DATE:
PAGE:

$$a_r = {}^{k+r-1}C_r \, a^r.$$

$$a_r = \left({}^{k-1}C_0, {}^{k}C_1 a, {}^{k+1}C_2 a^2, \ldots \right)$$

$\nearrow$ discrete numeric function.

Ex $f(x) = (1-ax)^{-1}$

$$= \sum_{r=0}^{\infty} \binom{-1}{r} (-ax)^r$$

$$= \sum_{r=0}^{\infty} \binom{-1}{r} (-a)^r x^r$$

Therefore,

$$a_r = \binom{-1}{r} (-a)^r$$

$$= (-1)^r \cdot {}^{1+r-1}C_r \, (-1)^r \cdot a^r$$

$$= {}^{r}C_r \, a^r$$

$$\boxed{a_r = a^r}$$

$\therefore a_r = (1, a, a^2, a^3, \ldots)$

Homework Questions

(1) find term independent of x in $\left(\sqrt{x} - \frac{2}{x^2}\right)^{10}$

r^{th} term $= \binom{10}{r} (\sqrt{x})^r \left(\frac{-2}{x^2}\right)^{10-r}$

DATE:
PAGE:

$$= \binom{10}{r} x^{\frac{r}{2}} \cdot (-2)^{10-r} (x^{-2})^{10-r}$$

$$= \binom{10}{r} (-2)^{10-r} \cdot x^{\frac{r}{2}+2r-20}$$

Now, $x^0 = x^{\frac{r}{2}+2r-20}$

$$\Rightarrow \frac{r}{2}+2r-20=0$$

$$\Rightarrow r+4r-40=0$$

$$\Rightarrow 5r=40 \Rightarrow \boxed{r=8}$$

$\therefore$ $\boxed{9^{th} \text{ term}}$ is independent from x.

(2) find r^{th} term in $\left(2x^2+\frac{3}{2x}\right)^8$

r^{th} term $= \binom{8}{r} (2x^2)^r \cdot \left(\frac{3}{2x}\right)^{8-r}$

$$= \binom{8}{r} 2^r \cdot x^{2r} \cdot 3^{8-r} (2x)^{r-8}$$

$$= \binom{8}{r} 2^{2r-8} \cdot 3^{8-r} \cdot x^{3r-8}$$

take $r=4$,

$$= \binom{8}{4} 2^0 \cdot 3^4 \cdot x^4 = \boxed{5670x^4}$$

(3) Find coeff. of x^9 in $(x^2+x^3+x^4+\dots)^3$

$$= \left[x^2(1+x+x^2+\dots)\right]^3$$

$$= x^6 (1+x+x^2+x^3+\dots)^3$$

$$= x^6 (1-x)^{-3}$$

$$= x^6 \sum_{r=0}^{\infty} \binom{-3}{r} (-x)^r$$

$$= x^6 \sum_{r=0}^{\infty} \binom{-3}{r} (-1)^r x^r$$

put $r=3$

coeff. of x^9 is $= \binom{-3}{3} (-1)^3$

$$= (-1)^3 \cdot {}^{3+3-1}C_3 (-1)^3$$

$$= {}^5C_3$$

$$= \boxed{10}$$

Ex. →

$$f(z) = (a-bz)^{-1}$$

$$= a^{-1}\left(1-\frac{bz}{a}\right)^{-1}$$

$$= a^{-1} \sum_{r=0}^{\infty} \binom{-1}{r} \cdot \left(\frac{-bz}{a}\right)^r$$

$$= a^{-1} \sum_{r=0}^{\infty} (-1)^r \, {}^rC_r \, (-1)^r \frac{b^r}{a^r} z^r$$

$$\boxed{f(z) = \sum_{r=0}^{\infty} \frac{b^r}{a^{r+1}} \cdot z^r}$$

$$\boxed{a_r = \frac{b^r}{a^{r+1}}} = \left\{\frac{1}{a}, \frac{b}{a^2}, \frac{b^2}{a^3}, \dots\right\}$$

