```
> with(LinearAlgebra):
> interface(rtablesize=20):
> VdotV := (x,y) -> x[1]*y[1]+x[2]*y[2]+x[3]*y[3];
```

$$VdotV := (x, y) \mapsto x_1 \cdot y_1 + x_2 \cdot y_2 + x_3 \cdot y_3 \quad (1)$$

UAV attitude estimation with bias on gyro

System model for prediction

```
> X := Vector[column](7,[q__0,q__1,q__2,q__3,b__wx,b__wy,b__wz])
  :<%>;
  U := Vector[column](3,[u__x,u__y,u__z]):<%>;
  q := Vector[column](4,[q__0,q__1,q__2,q__3]);
  b := Vector[column](3,[b__wx,b__wy,b__wz]);
  Epsilon__U := Vector[column](3,[epsilon__wx,epsilon__wy,
  epsilon__wz]):
  Epsilon__b := Vector[column](3,[epsilon__wbx,epsilon__wby,
  epsilon__wbz]):
  Epsilon__w := Vector[column](6,[Epsilon__U,Epsilon__b]);
  op(X)[1];
```

$$\begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \\ b_{wx} \\ b_{wy} \\ b_{wz} \end{bmatrix}$$

$$\begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix}$$

$$q := \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

$$b := \begin{bmatrix} b_{wx} \\ b_{wy} \\ b_{wz} \end{bmatrix}$$

$$E_w := \begin{bmatrix} \epsilon_{wx} \\ \epsilon_{wy} \\ \epsilon_{wz} \\ \epsilon_{wbx} \\ \epsilon_{wby} \\ \epsilon_{wbz} \end{bmatrix}$$

$$7 \tag{1.1.1}$$

```
> S := w -> <<0,w[1],w[2],w[3]>|<-w[1],0,-w[3],w[2]>|<-w[2],w[3],0,
  -w[1]>|<-w[3],-w[2],w[1],0>>;
  W := U-b+Epsilon__U-Epsilon__b;
  Sw := S(W);
  Emean_W := {seq(Epsilon__w[i] = 0,i=1..6)};
  F := Vector[column](7,[(Ts/2*Sw+ IdentityMatrix(4)).q,b+
  Epsilon__b]);
  subs(Emean_W,F);
```

$$S := w \mapsto \langle \langle 0, w_1, w_2, w_3 \rangle | \langle -w_1, 0, -w_3, w_2 \rangle | \langle -w_2, w_3, 0, -w_1 \rangle | \langle -w_3, -w_2, w_1, 0 \rangle \rangle$$

$$W := \begin{bmatrix} u_x - b_{wx} + \epsilon_{wx} - \epsilon_{wbx} \\ u_y - b_{wy} + \epsilon_{wy} - \epsilon_{wby} \\ u_z - b_{wz} + \epsilon_{wz} - \epsilon_{wbz} \end{bmatrix}$$

$$Sw := [[0, -u_x + b_{wx} - \epsilon_{wx} + \epsilon_{wbx}, -u_y + b_{wy} - \epsilon_{wy} + \epsilon_{wby}, -u_z + b_{wz} - \epsilon_{wz} + \epsilon_{wbz}],$$
$$[u_x - b_{wx} + \epsilon_{wx} - \epsilon_{wbx}, 0, u_z - b_{wz} + \epsilon_{wz} - \epsilon_{wbz}, -u_y + b_{wy} - \epsilon_{wy} + \epsilon_{wby}],$$
$$[u_y - b_{wy} + \epsilon_{wy} - \epsilon_{wby}, -u_z + b_{wz} - \epsilon_{wz} + \epsilon_{wbz}, 0, u_x - b_{wx} + \epsilon_{wx} - \epsilon_{wbx}],$$
$$[u_z - b_{wz} + \epsilon_{wz} - \epsilon_{wbz}, u_y - b_{wy} + \epsilon_{wy} - \epsilon_{wby}, -u_x + b_{wx} - \epsilon_{wx} + \epsilon_{wbx}, 0]]$$

