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Discrete Maths

Sem - IV

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Modeling Computation

→ set - unorderd collection of objects (distinct)

→ Representation of set
ex $\{9, 16, 25, 49\}$
$\{x \mid x \text{ is a perfect square } 5 < x < 50\}$

→ $\mathbb{N}$ - countable ~~definite~~ infinite
$\mathbb{R}$ — uncountable definite infinite

→ $R = \{x \mid \{a, b\} \subseteq x\}$
R is a set of set.
$R = \{\{2,3\}, \{1,2,3\}, \{1,2,3,9\}, \{8,2,3\}\}$
$= \{x \mid \{2,3\} \subseteq x\}$

→ $R = \{x \mid x \notin x\}$ $R \in R \rightarrow R \notin R$
$\{1,2\} \notin \{1,2\}$ $R \notin R \rightarrow R \in R$
$\{1,2\} \in \{1, 2, \{1,2\}\}$

→ $\Delta = \{\text{highest weight, highest height}\}$
$\{a,b\} \neq \{b,a\}$ ← ordered set
$\{a,a\}$
$\{\{\{a,b\}, c\}, d\} = \{a,b,c,d\}$

→ Languages:

* $A = \{a, b, c, \ldots, z\}$
strings/sentences of language are defined over A

L over $A^2 = \{aa, aj, ak, \ldots\}$ - known as 2-tuple
L over $A^5 = \{aaaaa, aabaa, \ldots\}$

A^+ = all possibe $A^1, A^2, \ldots$
(1 or more)

A^* = 0 or more accurence
↑alphabet

ex. $A = \{a, b, c\}$, L over A^*

$L_1 = \{a, aa, aaa, baa\}$ – finite lang

$L_2 = \{cab, ccb, bcc, \ldots\}$ – infinite lang.

$L_3 = \{a^i c b^i c^i \mid i \geq 1\}$

$L_4 = \{\ \}$ (Assume empty lang.)

ex. $B = \{a, b\}$, language M over B^*

M_1 (a is more than b)

M_2 (b is more than a)

M_3 (either a is higher or b is higher but they have not same)

→ To represent a language – we use grammar and that grammar is known as Phrase structure Grammar.

eg 1. $L = \{aaaa, bbbb, aabb, bbaa\}$

List of alphabets $\Sigma = \{a, b\}$

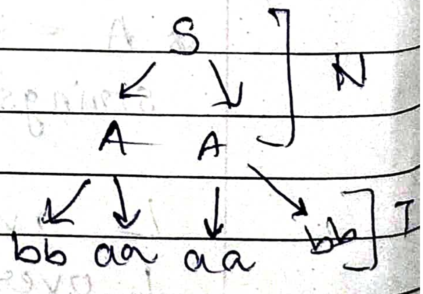
P ⇒ $S = AA$ (Beggining)
Production

$A \rightarrow aa \mid bb$
↖ or

Terminals T {a, b}

Non-terminal N {S, A}

S is a starting symbol



→ T – Terminals – makes up sentences
N – Non terminals – specifies the structure of a lang. (provide templete)
P – Productions Rules $\alpha \rightarrow \beta$
S – Special symbol (starting symbol)

eg. 2 Construct a grammar for the language having any no. of a's over the alphabet $\Sigma = \{a\}$

→ $L = \{\varepsilon, a, aa, aaa, \ldots\}$ $\boxed{\varepsilon = \text{empty set}}$

$T = \{a\}$ $N = \{S\}$

$P \Rightarrow$ ~~$S \rightarrow a \mid aS \mid \varepsilon$~~

$S \rightarrow aS \mid \varepsilon$

S is a starting symbol

eg. 3 $L = \{a^i b^{2i} / i \geq 1\}$

$L = \{abb, aabbbb, aaabbbbbb, \ldots\}$

$T = \{a, b\}$ N

$P \Rightarrow S \rightarrow aSA \mid \varepsilon$, $A \rightarrow bb$

$N \{S, A\}$

S is a starting symbol

eg. Grammar for the language having any no. of a's and any no. of b's. $\Sigma = \{a, b\}^*$

→ $L = \{\varepsilon, a, b, aa, bb, ab, baa, bab, \ldots\}$

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$S \rightarrow aS \mid bS \mid \varepsilon$ or (i) $S \rightarrow aS$
(ii) $S \rightarrow bS$
(iii) $S \rightarrow \varepsilon$

* eg. language having at least two a's, preceded and followed by any no. of a's and b's