The coeff. of z^r is discrete generating funⁿ

DATE:
PAGE:

Ans : $2(-1)^r + 3^r$

ex. $f(x) = \frac{8-5x}{1-2x-3x^2}$

$f(x) = \frac{5x-3}{3x^2+2x-1}$

$= \frac{5x-3}{(3x-1)(x+1)} = \frac{A}{x+1} + \frac{B}{3x-1}$

$\Rightarrow A(3x-1) + B(x+1) = 5x-3$

take $x=-1$

$-4A = 5(-1)-3$

$\boxed{A=2}$

take $x=0$

$2(-1)+B = -3$

$\boxed{B=-1}$

$$\boxed{f(x) = \frac{2}{x+1} - \frac{1}{3x-1}}$$

$= 2(1+x)^{-1} + (1-3x)^{-1}$

$= 2\sum_{r=0}^{\infty} \binom{-1}{r} x^r + \sum_{r=0}^{\infty} \binom{-1}{r} (-3x)^r$

$= \sum_{r=0}^{\infty} 2\cdot(-1)^r \cdot {}^rC_r \cdot x^r + \sum_{r=0}^{\infty} (-1)^r \cdot {}^rC_r \cdot (-1)^r \cdot 3^r \cdot x^r$

$= \sum_{r=0}^{\infty} 2\cdot(-1)^r x^r + \sum_{r=0}^{\infty} 3^r \cdot x^r$

therefore discrete numeric function is,

$$\boxed{a_r = 2\cdot(-1)^r + 3^r}$$

DATE:
PAGE:

→ Open form of eq^n $a_n = a_{n-1} + a_{n-2}$
closed form of eq^n $a_r = 2(-1)^r + 3^r$.

- Advantage of discrete ~~nur~~ ~~generating~~ numeric fun^c:
- we can get eq^n which is "closed form".

• Applications of generating functions

- combinatorial functions (to solve)
- solving counting problems.

Que

Find number of solution of
$e_1 + e_2 + e_3 = 17$
$2 \le e_1 \le 5$, $3 \le e_2 \le 6$, $4 \le e_3 \le 7$.

→ for e_1, $(x^2 + x^3 + x^4 + x^5)$ — (1)
for e_2, $(x^3 + x^4 + x^5 + x^6)$. — (2)
for e_3, $(x^4 + x^5 + x^6 + x^7)$ — (3)

Multiply 1 & 2 & 3

$x^2(1 + x + x^2 + x^3) \cdot x^3(1 + x + x^2 + x^3)\, x^4(1 + x + x^2 + x^3)$

$= x^9(1 + x + x^2 + x^3)^3$

$= x^9 \left[\frac{1(x^4 - 1)^3}{(x-1)^3}\right]$ $\left(\because a_r = \frac{a(r^{n+1} - 1)}{r - 1}\right)$

$= x^9 \frac{(1 - x^4)^3}{(1 - x)^3}$

$= x^9 (1 - x^4)^3 (1 - x)^{-3}$

DATE:
PAGE:

$$= x^9 \sum_{r=0}^{3} \binom{3}{r} (-x^4)^r \sum_{r=0}^{\infty} \binom{-3}{r} (-x)^r$$

$$= \sum_{r=0}^{3} {}^3C_r (-1)^r x^{4r+9} \cdot \sum_{r=0}^{\infty} (-1)^r \, {}^{r+3-1}C_r (-1)^r x^r$$

$$= \sum_{r=0}^{3} {}^3C_r (-1)^r x^{4r+9} \sum_{r=0}^{\infty} {}^{r+2}C_2 \, x^r$$

