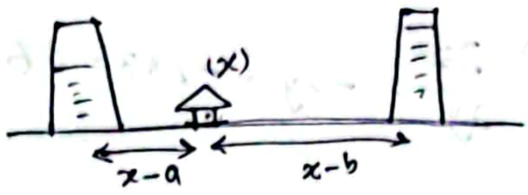
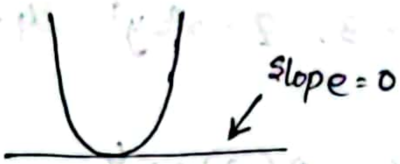
Lesson - 2 : Optimization

Description: Optimization is when you find the minimum value and maximum value of a function.



Optimization of squared loss:

$$f(x) = (x-a)^2 + (x-b)^2$$



$$\frac{d}{dx} f(x) = 2(x-a) + 2(x-b)$$

As minima = 0

$$\therefore \frac{dy}{dx} \quad 2(x-a) + 2(x-b) = 0$$

$$\Rightarrow (x-a) + (x-b) = 0$$

$$\Rightarrow 2x - a - b = 0$$

$$\Rightarrow x = \frac{a+b}{2}$$

[To understand the whole context need to go through W1, Lesson 2]

Similarly for $f(x) = (x-a)^2 + (x-b)^2 + (x-c)^2$

$$x \text{ would be } = \frac{a+b+c}{3}$$

And for $f(x) = (x-a_1)^2 + (x-a_2)^2 + \dots\dots (x-a_n)^2$

$$x \text{ would be } = \frac{a_1 + a_2 + \dots + a_n}{n}$$

Optimization of log loss:

Probability of getting heads 7 times in 10 coin tosses.

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p $1-p$

Chances of winning $g(p) = p^7 (1-p)^3$

Goal: maximize $g(p)$

$$\frac{dg}{dp} = \frac{d}{dp}\left(p^7 (1-p)^3\right)$$

$$= p^7 \frac{d}{dp}(1-p)^3 + (1-p)^3 \frac{d}{dp} p^7$$

$$= p^7 \cdot 3(1-p)^2 \cdot (-1) + 7p^6 (1-p)^3$$

$$= p^6 (1-p)^2 \left[7(1-p) - 3p\right]$$

$$= p^6 (1-p)^2 (7 - 10p) = 0$$

$\hookrightarrow p = 0$ $\hookrightarrow p = 1$ $\hookrightarrow p = \frac{7}{10} = 0.7$

$p = 0$, and $p = 1$ is not possible according to question.

So, $p = 0.7$

To make the process easier we can use the log loss method →

→ $\log(g(p))$ [maximizing $\log(g(p))$ is the same thing like maximizing $g(p)$]

$$\rightarrow \log(g(p)) = \log\left(p^7(1-p)^3\right)$$

$$= \log p^7 + \log\left((1-p)^3\right)$$

$$= 7\log p + 3\log(1-p)$$

$$= G(p) \longrightarrow \text{[The log loss function]}$$

Now,

$$\frac{d\,G(p)}{dp} = \frac{d}{dp}\left(7\log p + 3\log(1-p)\right)$$

$$= 7\cdot\frac{1}{p} + 3\,\frac{1}{1-p}\,(-1)$$

$$= \frac{7}{p} - \frac{3}{1-p}$$

$$= \frac{7(1-p) - 3p}{p(1-p)}$$

Why logarithm?

Because that makes the calculation more easier step for more complex function than doing derivative normally.

Now,

$$\frac{7(1-p) - 3p}{p(1-p)} = 0$$

$$\Rightarrow 7(1-p) - 3p = 0$$

$$\Rightarrow 7 - 7p - 3p = 0$$

$$\Rightarrow 7 - 10p = 0$$

$$\Rightarrow p = \frac{7}{10} = 0.7$$