Ollscoil na hÉireann
The National University of Ireland

Coláiste na hOllscoile, Corcaigh
University College, Cork

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CS4407 Algorithm Analysis

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Attempt all questions

Total marks: 100

90 minutes

Please answer all questions
Points for each question are indicated by [xx]

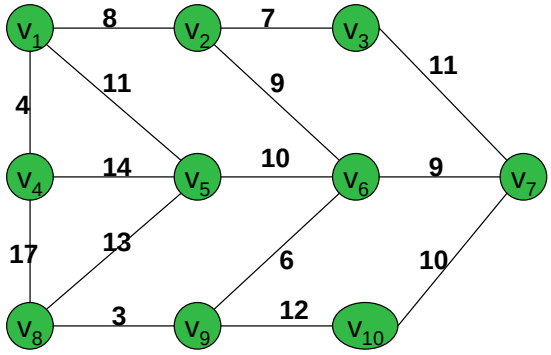
1. **[15]** Consider the *UniqueElements* problem, where we check whether all the elements in a given array are distinct.
   a. **[10]** Use the loop invariance approach to analyse this algorithm.
   b. **[5]** Use this approach to specify the complexity of the algorithm.

2. **[15]** Solve the following recurrence relation using repeated substitution. Do an inductive proof to show your formula is correct.

   T(1) = 1
   T(n) = T(n-1) + O(n)

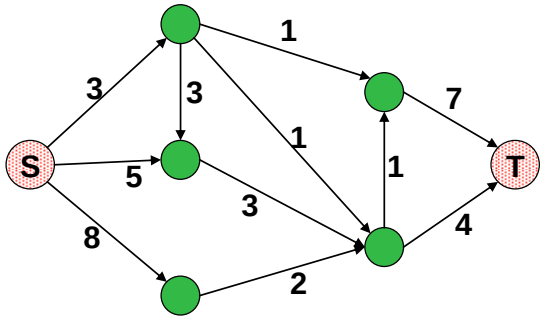
3. **[15]** Given the weighted graph *G* shown below,

   a. **[10]** Find a minimum spanning tree (MST) for *G*; show the steps of generating the MST.
   b. **[5]** What is the complexity of this algorithm?



4. **[15]** Consider a weighted graph *G(V,E)*, with source node *S* and sink node *T*.

   i. **[8]** For the instance of a flow network shown below, compute the maximum flow. Give the actual flow as well as its value.
   ii. **[4]** Justify why your answer is maximum.



iii. **[3]** Consider a decision problem defined for such a flow network: *Flow*:= {(*G*,*S*,*T*,*k*)|*G*(*V*,*E*) is a flow network, *S*,*T*∈*V*, and the value of a optimal flow from *S* to *T* in *G* is *k*}. What is the complexity of *Flow*?

5. **[20]** Prove that SET PACKING (SP) is NP-complete. We define SP as follows:

   INSTANCE: A collection *C* of finite sets over a universal set *U*, and integer $k \leq |C|$.
   QUESTION: Does *C* contain *k* disjoint sets?

   (Assume that you need to define a reduction from one of the following NP-complete problems: HAMILTON CIRCUIT, CLIQUE, INDEPENDENT SET, 3-SAT)

6. **[20]** Consider a class of graphs G(V,E) which contain an independent set of size ¾|V|. An independent set is a subset V' of vertices such that no two vertices in V' are connected by an edge of G.

   i. **[10]** Provide an approximation algorithm for G that can provably compute an independent set of size at least ½|V|.
   ii. **[10]** Prove that your algorithm can meet such bounds.

   (Hint: you may make use of the 2-approximation algorithm for vertex cover that was described in class, i.e., you may assume that this algorithm exists and can be called as a subroutine. A vertex cover of a graph G is a subset of vertices V' such that all edges in G are adjacent to at least one node of V'.)