$$\left({}^nC_r = {}^nC_{n-r} \right)$$

For x^{17},

i) $[0, 8]$, $\Rightarrow {}^3C_0 \cdot {}^{10}C_2$

$r=0$ in 1st part x^9 ; $r=8$ in 2nd part x^8

ii) $[1, 4] \Rightarrow {}^3C_1 (-1) \cdot {}^6C_2$

iii) $[2, 0] \Rightarrow {}^3C_2 \cdot (1) \cdot {}^2C_2$

$$\Rightarrow {}^3C_0 \cdot {}^{10}C_2 + {}^3C_1 \cdot (-1) \cdot {}^6C_2 + {}^3C_2 \cdot {}^2C_2$$
$$= (1) \cdot (45) - (3) \cdot (15) + (3) \cdot (1)$$
$$= 45 - 45 + 3 = \boxed{3}$$

Que. In how many diff. ways can 8 identical cookie be distributed among 3 distinct children if each child receives at least 2 cookies ~~no~~ and no more than 4 cookies.

DATE:
PAGE:

Que Find the no. of ways of selecting 'r' objects from 'n' objects with unlimited repetitions.

→ $e_1 + e_2 + e_3 + \dots + e_n = r$

$[0, 1, 2, \dots]$

$x^0 + x^1 + x^2 + \dots$

$f(x) = (x^0 + x^1 + x^2 + \dots)^n$ ← total no. of objects

$= (1 + x + x^2 + \dots)^n$

$= 1 \cdot \dfrac{(1 - x^{\infty})^n}{(1-x)^n}$ $\quad (\because 0 < |x| < 1)$

$= \dfrac{1}{(1-x)^n} = \boxed{(1-x)^{-n}}$

$= \sum_{r=0}^{\infty} \binom{-n}{r} (-x)^r$

$= \sum_{r=0}^{\infty} {}^{r+n-1}C_r (-1)^r \cdot (-1)^r x^r$

$= \sum_{r=0}^{\infty} \underline{{}^{r+n-1}C_r} \cdot x^r$

with repeatations

Que How many ways we choose a committee of 9 members from 3 political parties so that no party has absolute majority in the committee?

DATE:
PAGE:

Exponential Generating functions :

→ We want to choose 3 letters from 2 letters 'a' & 'b'.

___ ___ ___ 'a' & 'b'

$$(x^0+x^1+x^2+x^3)\,(x^0+x^1+x^2+x^3)$$

$$= \underset{bbb}{x^0x^3} + \underset{abb}{x^1x^2} + \underset{aab}{x^2x^1} + \underset{aaa}{x^3x^0}$$

4 ways to chose.

total possible words (order matters).

(1) bbb $= \dfrac{3!}{0!\,3!} = 1$

for a, for b.

(2) abb $= \dfrac{3!}{1!\,2!} = 3$

(3) aab $= \dfrac{3!}{2!\,1!} = 3$

(4) aaa $= \dfrac{3!}{3!\,0!} = 1$

$$= \frac{3!}{0!\,3!} + \frac{3!}{1!\,2!} + \frac{3!}{2!\,1!} + \frac{3!}{3!\,0!}$$

$$= 3!\left(\frac{1}{0!\,3!} + \frac{1}{1!\,2!} + \frac{1}{2!\,1!} + \frac{1}{3!\,0!}\right)$$

$$= \boxed{8}$$

ue. No. of words of length 3 from unlimited supply of 'a' & 'b'

DATE:
PAGE:

$$\left(\frac{x^0}{0!}+\frac{x^1}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}\right)\left(\frac{x^0}{0!}+\frac{x^1}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}\right)$$

$$= \frac{x^0x^3}{0!\,3!}+\frac{x^1x^2}{1!\,2!}+\frac{x^2x^1}{2!\,1!}+\frac{x^3x^0}{3!\,0!}$$

$$= x^3\left(\frac{1}{0!\,3!}+\frac{1}{1!\,2!}+\frac{1}{2!\,1!}+\frac{1}{3!\,0!}\right)$$