→ $L = \{aa, aaa, aba, baaba, \ldots\}$

~~S~~ $\cancel{(a+b)^* aa (a+b)^*}$ ← a or b
$(a+b)^* a (a+b)^* a (a+b)^*$

$S \rightarrow AaAaA$ $T = \{a, b\}$
$A \rightarrow aA \mid bA \mid \varepsilon$ $N = \{S, A\}$

eg $L = \{ wcw^T \mid w \in \{a+b\}^* \}$

$L = \{c, aabcbaa, baaacaaab, \ldots\}$

$\cancel{S \rightarrow ScS^T \mid \varepsilon}$
$\cancel{S \rightarrow \varepsilon}$
$\cancel{S \rightarrow aScSa \mid bScSb}$

$S \rightarrow c$ $T = \{a, b, c\}$
$S \rightarrow aSa$ $N = \{S\}$
$S \rightarrow bSb$

eg $S \rightarrow aB$ $N = \{S, A, B\}$
$S \rightarrow bA$ $T = \{a, b\}$
$A \rightarrow a \mid aS \mid bAA$ S is a starting symbol.
$B \rightarrow b \mid bS \mid aBB$

→ $L = \{ab, ba, baab, abba, bbaa, aabb, bbaabaab \ldots\}$

$L = \{a^i b^j,\ i = j\}$

eg $L = \{a^n b^m c^n \mid n, m \geq 1\}$

$L = \{abc, aabcc, aabbbcc, \ldots\}$

$S \rightarrow aSC$ $\quad T = \{a, b, c\}$

$S \rightarrow aAC$ $\quad N = \{S, A\}$

$A \rightarrow bA \mid b$

eg. $L = \{x \mid x \in \{a,b\}^*$, the member of a's in x is a multiple of 3$\}$

★

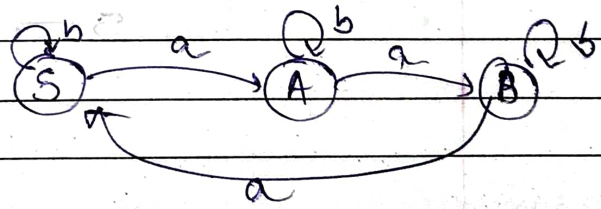
$L = \{aaab, abaa, ababa, aaa, \varepsilon, \ldots\}$

1 $S \rightarrow b \mid bS$ $\quad$ 5. $B \rightarrow a \mid aS$

2 $S \rightarrow aA$ $\quad$ 6. $B \rightarrow bB$

3 $A \rightarrow aB$

4 $A \rightarrow bA$



eg. $L = \{a^{m+n} b^m c^n \mid n, m \geq 1\}$

~~L = {aabc~~

$L = \{a^m a^n b^m c^n \mid n, m \geq 1\}$

$= \{a^n a^m b^m c^n \mid n, m \geq 1\}$

$S \to aSc$ $\qquad N = \{A, S\}$

$S \to aAc$ $\qquad T = \{a, b, c\}$

$A \to ab$

$A \to aAb$

<u>eg.</u> $L = \{a^i b^j \mid i, j \geq 1, i \neq j\}$

$L_1 = \{a^i b^j \mid i > j, \; i, j \geq 1\}$

$L_2 = \{a^i b^j \mid i < j, \; i, j \geq 1\}$

$L = L_1 \cup L_2$

For L_1,

$A \to aA$

$A \to aB$

$B \to ab$

$B \to aBb$

For L_2,

$A \to Ab$

$A \to Bb$

$B \to ab$

$B \to aBb$

Now, $S \to A$

$S \to B$

$A \to aA$

$A \to aB$

$B \to ab$

$B \to aBb$

$C \to cb$

$C \to Bb$

$T = \{a, b\}$

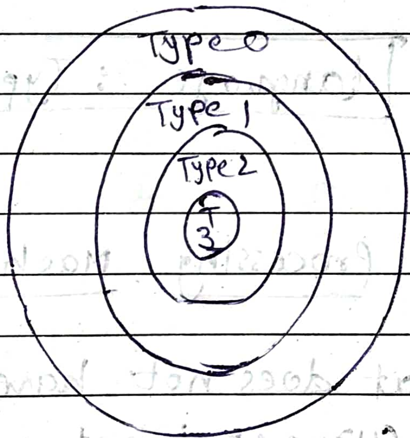
$N = \{S, A, B, C\}$

Types Of Grammar & languages

- Type 3 (regular gram.) $A \to a$, $A \to aB \mid Ba$ — a, b: Terminals; A, B: Non-Terminals; α, β: string of terminals & non-terminals
- Type 2 (context free gram.) CFG $A \to \alpha$
- Type 1 (context sensitive) CSG $\alpha \to \beta$, length of $\beta \geq$ length of α

ex. $\left.\begin{array}{l} AA \to a \\ AaB \to cC \end{array}\right\}$ → Not Valid

- Type 0 Universal (Recursivly enumerable gram.)



