Attitude Estimation Adaptively Compensating External Acceleration*

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This paper is concerned with attitude estimation using low cost, small-sized accelerometers and gyroscopes. A two step extended Kalman filter is proposed, which adaptively compensates external acceleration. External acceleration is the main source of estimation error. In the proposed filter, direction of external acceleration is estimated. According to the estimated direction, the accelerometer measurement covariance matrix of the two step extended Kalman filter is adjusted. The proposed algorithm is verified through experiments.

***Key Words*:** Attitude Estimation, Inertial Sensors, Kalman Filter, Adaptive Filter

1. Introduction

Attitude estimation is necessary in many different applications. Probably the most extensively studied area is attitude estimation in inertial navigation systems (INS)(1). In INS, attitude is very accurately estimated using expensive accelerometers and gyroscopes. Though the principle of attitude estimation in INS is relatively simple, until recently it has not been applied to more mundane applications such as robots. The reason is accelerometers and gyroscopes are too expensive and too large for most applications.

However, due to recent electro-mechanical technical advance, in particular due to microelectromechanical systems, low cost, small-sized accelerometers and gyroscopes have been developed(2). Subsequently these sensors are used in attitude estimation of many applications.

Basically attitude can be estimated using accelerometers only by measuring the gravitational field. However, due to disturbances (most notably external acceleration), gyroscopes are also used to reduce effects of disturbances. Thus the key issue is how to combine accelerometers and gyroscopes to obtain good attitude estimation. Almost all papers use the Kalman filter to do this: attitude estimation for mobile robots(3)−(5), for a walking robot(6), and for a head-tracker(7).

In our paper, we propose the two step extended Kalman filter, which adaptively compensates external acceleration. External acceleration, which affects attitude estimation based on accelerometers, is the major source of attitude estimation error. Similar approaches, which also adaptively compensate external acceleration, are used in Refs. (7) and (8). A brief comparison is made in section 4. We propose more sophisticated adaptive algorithm.

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2. Inertial Sensors for Attitude Estimation

Attitude in the paper means pitch angle (θ) and roll angle (ϕ) of the Euler angles. The heading (yaw angle) is not considered in the paper. The Euler angles are the angular rotation between the body axis (x_b, y_b, z_b) and the inertial axis (x_f, y_f, z_f): we follow the standard aeronautics convention in Ref. (9).

To estimate attitude, we use 6 measurement variables:

- (a_x, a_y, a_z): accelerometer outputs in the body axis (x_b, y_b, z_b)
- (g_x, g_y, g_z): gyroscope outputs, which measure angular rates (ω_x, ω_y, ω_z) around the body axis (x_b, y_b, z_b)

As an accelerometer, we used ADXL202. The ADXL202 is a low cost (about US$30), small-sized ($10.6\times 9.9\times 5.4\,\mathrm{mm}^3$) 2 axis accelerometer with a measurement range of $\pm 2g$. To measure 3 axis acceleration, two ADXL202 are used.

As a gyroscope, we used Murata Gyrostar (ENV-05F-03). The Gyrostar is a low cost (about US$70), small-sized ($11.5\times 19.6\times 27.2\,\mathrm{mm}^3$) piezo-type gyroscope with a measurement range of ±60 deg/sec. The bias drift (9 deg/sec) in the data sheet is very large: it means that the gyroscope integration error could be as large as 90 degrees after 10 seconds. However, our experiment shows that the actual error is far smaller: less than 1 degree after 10 seconds. Thus for short term attitude estimation, gyroscope integration can be used. To measure 3 axis rotation, three Gyrostars are used.

Attitude can be estimated using accelerometers only by measuring the gravitational acceleration. From simple geometry, we have

$$\theta = \sin^{-1}(a_x) \quad \text{and} \quad \phi = \sin^{-1}\left(\frac{a_y}{\cos\theta}\right) \tag{1}$$

where all accelerometer outputs are normalized with the gravitational acceleration constant g. For example, $a_x = 1$ means that $1\ g = 9.8\,\mathrm{m/sec}^2$ is sensed by the x_b axis accelerometer. Note that only a_x and a_y are needed for attitude estimation. Although a_z is not directly used for attitude estimation, a_z plays an important role in the proposed algorithm.

Attitude estimation error in Eq. (1) could be large when there are external acceleration: accelerometers cannot tell difference between the gravitational acceleration and external acceleration. To avoid this problem, gyroscopes are additionally used for attitude estimation. Attitude can be estimated by integrating gyroscope outputs. However, the integration error inevitably accumulates as time goes by; thus gyroscope-based attitude estimation is reliable only for the short time.

The key issue when we combine accelerometers and gyroscopes for attitude estimation is how to weight each sensor output. When the object is under external acceleration, we should rely more heavily on gyroscope outputs because accelerometer outputs contain unwanted external acceleration information. These issues will be considered in section 4.

