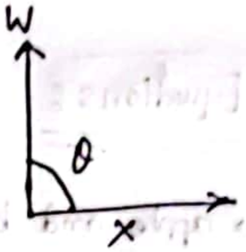
Now, $w^T x = |W| |x| \cos\theta$.



if $W^T x = 0$

$\Rightarrow |W| |x| \cos\theta = 0$

$\Rightarrow \theta = 90^\circ$

This means W is always perpendicular to the plane.

This next Notes are from Coursera

Deep Learning. AI → Linear Algebra.

Systems of Sentence:

System 1: The dog is black
The cat is orange → Complete system
(Non singular system)

System 2: The dog is black
The dog is black → It's a Redundant system
(Singular system)

System 3: The dog is black
The dog is orange → Contradictory system
(singular system)

Singular System: When a sentence is redundant or contradictory that is singular system.

Non-singular: When the system is complete.

System of Equations:

→ apple and banana cost 10$

→ apple and two banana cost 12 $

apple = x, banana = y   Question: How much does each fruit cost

$$\begin{aligned} x + 2y &= 12 \\ -\,(x + y &= 10) \\ \hline y &= 2 \end{aligned}$$

$\therefore x = 10 - y = 10 - 2 = 8$

So, apple cost 8 $, Banana cost 2 $

For

$\left.\begin{aligned} &\rightarrow x + y = 10 \\ &\rightarrow 2x + 2y = 20 \end{aligned}\right\}$ There is ~~most~~ many solution for this

x can be 8, y can be 2
x " " 2, y " " 8
x " " 6, y " " 4 and many more

So, it can be said that, not enough info given

For,

$\left.\begin{aligned} &\rightarrow x + y = 10 \\ &\rightarrow 2x + 2y = 24 \end{aligned}\right\}$ There is also no solution for this

because, if $x + y = 10$
$2x + 2y$ should be 20
not 24

There might be mistake.

Here 20 and 24 are contradictory.

For,
$$a + b = 10$$
$$a + 2b = 12$$
$$-b = -2$$
$$\therefore b = 2 \qquad (a,b) = (8,2)$$
$$\therefore a = 10 - 2 = 8$$

In this equation, we can find the exact solution.

