

# 1 Generalised Transition Rate Equation

$$T^{j \rightarrow i} = \left( u_i r_{i-1} \frac{n_{i-1}}{N} + (1 - u_{i+1}) r_i \frac{n_i}{N} \right) \frac{n_j}{\bar{r}} \quad (1)$$

When setting up the system,  $u_0 = 0$  and  $u_{N+1} = 0$  must be defined in order to stop cells mutating past the maximum number of mutations.

## 1.1 Diffusion with Drift Derivation

Taking the large  $N$  limit.

$$\dot{x}_i = \frac{1}{N} \left[ \sum_j T^{j \rightarrow i} - T^{i \rightarrow j} \right]$$

Using the generalised transition rate equation:

$$\begin{aligned} \bar{r} \dot{x}_i &= \sum_j u_i r_{i-1} x_{i-1} x_j - u_j r_{j-1} x_{j-1} x_i + x_j x_i [(1 - u_{i+1}) r_i - (1 - u_{j+1}) r_j] \\ \alpha &= u_i r_{i-1} \\ \beta &= u_j r_{j-1} \\ \gamma &= (1 - u_{i+1}) r_i - (1 - u_{j+1}) r_j \\ \bar{r} \dot{x}_i &= \sum_j \alpha x_j x_{i-1} - \beta x_{j-1} x_i + \gamma x_j x_i \end{aligned}$$

In a flat fitness and mutation landscape, away from the absorbing state  $u_i, r_i = 1 \forall i$ . From this  $\alpha = 1, \beta = 1, \gamma = 0, \bar{r} = 1$ .

$$\dot{\rho}_i = \sum_j \rho_j \rho_{i-1} - \rho_{j-1} \rho_i$$

We now want to expand this sum in order to simplify;

$$\begin{aligned} \dot{\rho}_i &= \rho_0 \rho_{i-1} + (\rho_1 \rho_{i-1} - \rho_0 \rho_i) + (\rho_2 \rho_{i-1} - \rho_1 \rho_i) + \dots + (\rho_{N-1} \rho_{i-1} - \rho_{N-2} \rho_i) + (\rho_N \rho_{i-1} - \rho_{N-1} \rho_i) \\ &= \rho_0 (\rho_{i-1} - \rho_i) + \rho_1 (\rho_{i-1} - \rho_i) + \dots + \rho_N \rho_{i-1} \\ &= \sum_{k=0}^{N-1} (\rho_{i-1} - \rho_i) \rho_k + \rho_N \rho_{i-1} \\ &= (\rho_{i-1} - \rho_i) (1 - \rho_N) + \rho_N \rho_{i-1} \\ &= \rho_{i-1} + (\rho_N - 1) \rho_i \\ \dot{\rho}_i &= \rho_{i-1} + (\rho_N - 1) \rho_i \end{aligned}$$

In the large mutation limit  $\rho_N \rightarrow 0$ .

$$\dot{\rho}_i = \rho_{i-1} - \rho_i \quad (2)$$

We might be breaking our previous assumptions, but for  $i = 0$ :

$$\begin{aligned} \dot{\rho}_0 &= 0 - \rho_0 \\ \rho_0 &= Ae^{-t} \\ A &= 1 \\ \rho_0 &= e^{-t} \end{aligned}$$

This gives us the first step in solving generally.

$$\begin{aligned} \dot{\rho}_1 &= \rho_0 - \rho_1 \\ &= e^{-t} - \rho_1 \\ \rho_1 &= te^{-t} \\ \dot{\rho}_2 &= \rho_1 - \rho_2 \\ \rho_2 &= \frac{1}{2}e^{-t}t^2 \end{aligned}$$

From this we can see that the general solution for  $\rho_i$ ;

$$\rho_i = \frac{t^i}{i!}e^{-t} \quad (3)$$