3. Standard Extended Kalman Filter

In this section, we introduce the standard Kalman filter for attitude estimation. In section 4, the standard Kalman filter will be modified so that external disturbance is adaptively compensated. The state $x(t)$ and the measurement $z(t)$ are defined as follows:

$$x(t) \triangleq \begin{bmatrix} \theta \\ \phi \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}, \quad z(t) \triangleq \begin{bmatrix} a_x \\ a_y \\ g_x \\ g_y \\ g_z \end{bmatrix}. \tag{2}$$

The system equation is given by

$$\begin{aligned} \dot{x}(t) &= A(t)x(t) + w(t) \\ z(t) &= f(x(t)) + v(t) \end{aligned} \tag{3}$$

where

$$A(t) \triangleq \begin{bmatrix} 0 & 0 & 0 & \cos\phi(t) & -\sin\phi(t) \\ 0 & 0 & 1 & \sin\phi(t)\tan\theta(t) & \cos\phi(t)\tan\theta(t) \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$f(x(t)) \triangleq \begin{bmatrix} \sin\theta \\ \sin\phi\cos\theta \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}.$$

Process and measurement noise $w(t)$ and $v(t)$ are assumed to be uncorrelated zero-mean white Gaussian processes satisfying

$$Q = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & q_1 & 0 & 0 \\ 0 & 0 & 0 & q_1 & 0 \\ 0 & 0 & 0 & 0 & q_1 \end{bmatrix} = \mathrm{E}\{w(t)w(t)'\}$$

$$R = \begin{bmatrix} r_1 & 0 & 0 & 0 & 0 \\ 0 & r_2 & 0 & 0 & 0 \\ 0 & 0 & r_3 & 0 & 0 \\ 0 & 0 & 0 & r_3 & 0 \\ 0 & 0 & 0 & 0 & r_3 \end{bmatrix} = \mathrm{E}\{v(t)v(t)'\}.$$

Once the system model Eq. (3) is chosen, the remaining thing is to select the covariance matrix Q and R. Since the estimation error depends on how Q and R are selected, it is important to choose correct Q and R values.

In Eq. (3), the first two rows represent the standard relationship between $(\dot{\theta}, \dot{\phi})$ and $(\omega_x, \omega_y, \omega_z)$. Since this relationship is exact, the first 2×2 block of Q is a zero matrix. The last three rows imply that we assume the derivatives of $(\omega_x, \omega_y, \omega_z)$ are uncorrelated white noises and its covariances are all q_1. This assumption is made because of simplicity although it is possible that the assumption is not true for many real situations. The covariance value q_1 reflects our knowledge about $(\dot{\omega}_x, \dot{\omega}_y, \dot{\omega}_z)$. For example, small q_1 value means that we assume that $(\omega_x, \omega_y, \omega_z)$ of the object is slowly changing.

The measurement noise covariance R is a diagonal matrix, which means that all sensors are assumed to be uncorrelated. Usually, R is chosen from sensor characteristics. In this case, r_i indicates how good or bad the given sensor is: for example, large r_3 value means that the gyroscope output noise is large.

Since the measurement output is sampled, a discretized system of Eq. (3) is used. Suppose the sampling period is T. An exact discretized system is a highly nonlinear system; thus to obtain a simplified discretized system, we assume that $A(t)$ is constant during the sampling period. Then the discretized system is given by Ref. (10).

$$\begin{aligned} x_{k+1} &= \Phi_k x_k + w_k \\ z_k &= f(x_k) + v_k \end{aligned} \tag{4}$$

where

$$x_k \triangleq x(kT) \quad \text{and} \quad z_k \triangleq z(kT)$$

$$\Phi_k \triangleq \exp(A(kT)T) = \begin{bmatrix} I & W(kT)T \\ 0 & I \end{bmatrix}$$

$$W(kT) \triangleq \begin{bmatrix} 0 & \cos\phi(kT) & -\sin\phi(kT) \\ 1 & \sin\phi(kT)\tan\theta(kT) & \cos\phi(kT)\tan\theta(kT) \end{bmatrix}.$$

Process noise covariance matrix Q_k of the discretized system is given by

$$\begin{aligned} Q_k &= \mathrm{E}\{w_k w_k'\} \\ &\approx \int_0^T \exp(A(kT)s) Q \exp(A(kT)'s)\, ds \\ &= \begin{bmatrix} \frac{1}{3} q_1 T^3 W(kT)W(kT)' & \frac{1}{2} q_1 T^2 W(kT) \\ \frac{1}{2} q_1 T^2 W(kT)' & q_1 T I \end{bmatrix}. \end{aligned}$$

