1 Generalised Transition Rate Equation

$$T^{j\to i} = \left(u_i r_{i-1}\frac{n_{i-1}}{N} + (1-u_{i+1})\, r_i \frac{n_i}{N}\right)\frac{n_j}{\bar{r}} \tag{1}$$

When setting up the system, $u_0 = 0$ and $u_{N+1} = 0$ must be defined in order to stop cells mutating past the maximum number of mutations.

1.1 Diffusion with Drift Derivation

Taking the large N limit.

$$\dot{x}_i = \frac{1}{N}\left[\sum_j T^{j\to i} - T^{i\to j}\right]$$

Using the generalised transition rate equation:

$$\begin{aligned}
\bar{r}\dot{x}_i &= \sum_j u_i r_{i-1} x_j - u_j r_{j-1} x_{j-1} + x_j x_i\left[(1-u_{i+1})r_i - (1-u_{j+1})r_j\right] \\
\alpha &= u_i r_{i-1} \\
\beta &= u_j r_{j-1} \\
\gamma &= (1-u_{i+1})r_i - (1-u_{j+1})r_j) \\
\bar{r}\dot{x}_i &= \sum_j \alpha x_j - \beta x_{j-1} + x_j x_i \gamma \\
&= \sum_j (\alpha + x_i\gamma)x_j - \beta x_{j-1}
\end{aligned}$$