.

In the scenario of tossing a coin →

Sample space = {HH, HT, TH, TT}

Event A = A Head appear = {HT, TH}

Event B = At least a Head appear = {HH, HT, TH}

Event C = At most 2 tail appear = {HH, HT, TH, TT}

Types of Event:

1) Impossible and Sure Event:

Sample space of dice = {1, 2, 3, 4, 5, 6}

Event A = "Number > than 6"

So, event A occurance is impossible for dice sample space.

Event B = "Number on dice is even or odd"

Every value in the sample space can relate with the event B.

So B is a sure event.

2) Simple Event:

Any event E is called simple if it has only one sample point from the sample space.

E = "Number in dice > 5" → {6}

3) **Compound Event:** Any event that contain at least 2 sample points from the sample space.

Example → Event A: "odd number in dice" → {1,3,5}

Event B: "number > 4 in dice" → {5,6}

Algebra of Event:

Complementary Event: Sample space = {1,2,3,4,5,6}

Event A = "Even number in dice" → {2,4,6}

Event A' = {1,3,5} → Numbers other than Event A.

$A \cup A' = S$ [S = Sample space]

Event A OR Event B: $(A \cup B)$ $\{x : x \in A \text{ OR } x \in B\}$

A = {3,6,9}, B = {2,3,8}

$A \cup B$ = {2,3,6,8,9} [Means Event A OR B OR Both]

Event A and Event B: $(A \cap B)$ $\{x : x \in A \text{ and } x \in B\}$

A = {3,6,9} & B = {2,3,6,10}

$A \cap B$ = {3,6}

Event A But not Event B: $(A-B)$ or $(A \setminus B)$

$A = \{2,4,6\}$ $B = \{1,2,3,4\}$

$A - B = \{6\}$, $B - A = \{1,3\}$

→ ~~Remove~~ keep only the unique elements

→ Remove the common elements.

Mutually exclusive Events: If there is no common element between two events such that $A \cap B = \emptyset$, then they called mutually exclusive events.

For A, B, C events, to be mutually ~~exclu~~ exclusive events →

1) $A \cap B = \emptyset$
2) $B \cap C = \emptyset$
3) $A \cap C = \emptyset$

seperately their pairs intersection should be $\emptyset$

Exhaustic events: When $A \cup B$ Becomes $= S$, then they are called exhaustic events

$A = \{1,3,5\}$, $B = \{2,4,6\}$ $S = \{1,2,3,4,5,6\}$

$\therefore A \cup B = \{1,2,3,4,5,6\}$

$= S$

So, A and B are Exhaustic events.

[Events can be more than two also]

The previous example events can also be called mutually exclusive and exhausted events.

Question 1: $E = \{4\}$, $F = \{2,4,6\}$. Are they mutually exclusive?

$E \cap F = \{4\}$ which is not $\emptyset$

So they are not mutually exclusive events.

Question 2: $S = \{1,2,3,4,5,6\}$

Events, A = number $< 7 \rightarrow \{1,2,3,4,5,6\}$

B = number $> 7 \rightarrow \{\emptyset\}$

C = multiple of $3 \rightarrow \{3,6\}$

D = number $< 4 \rightarrow \{1,2,3\}$

E = Even and $> 4 \rightarrow \{6\}$

F = not $< 3 \rightarrow \{4,5,6\}$

$F' = \{1,2,3\}$

$A \cup B = \{1,2,3,4,5,6\}$ $A \cap B = \{\emptyset\}$, $B \cup C = \{3,6\}$, $E \cap F = \{6\}$

$D \cap E = \{\emptyset\}$, $A - C = \{1,2,4,5\}$, $D - E = \{1,2,3\}$

$E \cap F' = \{\emptyset\}$.