So, in summeery →

System 1

$a + b = 10$

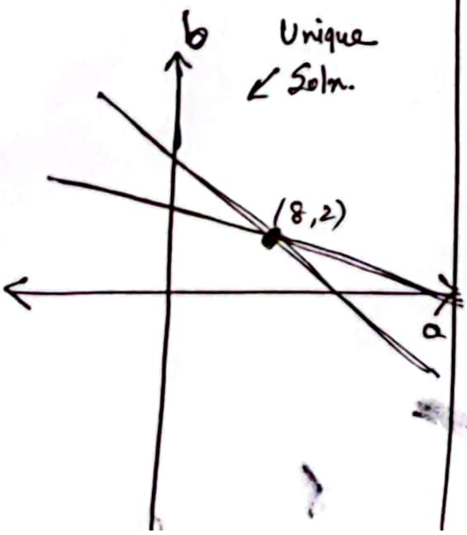
$a + 2b = 12$

We have unique solution

$a = 8, b = 2$

So, this is Complete

→ Non-Singular



System 2

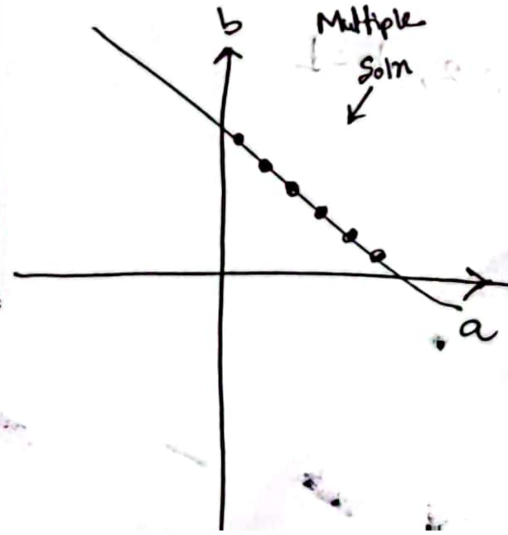
$a + b = 10$

$2a + 2b = 20$

Infinite solutions

So, this is redundant

→ Singular



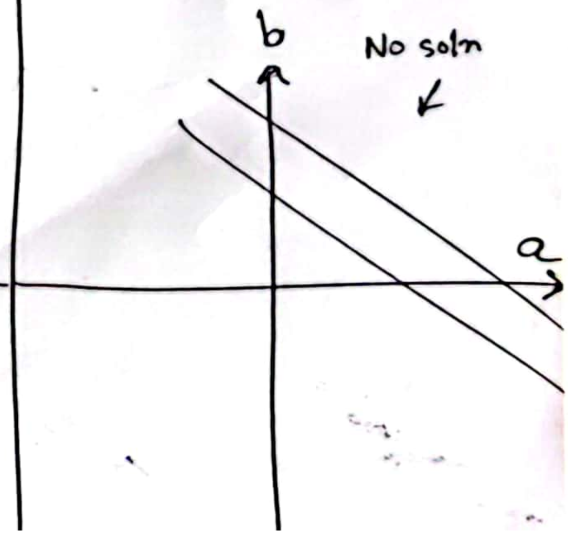
System 3

$a + b = 10$

$2a + 2b = 24$

equations are contradictory and no solution

→ Singular



$3a + 2b = 8$

$\frac{3}{8}a + \frac{1}{4}b = 0$

System of equations as lines :

