Ollscoil na hÉireann
The National University of Ireland

Coláiste na hOllscoile, Corcaigh
University College, Cork

End-of-Term Examination 2010
Practice Exam

CS4407 Analysis of Algorithms

Prof. G. Provan

Attempt all questions

Total marks: 100

60 minutes

Please answer all questions
Points for each question are indicated by [xx]

1. [20] f(n) and g(n) are asymptotically positive functions. Prove or disprove the following:
   $f(n) = O(g(n))$ implies $2^{f(n)} = O(2^{g(n)})$.

2. [20] Write the most efficient algorithm you can think of (in C, Java, pseudo-code) for the following:
   a. Given an array of n integers A[n], return a sorted sub-list of the k smallest integers, where $1<k<n$.
   b. What is the running time in terms of big-oh, big-theta, or big-omega? Explain your answer.

3. [20] Write out an algorithm that takes as input a directed acyclic graph $G=(V,E)$ and two vertices v and z, and returns a path from v to z in G. What is the complexity of this algorithm?

4. [20] Given a graph G and a minimum spanning tree T, suppose that we decrease the weight of one of the edges not in T. Give an algorithm for finding the minimum spanning tree in the modified graph. What is the complexity of this algorithm?

5. [20] The input consists of n skiers with heights $p_i, ..., p_n$, and n skis with heights $s_1, ..., s_n$. The problem is to assign each skier a ski to minimize the AVERAGE DIFFERENCE between the height of a skier and his/her assigned ski. That is, if the skier i is given the ski ai, then you want to minimize:
   $$\sum_{i=1}^{n} (|p_i - s_{ai}|)/n$$
   (a) Consider the following greedy algorithm. Find the skier and ski whose height difference is minimized. Assign this skier this ski. Repeat the process until every skier has a ski. Prove of disprove that this algorithm is correct.
   (b) Consider another greedy algorithm. Give the shortest skier the shortest ski, give the second shortest skier the second shortest ski, give the third shortest skier the third shortest ski, etc. Prove of disprove that this algorithm is correct.
   HINT: One of the above greedy algorithms is correct and one is incorrect for the other.