Properties of Probability:

Let, A is any event $A \subseteq S$

1) $0 \leq P(A) \leq 1$

2) $x_1, x_2, x_3, \ldots x_n \in S$, then $\sum_{i=1}^{n} P(x_i) = 1$

3) if $x_1, x_2, x_3 \ldots x_m \in A$

$$P(x_1) + P(x_2) + P(x_3) + \cdots P(x_m) = P(A) \quad [m \leq n]$$

4) $P(A) = 0$ means impossible event

5) $P(\phi) = 0$, $P(S) = 1$

Probability of Event A or Event B: $(A \cup B)$

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

Question 1: After throwing / tossing two coins →

Event A = At least one Head. ⇒ $A = \{HT, TH, HH\}$

Event B = At least one Tail → $B = \{TT, TH, HT\}$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{4}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{3}{4}$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{2}{4}$$

$$\therefore P(A \cup B) = \frac{3}{4} + \frac{3}{4} - \frac{2}{4}$$

$$= \frac{6-2}{4} = \frac{4}{4} = 1$$

If $A \cap B = \emptyset$, means both events are mutually exclusive, then $P(A \cap B) = 0$

So, $\boxed{P(A \cup B) = P(A) + P(B)}$

~~For~~ Probability of event not A : (A')

$$A' = S - A$$

$$P(A') = P(S) - P(A)$$

$$= 1 - \frac{n(A)}{n(S)}$$

$$\therefore \boxed{P(A') = 1 - P(A)}$$

Also → $\boxed{P(A \cup B)' = P(A' \cap B')}$

Question 2: A coin is tossed twice. What is the probability at least a tail occurs?

$S = \{HH, HT, TH, TT\}$

Event A = "Outcome at least one tail" $\longrightarrow \{HT, TH, TT\}$

$\therefore P(\text{get A}) = \frac{n(A)}{n(S)} = \frac{3}{4}$

Question 03: A dice is rolled.

Event A = 'A prime number will appear'

$S = \{1, 2, 3, 4, 5, 6\}$

$A = \{2, 3, 5\} \quad \therefore P(A) = \frac{3}{6} = \frac{1}{2}$

Event B = "A number > 2"

$S = \{1, 2, 3, 4, 5, 6\}$

$B = \{3, 4, 5, 6\} \quad \therefore P(B) = \frac{4}{6} = \frac{2}{3}$

Event C = 'A number ≤ 1 will appear'

$S = \{1, 2, 3, 4, 5, 6\}$

$C = \{1\} \quad \therefore P(C) = \frac{1}{6}$

~~Event D =~~

Question 04: Three coins are tossed.

$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

Event A = "3 Heads" $\rightarrow A = \{HHH\}$

$\therefore P(A) = \frac{1}{8}$

Event B = "2 Heads" $\rightarrow B = \{HHT, HTH, THH\}$

$\therefore P(B) = \frac{3}{8}$

Event C = "at least 2 Heads" $\rightarrow C = \{HHT, HTH, THH, HHH\}$

$P(C) = \frac{4}{8} = \frac{1}{2}$

Event D = "At most 2 Heads" $\rightarrow D = \{HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$P(D) = \frac{6}{8} = \frac{3}{4}$ $\frac{7}{8}$

Event E = "No Head" $\rightarrow E = \{TTT\}$

$P(E) = \frac{1}{8}$

Event F = "3 tails" $\rightarrow F = \{TTT\}$

$P(F) = \frac{1}{8}$

Question 05: If $\frac{2}{11}$ is the probability of an Event, What is the probability not A Event?

$$P(A') = 1 - P(A)$$
$$= 1 - \frac{2}{11}$$
$$= \frac{9}{11}$$ Ans

Question 06: A word is given → Assassination. Find the probability of vowel and consonant from the letter.

$$S = \{A, s, s, a, s, s, i, n, a, t, i, o, n\} \quad n(S) = 13$$

Event A = "set of vowel" → $\{A, a, i, a, i, o\}$ $n(A) = 6$

Event B = "set of consonant" → $\{s, s, s, s, n, t, n\}$ → $n(B) = 7$

$$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{6}{13}$$

$$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{7}{13}$$ Ans