

$$x \in (\tilde{x} + \ker(A))^\perp \stackrel{?}{\Leftrightarrow} x \in \ker(A)^\perp$$

$$\ker A = e_1 \Rightarrow \ker(A)^\perp = \text{span}\{e_2\}$$

$$x = \begin{pmatrix} 0 \\ r \end{pmatrix} \in \ker(A)^\perp$$

$$\text{was ist } L = (\tilde{x} + \ker(A))^\perp?$$

$$= \{x \in \mathbb{R}^2 : \langle x, \tilde{x} \rangle + r \cdot x_1 = 0 \forall r\}$$

$$= \{x \in \mathbb{R}^2 : \langle x, \tilde{x} \rangle = -r x_1 \forall r \in \mathbb{R}\}$$

$$= \{x \in \mathbb{R}^2 : x_1 = 0 = \langle x, \tilde{x} \rangle\} = \text{span } e_2 \cap \{x \in \mathbb{R}^2 : \langle x, \tilde{x} \rangle = 0\}$$

$\stackrel{\text{mit } \tilde{x} \neq 0}{=} \emptyset$

$$\tilde{x} + \ker(A)^\perp \cap \ker(A)^\perp \stackrel{\tilde{x}_2 \neq 0}{=} \emptyset$$