Ollscoil na hÉireann
The National University of Ireland

Coláiste na hOllscoile, Corcaigh
University College, Cork

Mid-Term Examination 2010

CS4407 Analysis of Algorithms

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Attempt all questions

Total marks: 100

60 minutes

Please answer all questions
Points for each question are indicated by [xx]

1. [25] Write the most efficient algorithm you can think of (in C, Java, pseudo-code) for the following:
   a. Given an array of n integers A[n], find the median (the number that divides A[n] into two equal halves).
   b. What is the running time in terms of big-oh, big-theta, or big-omega? Explain your answer.

2. **[25]** Solve the following recurrence relation using repeated substitution. Do an inductive proof to show your formula is correct.
   T(0) = 1
   T(n+1) = 2*T(n)

3. **[25]** Use the Master Theorem to compute the complexity of the MergeSort algorithm by defining a suitable recurrence.

Consider a recurrence of the form:

$$T(n) = a\,T(n/b) + f(n) \qquad \textbf{1)}$$

where a and b are constants subject to

$$a \geq 1 \qquad b > 1$$

and the function $f(n)$ is asymptotically positive.

The Master Theorem gives the solutions to recurrences of the form specified in **1)** in the table below (3 cases).

	Case 1	**Case 2**	**Case 3**
If *f(n)* is	$f(n) = O\left(n^{\log_b(a)-\varepsilon}\right)$	$f(n) = \Theta\left(n^{\log_b(a)} \lg^k n\right)$	$f(n) = \Omega\left(n^{\log_b(a)+\varepsilon}\right)$
Under the conditions	$\varepsilon > 0$		$\varepsilon > 0$ $a\,f(n/b) \leq c\,f(n)$ $c < 1$
Then the order of the solution is	$T(n) = \Theta\left(n^{\log_b(a)}\right)$	$T(n) = \Theta\left(n^{\log_b(a)} \lg^{k+1} n\right)$	$T(n) = \Theta\left(f(n)\right)$

4. **[25]** A string that contains only 0's, 1's and 2's is called a ternary string.
   a. State an algorithm to output the number of ternary strings that contain two consecutive symbols that are the same. For example, for n = 3, here are the strings: 001, 000, 100, 002, 200, 110, 011, 111, 112, 211, 220, 022, 122, 221, 222.
   b. Define a recurrence relation for the number of ternary strings that contain two consecutive symbols that are the same.