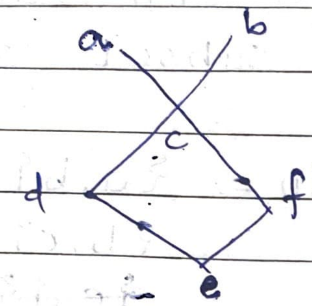
Lattice

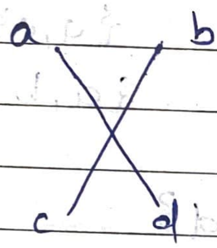
→ A POSET is called Lattice if for every two elements in the set we can find GLB and LUB

Eg



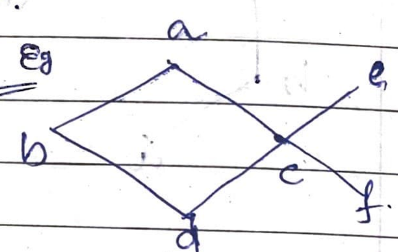
: whether this is lattice or not?
→ No.
Because No UB for a&b.

Eg



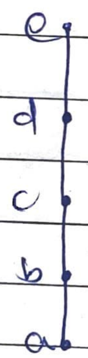
→ No

Eg



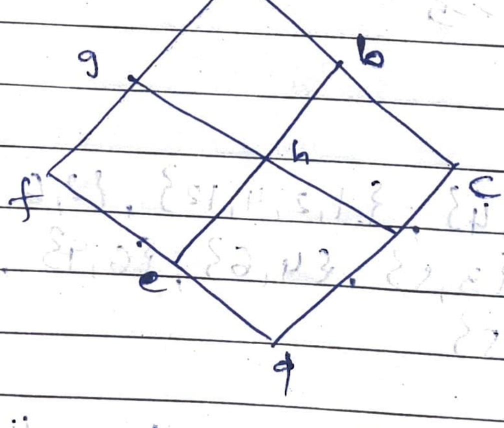
→ No

Eg



→ Yes. because for every two element we can find GLB & LUB.

Eg



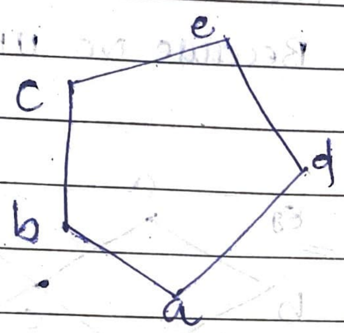
→ Yes.

DATE:
PAGE:

Chain and Anti-chain

Let (S, R) is a POSET. A subset of S is called chain if every two elements in the subset are related. and anti-chain if No two elements in the subset are related.

eg

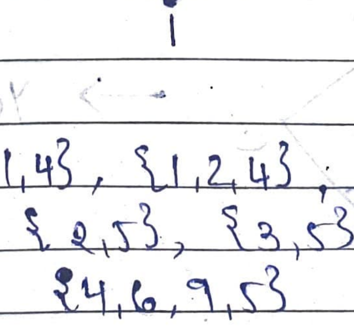


Chain = {a, b}, {b, c}, {c, e}, ~~{e, d}~~, {a, c}, {a, e}, {b, c, e}, {a, b, c, e}

Anti-chain = {c, d}, {b, d}

eg

S = {1, 2, 3, 4, 5, 6, 9, 12, 18}
R = divides relation



Chain = {1, 4}, {1, 2, 4}, {1, 2, 4, 12}, {2, 6, 18}
Anti-chain = {2, 5}, {3, 5}, {4, 6}, {6, 9}, {4, 5, 6}
{4, 6, 9, 5}

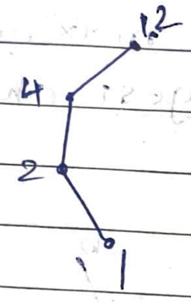
Longest chain & anti-chain length = 4

single element can be considered as chain or antichain.