The measurement noise covariance matrix of the discretized system is the same as that of Eq. (3). Now the standard extended Kalman filter for Eq. (4) can be used, where C_k is given by

$$\begin{aligned} C_k &= \left.\frac{\partial f(x)}{\partial x}\right|_{x=\hat{x}_k^-} \\ &= \begin{bmatrix} \cos\hat{\theta}(kT) & 0 & 0 & 0 & 0 \\ 0 & \cos\hat{\phi}(kT)\cos\hat{\theta}(kT) & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned} \tag{5}$$

4. Two Step Extended Kalman Filter

The main drawback of the standard extended Kalman filter is that the estimation error becomes large if the object is experiencing external acceleration. The Kalman filter contains the model Eq. (1), which is not valid when there is external acceleration. This problem cannot be avoided even if very accurate accelerometers are used. Thus when there is external acceleration, gyroscope outputs should be trusted more and this can be done by making r_1 and r_2 large. Similar ideas are employed in Refs. (7) and (8), though different system models are used.

In Ref. (7), if gyroscope outputs are zero and inclinometer outputs are not changing, then it is assumed that external acceleration does not exist and accelerometer measurement noise covariance (corresponding to r_1 and r_2) is adjusted to small value.

In Ref. (8), the following observation is used to detect existence of external acceleration.

Observation: A necessary condition for acceleration free movements is

$$a_x^2 + a_y^2 + a_z^2 = 1. \tag{6}$$

The above observation states that if external acceleration does not exists, then acceleration sensed by 3 axis accelerometer should be the gravitational acceleration only. Recall that the accelerometer outputs are normalized with the gravitational acceleration so that Eq. (6) is satisfied. In Ref. (8), if Eq. (6) is not satisfied, only gyroscope outputs are used to estimated attitude. In our framework, this can be interpreted as selecting very large r_1 and r_2 when Eq. (6) is not satisfied.

In this paper, we use the same method to check existence of external acceleration: it is assumed that there is external acceleration if

$$f(a_x, a_y, a_z) \triangleq \left| a_x^2 + a_y^2 + a_z^2 - 1 \right| > \delta \tag{7}$$

where δ is a scalar parameter depending on accelerometer measurement noise characteristics. When existence of external acceleration is detected by Eq. (7), we use more sophisticated method to adjust r_1 and r_2. The direction of external acceleration is estimated and according to the direction, r_1 and r_2 are adjusted. To do this, we propose the two step extended Kalman filter:

- Initialization
  - $\hat{x}_0$: Initial attitude
  - P_0: set 0
- Time Update

$$\hat{x}_{k+1}^- = \Phi_k \hat{x}_k$$
$$P_{k+1}^- = \Phi_k P_k \Phi_k' + Q_k$$

- Measurement Update Step 1: Gyroscope measurements only are updated

$$\begin{aligned} K_{k,g} &= P_k^- C_2' (C_2 P_k^- C_2' + R_2)^{-1} \\ \hat{x}_{k,g} &= \hat{x}_k^- + K_{k,g}(z_{k,2} - C_2 \hat{x}_k^-) \\ P_{k,g} &= (I - K_{k,g} C_2) P_k^- (I - K_{k,g} C_2)' + K_{k,g} R_2 K_{k,g}' \end{aligned} \tag{8}$$

where

$$C_2 \triangleq \begin{bmatrix} 0 & I_3 \end{bmatrix}, \quad R_2 \triangleq \begin{bmatrix} r_3 & 0 & 0 \\ 0 & r_3 & 0 \\ 0 & 0 & r_3 \end{bmatrix}, \quad z_k \triangleq \begin{bmatrix} z_{k,1} \\ z_{k,2} \end{bmatrix} \in \begin{bmatrix} R^2 \\ R^3 \end{bmatrix}$$

- Accelerometer noise covariance adjustment
  - if Eq. (7) is satisfied, then

$$\begin{bmatrix} r_{1,k} \\ r_{2,k} \end{bmatrix} = \max\left(\alpha_1 \begin{bmatrix} r_{1,k-1} \\ r_{2,k-1} \end{bmatrix} + \alpha_2 (z_{k,1} - C_{1,k}\hat{x}_{k,g}), \begin{bmatrix} r_{1,nom} \\ r_{2,nom} \end{bmatrix} \right) \tag{9}$$

where $\alpha_1 (|\alpha_1| < 1)$ and α_2 are scalar parameters and $C_{1,k}$ can be obtained from the following partition of C_k:

$$C_k = \begin{bmatrix} C_{1,k} \\ C_2 \end{bmatrix}.$$

  - if Eq. (7) is not satisfied, then

$$\begin{bmatrix} r_{1,k} \\ r_{2,k} \end{bmatrix} = \alpha_1 \begin{bmatrix} r_{1,k-1} \\ r_{2,k-1} \end{bmatrix} + \begin{bmatrix} r_{1,nom} \\ r_{2,nom} \end{bmatrix}. \tag{10}$$