$$Emean\_W := \{\epsilon_{wbx} = 0, \epsilon_{wby} = 0, \epsilon_{wbz} = 0, \epsilon_{wx} = 0, \epsilon_{wy} = 0, \epsilon_{wz} = 0\}$$

$$F := \Bigg[\Bigg[q_0 + \frac{Ts\,(-u_x + b_{wx} - \epsilon_{wx} + \epsilon_{wbx})\,q_1}{2} + \frac{Ts\,(-u_y + b_{wy} - \epsilon_{wy} + \epsilon_{wby})\,q_2}{2}$$
$$+ \frac{Ts\,(-u_z + b_{wz} - \epsilon_{wz} + \epsilon_{wbz})\,q_3}{2}\Bigg],$$

$$\left[\frac{Ts\left(u_x - b_{wx} + \epsilon_{wx} - \epsilon_{wbx}\right) q_0}{2} + q_1 + \frac{Ts\left(u_z - b_{wz} + \epsilon_{wz} - \epsilon_{wbz}\right) q_2}{2} + \frac{Ts\left(-u_y + b_{wy} - \epsilon_{wy} + \epsilon_{wby}\right) q_3}{2}\right],$$

$$\left[\frac{Ts\left(u_y - b_{wy} + \epsilon_{wy} - \epsilon_{wby}\right) q_0}{2} + \frac{Ts\left(-u_z + b_{wz} - \epsilon_{wz} + \epsilon_{wbz}\right) q_1}{2} + q_2 + \frac{Ts\left(u_x - b_{wx} + \epsilon_{wx} - \epsilon_{wbx}\right) q_3}{2}\right],$$

$$\left[\frac{Ts\left(u_z - b_{wz} + \epsilon_{wz} - \epsilon_{wbz}\right) q_0}{2} + \frac{Ts\left(u_y - b_{wy} + \epsilon_{wy} - \epsilon_{wby}\right) q_1}{2} + \frac{Ts\left(-u_x + b_{wx} - \epsilon_{wx} + \epsilon_{wbx}\right) q_2}{2} + q_3\right],$$

$$\left[b_{wx} + \epsilon_{wbx}\right],$$

$$\left[b_{wy} + \epsilon_{wby}\right],$$

$$\left[b_{wz} + \epsilon_{wbz}\right]\Big]$$

$$\left[\begin{array}{c}
q_0 + \frac{Ts\left(-u_x + b_{wx}\right) q_1}{2} + \frac{Ts\left(-u_y + b_{wy}\right) q_2}{2} + \frac{Ts\left(-u_z + b_{wz}\right) q_3}{2} \\
\frac{Ts\left(u_x - b_{wx}\right) q_0}{2} + q_1 + \frac{Ts\left(u_z - b_{wz}\right) q_2}{2} + \frac{Ts\left(-u_y + b_{wy}\right) q_3}{2} \\
\frac{Ts\left(u_y - b_{wy}\right) q_0}{2} + \frac{Ts\left(-u_z + b_{wz}\right) q_1}{2} + q_2 + \frac{Ts\left(u_x - b_{wx}\right) q_3}{2} \\
\frac{Ts\left(u_z - b_{wz}\right) q_0}{2} + \frac{Ts\left(u_y - b_{wy}\right) q_1}{2} + \frac{Ts\left(-u_x + b_{wx}\right) q_2}{2} + q_3 \\
b_{wx} \\
b_{wy} \\
b_{wz}
\end{array}\right] \qquad \mathbf{(1.1.2)}$$

```
> J__X := subs(Emean_W,VectorCalculus:-Jacobian(convert(F,list),
  convert(X,list)));
  J__Epsilon__U := VectorCalculus:-Jacobian(convert(F,list),convert
  (Epsilon__U,list) );
  J__Epsilon__b := VectorCalculus:-Jacobian(convert(F,list),convert
  (Epsilon__b,list) );
  J__Epsilon__w := VectorCalculus:-Jacobian(convert(F,list),convert
```

```
(Epsilon__w,list) );
```