(Relation)
- **Totally Ordered Set**

A POSET (S,R) is called totally ordered if S is a chain.

eg

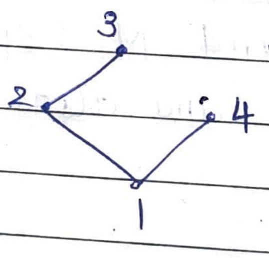


S = {1, 2, 4, 12}
R = divides
⟶ Totally ordered set.

Theorem : 1

⟶ Let (S,R) be a POSET if the length of longest chain in S is ~~or~~ 'n' then the elements of S can be partitioned into 'n' disjoint anti chains.

eg



n = 3
3 diff. antichains (disjoint)
{3, 4}, {2}, {1}

Proof

(i) Basis of Induction:
let's assume n = 1 (single elements)
if n=1 then we can find one antichain which contains all the elements of set

eg a . .b antichain = {a, b}

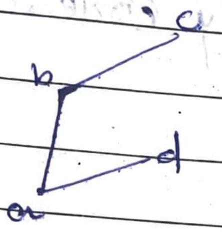
DATE:
PAGE:

(ii) **Induction Hypothesis**

Assume that the statement is true when the length of longest chain is $(n-1)$

(iii) **Induction step**

let's assume M is set of maximal elements so the set P−M has the longest chain of $n-1$ elements.



$M = \{c, d\}$

$n = 3$

$P-M$

$n = 2$

$= n-1$

So according to hypothesis the theorem holds for POSET P−M.

Now, $P = (P-M) \cup M$ and M represents set of maximal elements and also anti-chain

$$P = (P-M) \cup M$$

$$= \underbrace{n-1}_{\text{anti chain}} + \underbrace{1}_{\text{already anti-chain}} = n.$$

So, P can be partitioned into $(n-1)+1$ disjoi[nt] antichains. So theorem is true.

→ From this theorem,

Result: If a partial order set is containing $mn+$ no. of elements then either there is an antichain of $m+1$ element or there is a ch[ain] of n+1 elements.

Eg

$mn+1 = 8$

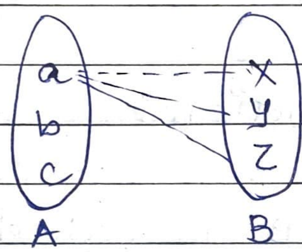
$mn = 7.$

$m=1, \quad n=7.$

Functions

→ A binary relation from $A \to B$ is called function if for every element in A there is an element in B such that (a, b) is in R.

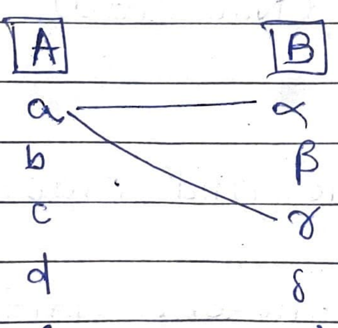
Ex



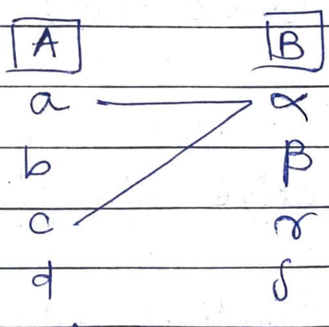
$R = \{(a,x), (b,y), (c,z)\}$ is called function of $A \times B$.

- fuction is special kind of relation.

→ In a general function one element of A never points to more than one element of B.
However reverse scenario is allowed. (if more than one element from A is pointing to ~~ele~~ same element of B is allowed.)



(Not allowed)

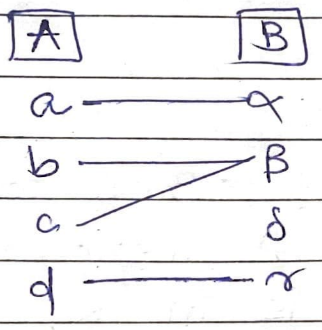


(Allowed)

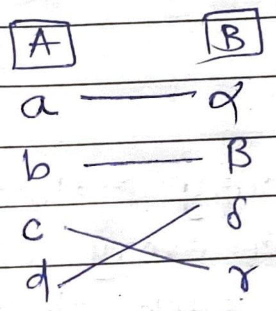
One - One (Injective) function :

A function from $A \to B$ is called one-one function if no two elements of A have the same image.

