

**Ollscoil na hÉireann  
The National University of Ireland**

**Coláiste na hOllscoile, Corcaigh  
University College, Cork**

In-Class Quiz #1, 2014

**CS4407 Algorithm Analysis**

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*Total marks: 50*

*50 minutes*

1. [20] Suppose you are choosing between the following three divide and conquer algorithms:
- Algorithm A solves problems of size  $n$  by dividing them into 4 sub-problems of size  $n/2$ , recursively solving each sub-problem, and then combining the solutions in linear time.
  - Algorithm B solves problems of size  $n$  by recursively solving 2 sub-problems of size  $n - 1$  and then combining the solutions in constant time.
  - Algorithm C solves problems of size  $n$  by dividing them into 9 sub-problems of size  $n/3$ , recursively solving each sub-problem, and then combining the solutions in  $O(n^2)$  time.
- (a) [15] Compute the running times of each of these algorithms in big-O notation.
- (b) [5] Which one is asymptotically fastest, and which one is slowest?

For each case, assume that  $T(0)=0$ . If you use the Master Theorem, use the version below:

**Master Theorem:**

Given the recurrence  $T(n) = aT(n/b) + \Theta(n^d)$ , let  $e = \log_b(a)$ . Then

- (a) Case 1:  $T(n) = O(n^d)$ , if  $e < d$ ,
- (b) Case 2:  $T(n) = O(n^d \log(n))$ , if  $e = d$ , and
- (c) Case 3:  $T(n) = O(n^e)$ , if  $e > d$ .

2. [10] Given an array  $A$  of  $n$  integers, we want to provide an algorithm to determine if  $A$  is *light*. An array is light if more than half the elements are less than the mean of  $A$ .
- a) Define an  $O(n \log n)$  algorithm to compute if an array  $A$  is light or not.
  - b) Derive the complexity of your algorithm.

3. [20] Let  $G=(V,E,w)$  be a directed weighted graph such that all the weights are positive. Let  $v$  and  $u$  be two vertices in  $G$  and  $k \leq |V|$  be an integer.

- (a) [15] Design an algorithm to find the  $k$  shortest paths from  $v$  to  $u$ , such that the complexity of the algorithm is strictly less than  $O(|V|^2)$ . Note that a path need not be simple. *Hint: Modify Dijkstra's algorithm.*
- (b) [5] Derive the complexity of your algorithm.