$$J_X := \left[\left[1, \frac{Ts\left(-u_x+b_{wx}\right)}{2}, \frac{Ts\left(-u_y+b_{wy}\right)}{2}, \frac{Ts\left(-u_z+b_{wz}\right)}{2}, \frac{Ts\, q_1}{2}, \frac{Ts\, q_2}{2}, \frac{Ts\, q_3}{2}\right],\right.$$

$$\left[\frac{Ts\left(u_x-b_{wx}\right)}{2}, 1, \frac{Ts\left(u_z-b_{wz}\right)}{2}, \frac{Ts\left(-u_y+b_{wy}\right)}{2}, -\frac{Ts\, q_0}{2}, \frac{Ts\, q_3}{2}, -\frac{Ts\, q_2}{2}\right],$$

$$\left[\frac{Ts\left(u_y-b_{wy}\right)}{2}, \frac{Ts\left(-u_z+b_{wz}\right)}{2}, 1, \frac{Ts\left(u_x-b_{wx}\right)}{2}, -\frac{Ts\, q_3}{2}, -\frac{Ts\, q_0}{2}, \frac{Ts\, q_1}{2}\right],$$

$$\left[\frac{Ts\left(u_z-b_{wz}\right)}{2}, \frac{Ts\left(u_y-b_{wy}\right)}{2}, \frac{Ts\left(-u_x+b_{wx}\right)}{2}, 1, \frac{Ts\, q_2}{2}, -\frac{Ts\, q_1}{2}, -\frac{Ts\, q_0}{2}\right],$$

$$\left[0, 0, 0, 0, 1, 0, 0\right],$$

$$\left[0, 0, 0, 0, 0, 1, 0\right],$$

$$\left.\left[0, 0, 0, 0, 0, 0, 1\right]\right]$$

$$J_{\mathrm{E}_U} := \begin{bmatrix} -\frac{Ts\, q_1}{2} & -\frac{Ts\, q_2}{2} & -\frac{Ts\, q_3}{2} \\ \frac{Ts\, q_0}{2} & -\frac{Ts\, q_3}{2} & \frac{Ts\, q_2}{2} \\ \frac{Ts\, q_3}{2} & \frac{Ts\, q_0}{2} & -\frac{Ts\, q_1}{2} \\ -\frac{Ts\, q_2}{2} & \frac{Ts\, q_1}{2} & \frac{Ts\, q_0}{2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$J_{E_b} := \begin{bmatrix} \frac{Ts\, q_1}{2} & \frac{Ts\, q_2}{2} & \frac{Ts\, q_3}{2} \\ -\frac{Ts\, q_0}{2} & \frac{Ts\, q_3}{2} & -\frac{Ts\, q_2}{2} \\ -\frac{Ts\, q_3}{2} & -\frac{Ts\, q_0}{2} & \frac{Ts\, q_1}{2} \\ \frac{Ts\, q_2}{2} & -\frac{Ts\, q_1}{2} & -\frac{Ts\, q_0}{2} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$J_{E_w} := \begin{bmatrix} -\frac{Ts\, q_1}{2} & -\frac{Ts\, q_2}{2} & -\frac{Ts\, q_3}{2} & \frac{Ts\, q_1}{2} & \frac{Ts\, q_2}{2} & \frac{Ts\, q_3}{2} \\ \frac{Ts\, q_0}{2} & -\frac{Ts\, q_3}{2} & \frac{Ts\, q_2}{2} & -\frac{Ts\, q_0}{2} & \frac{Ts\, q_3}{2} & -\frac{Ts\, q_2}{2} \\ \frac{Ts\, q_3}{2} & \frac{Ts\, q_0}{2} & -\frac{Ts\, q_1}{2} & -\frac{Ts\, q_3}{2} & -\frac{Ts\, q_0}{2} & \frac{Ts\, q_1}{2} \\ -\frac{Ts\, q_2}{2} & \frac{Ts\, q_1}{2} & \frac{Ts\, q_0}{2} & \frac{Ts\, q_2}{2} & -\frac{Ts\, q_1}{2} & -\frac{Ts\, q_0}{2} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \textbf{(1.1.3)}$$

