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Sessional - III.

- Experiment

outcome of exp1 — m   # outcome of exp2 — n

- (both happen) total # of outcomes = $m*n$ (rule of product)
- any one of them happen then total # of outcomes = $m+n$ (rule of sum)

Ex → **Permutation**

r distinctly coloured balls,
n distinctly numbered boxes.

Q. How many ways are possible?

$$n \cdot (n-1) \cdot \ldots \cdot (n-(r-1))$$
$$= n \cdot n-1 \cdot n-2 \cdots n-r+1$$

$$P(n,r) = \frac{n!}{(n-r)!}$$

Ex →
rooms = 7
officies of programmers = 4
offices of terminals = 3

Q. How many ways we can distribute these 7 rooms

→ 7 · 6 · 5 · 4 · 1

1st programmer can chose out of 7 rooms.
2nd
3rd
4th.
for terminals

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EX ⟹ no. of four-digit numbers without repeatitions

$\underline{9}\ \underline{9}\ \underline{8}\ \underline{7} = 9 \times 9 \times 8 \times 7$

~~exp~~ except 0 — one is used out of 10.

EX ⟹ # of exams = 3
# of days = 5
no constraints on # of exams on a day.

$\underline{5}\ \underline{5}\ \underline{5} = 5 * 5 * 5 = \boxed{125}$

1^{st} exam we can schedule on any of the day.

EX ⟹ (0,1,2,3,4)
How many r-digit quintary sequences have an even no. of 1's ?

- 0, 1, 2, 3, 4

2, 3, 4 = 3^r

$\underline{3}\ \underline{3}\ \underline{3}$ _ --- r digit

(2,3,4) (2,3,4)

total no. of sequences = 5^r

Now, $\underbrace{5^r - 3^r}$ — contain all digits

Therefore, $\boxed{\frac{5^r - 3^r}{2} + 3^r}$

even no. of 1's — only 2,3,4.

eg 234XX334X23
(odd) 28400334123
(even) 23401334123

half of them would be even and half is odd. (x can be 0 or 1)

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Ex ⟹ The total no. of distinct slips needed to print all five-digit numbers on slip of paper with one no. of on each slip of paper

- # numbers = 10^5

① 1 0 6 9 8 − 1 0 9 6 8

② 1 6 0 8 1 − 1 8 0 9 1 (we can use same slip)

③ 1 6 0 9 1 − 1 6 0 9 1

in 3^{rd} case

1 6 0 9 1 − 1 6 0 9 1

3 * 5 * 1 * 5 * 1 → same for 2^{nd} & 4^{th} place

Middle number can 1, 0, 8

first place we can have 6

last number ~~are~~ become fix

1 — 1, 8 — 8, 6 — 9,

0 — 0, 9 — 6

this is fix

$$= \boxed{10^5 - \frac{5^5 - 3 \cdot 5^2}{2}}$$

$5^5 - 3 \cdot 5^2$

we need 2 slips to represent numbers II^{nd} case.

Ex ⟹ r colored balls

n numbered boxes

q_1 of balls are of same color

q_2 of balls are of same color

How many ways you can distribute r balls in n boxes?

$$\boxed{\text{Ans.} = \frac{P(n, r)}{q_1!\, q_2!}}$$

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EX → 3 dash 2 dots

we want to identify message which contains 3 dashes & 2 dots

$$\boxed{\text{Ans}\quad \frac{5!}{3!\,2!}}$$

EX → r balls of same color

n numbered boxed

Combination problem

$$= \frac{n\cdot(n-1)\cdot(n-2)\ldots(n-(r-1))}{r!}$$

$$= \frac{n\cdot(n-1)\cdot(n-2)\ldots(n-r+1)}{r!}$$

$$\boxed{C(n,r) = \frac{n!}{(n-r)!\,r!}}$$

EX → 11 MLAs.

A committiee of 5.