$$3a + 2b = 8$$
$$2a - b = 3$$
$$\overline{5a + b = 11}$$

Now, $2a - b = 3$
$$5a + b = 11$$
$$\overline{7a = 14}$$

$\therefore a = \frac{8}{7} = 2$

$\cancel{\therefore b = 5 - \frac{8}{7}}$

$\cancel{= \frac{7}{3} = 2.3}$

$\cancel{= 15 - 1.1}$

$\cancel{= 9.9}$

1st

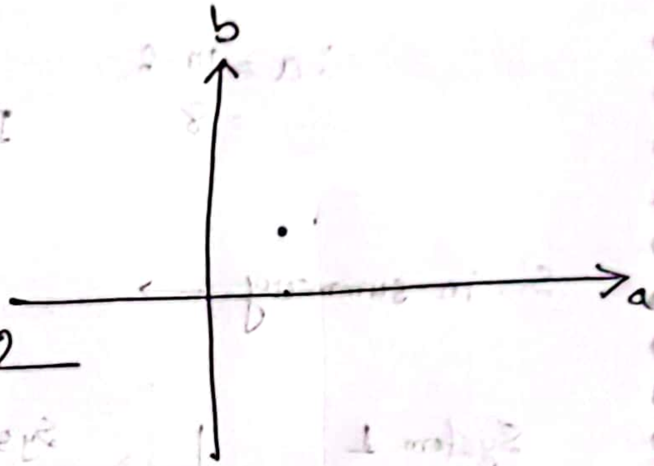
$2a - b = 3$

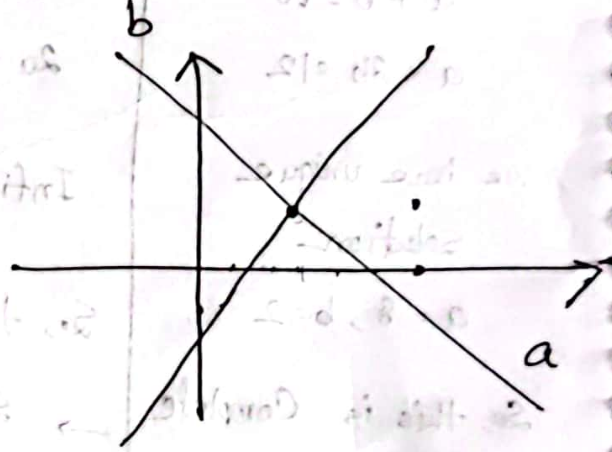
$\Rightarrow 2 \times 2 - b = 3$

$\Rightarrow -b = -1$

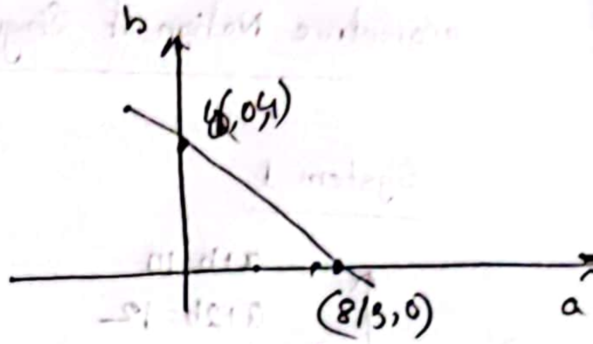
$\therefore b = 1$

$\therefore a = 2, b = 1$





How can we plot an equation,



→ $3a + 2b = 8$

if, $b = 0$, $3a = 8$, $a = \frac{8}{3}$

if, $a = 0$, $b = 4$

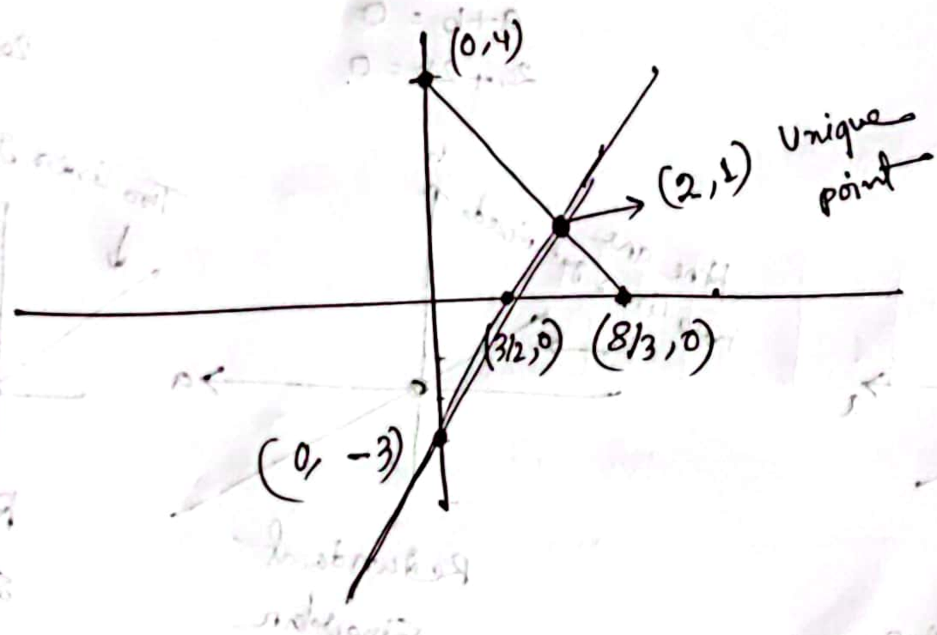
So, a straight line plot can be $(8/3, 0)$ and $(0, 4)$

→ $2a - b = 3$

if $a = 0$, $b = -3$

if $b = 0$, $a = \frac{3}{2}$

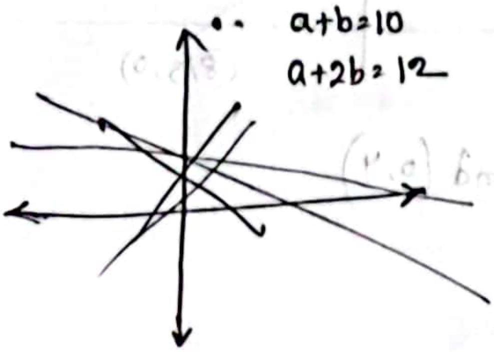
a straight line plot can be $(3/2, 0)$ and $(0, -3)$

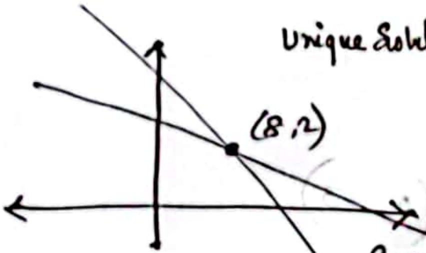


Geometric Notion of Singularity:

System 1

$a+b=10$
$a+2b=12$



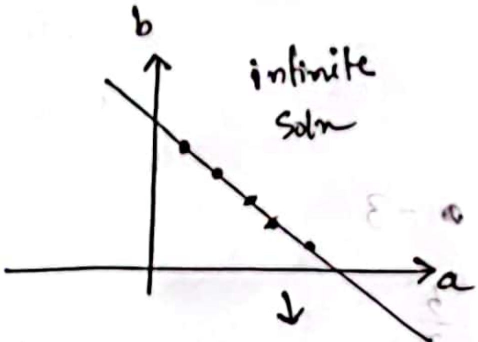


Unique Solution

Complete (Non Singular)

System 2

$a+b=10$
$2a+2b=20$

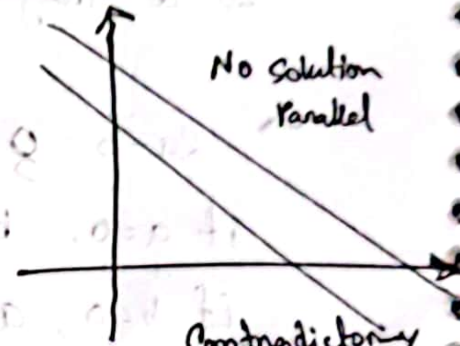


infinite soln

Redundant (Singular)

System 3

$a+b=10$
$2a+2b=24$

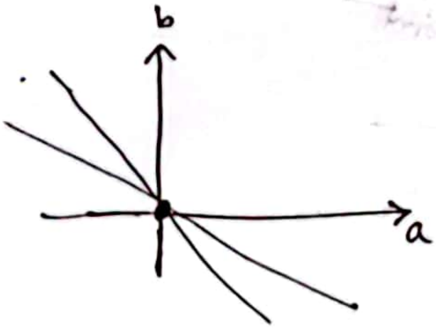


No solution Parallel

Contradictory (Singular)

if we remove the constants

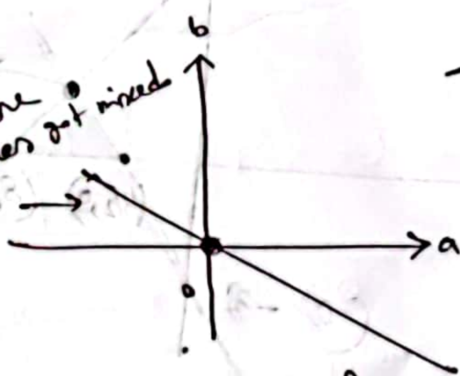
$a+b=0$
$a+2b=0$



Complete
Non Singular

$a+b=0$
$2a+2b=0$

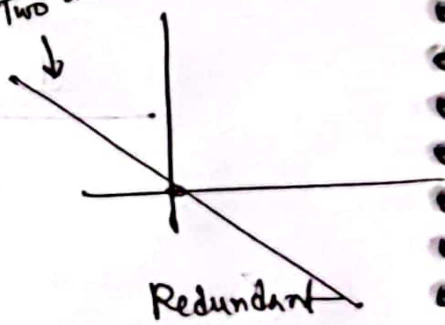
Here are two lines got mixed



Redundant
Singular

$a+b=0$
$2a+2b=0$

Two lines got mixed



Redundant
Singular

→ So, we can say that the constants are not required to define a system is Singular or Non singular.

→ Also, contradictory solutions turns into redundant if we remove constants from the equations.