PREDICTION

X(k+1) = F(X(k),U(k),Ew=0,Ewb=0)

P^(k+1) = Jx.P(k).Jx^T + Jeu.Ru.Jeu^T + Jeb.Rb.Jeb^T

Measurement model for update

Accelerometer

```
> A := Vector[column](3,[a__x+epsilon__ax,a__y+epsilon__ay,a__z+
  epsilon__az]);
  Epsilon__A := Vector[column](3,[epsilon__ax,epsilon__ay,
  epsilon__az]);
  R__A := DiagonalMatrix(Epsilon__A);
```

$$A := \begin{bmatrix} a_x + \epsilon_{ax} \\ a_y + \epsilon_{ay} \\ a_z + \epsilon_{az} \end{bmatrix}$$

$$E_A := \begin{bmatrix} \epsilon_{ax} \\ \epsilon_{ay} \\ \epsilon_{az} \end{bmatrix}$$

$$R_A := \begin{bmatrix} \epsilon_{ax} & 0 & 0 \\ 0 & \epsilon_{ay} & 0 \\ 0 & 0 & \epsilon_{az} \end{bmatrix} \tag{1.2.1.1}$$

Normalization

```
> Z__A := map(x->x/g,A);
  J__A := VectorCalculus:-Jacobian( convert(Z__A,list),convert
  (Epsilon__A,list) );
  R__ZA := simplify(J__A.R__A.Transpose(J__A));

  A__N := Vector[column](3,[a__xN+epsilon__axN,a__yN+epsilon__ayN,
  a__zN+epsilon__azN]);
  Epsilon__AN := Vector[column](3,[epsilon__axN,epsilon__ayN,
  epsilon__azN]);
  Emean_AN := {seq(Epsilon__AN[i] = 0,i=1..3)};
  R__AN := DiagonalMatrix(Epsilon__AN);
```

$$Z_A := \begin{bmatrix} \frac{a_x + \epsilon_{ax}}{g} \\ \frac{a_y + \epsilon_{ay}}{g} \\ \frac{a_z + \epsilon_{az}}{g} \end{bmatrix}$$

$$J_A := \begin{bmatrix} \frac{1}{g} & 0 & 0 \\ 0 & \frac{1}{g} & 0 \\ 0 & 0 & \frac{1}{g} \end{bmatrix}$$

$$R_{ZA} := \begin{bmatrix} \frac{\epsilon_{ax}}{g^2} & 0 & 0 \\ 0 & \frac{\epsilon_{ay}}{g^2} & 0 \\ 0 & 0 & \frac{\epsilon_{az}}{g^2} \end{bmatrix}$$

$$A_N := \begin{bmatrix} a_{xN} + \epsilon_{axN} \\ a_{yN} + \epsilon_{ayN} \\ a_{zN} + \epsilon_{azN} \end{bmatrix}$$

$$\mathrm{E}_{AN} := \begin{bmatrix} \epsilon_{axN} \\ \epsilon_{ayN} \\ \epsilon_{azN} \end{bmatrix}$$

$$Emean\_AN := \{\epsilon_{axN} = 0, \epsilon_{ayN} = 0, \epsilon_{azN} = 0\}$$

$$R_{AN} := \begin{bmatrix} \epsilon_{axN} & 0 & 0 \\ 0 & \epsilon_{ayN} & 0 \\ 0 & 0 & \epsilon_{azN} \end{bmatrix} \quad \textbf{(1.2.1.2)}$$

an := a/g
Ran := Ra/g^2

Magnetometer

```
> M := Vector[column](3,[m__x+epsilon__mx,m__y+epsilon__my,m__z+
  epsilon__mz]);
  Epsilon__M := Vector[column](3,[epsilon__mx,epsilon__my,
  epsilon__mz]);
  R__M := DiagonalMatrix(Epsilon__M);
```

$$M := \begin{bmatrix} m_x + \epsilon_{mx} \\ m_y + \epsilon_{my} \\ m_z + \epsilon_{mz} \end{bmatrix}$$

$$\mathrm{E}_M := \begin{bmatrix} \epsilon_{mx} \\ \epsilon_{my} \\ \epsilon_{mz} \end{bmatrix}$$

$$R_M := \begin{bmatrix} \epsilon_{mx} & 0 & 0 \\ 0 & \epsilon_{my} & 0 \\ 0 & 0 & \epsilon_{mz} \end{bmatrix} \quad \textbf{(1.2.2.1)}$$

