

**Ollscoil na hÉireann
The National University of Ireland**

**Coláiste na hOllscoile, Corcaigh
University College, Cork**

Practice Quiz1, 2014

CS4407 Algorithm Analysis

Prof. G. Provan
Prof. J. Bowen (HoD)
Prof. Ian Gent (extern)

Attempt all questions

Total marks: 100

50 minutes

Please answer all questions
Points for each question are indicated by [xx]

1. [20] Assume that we are given as input two arrays X and Y of integers, with lengths m and n respectively. We want an algorithm A that computes the intersection of X and Y .
 - a. [5] Does there exist an algorithm A that runs in time strictly less than $O(mn)$?
 - b. [15] If you argue that the required algorithm A exists, show the pseudo-code for A and specify the complexity of the algorithm you describe. If you argue no such algorithm exists, prove that no such algorithm can exist.
2. [20] Consider a recursive algorithm A that takes $O(1)$ time to initialize the input array X of n items at the first step; then, at each subsequent step when the array has $k \leq n$ items, it takes $O(k)$ time to compute an item V in X , and recursively processes the array minus the item V .
 - a. Define a recurrence relation for the complexity of A .
 - b. Solve the recurrence relation using repeated substitution to determine the complexity of A . Hint: $\sum_{i=1}^n i = \frac{1}{2}n(n-1)$.
3. [20] A *bipartite graph* is a graph where the vertices V can be divided into two disjoint sets V_1 and V_2 such that every edge has one vertex in V_1 and the other in V_2 . For example, we need a colouring of the nodes V as either red or blue.
 - a. [12] Show how a Breadth-First-Search algorithm can be adapted to compute if a graph $G(V,E)$ is bipartite or not by labelling the vertices either red or blue.
 - b. [8] Apply your algorithm to the graph below (Figure 1), showing the steps you choose and the final labelling of the vertices.

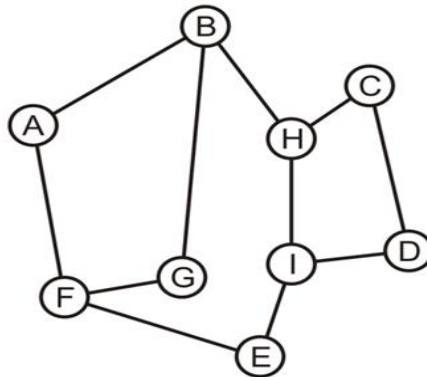


Figure 1: Graph to be studied for being bipartite or not