Eg



(Not one-one)

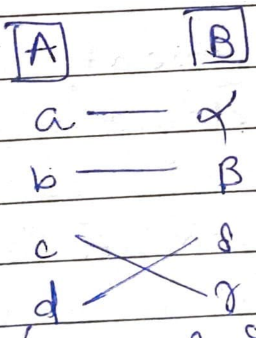


(one-one function)

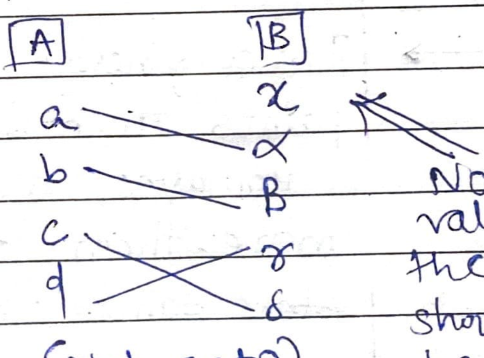
Onto (Surjective) function :

A function from $A \to B$ is called Onto if every element of B is the image of one or more elements of A.

Eg



(onto fun^c)



(Not onto)

Not valid there should be corresponding element in A

Bijective function : (one-one and onto)

A function which is both one-one and onto is called Bijective function.

(i) → To show any function is one-one we have to assume $f(\alpha) = f(\beta)$ and prove that $\alpha = \beta$.

Eg. 1 A function is defined on R $f : R \to R$. $f(x) = 2x+3$. Prove that this function is one-one

→ $f(x) = 2x+3$
$f(1) = 5$, $f(2) = 7$.

Assume, $f(\alpha) = f(\beta)$
$\Rightarrow 2\alpha + 3 = 2\beta + 3$
$\Rightarrow 2(\alpha - \beta) = 0$
$\Rightarrow \boxed{\alpha = \beta}$

Hence, ~~it~~ ~~is~~ this fun^c is one to one function.

Eg. 2 $f : R \to R$
$f(x) = x^2$. Determine one-one or not?

→ Assume, $f(\alpha) = f(\beta)$
$\Rightarrow \alpha^2 = \beta^2$
$\Rightarrow \boxed{\alpha = \pm \beta}$

Hence, it is not one-one function.

(ii) → To verify whether the function is onto or not check that for every y in codomain there must exist some x in the domain such that $f(x) = y$

$f : A \to B$ (B is codomain of A)

Eg.3 $f : N \to R$

$f(x) = 2x+3$

→ let $f(x) = y \Rightarrow 2x+3 = y$

$$\boxed{x = \frac{y-3}{2}}$$

Since $y \in R$, $\left(\frac{y-3}{2}\right) \in R$

So, for every y it will not be possible to find corresponding $x \in N$.

Eg.4 $f : R^+ \to N$, $f(x) = \lfloor x \rfloor$

→ ~~let $f(\alpha) = f(\beta)$~~ Given funⁿ is not one-one but ~~it~~ is onto function.

Graph

→ Graph : Graph is a collection of nodes & edges. where nodes are called vertices and edges are connection between the vertices.

→ node can be any object.

→ Graph

① Directed graph
- always represent one way relation

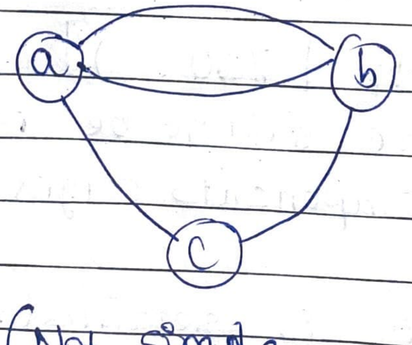
② Undirected graph
- It is two way relation.