- Measurement Update 2: Acceleration measurements are now updated

$$\begin{aligned} K_{k,a} &= P_{k,g} C_{1,k}' (C_{1,k} P_{k,g} C_{1,k}' + R_{1,k})^{-1} \\ \hat{x}_k &= \hat{x}_{k,g} + K_{k,a}(z_{k,1} - C_{1,k}\hat{x}_{k,g}) \\ P_k &= (I - K_{k,a} C_{1,k}) P_{k,g} (I - K_{k,a} C_{1,k})' + K_{k,a} R_{1,k} K_{k,a}' \end{aligned} \tag{11}$$

where

$$R_{1,k} = \begin{bmatrix} r_{1,k} & 0 \\ 0 & r_{2,k} \end{bmatrix}.$$

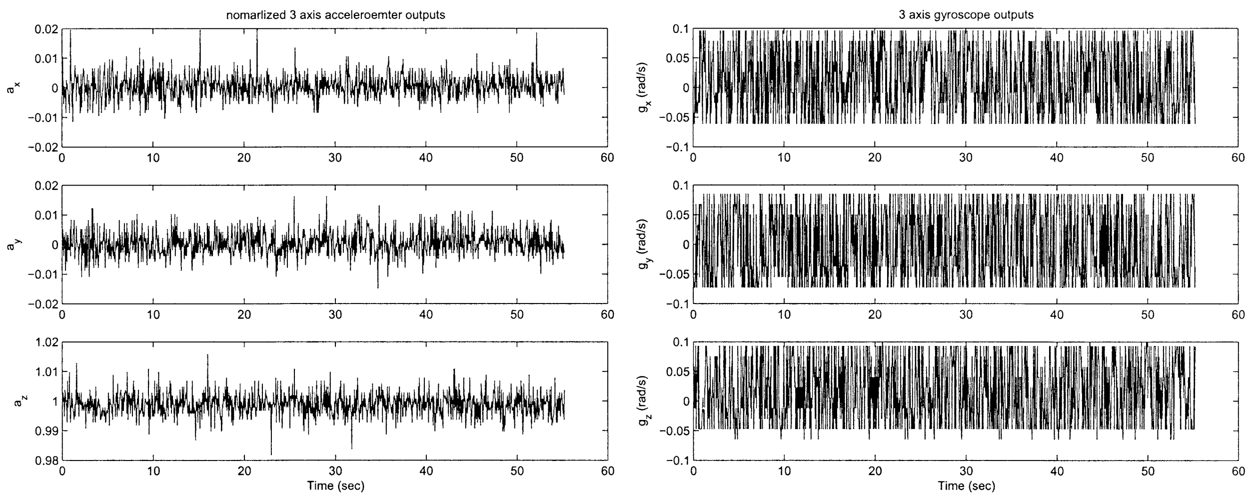

Fig. 1 Experiment 1: accelerometer and gyroscope outputs when there is neither attitude change nor external acceleration

In Eq. (8), only gyroscope outputs are used to estimate the state. Note that Eq. (8) is nothing but the standard Kalman filter equation when only gyroscope outputs ($z_{k,2}$) are available. In Eq. (9), $r_{1,k}$ and $r_{2,k}$ are adjusted if there is external acceleration. Note that $z_{k,1}$ is the accelerometer output and $C_{1,k}\hat{x}_{k,g}$ is the estimated accelerometer output using only gyroscope outputs. The difference between these values should be small when there is no external acceleration. When there is external acceleration, $z_{k,1} - C_{1,k}\hat{x}_{k,g}$ is proportional to external acceleration. Thus the role of Eq. (9) is that accelerometer output in the direction of external acceleration is not trusted. The level of trust is reflected in $r_{1,k}$ and $r_{2,k}$. Note that in Eqs. (9) and (10), a low pass filter is used so that $r_{1,k}$ and $r_{2,k}$ are not changed abruptly. Also note that $r_{1,k}$ and $r_{2,k}$ are always greater than $r_{1,nom}$ and $r_{2,nom}$ since too small r_1 and r_2 values may cause the Kalman filter divergence problem. In the measurement update 2, acceleration measurements only are used to estimate state.

We note that if the same C_k is used and R_k is constant (i.e., the adaptive algorithm is not used), the standard Kalman filter and the two step Kalman filter Eqs. (8) and (11) are identical. In the two step extended Kalman filter, the standard extended Kalman filter is divided into two step to estimate and compensate external acceleration.

