Multiple sclerosis

Demyelination patterns

Matheus Avila Moreira de Paula

UFJF

Summary

Model

$$
\begin{aligned}
\frac{\partial m}{\partial t} &= \Delta m + m(1-m) - \nabla \cdot (\chi(m)\nabla c) && (1)\\
\frac{\partial c}{\partial t} &= \frac{1}{\tau}\left[\epsilon \Delta c + \delta d - c + \beta m\right] && (2)\\
\frac{\partial d}{\partial t} &= rF(m)m(1-d) && (3)\\
\chi(m) &= \chi \frac{m}{1+m}\\
F(m) &= \frac{m}{1+m}\\
\frac{\partial m}{\partial \mathbf{x}} &= \frac{\partial c}{\partial \mathbf{x}} = 0,\ \mathbf{x} \in \partial\Omega\\
c(\mathbf{x},0) = d(\mathbf{x},0) &= 0\\
m(\mathbf{x},0) &= 1,\ \mathit{if}\ \mathbf{x}\ \in C\\
m(\mathbf{x},0) &= 0,\ \mathit{else}
\end{aligned}
$$

C : circle with radius $\sqrt{20}$ centered in the middle of mesh

Model

- m: Comparative density of magrophages
- c: Comparative density of cytokines
- d: Comparative density of destroyed oligodendrocytes
- χ: Chemoattraction
- τ: Time scale
- ϵ: Diffusion of cytokines
- β: production rate per magrophages
- δ: production rate per destroyed oligodendrocytes
- *r*: destructive strength

Numerical solution

- $h_t = 0.001\ day$
- $T_f = 7\ days$
- Mesh 100 x 100
- $hx = hy = h = 1$
- Explicit method.
- Centered difference for Δm e Δc
- Up wind e down wind for Chemotaxis. $\nabla \chi$
- Centered difference for ∇c
-
- $\nabla \cdot (\chi(m)\nabla c) \implies \nabla c \cdot \nabla \chi(m)$

Numerical solution

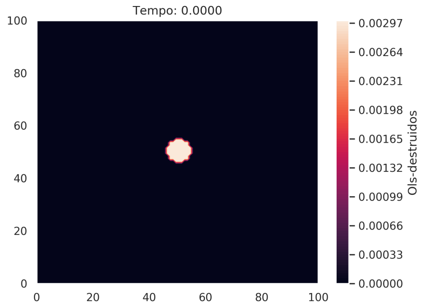
$$
\begin{aligned}
m_{i,j}^{n+1} &= m_{i,j}^{n} + h_t[\Delta m + m(1-m) - \nabla c \cdot \nabla \chi(m)] \\
\Delta m &= \frac{\partial^2 m}{\partial x^2} + \frac{\partial^2 m}{\partial y^2} = \frac{1}{h^2}(m_{i+1,j}^{n} + m_{i-1,j}^{n} - 4m_{i,j}^{n} + m_{i,j+1}^{n} + m_{i,j-1}^{n}) \\
\nabla c = \left[\frac{\partial c}{\partial x}, \frac{\partial c}{\partial y}\right] &= \left[\frac{c_{i+1,j} - c_{i-1,j}}{2h}, \frac{c_{i,j+1} - c_{i,j-1}}{2h}\right] \\
\nabla \chi(m) &= \left[\frac{\partial \chi(m)}{\partial x}, \frac{\partial \chi(m)}{\partial y}\right] \\
If\ \frac{\partial c}{\partial x} > 0 : \frac{\partial \chi(m)}{\partial x} &= \frac{\chi(m)_{i,j} - \chi(m)_{i-1,j}}{h} \\
If\ \frac{\partial c}{\partial x} \leq 0 : \frac{\partial \chi(m)}{\partial x} &= \frac{\chi(m)_{i+1,j} - \chi(m)_{i,j}}{h} \\
If\ \frac{\partial c}{\partial y} > 0 : \frac{\partial \chi(m)}{\partial y} &= \frac{\chi(m)_{i,j} - \chi(m)_{i,j-1}}{h} \\
If\ \frac{\partial c}{\partial y} \leq 0 : \frac{\partial \chi(m)}{\partial y} &= \frac{\chi(m)_{i,j+1} - \chi(m)_{i,j}}{h}
\end{aligned}
$$

Numerical Solution

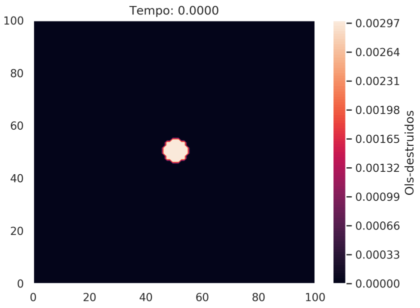
$$
\begin{aligned}
c_{i,j}^{n+1} &= c_{i,j}^{n} + \frac{h_t}{\tau}\left[\epsilon\Delta c + \delta d - c + \beta m\right] \\
\Delta c &= \frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2} = \frac{1}{h^2}\left(c_{i+1,j}^{n} + c_{i-1,j}^{n} - 4c_{i,j}^{n} + c_{i,j+1}^{n} + c_{i,j-1}^{n}\right) \\
d_{i,j}^{n+1} &= d_{i,j}^{n} + h_t\left(rF(m)(1-d)\right)
\end{aligned}
$$

Table: values of parameters.