→ Graph is mathematically represented as (V, E) where V is set of vertices and E is binary relation if the graph is directed. However E is also set of multisets if the graph is undirected.

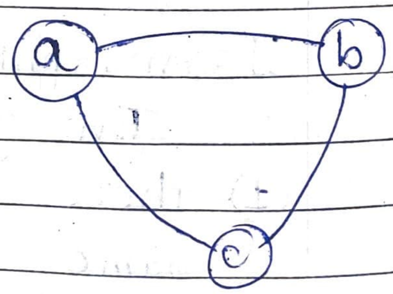
(i) Simple Graph.

A simple graph does not have more than two edges bet^{n} any two vertices and it has no self loops.

Eg



(Not simple graph)

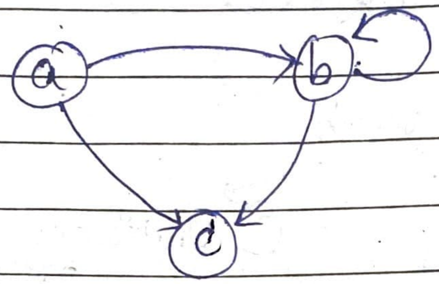


(Simple graph)

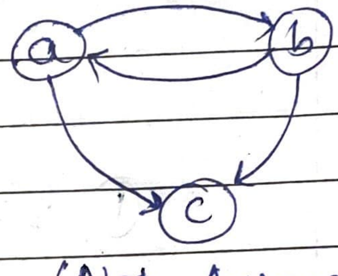
(ii) Asymmetric Directed Graph.

A directed graph that has at most one directed edge betⁿ pair of vertices is called Asymmetric digraph. (directed graph)

Eg.



(Asymmetric)



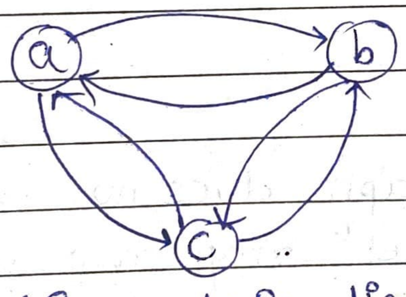
(Not Asymmetric)

- self loops are allowed

(iii) Symmetric directed graph

In this kind of graph for every edge $a \to b$, there is also an edge from $b \to a$.

Eg



(Symmetric digraph)

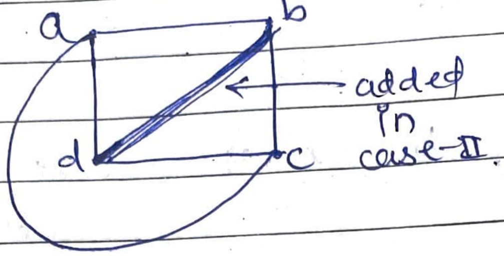
(iv) Isomorphic Graph [ISO( ).]

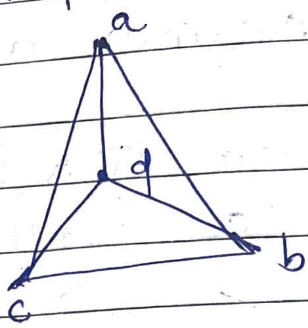
Two graphs are said to be isomorphic if

(1) there no. of components (edges & vertices) are same

(2) edge connectivity is retained.

Eg. Graph-1 Graph-2





Case-I Nodes = 5 Nodes = 5
edges = 5 edges = 6

Hence Not Isomorphic.

Case-II Nodes = 5 Nodes = 5
edges = 6 edges = 6

Hence Isomorphic

* Continue in function.

Eg $f : Z \times Z \to Z \times Z$
~~f(A, B)~~ $f(a,b) = (a+b, a-b)$.