Systems of equations as Matrix:

System 1

$a+b=0$
$a+2b=0$ → $\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$

Non singular System — Non-singular Matrix

(Unique Solution)

System 2

$a+b=0$
$2a+2b=0$ → $\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$

Singular System — Singular Matrix

(Infinitely Many solution)

Linear Dependence between rows:

In a singular system →

1) Second equation is the multiple of first equation
2) In terms of matrix, Second rows are multiple of first row
3) Second row depends on first row, or viseversa

$a+b=0$
$2a+2b=0$ | $\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$

→ In Non Singular system

1) Rows or equations are independent from each other

$$a+b=0 \qquad a+2b=0 \qquad \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

The determinant rule: (To find singular and non singular quickly)

A Matrix is singular if → $\begin{bmatrix} a & b \end{bmatrix} * K = \begin{bmatrix} c & d \end{bmatrix}$

$$aK = c$$
$$bK = d$$

$$\Rightarrow \frac{c}{a} = \frac{d}{b} = K$$

$$\Rightarrow ad = bc.$$

$$\Rightarrow ad - bc = 0$$

Determinant $\boxed{ad - bc = 0}$

So, $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \Rightarrow 1\times 2 - 1\times 1 = 1$

So, determinant $= 1 \neq 0$

Means non singular Matrix

$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \Rightarrow 1\times 2 - 1\times 2 = 0$

So, determinant $= 0$

means Singular Matrix

System of equations (3X3)

① System 1

$x+y+z=10$

$x+2y+z=15$

$x+y+2z=12$

$\rightarrow$

$x+y+2z=12$

$x+y+z=10$

$z=2$

$\therefore z=2$

$\rightarrow$ $\cancel{x}+\cancel{y}+\cancel{3}=\cancel{12}$, $x+\cancel{y}+\cancel{1}=10$ $\downarrow$ $x+\cancel{y}=$

Now, $x+2y+z=15$, $x+y+z=10$ $\rightarrow$ $x+2y=13$, $x+y=8$; $(-)\ (-)$; $y=5$

Now, $x+y+z=10$

$\Rightarrow x+5+2=10$

$\Rightarrow x=3$

$\therefore (x,y,z)=\cancel{(1,5,1)}\ (3,5,2)$

As we found the unique points,
The system is complete and non singular

System 2

② $\rightarrow$

$a+b+c=10$

$a+b+2c=15$

$a+b+3c=20$

$\rightarrow$

$a+b+2c=15$

$a+b+c=10$

$c=5$

$\rightarrow$

Only we can ~~found~~ find the value of c.

a, b values cannot be find from the equation.

In terms of assumption, a and b can have infinite solutions.

$\rightarrow$ Redundant

~~Non~~ Singular System

System 3

(3)

$$a+b+c=10$$
$$a+b+2c=15$$
$$a+b+3c=18$$

The third equation is not relying on the other two equations for $a+b+3c$ the constant value should be ~~18~~ 20

So, it is a contradictory singular system.

Now, if remove constants from the 3 systems

(1)

$x+y+z=\cancel{10}\ 0$	$x+y+z=0$	$x+y+z=0$
$x+2y+z=\cancel{15}\ 0$	$x+y+2z=0$	$x+y+2z=0$
$x+y+2z=\cancel{12}\ 0$	$x+y+3z=0$	$x+y+3z=0$

Unique soln:

$a=0$ $x=0$
$b=0$ $y=0$
$c=0$ $z=0$

Complete
Non-singular

Redundant soln

Singular

Infinite soln:

$c=0$ $z=0$
$a+b=0$ $x+y=0$

(watch system of equations as planes (3x3) to visualize.)

Linear dependence and independence:

if in a 3D plane equations or in a 3D matrix if the third equation or row depends on the other two rows, then the 3rd row is linearly dependent to the others. There fore ~~they will~~ the equation & matrix will be singular.

Example can be →

$$a + b + c = 10$$
$$2a + 2b + 2c = 20 \quad \rightarrow \text{ This the singular eqn}$$
$$3a + 3b + 3c = 30$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix} \rightarrow \text{ This the singular matrix}$$

It's not that only row 3, has to be dependent on Row 1 and Row 2

So Row 2 can be dependent on Row 1 and Row 3, Row 1 can be dependent on Row 2 and Row 3.

For example

$$a+b+c = 0$$
$$a+b+2c = 0$$
$$a+b+3c = 0$$

$$\begin{aligned} & a+b+c = 0 \\ + \; & a+b+3c = 0 \\ \hline & 2a+2b+4c = 0 \end{aligned}$$

$$\Rightarrow a+b+2c = 0$$

which is eqn(2)

For this equation →

$$\left.\begin{aligned} a+b+c&=0 \\ a+2b+c&=0 \\ a+b+2c&=0 \end{aligned}\right\}$$ no such relations between equations.

