Question 27: A box contains 5 white and 7 black balls. Two successive draws of 3 balls are made with replacement. The probability that the first draw would produce white balls and the second draw would produce black balls are?

Three balls are drawn together before replacement. And each time 3 balls were picked.

~~So, P(white) = $\frac{5}{12}$~~

~~P(black) = $\frac{7}{11}$~~

~~$\therefore$ P(favourable outcome) = $\frac{5}{12} \times \frac{7}{11}$~~

So, total no of outcome = $^{5+7}C_3 = 220$

$$P(\text{White}) = \frac{^5C_3}{220} = \frac{10}{220}$$

$$P(\text{black}) = \frac{^7C_3}{220} = \frac{35}{220}$$

$\therefore P(\text{white} \cap \text{black}) = P(\text{white}) \times P(\text{Black})$ [They are independent events]

$$= \frac{10}{220} \times \frac{35}{220}$$

$$= 0.00723$$

Question 28: A box contains 5 white and 7 black balls. Two successive draws of 3 balls are made without replacement. The probability that the first draw would produce white balls and the second draw would produce respectively

~~Total no of outcomes =~~

$$P(\text{white}) = \frac{{}^5C_3}{{}^{12}C_3} = \frac{10}{220}$$

$$P(\text{black}) = \frac{{}^7C_3}{{}^9C_3} = \frac{35}{84}$$

$$\therefore P(\text{white} \cap \text{Black}) = P(\text{white}) \times P(\text{black})$$

$$= \frac{10}{220} \times \frac{35}{84}$$

$$= 0.018939$$ Ans

Question 29: There are two Boxes. The first box contains 3 red and 5 white balls where the second Box contains 4 red and 6 white balls. A ball is taken at random from the first box and is transferred to the second box. Now another ball selected from the second box. The probability that the second ~~box~~ ball would be red is?

Here 2 cases will be formed →

Case 1: Ball picked up from Box 1 is Red.

$$P(R_1) = \frac{3}{8} \quad [\text{From first box}]$$

Then it (Red ball) transfered to Box 2.

So, Box 2 has Now 5 red, ~~4 black~~ 6 white balls.

$$\therefore P(R_2) = \frac{5}{11}$$

$$\therefore \text{For case 1} \rightarrow P(R_1 \cap R_2) = P(R_1) \times P(R_2)$$

$$= \frac{3}{8} \times \frac{5}{11} = \frac{15}{88}$$

Case 2: Ball picked up from Box 1 is white.

$$P(W) = \frac{5}{8}$$

Then it (white ball) transfered to Box 2

So, Box 2 has now 4 red and 7 white balls.

$$P(R) = \frac{4}{11}$$

$$\text{for, case 2, } \rightarrow P(W \cap R) = P(W) \times P(R)$$

$$= \frac{5}{8} \times \frac{4}{11}$$

$$= \frac{20}{88}$$

$$\text{Total Probability} \rightarrow \frac{15}{88} + \frac{20}{88} = \frac{35}{88}$$

Ans

Question - 30: What is the probability of having at least one 'six' from 3 throws of a perfect dice?

$P(\text{at least 1 six}) = 1 - P(\text{having no six at all})$

$P(\text{not getting 6}) = \frac{5}{6}$ [For 1 throw]

$P(\text{not getting 6}) = (\frac{5}{6})^3$ [For 3 throws]

$\therefore P(\text{at least 1 six}) = 1 - (\frac{5}{6})^3$

Ans

Question 31: Tom speak truth in 30% cases and Dick speak truth in 25% cases. What is the probability that they would contradict each other?

$P(\text{Tom}) = 30\% = 0.3$ $\quad P(\text{Dick}) = 0.25$

$P(\text{Tom}') = 70\% = 0.7$ $\quad P(\text{Dick}') = 0.75$

$P(\text{Tom} \cap \text{Dick}') = 0.30 \times 0.75$

$P(\text{Tom}' \cap \text{Dick}) = 0.7 \times 0.25$

Total probability $= (0.30 \times 0.75) + (0.70 \times 0.25)$

$= 0.400$ Ans

(A,B,C)

Question 32: There are three persons aged 60, 65 and 70 years old. The survival probabilities for these three persons fore another 5 years are 0·7, 0·4 and 0·2 respectively. What is the probability that, at least 2 of them would survive another 5 years?

	A	B	C
Probability of surviving	0·70	0·40	0·20
Probability of not surviving	0·30	0·60	0·80

There can be 4 cases for at least 2 of their survivals.

