Ollscoil na hÉireann
The National University of Ireland

Coláiste na hOllscoile, Corcaigh
University College, Cork

In-Class Quiz 2
2014

CS4407 Algorithm Analysis

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Attempt all questions

Total marks: 50

50 minutes

Please answer all questions
Points for each question are indicated by [xx]

1. **[15]** A directed graph G = (V, E) is singly-connected if there is at most one directed path from u to v for all vertices u, v ∈ V .
   a. Give an efficient algorithm to determine whether or not a directed graph is singly connected.
   b. Define the complexity of your algorithm.

2. **[15]** Consider a set of m people and n jobs, $m<n$, where each person ranks the subset of $k \leq n$ jobs for which she is suitable. A matching is an assignment of a person to a job, and a maximum-weight matching is a matching whose ranking is highest among all ranked matchings.
   a. [**10**] Show the pseudo-code for an O(n log n) time greedy algorithm to solve this problem.
   b. [**5**] Either show that this algorithm is optimal, or provide a counter-example to show that it is not optimal.

3. **[20]** Given a graph G and a minimum spanning tree T, suppose that we decrease the weight of one of the edges not in T. Give an algorithm for finding the minimum spanning tree in the modified graph. What is the complexity of this algorithm?

4. [20] Consider the following scheduling problem:
   INPUT: A set $S = \{(x_i, y_i) | 1 \leq i \leq n\}$ of intervals over the real line.
   OUTPUT: A maximum cardinality subset T of S such that no pair of intervals in T overlap.
   Consider the following greedy algorithm:

```
1. T=∅.
2. Repeat until S is empty.
3.     Select the interval I that overlaps the least number of other intervals in S.
4.     Add I to the initial solution set T: T ←T ∪ I
5.     Remove all intervals from S that overlap with I.
6. Return T.
```

   a) [5] What is the invariant for this problem?
   b) [5] What is the complexity of the algorithm?
   c) [10] Prove or disprove that this algorithm solves the problem correctly.