Normalization

```
> modM := sqrt(M[1]^2+M[2]^2+M[3]^2);
  m__N := map(x->x/modM,M);
  Emean_M := {seq(Epsilon__M[i] = 0,i=1..3)};
  J__N := simplify(subs(Emean_M,VectorCalculus:-Jacobian( convert
  (m__N,list),convert(Epsilon__M,list) )));
  R__mN := simplify(J__N.R__M.Transpose(J__N));

  M__N := Vector[column](3,[m__xN+epsilon__mxN,m__yN+epsilon__myN,
  m__zN+epsilon__mzN]);
  Epsilon__MN := Vector[column](3,[epsilon__mxN,epsilon__myN,
  epsilon__mzN]);
  Emean_MN := {seq(Epsilon__MN[i] = 0,i=1..3)};
  R__MN := DiagonalMatrix(Epsilon__MN);
```

$$modM := \sqrt{\left(m_x+\epsilon_{mx}\right)^2+\left(m_y+\epsilon_{my}\right)^2+\left(m_z+\epsilon_{mz}\right)^2}$$

$$m_N := \begin{bmatrix} \dfrac{m_x+\epsilon_{mx}}{\sqrt{\left(m_x+\epsilon_{mx}\right)^2+\left(m_y+\epsilon_{my}\right)^2+\left(m_z+\epsilon_{mz}\right)^2}} \\ \dfrac{m_y+\epsilon_{my}}{\sqrt{\left(m_x+\epsilon_{mx}\right)^2+\left(m_y+\epsilon_{my}\right)^2+\left(m_z+\epsilon_{mz}\right)^2}} \\ \dfrac{m_z+\epsilon_{mz}}{\sqrt{\left(m_x+\epsilon_{mx}\right)^2+\left(m_y+\epsilon_{my}\right)^2+\left(m_z+\epsilon_{mz}\right)^2}} \end{bmatrix}$$

$$Emean\_M := \left\{\epsilon_{mx}=0, \epsilon_{my}=0, \epsilon_{mz}=0\right\}$$

$$J_N := \begin{bmatrix} \dfrac{m_y^2+m_z^2}{\left(m_x^2+m_y^2+m_z^2\right)^{3/2}} & -\dfrac{m_x m_y}{\left(m_x^2+m_y^2+m_z^2\right)^{3/2}} & -\dfrac{m_x m_z}{\left(m_x^2+m_y^2+m_z^2\right)^{3/2}} \\ -\dfrac{m_x m_y}{\left(m_x^2+m_y^2+m_z^2\right)^{3/2}} & \dfrac{m_x^2+m_z^2}{\left(m_x^2+m_y^2+m_z^2\right)^{3/2}} & -\dfrac{m_y m_z}{\left(m_x^2+m_y^2+m_z^2\right)^{3/2}} \\ -\dfrac{m_x m_z}{\left(m_x^2+m_y^2+m_z^2\right)^{3/2}} & -\dfrac{m_y m_z}{\left(m_x^2+m_y^2+m_z^2\right)^{3/2}} & \dfrac{m_x^2+m_y^2}{\left(m_x^2+m_y^2+m_z^2\right)^{3/2}} \end{bmatrix}$$

$$R_{mN} := \left[\left[\frac{\left(m_y^2+m_z^2\right)^2 \epsilon_{mx}+m_x^2\left(m_y^2 \epsilon_{my}+m_z^2 \epsilon_{mz}\right)}{\left(m_x^2+m_y^2+m_z^2\right)^3},\right.\right.$$