Case 1: A would die, and B and C survived → P(0·30 × 0·40 × 0·20) =

Case 2: B would die, and A and C survived → P(0·70 × 0·60 × 0·20)

Case 3: C would die, and A and B survived → P(0·70 × 0·40 × 0·80)

Case 4: A, B, C all are survived. → P(0·70 × 0·40 × 0·20)

Total probability → All of the cases summation

= 0·388

Question 33: A problem in probability was given to three CA students A, B, C whose chances of solving it are 1/3, 1/5 and 1/2 respectively. What is the probability that the problem would be solved?

$P(A) \to \frac{1}{3}$ $P(B) \to \frac{1}{5}$ $P(C) \to \frac{1}{2}$

$P(A') \to \frac{2}{3}$ $P(B') \to \frac{4}{5}$ $P(C') \to \frac{1}{2}$

$\therefore P(A' \cap B' \cap C') = \frac{2}{3} \times \frac{4}{5} \times \frac{1}{2}$

$= \frac{20 \times 24 \times 15}{300} = 0.26$

$P(A \cap B \cap C) = 1 - P(A' \cap B' \cap C')$

$= 1 - 0.26$

$= 0.74$

Ans

Question 34: If 8 balls are distributed at random among three boxes, what is the probability that the first box would contain 3 balls?

Total number of outcome = $3^8 = 6561$

(Because we can put 1 ball in 3 bags any of the 3 bags)

No. of favourable outcome = ${}^8C_3 \times 2^5$

(We are selecting 3 balls from the 8 balls for first box $({}^8C_3)$ ways)

(Then the remaining balls are 5 which have to be put in 2 box

So every ball can be put either on second box or in third box

so outcome $\rightarrow 2^5$)

$$P = \frac{n(A)}{n(S)} = \frac{{}^8C_3 \times 2^5}{6561} = 0.2731 \text{ Ans}$$

Question 35: For 2 events A and B, $P(B) = 0.3$, $P(A-B) = 0.4$ and $P(A') = 0.6$

Events A and B are? $\rightarrow (A \cap B)$?

Given, $P(B) = 0.3$, $P(A-B) = 0.4$, $P(A') = 0.6$

$$P(A-B) = P(A) - P(A \cap B)$$

$$\Rightarrow P(A \cap B) = P(A) - P(A-B) \quad [P(A) = 1 - 0.6 = 0.4]$$

$$\Rightarrow P(A \cap B) = 0.4 - 0.4$$

$$= 0$$

As, $P(A \cap B) = 0$, The events are (A, B) mutually exclusive.

Question 36: For two independent events, A and B. What is $P(A+B)$?

Given, $P(A) = 3/5$ and $P(B) = 2/3$?

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= P(A) + P(B) - (P(A) \times P(B)) \quad [\text{As they are independent event}]$$
$$= \frac{3}{5} + \frac{2}{3} - \left(\frac{3}{5} \times \frac{2}{3}\right)$$
$$= \frac{3}{5} + \frac{2}{3} - \frac{6}{15}$$
$$= 0.867$$

Ans

Question 37: Mr. Roy is selected for three separate posts. For the first post, there are three candidates, for the second post, there are 5 candidates, for the third post, there are 10 candidates. What is the probability that Mr. Roy would be selected?

~~P(Roy selecting) = P(first~~

A = first Job
B = second Job
C = Third Job

$\therefore P(A) = \frac{1}{3}$, $P(A') = \frac{2}{3}$

$P(B) = \frac{1}{5}$, $P(B') = \frac{4}{5}$

$P(C) = \frac{1}{10}$, $P(C') = \frac{9}{10}$

$\therefore P(\text{not getting selected}) = \frac{2}{3} \times \frac{4}{5} \times \frac{9}{10}$ $(P(A' \cap B' \cap C'))$

$= \frac{72}{150}$

$\therefore P(\text{getting selected}) = 1 - \frac{72}{150}$

$= 0.52$

Ans

Question 38: The independent probabilities that the three sections of a costing department will encounter a computer error 0·2, 0·3 and 0·1 per week respectively. What is the probability that there would be at least one 1 computer error per week?

$P(A) = 0.2, P(B) = 0.3, P(C) = 0.1$

$P(A') = 0.8, P(B') = 0.7, P(C') = 0.9$

$P(\text{not getting any error}) = P(A' \cap B' \cap C') = 0.8 \times 0.7 \times 0.9$

$= 0.504$

$\therefore P(\text{getting at least 1 error}) = 1 - 0.504$

$= 0.496$ Ans

Question 39: Same question. But what is the probability of getting one and only one error per week?