Ans $C(11,5)$

- MLA – A is always there
  $C(10,4)$
- MLA → A is not there always
  $C(10,5)$
- MLA – A, B (MLA A or MLA B can be there)

$C(9,4) + C(9,4) + C(9,3)$

either A or B can be there

both are ~~not~~ there

$C(9,4)$ = A is fixed, B is not there

∴ remaining is 9.

~~we~~ have to select 4 out of 9.

A is fixed.

sess-II

(f) $\binom{-1/2}{n} = \dfrac{\binom{2n}{n}}{(-4)^n}$

$$\text{L.H.S.} = \binom{-1/2}{n} = \frac{(-1/2)(-3/2)(-5/2)\cdots(-1/2-n+1)}{n!}$$

$$= \frac{(-1/2)(-3/2)(-5/2)\cdots\left(\frac{1-2n}{2}\right)}{n!}$$

$$= \frac{(-1)^n\,(1\cdot3\cdot5\cdots(2n-1))}{2^n\,n!}$$

Now, multiply-divide $(2\cdot4\cdot6\cdots2n)$

$$= \frac{(-1)^n(2n)!}{2^n\cdot n!\;2^n\cdot n!}$$

take 2 common

$$= \frac{\binom{2n}{n}}{(-4)^n} = \text{R.H.S.}$$

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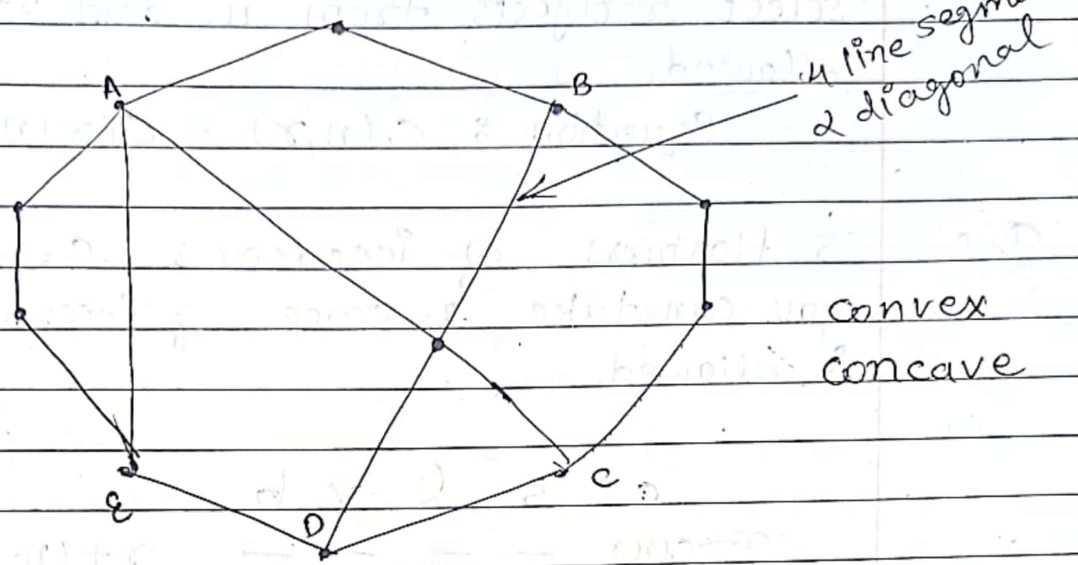

Decagon

→ 3 diagonals never pass through single intersection point

Que → How many line segments we have when diagonals intersect with eachother?

- total diagonals = C(10,2) − 10 → sides.

= $\boxed{35}$

$\boxed{\text{Total diagonals} = {}^nC_2 - n}$

k intersection can create k+1 segments.

- # intersection points = C(10,4) = 210
↳ 1 intersection needs 4 points (ABCD)

# of line segments = 210 * 2 + 35 → one diagonal is not intersecting with any other (AE)

K+1 on one diagonal

K+1 on second diagonal

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select r objects from n. and repetation is allowed.