So, they are linearly independent.

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \rightarrow$$ non singular matrix

Some Excercise to check Linear, dependence or independence

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array}\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 3 & 2 & 3 \end{bmatrix} \rightarrow$$

$3 \times \text{Row1} = 3 \quad 0 \quad 3$

$2 \times \text{Row2} = 0 \quad 2 \quad 0$

$\overline{\qquad 3 \quad 2 \quad 3}$

↳ which is Row3

So, they are linearly dependent

(2)

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ 0 & 0 & -1 \end{bmatrix}$$

$R_1 - R_2 = R_3$

So, they are also linearly dependent

(3)

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array}\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 3 & 3 \end{bmatrix}$$

→ No relations

So, they are linearly independent,

(4)

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array}\begin{bmatrix} 1 & 2 & 5 \\ 0 & 3 & -2 \\ 2 & 4 & 10 \end{bmatrix}$$

$R_1 \times 2 = 2 \quad 4 \quad 10$

$= R_3$

So, they are linearly dependent

The determinant:

For 3X3 matrix →

1 +	1 −	1 ×
1 ×	2 +	1 −
1 −	1 ×	2 +

→ The diagonals are multiple and ~~subtrac~~ added cording to the signs

$(1\times2\times2) + (1\times1\times1) + (1\times1\times1)$

$= 4+1+1 = 6$

Then we have to subtract it with the anti diagonals

1 ×	1 −	1 +
1 −	2 +	1 ×
1 +	1 ×	2 −

→ $(1\times2\times1) + (1\times1\times1) + (1\times1\times2)$

$= 2+1+2 = 5$

$\therefore 6-5 = 1$

result is 1 $\neq 0$, So Non singular system/ matrix

if determinant is $= 0$, then singular Matrix

if 〃 〃 $\neq 0$, then non singular Matrix

Excercises:

$$\begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 3 & 3 & 3 \end{vmatrix} \rightarrow (1\times1\times3)+(0\times0\times3)+(1\times0\times3)$$

$$= 3$$

$$\begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 3 & 3 & 3 \end{vmatrix} \rightarrow (1\times1\times3)+(0\times0\times3)+(0\ 1\times0\times3)$$

$$= 3$$

$$\text{det} = 3-3 = 0$$

So, singular matrix!

(2)

$$\begin{array}{|c|c|c|}\hline 1^{+} & 1^{-} & 1^{\times} \\ \hline 1^{\times} & 1^{+} & 2^{-} \\ \hline 0^{-} & 0^{\times} & -1^{+} \\ \hline \end{array} \rightarrow (1\times1\times(-1)) + (1\times2\times0) + (1\times1\times0)$$

$$= -1$$

$$\begin{array}{|c|c|c|}\hline 1^{\times} & 1^{-} & 1^{+} \\ \hline 1^{-} & 1^{+} & 2^{\times} \\ \hline 0^{+} & 0^{\times} & -1^{-} \\ \hline \end{array} \rightarrow 0 + (-1) + 0$$

$$= -1$$

$$\det = -1 - (-1)$$

$$= 0$$

(3)

$$\begin{array}{|c|c|c|}\hline 1 & 2 & 5 \\ \hline 0 & 3 & -2 \\ \hline 2 & 4 & 10 \\ \hline \end{array} \rightarrow \cancel{30} + 30 + (-8) + 0$$

$$= 22$$

$$\begin{array}{|c|c|c|}\hline 1 & 2 & 5 \\ \hline 0 & 3 & -2 \\ \hline 2 & 4 & 10 \\ \hline \end{array} \rightarrow 30 + 0 + (-8)$$

$$= 22$$

$$\det = 22 - 22$$

$$= 0$$

General structure

(Here, sign representing values that has to multiply together)

Left diagonal

+	−	×
×	+	−
−	×	+

Multiply the signs, and add them

Right Diagonal

×	−	+
−	+	×
+	×	−

Multiply the signs and add them

Determinant = Left diagonal − Right diagonal.