

Performance metrics used in Regression: To calculate the accuracy of a Regression model we use performance metrics.

There are two techniques:

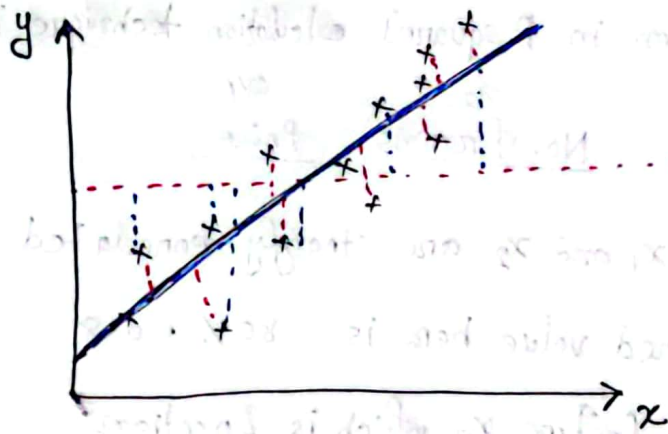
- ① R squared
- ② Adjusted R squared

R squared Formula:

$$R_{\text{squared}} = 1 - \frac{SS_{\text{Res}}}{SS_{\text{Total}}}$$

$\{ \text{Error} \}$
 $SS_{\text{Res}} = \text{Sum of square residual}$
 $SS_{\text{Total}} = \text{Sum of square total}$

SS_{Res} = The total error $\sum (y_i - \hat{y}_i)^2$ $[\sum (\text{actual point} - \text{predicted point})^2]$



SS_{Total} → We take a line (straight) from the average value of y .

Now the summation of $(y_{\text{avg}} - \text{actual points})^2$ would be SS_{Total}

$$\sum (y_{\text{avg}} - y_i)^2$$

So, we can write the R squared Formula like below →

$$R \text{ squared} = 1 - \sum_{i=1}^n \frac{(y_i - h_{\theta}(x_i))^2}{(y_i - \bar{y}_i)^2} \quad \left| \begin{array}{l} h_{\theta}(x) = \hat{y}_i \text{ (predicted point)} \\ \bar{y}_i = y_{\text{mean}} \end{array} \right.$$

$$= 1 - \sum_{i=1}^n \frac{(y_i - \hat{y}_i)^2}{(y_i - \bar{y})^2}$$

R squared ranges between 0 to 1.

② Adjusted R squared:

One of the problem in R squared calculation technique is. Suppose

x_1	x_2	O/P
<u>House size</u>	<u>No. of rooms</u>	<u>Price</u>

Both the feature x_1 and x_2 are strongly correlated with output (Price)

Suppose, R squared value here is = 80% = 0.8

Let's add another feature x_3 which is Location

x_1	x_2	x_3	O/P
<u>House Price</u>	<u>No. of rooms</u>	<u>Location</u>	<u>Price</u>

Here 3 of features are strongly correlated with output. For that R squared will increase even more. Suppose that become → 90%.

Now, add another feature, which is gender (x_4)

x_1	x_2	x_3	x_4	O/P
<u>House size</u>	<u>No. of rooms</u>	<u>Location</u>	<u>Gender</u>	<u>Price</u>

In the case of gender, this feature is not highly correlated or important for output prediction. Although for this feature also R^2 increased a bit. Let's say it increased 1% and became 91%.

So, no matter what feature we are adding, R^2 is increasing. That is not right for the model accuracy. For gender feature R^2 should not increase. Adjusted R^2 solve this specific problem.

Formula of Adjusted R^2 squared:

$$1 - \frac{(1 - R^2)(N - 1)}{N - P - 1}$$

N = Num of datapoints

R^2 = R^2 squared

P = Num of independent features.

We should perform both R^2 and adjusted R^2 to know that not every feature is important.

Cost Functions (MSE, MAE, RMSE)

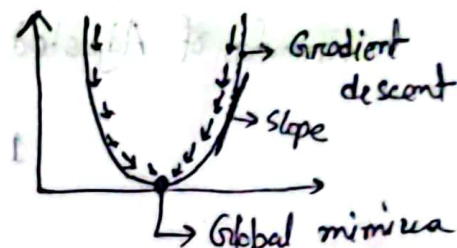
- ① Mean Squared Error (MSE)
- ② Mean Absolute Error (MAE)
- ③ Root Mean Squared Error (RMSE)

Previously we already discussed about Mean squared Error whose Formula

$$\text{was } \rightarrow J(\theta_0, \theta_1) = \frac{1}{n} \sum_{i=1}^n (y_i - h_{\theta}(x_i))^2$$
$$= \frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2 \rightarrow \text{quadratic Equation}$$

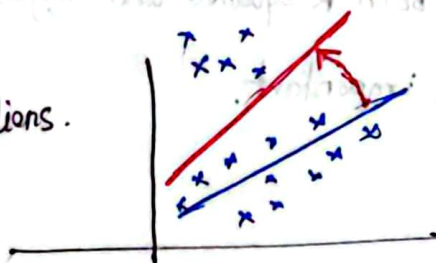
Advantage of MSE:

- ① Equation is differentiable.
- ② It has only one local/global minima



Disadvantage of MLE:

- ① Not Robust to outliers.



If dataset has outliers, Best fit line moves away from where it should be to the side of outliers a bit. (can't handle situation)

② It is not in the same unit.

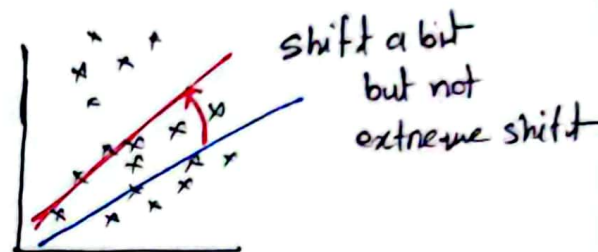
As we are squaring the errors, then the unit will also be squared.

⑪ Mean Absolute Error (MAE):

$$\text{Formula: } \frac{1}{n} \sum_{i=1}^n |y_i - \hat{y}_i|$$

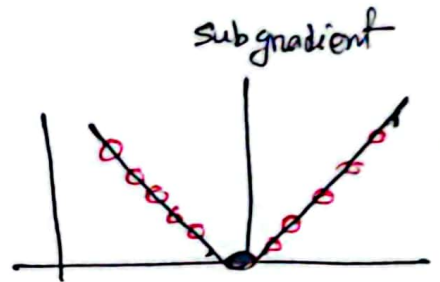
Advantage:

- ① Robust to outliers
- ② It will be in the same unit



Disadvantage:

- ① Convergence usually take more times.



⑫ Root Mean squared Error (RMSE):

$$\begin{aligned} \text{Formula, RMSE} &= \sqrt{\text{MSE}} \\ &= \sqrt{\frac{1}{n} \sum_{i=1}^n (y_i - h(x)_i)^2} \end{aligned}$$

Advantages:

- ① Same Unit
- ② Differentiable
- ③ 1 Global minima

Disadvantage:

- ① Not Robust to outliers

When you have outliers → USE MAE

When you don't have outliers → USE MSE, RMSE