5. Experiments

To verify the proposed filter, the sensor system consisting of 3 accelerometers and 3 gyroscopes is made and experiments are performed. In the first experiment, sensor outputs are measured while the sensor system is still: i.e., there is neither attitude change nor external acceleration. This experiment is to test how good the sensors are. Ideally, all sensor outputs should be zero. The accelerometer outputs (a_x, a_y, a_z) and gyroscope outputs (g_x, g_y, g_z) are given in Fig. 1. All outputs are sampled at 20 Hz (i.e., the sampling period $T = 0.05$ sec).

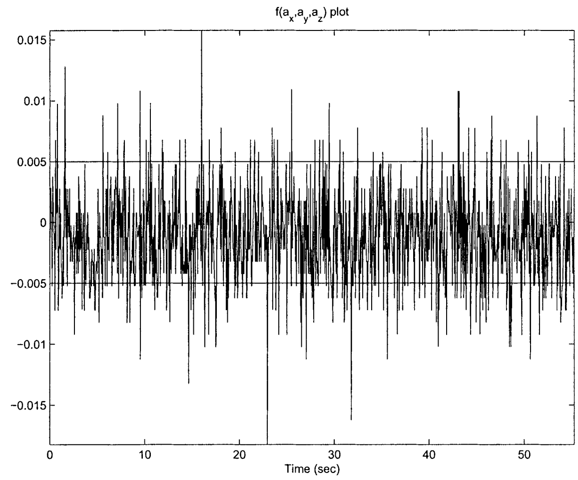

Fig. 2 Experiment 1: $f(a_x,a_y,a_z)$ plot when there is neither attitude change nor external acceleration

To determine r_1, r_2, and r_3, sample variance of a_x, a_y, w_x, w_y, w_z are computed: 0.000 016 8, 0.000 016 7, 0.003 2, 0.004 0, 0.003 1. Note that not the absolute value but the ratio between (r_1, r_2) and r_3 is important. The ratio of accelerometer sample variances to gyroscope sample variances is about 200. Since the accelerometers are more sensitive to disturbances, we decrease ratio to 20 and (r_1, r_2) and r_3 are chosen as follows:

$$r_{1,nom} = r_{2,nom} = 0.05, \quad r_3 = 1$$

where $r_{1,nom}$ and $r_{2,nom}$ are r_1 and r_2 values for the standard Kalman filter and also used in Eqs. (9) and (10).

The value $f(a_x,a_y,a_z)$ in Eq. (7) is given in Fig. 2.

Note that δ in Eq. (7) is a parameter, which determines existence of external acceleration. If δ is too small, external acceleration is falsely detected even though there is no external acceleration. If δ is too large, external acceleration is not detected even though there is external accelera-