$$= x^3 \underbrace{A_3}_{\text{coeff. of } x^3}$$

$\longrightarrow$ $f(x) = x^0A_0 + x^1A_1 + x^2A_2 + \dots$

$$= \sum_{r=0}^{\infty} A_r x^r$$

$$= \sum_{r=0}^{\infty} r!\,A_r \frac{x^r}{r!}$$

$$\boxed{f(x) = \sum_{r=0}^{\infty} a_r \frac{x^r}{r!}}$$

Defn: Let $(a_0, a_1, a_2, \dots)$ be a symbolic representation of a sequence of an event or let it be a sequence of a numbers, the function

$$f(x) = a_0\frac{x^0}{0!} + a_1\frac{x^1}{1!} + a_2\frac{x^2}{2!} + \dots$$

called exponential generating function of the sequence $(a_0, a_1, a_2, \dots)$

Ex. Find the exponential generating func^{c} for a_r, the no. of diff. arrangments of r objects from 4 diff. types of objects with each type of object available at least 2 times and no more than 5 times.

$\rightarrow$

$$O_1 + O_2 + O_3 + O_4 = r$$

$$[2, 3, 4, 5]$$

$$\boxed{\left(\frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!}\right)^4}$$

Ex. Find the ~~exp.~~ generating function for the no. of ways to place r distinct people into 3 rooms with at least one person in each room.

$\rightarrow$

$$\begin{array}{c|ccc} & r_1 & r_2 & r_2 \\ \hline \text{persons} & a & d & c \\ & d & a & c \end{array}$$

$\Longrightarrow$ diff. arrangement. Therefore it is permutation problem.

$$r_1 \quad r_2 \quad r_3 = r$$

$$[1, 2, 3, \ldots]$$

Exponential generating function is

$$\left(\frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\right)^3$$

- What if we want an even no. of people in each room?

$r_1 \quad r_2 \quad r_3 \ldots = r$

$[2, 4, 6, \ldots]$

Exp. generating fun^c is,

$$\boxed{\left(\frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \ldots\right)^n}$$

→ Expansion of e^x.

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$$

$$e^{nx} = 1 + nx + \frac{n^2x^2}{2!} + \frac{n^3x^3}{3!} + \ldots$$

$$= \boxed{\sum_{r=0}^{\infty} \frac{n^r x^r}{r!}}$$

$$\frac{1}{2}\left[e^{-x} + e^{x}\right] = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots$$

$$\frac{1}{2}\left(e^{x} - e^{-x}\right) = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots$$

Ex. Find the no. of diff. r arrangements of objects chosen from an unlimited supply of n types of objects.

→ $\underbrace{n \; n \; n \; n \; n}_{}$ r places $= \boxed{n^r}$

$$O_1 + O_2 + O_3 + \ldots + O_n = r$$

$[0, 1, \ldots, r]$

DATE:
PAGE:

$$\left(\frac{x^0}{0!}+\frac{x^1}{1!}+\frac{x^2}{2!}+\dots\right)^n$$

$$=\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\dots\right)^n$$

$$=e^{xn}$$

$$=\sum_{r=0}^{\infty} n^r \cdot \frac{x^r}{r!} \qquad \boxed{Ans=n^r}$$

Ex Find the no. of r digit quaternary $(0,1,2,3)$ sequences with an even no. of 0's and odd no. of 1's.