→ If a language is specified by type i grammar but not by type (i+1) grammar, it is type i language.

$S \to ABC$
$A \to a$
$A \to b$
$aB \to b$
$bB \to a$
$bC \to a$, $aC \to b$

- Grammar is Type 0. | - Language is Type 3.

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$$S \to ABC \begin{cases} \to aBC \to bC \to a \\ \to bBC \to aC \to b \end{cases}$$

$$\boxed{\begin{array}{l} S \to a \\ S \to b \end{array}}$$
Type 3

eg. $S \to AB$
$AB \to BA$
$A \to a$
$B \to b$

$\boxed{\text{Grammar is of type 1.}}$

$S \to ab \mid ba$: Type 3
$A \to b$: Type 2
$B \to a$
$S \to AB$: Type 1
$AB \to ba$

$\boxed{\text{language : Type 3}}$

Information Processing Machine

Machine that does not have memory, output depends on current input.

A finite state machine is composed of following

1. A finite ~~o~~ set of states $S = \{s_0, s_1, s_2, \ldots\}$
2. A special state s_0, initial state
3. A set of input letters
   $I = \{i_1, i_2, i_3, \ldots\}$
4. A set of output letters
   $O = \{O_1, O_2, O_3, \ldots\}$

5. A function f from $S \times I$ to S, a transition function.

6. A function g from S to O, on output function

Eg. Make Modulo 3 sum of input letters {0,1,2}

→ Sum of input $= 3\%3 = 0 = 3K$
if sum $= 3K+1$, mod 3 ⇒ 1
$= 3K+2$, mod 3 ⇒ 2

Tabular way:

f State	Inputs 0	1	2	Outputs O
A	A	B	C	0
B	B	C	A	1
C	C	A	B	2

(2+2) % 3 = 1 (B).

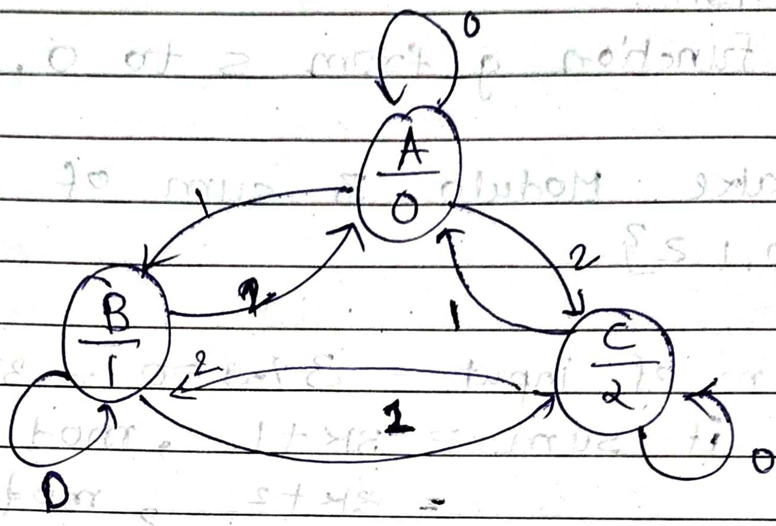
transition f^n:

$f(B, 2) = A$
$f(C, 0) = C$

Output f^n:

$g(A) = 0$
$g(B) = 1$
$g(C) = 2.$

Graphical way :



ATM, vending machine, calculator are example of finite state machine.

Two machines are equivalent, if we provide them same set of input signal they produce same output then Both machine are said to be equivalent.

Que

state	Input 1	Input 2	o/p
⇒ A	B	C	0
B	F	D	0
C	G	E	0
D	H	B	0
E	B	F	1
F	D	H	0
G	E	B	0
H	B	C	1

A

state	Input 1	Input 2	o/p
⇒ A	B	C	0
B	C	D	0
C	D	E	0
D	E	B	0
E	B	C	1

B

i/p : 11222122212

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E & H are equivalent multiple of 4.
C & F are equivalent multiple of 4+2.
B 4+1
D & G are equivalent multiple of 4+3

→ Machine A :

Exhaustive Approach.

State	Input 1	Input 2	Output
→ A	B	C	0
B	C	D	0
C	D	E	0
D	E	B	0
E	B	C	1

Therefore, machine A & B are equivalents.