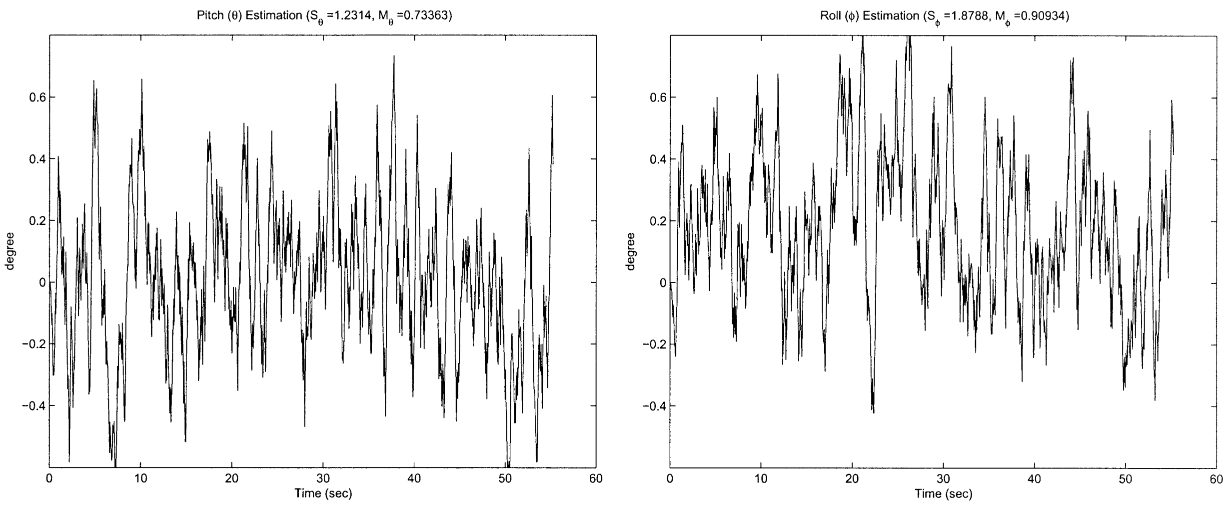

Fig. 3 Experiment 1: attitude estimation by the proposed filter

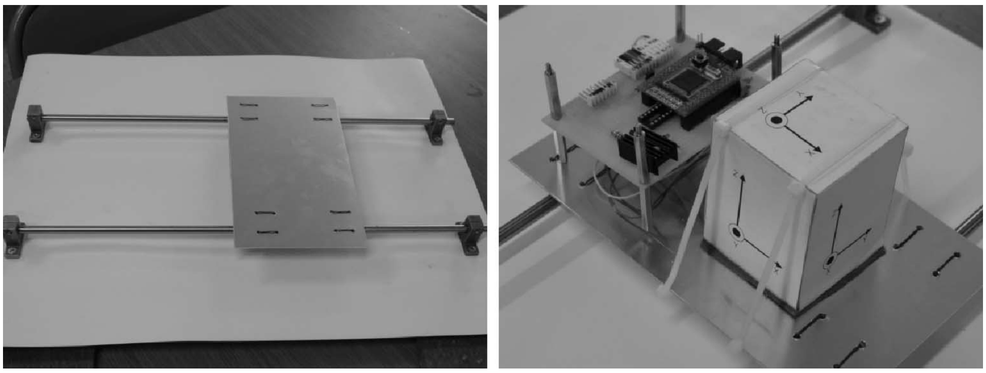

Fig. 4 Experiment 2: a cart and rails for external acceleration generation

tion. From Fig. 2, δ is set to be 0.005. The parameters for the two step extended Kalman filter are as follows:

$$q_1 = 1, \quad \alpha_1 = 0.4, \quad \alpha_2 = 10.$$

These parameters are rather heuristic choices based on trial-and-error. As performance indices of estimation, we use average absolute errors and maximum errors as follows:

$$\begin{aligned}
S_\theta &\triangleq \frac{1}{N}\sum_{k=1}^{N}(\hat{\theta}(kT)-\theta(kT))^2,\\
S_\phi &\triangleq \frac{1}{N}\sum_{k=1}^{N}(\hat{\phi}(kT)-\phi(kT))^2\\
M_\theta &\triangleq \max_k\left|\hat{\theta}(kT)-\theta(kT)\right|,\\
M_\phi &\triangleq \max_k\left|\hat{\phi}(kT)-\phi(kT)\right|. \qquad (12)
\end{aligned}$$

Attitude estimation by the proposed filter is given in Fig. 3. Since there is no external acceleration, the result is almost the same as the standard Kalman filter.

Since the sensor system is still, the true attitude values are $\theta = 0$ and $\phi = 0$. We can see the maximum estimation errors for $\hat{\theta}$ and $\hat{\phi}$ are 0.733 and 0.909 degree, respectively. These errors may be attributed to sensor noises, misalignment (sensors are not located exactly orthogonal) and A/D conversion errors.

In the second experiment, the sensor system is attached on the cart, which can move along the rails (Fig. 4). By moving the cart with hands, external acceleration is applied to the sensor system. The sensor system is located so that there is only y_b axis direction external acceleration.

Attitude is estimated using 3 different methods: the standard extended Kalman filter in section 3, the simple adaptive extended Kalman filter, and the proposed two step extended Kalman filter in section 4. In the simple adaptive extended Kalman filter, r_1 and r_2 are set to large values (5 in this experiment) when Eq. (7) is satisfied. The simple adaptive filter have a similar method to that in Ref. (8), where gyroscope outputs only are integrated when there is external acceleration. Note that attitude is not changed in the second experiment; that is, true attitude values are $\theta = 0$ and $\phi = 0$.

Attitude estimation by 3 different filters is given in Fig. 5. The three filters show similar performance in pitch estimation. Recall that there is no external acceleration in the direction of x_b; thus pitch estimation is not affected by external acceleration in the second experiment. On the other hand, the three filters show different performance in