Name	1° set	2° set	phisical interpretation
τ	1	1	Time scale of citokines
ϵ	0.5	0.5	Diffusion of citokines
β	1	1	production rate per magrophages
δ	1	1	Release of citokines per OL
χ	4	15	Chemoattraction
r	6	6	Destructive strength

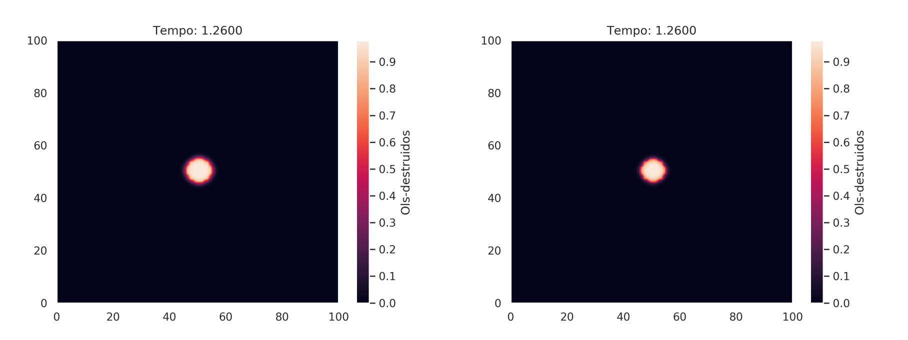


(a) $\chi = 4$



(b) $\chi = 15$

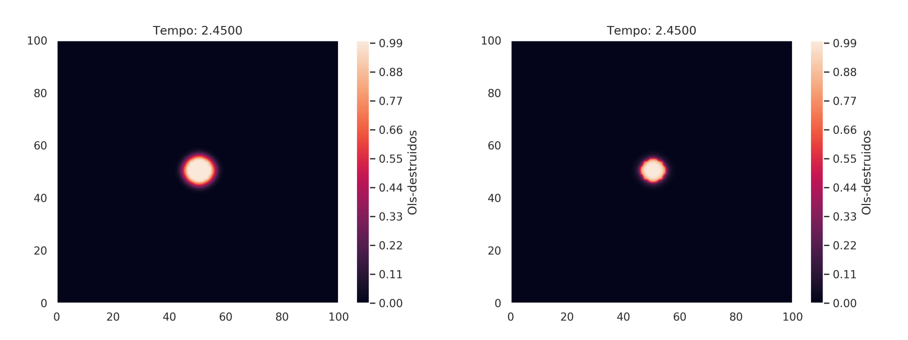
Figure: Comparative density of destroyed oligodendrocytes t = 0 day



(a) $\chi = 4$

(b) $\chi = 15$

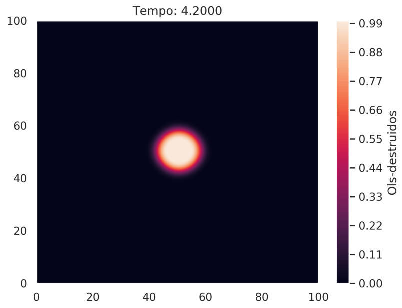
Figure: Comparative density of destroyed oligodendrocytes t = 1.26 day



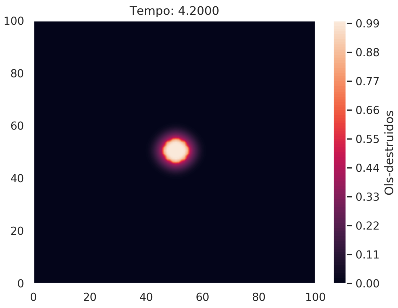
(a) $\chi = 4$

(b) $\chi = 15$

Figure: Comparative density of destroyed oligodendrocytes t= 2.45 days

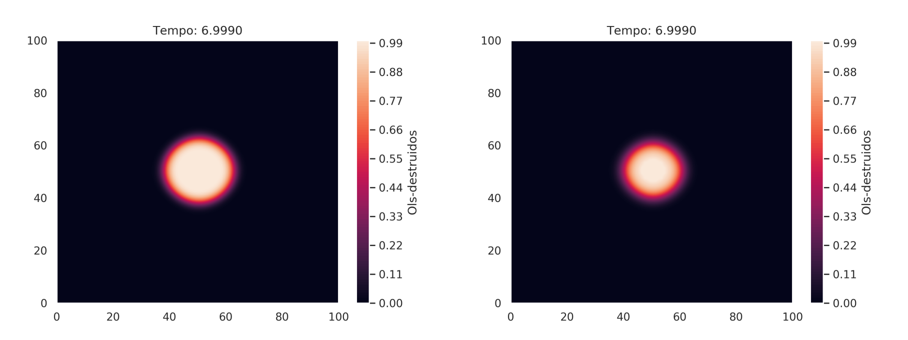


(a) $\chi = 4$



(b) $\chi = 15$

Figure: Comparative density of destroyed oligodendrocytes t= 4.2 days



(a) $\chi = 4$ (b) $\chi = 15$

Figure: Comparative density of destroyed oligodendrocytes t= 7 days