Equation : $C(n,r) = C(r+n-1, r) = \frac{(r+n-1)!}{r!\,(n-1)!}$

Que 5 flavours of icecreams c, s, l, v, b
you can take 3 scoops of Icecream and repeatation is allowed.

c s l v b

$r \rightarrow \underline{000}$ — — — — $r+(n-1)$.

— 0 — — 0 — 0

— — 00 — 0 —

When we have repeatation ~~is allowed~~
choices increase from n to $r+(n-1)$,

Que types of cookies $= 4$
6 cookies to select

$\longrightarrow$ $C(r+n-1, r)$ $n=4, r=6$ Here $n<r$.
therefore repetition we need to consider

$= C(6+4-1, 6)$
$= C(4,6)$.

Que 5 girls, 12 chairs.
5 girls to sit in 12 chairs. How many ways are possible?

$\longrightarrow$? # ? # ? # ? # ? # ? ?= places

5 chairs and 7 are left.

Here $r=7$, $n=6$.

No. of ways $= 5! * C(6+7-1, 7)$

5 girls can exchange places

Que In same que above, how no girls can ~~seat~~ sit next to each other.

\# - \# - \# - \# - \#

\# → girls (5)
\- → chairs (4)
∴ remaining places $r=3$
$n=6$

No. of ways $= 5! * C(3+6-1, 3)$

Que $2t+1$ → 3 distinguisable balls put them 3 boxes such that 2 boxes will contain ~~no~~ more balls than 3rd one.

→ distribution of $2t+1$ into 3 boxes

$C(r+n-1, r) = C(2t+1+3-1, 2t+1)$
$= C(2t+3, 2t+1)$

Sub Que In same, 1 box will contain more balls than remaining 2 boxes.

$2t+1 = t + (t+1)$ distribute t in all boxes

$3 \cdot C(t+3-1, t) \Rightarrow 3C(t+2, t)$

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$\therefore$ 2 boxes $\geq$ 1 box

$= \boxed{C(2t+3, 2t+1) - 3 \cdot C(t+2, t)}$

- Generation of Permutation and combination
  (order matters)

- combination and subset are same thing.

- TSP (Travelling Sales Person Problem).
  we generate all possible combinations and also follow "lexicographical order".
  eg 1, 10, 11, 2. ← lexicographical order.

⟹ Algorithm to generate all possible lexicographic permutation:

given : $a_1, a_2, \ldots, a_n$
next : $b_1, b_2, \ldots, b_n$

1. $a_i = b_i$, $1 \leq i \leq m-1$ and $a_m < b_m$ for the largest possible m

2. b_m is the smallest element among $a_{m+1}, a_{m+2}, \ldots$ that is larger than a_m.

3. $b_{m+1} < b_{m+2} < \ldots < b_n$.

eg We want to find the next sequence of 124653

am = from this element start we change the sequence

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Need to find am from right side (LSB)

am

12 4 6 5 3 ← Here 4 < 6 Hence it is am.

1 2 5 3 4 6 put all am-1 elements as it is.

1 2 5 ~~3~~ ~~4~~ 6 4 Now, maximum than am, write it

1 2 5 4 3 6

1 2 5 4 6 3

eg. {1, 2, 3, 4} → 1234, 1243, ~~1324~~, ⋮ 1342, 1423, 1432, 2134, 2143

• Generating r combinations / k-subsets

$S = \{1, 2, \ldots, n-1, n\}$

Begin with $12\ldots r$

while $a_1 a_2 \ldots a_r \; != (n-r+1) \ldots (n-1)n$

do

1. Find the largest integer k such that $a_k < n$ and a_{k+1} is not in the $a_1 a_2 \ldots a_r$

2. Replace $a_1 a_2 \ldots a_r$ with
$a_1 a_2 \ldots a_{k-1} (a_k+1)(a_k+2) \ldots (a_k + r - k + 1)$

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eg $\{1,2,3,4,5,6\}$ How many ways 4 elements can be chosen from $\{1,2,3,4,5,6\}$
$n=6$, $r=4$
- $^6C_4 = 15$ subsets
we find.