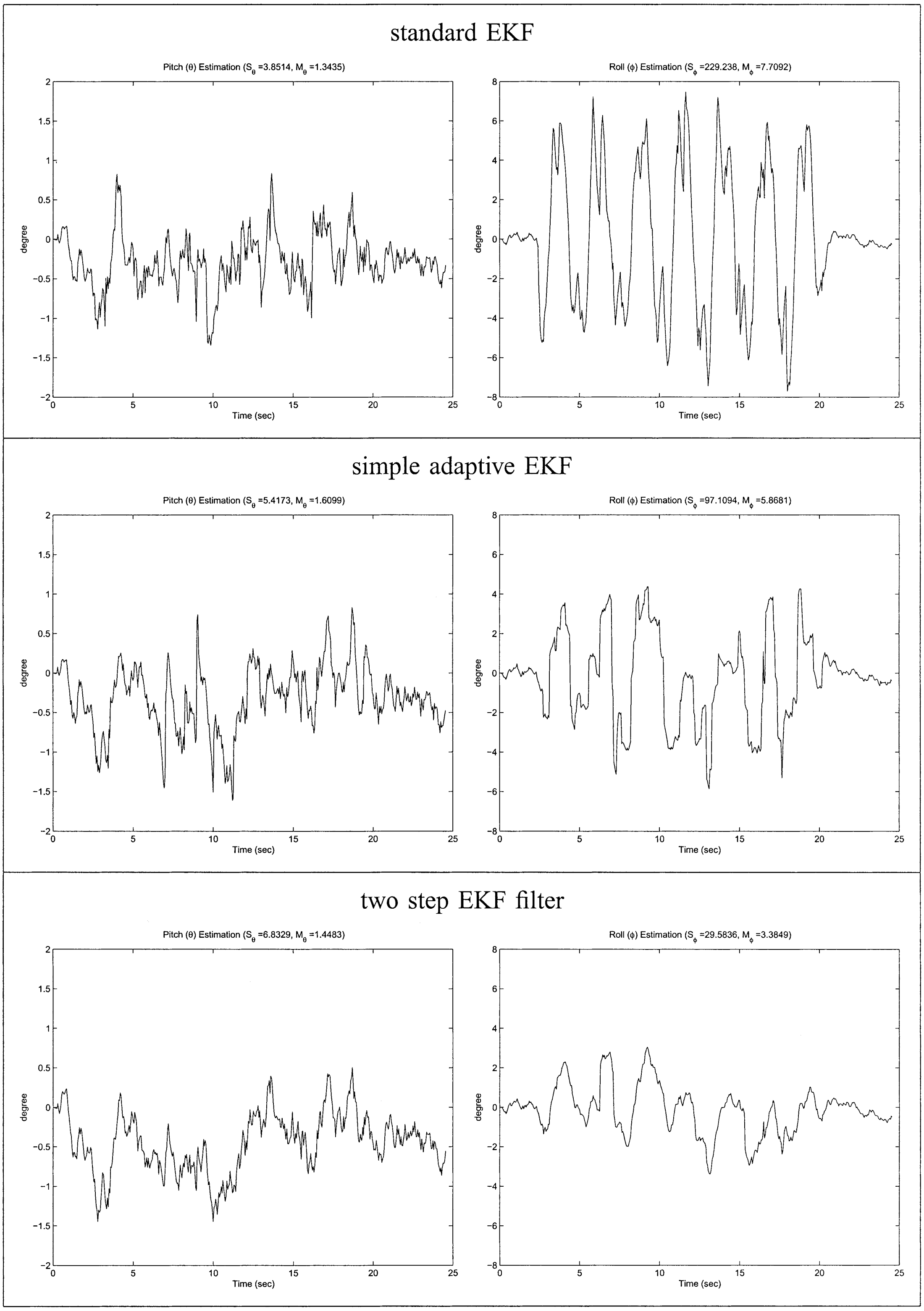

Fig. 5 Experiment 2: attitude estimation by 3 different filters

roll estimation. In the standard Kalman filter, external acceleration is not effectively rejected, which results in large estimation error. In the simple adaptive and the two step filter, effects of external acceleration are reduced and the performance of the two step filter is better than that of the simple adaptive filter (compare S_ϕ and M_ϕ).

The r_1 and r_2 variations in the simple adaptive extended Kalman filter and the proposed two step extended Kalman filter are given in Fig. 6. In the two step filter, $r_{2,k}$ becomes significantly larger that $r_{1,k}$, which reflects that