$$
-\frac{m_y m_x \left(\left(\epsilon_{mx}+\epsilon_{my}-\epsilon_{mz}\right) m_z^2+m_x^2 \epsilon_{my}+m_y^2 \epsilon_{mx}\right)}{\left(m_x^2+m_y^2+m_z^2\right)^3},
$$

$$
\left.-\frac{m_x \left(\left(\epsilon_{mx}-\epsilon_{my}+\epsilon_{mz}\right) m_y^2+m_x^2 \epsilon_{mz}+m_z^2 \epsilon_{mx}\right) m_z}{\left(m_x^2+m_y^2+m_z^2\right)^3}\right],
$$

$$
\left[-\frac{m_y m_x \left(\left(\epsilon_{mx}+\epsilon_{my}-\epsilon_{mz}\right) m_z^2+m_x^2 \epsilon_{my}+m_y^2 \epsilon_{mx}\right)}{\left(m_x^2+m_y^2+m_z^2\right)^3},\right.
$$

$$
\frac{\left(m_x^2+m_z^2\right)^2 \epsilon_{my}+m_y^2 \left(m_x^2 \epsilon_{mx}+m_z^2 \epsilon_{mz}\right)}{\left(m_x^2+m_y^2+m_z^2\right)^3},
$$

$$
\left.\frac{m_y \left(\left(\epsilon_{mx}-\epsilon_{my}-\epsilon_{mz}\right) m_x^2-m_y^2 \epsilon_{mz}-m_z^2 \epsilon_{my}\right) m_z}{\left(m_x^2+m_y^2+m_z^2\right)^3}\right],
$$

$$
\left[-\frac{m_x \left(\left(\epsilon_{mx}-\epsilon_{my}+\epsilon_{mz}\right) m_y^2+m_x^2 \epsilon_{mz}+m_z^2 \epsilon_{mx}\right) m_z}{\left(m_x^2+m_y^2+m_z^2\right)^3},\right.
$$

$$
\frac{m_y \left(\left(\epsilon_{mx}-\epsilon_{my}-\epsilon_{mz}\right) m_x^2-m_y^2 \epsilon_{mz}-m_z^2 \epsilon_{my}\right) m_z}{\left(m_x^2+m_y^2+m_z^2\right)^3},
$$

$$
\left.\left.\frac{\left(m_x^2+m_y^2\right)^2 \epsilon_{mz}+m_z^2 \left(m_x^2 \epsilon_{mx}+m_y^2 \epsilon_{my}\right)}{\left(m_x^2+m_y^2+m_z^2\right)^3}\right]\right]
$$

$$
M_N := \begin{bmatrix} m_{xN}+\epsilon_{mxN} \\ m_{yN}+\epsilon_{myN} \\ m_{zN}+\epsilon_{mzN} \end{bmatrix}
$$

$$
\mathrm{E}_{MN} := \begin{bmatrix} \epsilon_{mxN} \\ \epsilon_{myN} \\ \epsilon_{mzN} \end{bmatrix}
$$

$$
Emean\_MN := \left\{\epsilon_{mxN}=0, \epsilon_{myN}=0, \epsilon_{mzN}=0\right\}
$$

$$
R_{MN} := \begin{bmatrix} \epsilon_{mxN} & 0 & 0 \\ 0 & \epsilon_{myN} & 0 \\ 0 & 0 & \epsilon_{mzN} \end{bmatrix} \qquad \textbf{(1.2.2.2)}
$$