$\rightarrow$

#0	#1	#2	#3
c_1	c_2	c_3	c_4
$[0,2,4,\dots]$	$[1,3,5,\dots]$	$[0,1,2,\dots]$	

$$\left(\frac{x^0}{0!}+\frac{x^2}{2!}+\frac{x^4}{4!}+\dots\right)\left(\frac{x^1}{1!}+\frac{x^3}{3!}+\frac{x^5}{5!}+\dots\right)\left(\frac{x^0}{0!}+\frac{x^1}{1!}+\frac{x^2}{2!}+\dots\right)^2$$

$$=\frac{1}{2}(e^x+e^{-x})\cdot\frac{1}{2}(e^x-e^{-x})\cdot(e^x)^2$$

$$=\frac{1}{4}(e^{2x}-e^{-2x})\cdot e^{2x}=\frac{1}{4}\left[e^{4x}-1\right]$$

$$=\frac{1}{4}\left(\sum_{r=0}^{\infty}4^r\frac{x^r}{r!}-1\right)=\sum_{r=0}^{\infty}4^{r-1}\frac{x^r}{r!}-\frac{1}{4}.$$

$$\boxed{Ans=4^{r-1}}$$

DATE:
PAGE:

closed
(compact form of eq^n)

Que. Solve the recurrence relation using generating function

$$a_{n+2} - 3a_{n+1} + 2a_n = 0 \; ; \; a_0 = 2, a_1 = 3$$

→ Multiply with x^n and $\sum_{n=0}^{\infty}$ → therefore we can write a_0 putting $n=0$

$$\sum_{n=0}^{\infty} a_{n+2} x^n - 3 \sum_{n=0}^{\infty} a_{n+1} x^n + 2 \sum_{n=0}^{\infty} a_n x^n = 0.$$

: Expanding we ~~can~~ get

$$= (a_2 + a_3 x + a_4 x^2 + \dots) - 3(a_1 + a_2 x + a_3 x^2 + \dots)$$
$$+ 2(a_0 + a_1 x + a_2 x^2 + \dots) = 0.$$

$$\Big[a_2 + a_3 x + a_4 x^2 + \dots$$
$$= \frac{a_2 x^2 + a_3 x^3 + a_4 x^4 + \dots}{x^2}$$
$$= \frac{a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots - a_0 - a_1 x}{x^2}$$
$$= \frac{\alpha(x) - a_0 - a_1 x}{x^2} \Big]$$

= Rewrite eq^n in form of $\alpha(x)$

$$= \frac{\alpha(x) - a_0 - a_1 x}{x^2} - 3 \cdot \left[\frac{\alpha(x) - a_0}{x} \right] + 2\alpha(x) = 0$$

using initial values,

$$= \frac{\alpha(x) - 2 - 3x}{x^2} - 3 \frac{\alpha(x) - 2}{x} + 2\alpha(x) = 0$$

$$= \alpha(x) - 2 - 3x - 3x\alpha(x) - 6x + 2x^2\alpha(x) = 0$$

DATE:
PAGE:

$\Rightarrow G(x)\left[2x^2-3x+1\right] = 2-3x$

$\Rightarrow \quad G(x) = \dfrac{2-3x}{2x^2-3x+1} = \dfrac{2-3x}{2x(x-1)-1(x-1)}$

$\Rightarrow \quad \dfrac{2-3x}{(2x-1)(x-1)} = \dfrac{A}{x-1} + \dfrac{B}{2x-1}$

$\Rightarrow \quad A(2x-1) + B(x-1) = 2-3x.$

take $x = 1/2$

$B(-1/2) = 2-3/2 \Rightarrow \boxed{B=-1}$

$A(2x-1) + (1-x) = 2-3x$

$A(2x-1) = 1-2x \quad \Rightarrow \boxed{A=-1}$

$G(x) = \dfrac{1}{1-x} + \dfrac{1}{1-2x}$

$= (1-x)^{-1} + (1-2x)^{-1}$

$= \sum_{r=0}^{\infty} \binom{-1}{r}(-x)^r + \sum_{r=0}^{\infty} \binom{-1}{r}(-2x)^r.$

$= \sum_{r=0}^{\infty} (-1)^r \cdot (-1)^r \cdot x^r + \sum_{r=0}^{\infty} (-1)^r \cdot (-1)^r \cdot 2^r \cdot x^r$

$\boxed{G(x) = 1+2^r}$

Que Solve the recurrence relation using Generating function $u_n = 6u_{n-1} + 2^{n-1}$; $u_0 = 1$

$\longrightarrow$ Multiply with x^n and $\sum_{n=1}^{\infty}$

$\Rightarrow \sum_{n=1}^{\infty} u_n x^n = 6 \sum_{n=1}^{\infty} u_{n-1} x^n + \sum_{n=1}^{\infty} 2^{n-1} x^n$

Keep exponent as same as subscript of coefficient inside Σ.