→ for one to one,
$f(a,b) = f(c,d)$ // Assume.
$\Rightarrow (a+b, a-b) = (c+d, c-d)$
$\Rightarrow \boxed{(a+b) = c+d}$ — ① and $\boxed{a-b = c-d}$ — ②

from (1) & (2),
$a = c$, $b = d$. ; $a, b \in Z$
(add both) (sub both)
So, function is one-one.

DATE:

PAGE:

for onto,

Assume, $a+b=x$ —①, $a-b=y$ —②

from ① & ②

$a = \frac{x+y}{2}$, $b = \frac{x-y}{2}$ which does not belongs to Z

so, a, b does not always belong to Z

so, function is not onto function.

Eg

$A - |m|^3$ $B - |n|^2$

A	B
a	x
b	y
c	

How many functions from A to B can be possible?

- a has 2 options
b has 2 options
c has 2 options

$2 \times 2 \times 2 = \boxed{8}$

Generalized function : $\boxed{n^m}$ ∴ $2^3 = 8$.

Eg

A	B
a	1
b	2
c	3
d	4
e	5
	6
	7

$m=5$, $n=7$.

How many fun^s from A to B are possible which are one-one

- a has 7 options
b has 6 options
c has 5 options
d has 4 options
e has 3 options.

$= 7 \times 6 \times 5 \times 4 \times 3$

DATE:
PAGE:

- Generalized formula : $\boxed{{}^nP_m} = \frac{n!}{(n-m)!}$

where $n \geq m$.

Eg

A	B
a	x
b	y
c	
$m=3$	$n=2$

How many fun^c s from A to B are possible which are onto function

$$= \boxed{\sum_{r=1}^{n} (-1)^{n-r} \cdot {}^nC_r \cdot r^m}$$

$$= \sum_{r=1}^{2} (-1)^{2-r} \cdot {}^2C_r \cdot r^3$$

- **Pigeonhole Principle**

Container : □ □ □ $\quad i = \left\lceil \frac{4}{3} \right\rceil$

Items : △ △ △ △

$|\text{Container}| < |\text{Items}|$

→ Pigeonhole principle ~~says~~ states that if n items are put in m containers $(n>m)$, then at least one container must contain more than one item.

let D and R are finite ~~states~~ sets where $|D| > |R|$ then any function $f: D \to R$ there exist elements $d_1, d_2, \ldots, d_i$ such that $f[d_1] = f[d_2] = \ldots = f[d_i]$.

and $i = \left\lceil \frac{|D|}{|R|} \right\rceil$.

DATE:
PAGE:

at least.
i = no. of elements which will be mapped into container

eg 50 objects are to be coloured with 7 diff. colours. then at least how many objects have the same colors.

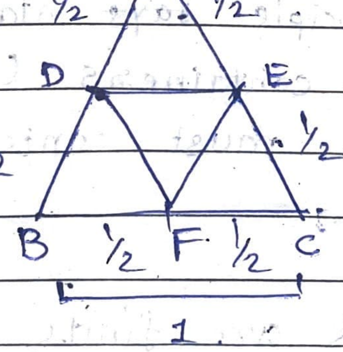
→ Here, $|D| = 50$, $|R| = 7$

∴ let i be No. of elements which will ~~be~~ belongs to same box.

$$\therefore \; i = \left\lceil \frac{|D|}{|R|} \right\rceil = \left\lceil \frac{50}{7} \right\rceil = \boxed{8}$$

eg. Consider an equilateral triangle whose size are equal to 1 unit. Now, so ~~show~~ that if any 5 points are chosen either lying on or inside the triangle. then prove that ~~any two~~ points then two of them must be no more than 0.5 units apart.

→



Now, the original triangle is divided into 4 equilateral triangles.

DATE:
PAGE:

Here, 5 points and 4 triangles.
∴ According to pigeonhole principle,
5 points are to be accomodated in 4 triangles. So,
at least $\lceil 5/4 \rceil = 2$ points must belong to same triangle and distance betn those 2 points are always less than 0.5(1/2).

eg. Single day 1 game minimum. no more than 132 games. 77 days are there.
Prove that there is a period of consecutive days the player will exactly play 21 games.