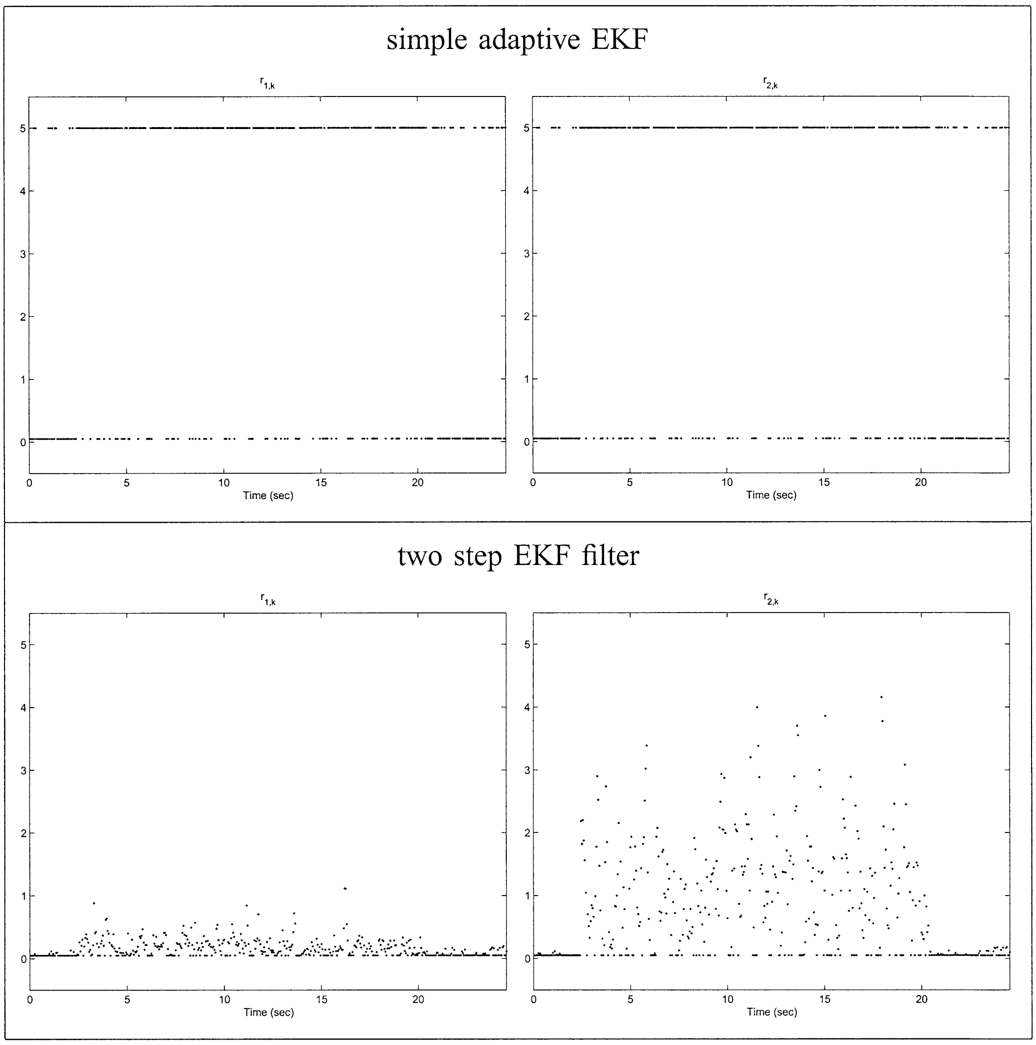

Fig. 6 Experiment 2: ($r_{1,k}$, $r_{2,k}$) variations

there is external acceleration in the direction of y_b but not x_b. In the ideal situation, $r_{1,k}$ should stay at its nominal value; it can be conjectured that vibration (from the cart and the rails) and misalignments increase noise level of a_x output.

6. Conclusion

In this paper, we have proposed the two step extended Kalman filter for general purpose attitude estimation. The main contribution is external acceleration, which is the main source of estimation error, is estimated and compensated. To verify the proposed algorithm, the sensor system consisting of 3 axis accelerometers and 3 axis gyroscopes is constructed and tested while intentional external acceleration is generated. The experiment results show that the direction of external acceleration is detected and the estimation performance is less disturbed by external acceleration.

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References

(1) Lawrence, A., Modern Inertial Technology, (1998), Springer Verlag.

(2) Barbour, N. and Schmidt, G., Inertial Sensor Technology Trends, IEEE Sensors Journal, Vol.1, No.4 (2001), pp.332–339.

(3) Vaganay, J., Aldon, M.J. and Fournier, A., Mobile Robot Attitude Estimation by Fusion of Inertial Data, Proceedings of IEEE International Conference on Robotics and Automation, (1993), pp.277–282.

(4) Barshan, B. and Durrant-Whyte, H.F., Evaluation of a Solid-State Gyroscope for Robotics Application, IEEE Trans. on Instrumentation and Measurement, Vol.44, No.1 (1994), pp.61–67.

(5) Barshan, B. and Durrant-Whyte, H.F., Inertial Navigation System for Mobile Robots, IEEE Trans. Robot. Automat., Vol.11, No.3 (1995), pp.328–342.

(6) Rehbinder, H. and Hu, X., Nonlinear Pitch and Roll Estimation for Walking Robots, Proceedings of the 2000 IEEE International Conference on Robotics & Automation, (2000), pp.2617–2622.

(7) Foxlin, E., Inertial Head-Tracker Sensor Fusion by a Complementary Separate-Bias Kalman Filter, Proceedings of the IEEE 1996 Virtual Reality Annual International Symposium, (1996), pp.185–194.

(8) Rehbinder, H. and Hu, X., Drift-Free Attitude Estimation for Accelerated Rigid Bodies, Proceedings of the 2001 IEEE International Conference on Robotics & Automation, (2001), pp.4244–4249.

(9) Nelson, R.C., Flight Stability and Automatic Control, (1998), McGraw-Hill.

(10) Brown, R.G. and Hwang, P.Y.C., Introduction to Random Signals and Applied Kalman Filtering, (1997), John Wiley & Sons.