

**Ollscoil na hÉireann
The National University of Ireland**

**Coláiste na hOllscoile, Corcaigh
University College, Cork**

In-Class Quiz #3, 2014

CS4407 Algorithm Analysis

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Total marks: 50

50 minutes

1. [25] Prove that the *Hitting Set* problem, described below, is NP-complete.

Problem: Hitting Set (S, C, k)

Input: A collection C of subsets of a set $S = \{s_1, s_2, \dots\}$, and a positive integer $k \leq |S|$.

Question: Does S contain a subset S' such that $|S'| \leq k$ and each subset in C contains at least one element from S' ?

You may choose to reduce the NP-complete problem, Set Cover, to *Hitting Set*:

Problem: Set Cover (X, F, k)

Input: A base set $X = \{x_1, x_2, \dots\}$, a collection F of subsets of X , and an integer $k \leq |F|$.

Question: Is there a subset F' of F of size at most k such that every element in X is

covered, i.e., such that $X = \bigcup_{x_j \in F'} x_j$?

Required steps:

- i. [5] Show *Hitting Set* is in NP.
- ii. [10] Show the poly-time reduction.
- iii. [10] Show the correctness of the poly-time reduction (using an *if-and-only-if* proof).

2. [25] Consider the task of defining an Approximation Algorithm for Hitting Set, using the Approximation for Set Cover described in Class.

- a) [5] Define pseudo-code for the greedy approximation algorithm for Set Cover (as shown in class).
- b) [15] Modify the Approximation Algorithm for Set Cover to be an Approximation Algorithm for Hitting Set.
- c) [5] Show that the Approximation Algorithm for Hitting Set is correct. (Assume that the Set Cover approximation algorithm as defined in class is correct.)