1234 (a_k)
1235
1236 Here $a_k = 3 < n=6$.
1245 (a_k, a_{k+1})
1246 Here $a_k = 4$ and a_{k+1} is not in sequence.
1256
1345 $a_k=2$, $a_{k+1}=3$ and remaining a_{k+1}, a_{k+2}
1346
1356
1456 $a_k=1$
2345
2346
2356
2456
3456.

- **Discrete Probability**

→ Outcomes can be mutually exclusive (all outcome are not there)
mutually exhaustive (either of o/come are there)

→ Experiment : Something which has outcome. eg. toss a coin

→ sample space : It is a space where all possible outcomes are listed.

eg $S = \{HH, HT, TH, TT\}$

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→ Discrete Sample space :
Sample space where we have finite no. of outcomes or ~~com countabilly~~ countably infinite no. of outcomes.

eg $S = \{HH, HT, TH, TT\}$ → finite.
$S = \{h, mh, mmh, mmmh, \ldots\}$ → countably infinite.
↳ hit & miss

→ Probability is a real number which is allocated to every outcome.

→ probability signifies frequency.

→ (i) $\sum_{x_i \in S} P(x_i) = 1$
(ii) $0 \le P(x_i) \le 1$
→ Pillars

ex $S = \{hh, ht, th, tt\}$ Unfair coin.
$P(hh) = \frac{4}{9}$, $P(tt) = \frac{2}{9}$.
$P(ht) = \frac{1}{9}$, $P(th) = \frac{2}{9}$

ex $S = \{h, mh, mmh, mmmh, \ldots\}$
$P(h) = \frac{1}{2}$, $P(mh) = \frac{1}{4}$
either miss or hit
$P(\underbrace{mmm\ldots h}_{K}) = 2^{-K}$.

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→ Event - subset of a sample space

$$\text{Event} \subseteq S$$

simple	compound
- only one outcome is guranty.	- more then one sample.

Eg. 2 dice.

$S = \{ (1,1), \ldots, (1,6)$
$(2,1), \ldots, (6,6) \}$

$|S| = 36$

Event : The sum of the rolling dice is 9.

$$P(E) = \frac{4}{36}$$

Eg 8 students

I II III IV

two of them are belonging in I^{st} year. Similar
two stu → II^{nd} year
two stu → III^{rd} year
two stu → IV^{th} year

$$P(E) = \frac{8!}{\underbrace{2!}_{\text{two stu. } I^{st}\text{ year}} \times 2! \times 2! \times 2! \times \underbrace{4^8}_{\text{sample space}}}$$

_ _ _ _ _ _ _ _

1^{st} stu. can belong to I, II, III, IV year

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eg People $P = 100,000$ find, $P(fb)$
Female $F = 51,500$ $P(mb)$
Male $M = 48,500$ $P(mh)$
$FB = 9000$ $P(fh)$
$MB = 30200$
$FH = 42500$
$MH = 18300$

→ $P(fb) = \frac{9000}{100,000} = 0.09$

$P(mb) = \frac{30200}{100,000} = 0.302$

$P(mh) = \frac{18300}{100,000} = 0.183$

$P(fh) = \frac{42,500}{100,000} = 0.425$

eg A : A bald person was chosen.
B : A female was chosen

$P(A) = P(mb) + P(fb)$
$= 0.302 + 0.09 = \boxed{0.392}$

$P(B) = P(fb) + P(fh)$
$= 0.09 + 0.425 = \boxed{0.515}$

$P(A \cup B) = P(mb) + P(fb) + P(fh)$
$P(A \oplus B) = A\bar{B} + \bar{A}B = P(mb) + P(fh)$
$P(A \cap B) = P(fb)$
$P(A - B) = P(mb)$
$P(B - A) = P(fh)$

Cumilitive $dist^n$ fun^n → also known as ... dist fun
Probability $dist^n$ fun^n → also known as
→ Prob. fun^c

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- **Discrete Random Variable**

Discrete random number is defined over discrete ~~random~~ sample space.