$G(x) = a_0 + a_1 x + a_2 x^2 + \cdots$

DATE:
PAGE:

$$\Rightarrow \sum_{n=1}^{\infty} u_n x^n = 6x \sum_{n=1}^{\infty} u_{n-1} x^{n-1} + x \cdot \sum_{n=1}^{\infty} 2^{n-1} \cdot x^{n}$$

Expanding ... we get :

$$\Rightarrow (u_1 x + u_2 x^2 + \dots) = 6x\,(u_0 + u_1 x + u_2 x^2 + \dots) + x\,(1 + 2x + 2^2 x^2 + \dots)$$

Rewrite in terms of $G(x)$ → using G.P. series

$$G(x) - u_0 = 6x\left(G(x) + x \cdot \frac{1}{1-2x}\right)$$

$$\Rightarrow G(x) - 1 = 6x \cdot G(x) + \frac{x}{1-2x}$$

$$\Rightarrow G(x)(1-6x) = 1 + \frac{x}{1-2x}$$

$$\Rightarrow G(x) = \frac{1}{1-6x} + \frac{x}{(1-2x)(1-6x)}$$

$$\therefore \boxed{G(x) = 6^n + 2^n}$$

Que $u_n = u_{n-1} + 2u_{n-2}$; $u_0 = 3$, $u_1 = 7$

Que $a_n - 7a_{n-1} + 12a_{n-2} = 0$

→ Multiply with x^n and $\sum_{n=2}^{\infty}$

$$\Rightarrow \sum_{n=2}^{\infty} a_n x^n - 7 \sum_{n=2}^{\infty} a_{n-1} x^n + 12 \sum_{n=2}^{\infty} a_{n-2} x^n = 0$$

Expanding we get,

$$\Rightarrow (a_0 + a_1 x + a_2 x^2 + \dots) - 7\,(a_1 + a_2 x^2 + \dots$$

DATE:
PAGE:

$$\Rightarrow (a_2 x^2 + a_3 x^3 + \dots) - 7(a_1 x^2 + a_2 x^3 + \dots) + 12(a_0 x^2 + a_1 x^3 + \dots) = 0$$

$\Rightarrow$ Rewrite eq^n in form of $G(x)$

$$\Rightarrow (G(x) - a_0 - a_1 x) - 7x(G(x) - a_0) + 12x^2(G(x)) = 0$$

$$\Rightarrow G(x) - a_0 - a_1 x - 7x G(x) + 7x a_0 + 12x^2 G(x) = 0$$

$$\Rightarrow G(x)[12x^2 - 7x + 1] + (7x-1)a_0 - a_1 x = 0$$

$$\Rightarrow G(x)(3x-1)(4x-1) + (7x-1)a_0 - a_1 x = 0$$

$$\Rightarrow \boxed{G(x) = \frac{a_1 x - (7x-1)a_0}{(3x-1)(4x-1)}}$$

Sessional - III.

- **Experiment**

outcome of exp1 m # outcome of exp2 n

- (both happen.) total # of outcomes = $m * n$ (rule of product)
- any one of them happen then total # of outcomes = $m + n$ (rule of sum)

Ex →

Permutation

r distinctly coloured balls,
n distinctly numbered boxes.

