Ollscoil na hÉireann
The National University of Ireland

Coláiste na hOllscoile, Corcaigh
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Summer Examination 2013

CS4407 Algorithm Analysis

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Attempt all questions

Total marks: 80

90 minutes

Please answer all questions
Marks for each question are indicated by [xx]

1. **[15]** We are given as input two sorted arrays A and B, each of length *n* and containing integers. Our task is to find if there exists *i* and *j* such that A[*i*] + B[*j*] = c, for some integer c.
    a. **[5]** Define an algorithm to solve this problem in time O(*nlogn*).
    b. **[5]** Define an algorithm to solve this problem in time O(*n*).
    c. **[5]** Prove that your O(*n*) algorithm is correct.

2. **[20]** Consider a knapsack problem where we want to pack a collection of different-sized items, each with integer-valued volume *V* and weight *W*, into a knapsack of volume V_{max} such that the weight of the knapsack is maximal.
    a. **[4]** Define a greedy algorithm to solve the knapsack problem if we apply the heuristic of taking, at each step, an item that maximizes the ratio of weight/volume. (At the last step, you may take a fraction of an item to make up the volume V_{max}).
    b. **[6]** Is this algorithm optimal? Show it is optimal or provide a counter-example.
    c. **[4]** If we must take entire items, describe an algorithm where we must use the greedy heuristic of taking the maximum-weight item.
    d. **[6]** Is this second greedy algorithm optimal? Show it is optimal or provide a counter-example.

3. **[20]** Prove that the problem FEEDBACK VERTEX SET (FVS) is NP-complete. We define FVS as follows:

    INSTANCE: A directed graph G = (*V*, *E*) and positive integer $k \leq |V|$.
    QUESTION: Is there a subset $V' \subseteq V$ with $|V'| \leq k$ such that V' contains at least one vertex from every directed cycle in G?
    (Assume that you need to define a reduction from one of the following NP-complete problems studied in class: VERTEX COVER, CLIQUE, 3-SAT)

4. **[10]** Suppose that A is a polynomial-time randomized algorithm for Problem X, whose "yes" answers are always correct, and that on any member of X, A answers "yes" with probability at least $1/n^2$.
    a. [**10**] Show that there exists a polynomial-time randomized decision procedure B for X that is correct with probability at least 3/4 on any input, and B may be built from A without any unproven assumptions.

5. [**15**] A graph G(V,E) is *k*-colourable, for some integer $k>0$, if we can colour the vertices with the *k* colours such that no two vertices that share an edge have the same colour.

    a. [**5**] State a polynomial-time algorithm to decide if an undirected graph is 2-colorable lies in P. [Hint: Use a greedy algorithm or Breadth First Search.]

    b. [**5**] Prove that an undirected graph is 2-colorable iff it does not have a cycle with an odd number of vertices. [Hint: use a proof by contradiction.]

    c. [**5**] Using the above two parts, prove that your algorithm is correct.