Eg. $rv = X$ ← no. of heads on 4 tosses of coin
$S = 2^4$

$\therefore X = \{0, 1, 2, 3, 4\}$
1 head, 2 head

- random variable represents sum of numbers when 4 ~~coins~~ dice are rolled together

$|S| = 6^4$

- $y = \{4, \ldots, 24\}$

Eg. 3 dice are rolled together. sum should be even.

random variable $X = \{4, \ldots, 18\}$ (6+6+6)
$|S| = 6^3$
minimum (1+1+2)
list all in exam

- **Probability Distribution Function**

If x is a drv with sample space s then a function denoted by $f(x)$ or $P(X=x)$ and defined as $f(x) = P(X=x)$ = Probability for the random variable $x = x$ is called the probability distribution function or pdf or probability function

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density → distribution is continuous

eg. 4 coins are toss

X = {no. of heads observer over 4 tosses of coins}

$X = \{0, 1, 2, 3, 4\}$, $|S| = 2^4$

Now, Prob. distⁿ func.,

$f(0) = P(X=0) = \frac{4C_0}{16}$

$f(1) = P(X=1) = \frac{4C_1}{16}$

$f(2) = P(X=2) = \frac{4C_2}{16}$

$f(3) = P(X=3) = \frac{4C_3}{16}$

Now, $f(x) \geq 0$

$$\sum_{x_i \in X} f(x) = 1$$

eg. 2 dice once , sum = 2, 3, 4, 5 & 6

So,

X = sum resulting from rolling of 2 dice once

$= \{2, \dots, 12\}$

$|S| = 6^2$

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$P(X=2) = \frac{1}{36}$ // (1,1)

$f(X=3) = \frac{2}{36}$ // (1,2) (2,1)

$f(X=4) = \frac{3}{36}$ // (1,3), (2,2), (3,1)

$f(5) = \frac{4}{36}$ // (1,4), (2,3), (3,2), (4,1)

$f(6) = \frac{5}{36}$ // (1,5), (2,4), (3,3), (4,2), (5,1)

Eg $2P(X=1) = 3P(X=2) = P(X=3) = 5(P(X=4))$

$\cancel{\alpha}$ X $= \{1,2,3,4\}$

LCM $(2,3,1,5) = 30$

$2P(X=1) = 30K$ $\therefore P(X=2) = 10K$

$\Rightarrow \boxed{P(X=1) = 15K}$ $\therefore P(X=3) = 30K$

$\therefore P(X=4) = 6K$

WKt, $\sum_{x_i \in X} f(x_i) = 1 \Rightarrow 61K = 1 \Rightarrow \boxed{K = \frac{1}{61}}$

So, $P(1) = \frac{15}{61}$, $P(2) = \frac{10}{61}$, $P(3) = \frac{30}{61}$, $P(4) = \frac{6}{61}$

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- Discrete Distribution function (CDF)

If X is drv defined over the sample space S with pdf $f(x)$ then a function given by,

$$F(x) = P(X \leq x) = \sum_{x_i \leq x} f(x_i)$$

is called the distribution function.

Eg Prev. Example

$$F(1) = \sum_{x \leq 1} f(x) = f(1) = \frac{15}{61}$$

$$F(2) = \sum_{x \leq 2} f(x) = f(1) + f(2) = \frac{25}{61}$$

$$F(3) = \sum_{x \leq 3} f(x) = f(1) + f(2) + f(3) = \frac{55}{61}$$

$$F(4) = \sum_{x \leq 4} f(x) = f(1) + f(2) + f(3) + f(4) = \frac{61}{61} = 1$$

Eg.