	Section 1	Section 2	Section 3
P(error)	0·2	0·3	0·1
P(no error)	0·8	0·7	0·9

So as only one error can happen,

So, there are 3 cases now,

Case 1 → Error happen from section 1 and not from section 2,3

Case 2 → Error happen from section 2 and not from section 1,3

Case 3 → Error happen " " 3 and " " " 1,2

$P(\text{Case 1}) \rightarrow 0.2 \times 0.7 \times 0.9$

$P(\text{case 2}) \rightarrow$ ~~$0.3 \times 0.8 \times 0.9$~~ $0.8 \times 0.3 \times 0.9$

$P(\text{case 3}) \rightarrow$ ~~$0.1 \times$~~ $0.8 \times 0.7 \times 0.1$

Total probability = P(case 1) + P(case 2) + P(case 3)

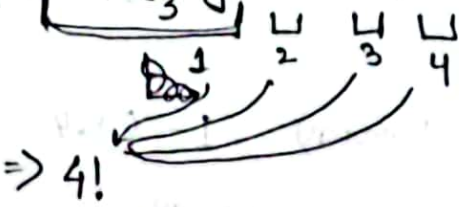
Question 40: x and y are in a line with 6 other people. What is the probability that there are 3 persons between them?

Total → 8 people

Total no of outcome = 8!

No. of ways of selecting 3 persons = $^6C_3$ [because except x and y there are 6 more people]

Here is the arrangement → x, $^6C_3$, y, 6, 7, 8



$\Rightarrow 4!$

$\Rightarrow 4! \times {}^6C_3 \times 3! \times 2!$ → [x, y can be arranged 2!]

→ [3 persons can be arranged in 3!]

Question 41: In connection with a random experiment, it is found that $P(A \cup B) = \frac{5}{6}$, $P(A) = \frac{2}{3}$, $P(B) = \frac{3}{5}$. Evaluate, $P(A|B')$?

$$P(A|B') = \frac{P(A \cap B')}{P(B')}$$

$$= \frac{P(A) - P(A \cap B)}{P(B')}$$

$$= \frac{\frac{2}{3} - \left\{\frac{2}{3} \times \frac{3}{5}\right\}}{\frac{2}{5}}$$

$$= \frac{\frac{2}{3} - \frac{13}{30}}{\frac{2}{5}} = 0.5833$$

$$= 0.277 \text{ Hz}$$

$\therefore P(B') = 1 - \frac{3}{5}$

$= \frac{2}{5}$

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= \frac{2}{3} + \frac{3}{5} - \frac{5}{6}$

$= \frac{20 + 18 - 25}{30}$

$= \frac{13}{30}$

Question 42: In connection with a random experiment, it is found that, $P(A) = \frac{2}{3}$, $P(B) = \frac{3}{5}$ and $P(A \cup B) = \frac{5}{6}$. Evaluate $P(A'/B')$

$$P(A'|B') = \frac{P(A' \cap B')}{P(B')}$$

$$= \frac{P(A \cup B)'}{P(B')} \qquad [P(A' \cap B') = P(A \cup B)']$$

$$= \frac{1 - P(A \cup B)}{P(B')}$$

Ans

Question 43: For a group of students, 30%, 40% and 50% failed in physics, chemistry, and at least one of the two subjects respectively. If an examinee is selected at random, what is the probability that he passed in phisics if it is known that he failed in chemistry?

$P(P) = 0.3$, $P(C) = 0.4$ $P(P \cup C) = 0.5$ $\therefore P(P \cap C) = 0.3 + 0.4 - 0.5$

$P(P') = 0.7$ $P(C') = 0.6$ $= 0.2$

$$P(P'|C) = \frac{P(C) - P(P \cap C)}{P(P')}$$

$$= \frac{0.4 - 0.2}{0.7}$$

$[P(P' \cap C) = P(C) - P(P \cap C)]$

$[P(x \cap y') = P(x) - P(x \cap y)]$

Question 44: In a business venture, a man can make a profit of 50,000 or incur a loss of 20000. The probabilities of making profits or incur loss, from the past are known to be 0.75 and 0.25. What is the expected profit?

$$\text{Expected profit} = (0.75 \times 50000) + (0.25 \times (-20000))$$

$$= 32500.$$

Question 45: A bag contains 6 white and 4 red balls. If a person draws 2 balls and recieves 10$ and $20 for a white and red ball respectively, then his expected amount is?

There can be 3 cases.