Q. How many ways are possible ?

$$n \cdot (n-1) \cdot \ldots (n-(r-1))$$

$$= n \cdot n-1 \cdot n-2 \cdots n-r+1$$

$$P(n, r) = \frac{n!}{(n-r)!}$$

Ex →

rooms = 7
officies of programmers = 4
officies of terminals = 3

Q. How many ways we can distribute these 7 rooms

→ $7 \cdot 6 \cdot 5 \cdot 4 \cdot 1$

2nd, 3rd, 4th

for terminals

1st programmer can chose out of 7 rooms.

DATE:
PAGE:

Ex → no. of four digit numbers without repeatitions

$\underline{9}\ \underline{9}\ \underline{8}\ \underline{7} = 9 \times 9 \times 8 \times 7$.

~~exp~~ except 0

one is used out of 10.

Ex → # of exams = 3

# of days = 5

no constraints on # of exams on a day.

$\underline{5}\ \underline{5}\ \underline{5} = 5 * 5 * 5 = \boxed{125}$

1st exam we can schedule on any of the day.

Ex →

(0,1,2,3,4)

How many r-digit quintary sequences have an even no. of 1's ?

- 0, 1, 2, 3, 4

2, 3, 4 = 3^r

$\underline{3}\ \underline{3}\ \underline{3}\ \underline{\ }$ --- r digit

(2,3,4) (2,3,4)

total no. of sequences = 5^r

Now, $5^r - 3^r$

contain all digits

Therefore,

$$\boxed{\frac{5^r - 3^r}{2} + 3^r}$$

even no. of 1's

only 2,3

eg 234XX334X23

(odd) 2840033̶4123

(even) 284013341̇23

Ex ⟹

The total no. of distinct slips needed to print all five-digit numbers on slip of paper with one no. ~~of~~ on each slip of paper

- # numbers = 10^5

① 1 0 6 9 8 - 1 0 9 6 8

② 1 6 0 8 1 - 1 8 0 9 1 (we can use same slip)

③ 1 6 0 9 1 - 1 6 0 9 1

in 3^{rd} case

1 6 0 9 1 - 1 6 0 9 1

$3 * 5 * 1 * 5 * 1$ → same for 2^{nd} & 4^{th} place

Middle number can 1, 0, 8

first place we can have 6

last number ~~are~~ become fix

1 - - - 1, 8 - - - 8, 6 - - - 9,
0 - - - 0, 9 - - - 6

this is fix

$$= \boxed{10^5 - \frac{5^5 - 3\cdot 5^2}{2}}$$

$5^5 - 3\cdot 5^2$

we need 2 slips to represent numbers II^{nd} case.

Ex ⟹

r colored balls

n numbered boxes

r_1 of balls are of same color

r_2 of balls are of same color

$$\boxed{\text{Ans.} = \frac{P(n, r)}{r_1!\, r_2!}}$$

How many ways you can distribut r balls ...

DATE:
PAGE:

Ex $\Rightarrow$

3 dash 2 dots

we want to identify message which contains 3 dashes & 2 dots

$$\boxed{\text{Ans} \quad \frac{5!}{3!\,2!}}$$

Ex $\Rightarrow$

r balls of same color

n numbered boxed

Combination problem

$$= \frac{n\cdot(n-1)\cdot(n-2)\ \ldots\ (n-(r-1))}{r!}$$

$$= \frac{n\cdot(n-1)\cdot(n-2)\ \ldots\ (n-r+1)}{r!}$$

$$\boxed{C(n,r) = \frac{n!}{(n-r)!\,r!}}$$

Ex $\Rightarrow$

11 MLAs

A committiee of 5

Ans $C(11,5)$

- MLA – A is always there
  $C(10,4)$
- MLA – A is not there alway
  $C(10,5)$
- MLA – A, B (MLA A or MLA B can be there)
  $C(9,4) + C(9,4) + C(9,3)$

either A or B can be there

both are not there

$C(9,4)$ = A is fixed, B is not there

$\therefore$ remaining is 9.

we have to select 4 out of 9.