# OPTIMAL CONGRESS SIZE

*Manon Revel, Daniel Halpern, Tao Lin*

*Group Meeting May 7*

James Madison
Fourth president of the United States.

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However small the Republic may be, the Representatives must be raised to a certain number, in order to guard against the *cabals of a few*; and however large it may be, they must be divided to certain number, in order to guard against the *confusion of a multitude*.

Federalist Paper #10

# AGENDA

- In the epistemic world, the optimal congress size is *linear*.
- Seminal work has shown the size of congress ought to be the *cube-root* of the population's — and it is.
- However, all is not lost for small congresses — they can still be *legitimate*.

# THE EPISTEMIC WORLD

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If $\mathbb{E}[\mathcal{D}] > \frac{1}{2}$, $\quad lim_{n\to\infty}\mathbb{P}(\bar{X}_n > \frac{1}{2}) = 1$

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# LITERATURE

- R. Taagepera. ***The size of national assemblies***. Social science research, 1972.
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Note, q* decreases with K.

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- A **congress of size** $k$ is the $k$ best experts $X_{(1)}, \ldots, X_{(k)}$
- A congress is correct if at least half of the experts vote for 1

# WHAT IS THE OPTIMAL CONGRESS SIZE?

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$$K^\star = \operatorname*{argmax}_{1 \le k \le n} \mathbb{P}\left[\sum_{i=1}^{k} X_{(i)} > k/2\right]$$

# UNIFORM[0, 1]

$$K^\star = \underset{1 \le k \le n}{\operatorname{argmax}} \, \mathbb{P}\left[\sum_{i=1}^{k} X_{(i)} > k/2\right]$$

Theorem 1

Let $\mathscr{D} = \mathscr{U}[0,1]$, and suppose the competence levels are deterministically their expectation, so $p_{(i)} = 1 - \dfrac{i}{n+1}$. Then:

$$\frac{1}{6} < \frac{K^\star}{n} < \frac{1}{2}$$

# DISTRIBUTIONS BOUNDED AWAY FROM 1

**Theorem 2**

Let $\mathscr{D}$ be a distribution supported by $[l, h]$ with $l > 0$ and $h < 1$ whose inverse CDF is Lipschitz continuous. Then, w.p. $\geq 1 - \frac{1}{n}$:

$$1 - F\left(\frac{1}{1 + \sqrt{\frac{1-H}{H}}} + o(1)\right) - o(1) < \frac{K^\star}{n} < 1 - F\left(\frac{1}{1 + \sqrt{\frac{1-L}{L}}} - o(1)\right) - o(1)$$

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  - exactly $\frac{k-1}{2}$ out of the $k$ initial experts are *correct*, so adding two *correct* experts reverses the majority decision from *incorrect* to *correct*
- Then, compare the probabilities of these two scenarios.

# MORE REALISTIC SAMPLING METHODS

$$K_S^\star = \underset{1 \leq k \leq n}{\operatorname{argmax}} \, \mathbb{P}\left[ \sum_{i \in S_k} X_{(i)} > k/2 \right]$$

Conjecture

Let's S be a sampling method that defines a sequence $S_1, \ldots, S_n$ of congresses of increasing size, that allows to sample any agent i with $p_{(i)} > \frac{1}{2}$, then $K_S^\star > K^\star$.

# TAKE AWAY

- In the epistemic world, even when the top experts' competence approaches 1, the optimal congress size is *linear*.
- This holds when giving the congress its best shot.

# CUBE ROOT LAW AND REAL CONGRESSES

# REAL-WORLD CONGRESSES

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- Real-world congresses are of size $\approx n^{0.36}$

# AN ACTIVE DEBATE

- Italian Referendum 2020: Should the congress size be reduced?
    - House: 630 to 400
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by Giuseppe De Bartolo — 07/10/2019 in Società e Demografia

IL FOGLIO
quotidiano

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Segnalo infine una curiosità, per quel che può valere: un importante politologo estone, **Rein Tageepera**, ha teorizzato che il numero di rappresentanti “ideale” sarebbe **la radice cubica della popolazione rappresentata**: ebbene applicando la sua “legge”, i 60.400.000 italiani dovrebbero esprimere esattamente 392,14 deputati, un numero straordinariamente vicino ai 392 (più gli 8 che rappresentano i circa quattro milioni di italiani all’estero) che effettivamente essi eleggeranno se la riforma verrà confermata.

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Taagepera 72
Auriol & Gary-Bobo 07-12
Zhao & Peng 20
Margaritondo 21

# LEGITIMATE CONGRESS

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- Under which conditions is a sub-optimally sized congress still better than majority? That is, under which conditions on $\mathscr{D}$ do we have:

$$\Gamma(k) = \mathbb{P}\left[\sum_{i=1}^{k} X_{(i)} > k/2\right] - \mathbb{P}\left[\sum_{i=1}^{n} X_{(i)} > n/2\right] > 0\,?$$

# DICTATORSHIP

Theorem 3

Let $\mathscr{D} = \mathscr{U}[\varepsilon_n, 1]$.

- If $\varepsilon_n = o\left(\sqrt{\frac{\log n}{n}}\right)$, then $\Gamma(1) > 0$.
- If $\varepsilon_n = \omega\left(\sqrt{\frac{\log n}{n}}\right)$, then $\Gamma(1) < 0$.

# CUBE-ROOT CONGRESS

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- There exists a critical population size for large biases.
- For population sizes too small in comparison to the society biases, majority beats a cube-root congress.

# CONCLUSION

- Epistemic congresses ought to grow *linearly* with the population's size.
- Yet, other desiderata should be considered (e.g. costs, representativity, practicality).
- *Cube-root* sized congresses can still beat majority under mild conditions.

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