$$F(X) = \begin{cases} 0 & ; x < 1 \\ 15/61 & ; 1 \leq x \leq 2 \\ 25/61 & ; 2 \leq x < 3 \\ 55/61 & ; 3 \leq x < 4 \\ 1 & ; 4 \leq x < 5 \\ 1 & ; x > 4 \end{cases}$$

Above is cumilitive dist^n fun^c

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eg $f(0) = f(1) = \frac{1}{3}$ & $f(2) = f(3) = \frac{1}{6}$

$\xrightarrow{}$ $X = \{0, 1, 2, 3\}$ → rv.

$$F(0) = \sum_{x \le 0} f(x) = f(0) = \frac{1}{3}$$

$$F(1) = \sum_{x \le 1} f(x) = f(0) + f(1) = \frac{2}{3}$$

$$F(2) = \sum_{x \le 2} f(x) = f(0) + f(1) + f(2) = \frac{1}{3} + \frac{1}{3} + \frac{1}{6} = \frac{5}{6}$$

$$F(3) = \sum_{x \le 3} f(x) = f(0) + f(1) + f(2) + f(3) = 1$$

Cdf :
$$F(X) = \begin{cases} 0 & ; \ x < 0 \\ 1/3 & ; \ 0 \le x < 1 \\ 2/3 & ; \ 1 \le x < 2 \\ 5/6 & ; \ 2 \le x < 3 \\ 1 & ; \ 3 \le x < 4 \\ 1 & ; \ x \ge 4 \end{cases}$$

Eg Suppose when a rv can take only two values each with prob. 1/2. Find its distⁿ funᶜ

- Conditional Probability

$$P(4) = \frac{1}{6}$$

$$P(4 \mid \text{even}) = \frac{1}{3}$$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

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eg. coin was chosen at random. prob. that chosen is a fair one and it shows head. is 1/5.
~~given that fair coin was chosen~~
A fair coin was chosen and tail shows 1/3. Prob. that unfair was chosen & head shows 1/12. & unfair was chosen & tail shows 1/4. find P (unfair coin | head) & P (head | unfair)

→ $P(\text{fair} \cap \text{head}) = 1/3$

$P(\text{fair} \cap \text{tail}) = 1/3$

$P(\text{unfair} \cap \text{head}) = 1/12$

$P(\text{unfair} \cap \text{tail}) = 1/4$

$P(\text{head}) = \frac{1}{3} + \frac{1}{12}$

$P(\text{unfair}) = \frac{1}{12} + \frac{1}{4}$

i) $P(\text{unfair} \mid \text{head}) = \frac{P(\text{unfair} \cap \text{head})}{P(\text{head})}$

$= \frac{1/12}{1/3 + 1/12}$

ii) $P(\text{head} \mid \text{unfair}) = \frac{P(\text{unfair} \cap \text{head})}{\frac{1}{12} + \frac{1}{4} \quad P(\text{unfair})}$

$= \frac{1/12}{\frac{1}{4} + \frac{1}{12}}$.

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Eg. 3 dice given that no two ~~phase~~ faces were same. What is the prob. that there was an ace?

→ $P\left(\begin{matrix}\text{there is an} \\ \text{ace}\end{matrix} \middle| \begin{matrix}\text{no two} \\ \text{faces are same}\end{matrix}\right) = P(A|B)$

$P(\text{No two faces are same}) = \dfrac{P(6,3)}{6^3}$

$\therefore P\underbrace{(\text{there is an ace})}_{\text{and no two are same}}^{(A \cap B)} = \dfrac{\overset{3\text{ ways}}{3} \cdot \overset{\text{ace}}{1} \cdot P(5,2)}{6^3}$ → remaing 2 select from 5

Topic: Information

$\therefore P\left(\begin{matrix}\text{there is an} \\ \text{ace}\end{matrix} \middle| \begin{matrix}\text{no two} \\ \text{faces are same}\end{matrix}\right) = \boxed{\dfrac{3 \cdot 1 \cdot P(5,2)}{P(6,3)}}$

Here, A: There is an ace.
B: No two faces of the rolled dice are same