Conditional Probability: P(A) Given that B already occured.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Properties: If E and F are two events and S is the sample space.

① $P(S|F) = P(F|F) = 1$

② If A, B and F are 3 events and S = sample space,

$$P((A \cup B)|F) = P(A|F) + P(B|F) - P(A \cap B|F) \quad [P(F) \neq 0]$$

③ $P(A'|B) = 1 - P(A|B)$

Question 1: After tossing three coins, what is the probability of getting 2 heads, given that last one is tail.

Event A = "2 heads" → {HHT, HTH, THH}

Event B = "last one tail" → {HHT, HTT, THT, TTT}

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \Rightarrow P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{8}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{4}{8} = \frac{1}{2}$$

$$\therefore P(A|B) = \frac{1}{8} \times 2$$

$$= \frac{1}{4}$$

Ans

Independent Event :

Let A anB are two event such that—

$$P(A \cap B) = P(A) \cdot P(B) \qquad P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$$

Then A and B are called independent events.

Mutually exclusive events A and B Don't have a relation with being independent.

A and B events which are mutually exclusive, they can be independent or not independent.

→ For being mutually exclusive → $(A \cap B) = \emptyset$

→ For being independent → $P(A \cap B) = P(A) \cdot P(B)$

→ If A and B are two independent event

Then A' and B', A and B', A' and B' they are also independent.

$$\therefore P(A' \cap B) = P(A') \cdot P(B)$$

$$P(A \cap B') = P(A) \cdot P(B')$$

$$P(A' \cap B') = P(A') \cdot P(B')$$

Question 01: From a pack of 52 playing cards, find the probability that both the cards are black

Let, Event A = "1st card is black" $\rightarrow P(A) = \frac{26}{52}$

Event B = "2nd card is black" $\rightarrow P(B|A) = \frac{25}{51}$

As Event A an B Both are independent

$P(A \cap B) = P(A) \cdot P(B|A)$

$= \frac{26}{52} \cdot \frac{25}{51} = \frac{25}{102}$

Question 03: A box of oranges examined, 3 randomly selected oranges drawn without replacement. If all 3 oranges are good, then the box is ready for sell Out of which 12 oranges are good and 3 are bad, the box would be sent for sell. find the probability, Out of 15 oranges, 12 are good and 3 are bad, that box is packed and sent for sell.

Event A = "First orange is good" $\rightarrow P(A) = \frac{12}{15}$

Event B = "Second orange is good" $\rightarrow P(B|A) = \frac{11}{14}$ [without replacement]

Event C = "Third orange is good" $\rightarrow P(C|A \text{ and } B) = \frac{10}{13}$ [without replacement]

$P(A \cap B \cap C) = P(A) \cdot P(B|A) \cdot P(C|A, \text{and } B)$

$= \frac{12}{15} \times \frac{11}{14} \times \frac{10}{13}$

$= \frac{44}{91}$

Question 04: You toss a coin and roll a dice. What is the probability of getting heads on the coin and an even number on the dice.

Let ~~P(A~~ Event A = "Getting heads on the coin"

$$P(A) = \frac{1}{2}$$

Event B = "even number on the dice"

$$P(B) = \frac{n(B)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

As they both are independent event;

$$\therefore P(A \cap B) = P(A) \cdot P(B)$$

$$= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \quad \text{Ans.}$$

Question 05: A deck of 52 cards used. If you draw a card, note its value and put it back in the deck before picking up again, What is the probability of drawing a red card on the first draw and a face card (Jack, king, queen) on the second draw.

Event A = "Picking up red card"

$$\therefore P(A) = \frac{26}{52} = \frac{1}{2}$$

Event B = "A face card"

There are $(3 \times 4) \rightarrow 12$ face cards in a deck

$$\therefore P(B) = \frac{12}{52} = \frac{3}{13}$$

As each card is getting replaced again. So it can be said that, Both events are independent.

$$\therefore P(A \cap B) = P(A) \cdot P(B)$$
$$= \frac{1}{2} \times \frac{3}{13} = \frac{3}{26}$$

Question 6: A bag contains 4 red marbels and 6 blue marbles. If you draw 2 marbles without replacement, What is the probability of drawing a red marble on the first draw and the blue marble on the second ~~draw~~ draw?

Event A = "first one is a red marble"

$$P(A) = \frac{4}{10} \quad \frac{n(\pi)}{n(S)} = \frac{4}{10} = \frac{2}{5}$$

Event B|A ~~Event B~~ = "second one is a ~~red~~ Blue marble"

$$P(B|A) = \frac{6}{9} = \frac{2}{3}$$

[The events are dependent]

$$\therefore P(A \cap B|A) = \frac{2}{5} \times \frac{2}{3} = \frac{4}{15}$$

~~Ans~~

Question 07: You go bowling and simultaneously flip a coin. In order to win a special price, you need to knock down all pins in the first strike and get Heads on the coin. The probability of a strike is 0.2, and probability of getting heads on the coin = 0.5. What is the probability of winning the special price?

Event P(A) = "knock down all pins"

$$P(A) = 0.2$$

Event (B) = "Heads on the coin"

$$P(B) = 0.5$$

$$\therefore P(A \cap B) = P(A) \cdot P(B)$$

$$= 0.2 \times 0.5 = 0.1$$

Question 08: You are drawing cards from a deck without replacement and rolling a fair dice.

a) You draw card from the 52 ~~deck~~ cards. What is the probability of your card being red?

b) After that you roll the dice. What is the probability of getting even number?

c) If the drawn card was a heart, what is the probability of getting an odd number in the dice?

d) If the drawn card was a diamond, what is the probability of getting a number >3 on the dice.

a) $P(A) = \frac{26}{52} = \frac{1}{2}$

b) $P(B) = \frac{3}{6} = \frac{1}{2}$

c) $P(\text{Number of odd number in dice} \mid \text{Heart in the card}) = \frac{3}{6} = \frac{1}{2}$

d) $P(\text{Number of } > 3 \text{ in the dice} \mid \text{diamond in the card}) = \frac{3}{6} = \frac{1}{2}$