Case 1 → Draw 2 white balls → $\frac{{}^6C_2}{{}^{10}C_2} = \frac{15}{45}$ (Probability)

He will get = $10 \times 2 = 20$$

Case 2 → Draw 2 red balls → $\frac{{}^4C_2}{{}^{10}C_2} = \frac{6}{45}$ (Probability)

He will get = $20 \times 2 = \$40$

Case 3 → 1 red and 1 white draw → $\frac{{}^6C_1 \times {}^4C_1}{{}^{10}C_2} = \frac{24}{45}$ (Probability)

He will get → $10 \times 1 + 20 \times 1 = 30$$

Expected value → $\left(\frac{15}{45} \times 20 + \frac{6}{45} \times 40 + \frac{24}{45} \times 30\right)$

$= \$28$

Example 46: Moidul draws 2 balls from a bag containing 3 white and 5 Red balls. He gets $500 if he draws a ~~red ball. What is his expectation?~~ white ball and $200 if he draws a red ball. What is his expectation? If he is asked to pay $400 for participating in the game, would he consider it a fair game and participate?

There can be 3 cases of picking 2 balls

Case 1: Both are red, $\frac{{}^5C_2}{{}^8C_2} = \frac{10}{28}$

He will get = $200 \times 2 = \$400$

Case 2: Both are white, $\frac{{}^3C_2}{{}^8C_2} = \frac{3}{28}$

He will get = $500 \times 2 = \$1000$

Case 3: One red and one white, $\frac{{}^3C_1 \times {}^5C_1}{{}^8C_2} = \frac{3 \times 5}{28}$

He will get = $200 + 500 = \$700$

Expected value = $\left(\left(\frac{10}{28} \times 400\right) + \left(\frac{3}{28} \times 1000\right) + \left(\frac{15}{28} \times 700\right)\right)$

$= 625$

Expected value > payment. So he will obviously pay the game.

Question 47: A box contains 12 electric lamps of which 5 are defectives. A man selects 3 lamps at random. What is the expected number of defective lamps in his selection?

As the man selects 3 lamps at random, so there can be 4 cases →

x (Number of defective)

Case 1: No lamp is defective.

$$P_1 = \frac{{}^7C_3}{{}^{12}C_3} = \frac{35}{220} \quad ; \quad 0$$

Case 2: 1 lamp is defective

$$P_1 = \frac{{}^5C_1 \times {}^7C_2}{{}^{12}C_3} = \frac{105}{220} = 1$$

Case 3: 2 lamps are defective

$$P_2 = \frac{{}^5C_2 \times {}^7C_1}{{}^{12}C_3} = \frac{70}{220} = 2$$

Case 4: 3 lamps are defective.

$$P_3 = \frac{{}^5C_3}{{}^{12}C_3} = \frac{10}{220} = 3$$

$\therefore$ Expected Number of defective lamps $= \left(\frac{35}{220} + \frac{105}{220} + \frac{70}{220} + \frac{10}{220}\right)$

$= 1.25$

Ans

Question 48: The probability that there is at least one error in an account statement prepared by 3 persons A, B and C are 0·2, 0·3 and 0·1 respectively. If A, B and C are prepare 60, 70 and 90 such statements, then the expected number of current statement ?

No of correct statements →

for A → $(1-0.2) \times 60 = 48$

for B → $(1-0.3) \times 70 = 49$

for C → $(1-0.1) \times 90 = 81$

Expected value $E(x) = (48+49+81)$

$= 178$

Ans

Question 49 A random variable x has the following probability distribution

x	0	1	2	3	4	5	6	7
P(x)	0	2K	3K	K	2K	$2K^2$	$7K^2$	$2K^2+K$

find the value of K ?

$0+2K+3K+K+2K+K^2+7K^2+2K^2+K = 1$

$\Rightarrow 10K^2+9K = 1$

$\Rightarrow 10K^2+9K-1 = 0$

$\Rightarrow 10K^2+10K-K-1 = 0$

$\Rightarrow 10K(K+1)-1(K+1) = 0$

$\Rightarrow (K+1)(10K-1) = 0$

either $K = 0-1$ or, $K = \frac{1}{10}$

But -1 can't be a value

So, $K = \frac{1}{10}$ Ans

Question 50: The following data relate to the distribution of wages of a group of workers

Wages	50-60	60-70	70-80	80-90	90-100	100-110	110-120
No. of workers	15	23	36	42	17	12	5

If a worker is selected at random from the entire group of workers, what is the probability that his wage would be less than 50?

Total number of outcomes = $15+23+36+42+017+12+5$

$= 150$

$$\text{Probability}(\text{wage} < 50) = \frac{0}{150} = 0$$

Because there is no wage range < 0 or 50.

Ans