mn = m/norm(m)
Rmn = Jn.Rm.Jn^T

TRIAD algorithm

```
> m__D := VdotV(A__N,M__N);
  m__N := sqrt(1-m__D^2);
  N__ZM := A__N[2]*M__N[3]-A__N[3]*M__N[2];
  Z__M := N__ZM/m__N;
  Z := Vector[column](4,[A__N,Z__M]);
  eqns := subs(Emean_AN,Emean_MN ,{m__D = mD,N__ZM = nZM});
  J__ZA := subs(Emean_AN,Emean_MN ,VectorCalculus:-Jacobian(
  convert(Z,list),convert(Epsilon__AN,list) ));
  J__ZM := subs(Emean_AN,Emean_MN ,VectorCalculus:-Jacobian(
  convert(Z,list),convert(Epsilon__MN,list) ));
  simplify(%, eqns):
  J__Zsimp := <<1,0,0,nZM*mD*m__xN/mN^3>|<0,1,0,m__zN/mN+nZM*mD*
  m__yN/mN^3>|<0,0,1,-m__yN/mN+nZM*mD*m__zN/mN^3>|<0,0,0,nZM*mD*
  a__xN/mN^3>|<0,0,0,-a__zN/mN+nZM*mD*a__yN/mN^3>|<0,0,0,a__yN/mN+
  nZM*mD*a__zN/mN^3>>;
```

$$m_D := (a_{xN} + \epsilon_{axN})(m_{xN} + \epsilon_{mxN}) + (a_{yN} + \epsilon_{ayN})(m_{yN} + \epsilon_{myN}) + (a_{zN} + \epsilon_{azN})(m_{zN} + \epsilon_{mzN})$$

$$m_N := \left(-\left((a_{xN} + \epsilon_{axN})(m_{xN} + \epsilon_{mxN}) + (a_{yN} + \epsilon_{ayN})(m_{yN} + \epsilon_{myN}) + (a_{zN} + \epsilon_{azN})(m_{zN} + \epsilon_{mzN})\right)^2 + 1\right)^{1/2}$$

$$N_{ZM} := (a_{yN} + \epsilon_{ayN})(m_{zN} + \epsilon_{mzN}) - (a_{zN} + \epsilon_{azN})(m_{yN} + \epsilon_{myN})$$

$$Z_M := \left((a_{yN} + \epsilon_{ayN})(m_{zN} + \epsilon_{mzN}) - (a_{zN} + \epsilon_{azN})(m_{yN} + \epsilon_{myN})\right) \Big/ \left(-\left((a_{xN} + \epsilon_{axN})(m_{xN} + \epsilon_{mxN}) + (a_{yN} + \epsilon_{ayN})(m_{yN} + \epsilon_{myN}) + (a_{zN} + \epsilon_{azN})(m_{zN} + \epsilon_{mzN})\right)^2 + 1\right)^{1/2}$$

$$Z := \Bigg[\left[a_{xN} + \epsilon_{axN}\right], \left[a_{yN} + \epsilon_{ayN}\right], \left[a_{zN} + \epsilon_{azN}\right], \Big[\left((a_{yN} + \epsilon_{ayN})(m_{zN} + \epsilon_{mzN}) - (a_{zN} + \epsilon_{azN})(m_{yN} + \epsilon_{myN})\right) \Big/ \left(-\left((a_{xN} + \epsilon_{axN})(m_{xN} + \epsilon_{mxN}) + (a_{yN} + \epsilon_{ayN})(m_{yN} + \epsilon_{myN}) + (a_{zN}\right.\right.$$

$$+ \epsilon_{azN}) \left(m_{zN} + \epsilon_{mzN})\right)^2 + 1\Big)^{1/2}\Big]\Big]$$

$$eqns := \{a_{yN} m_{zN} - a_{zN} m_{yN} = nZM, a_{xN} m_{xN} + a_{yN} m_{yN} + a_{zN} m_{zN} = mD\}$$

$$J_{ZA} := \left[\left[1, 0, 0\right],\right.$$

$$\left[0, 1, 0\right],$$

$$\left[0, 0, 1\right],$$

$$\left[\frac{\left(a_{yN} m_{zN} - a_{zN} m_{yN}\right) \left(a_{xN} m_{xN} + a_{yN} m_{yN} + a_{zN} m_{zN}\right) m_{xN}}{\left(-\left(a_{xN} m_{xN} + a_{yN} m_{yN} + a_{zN} m_{zN}\right)^2 + 1\right)^{3/2}},\right.$$

$$\frac{m_{zN}}{\sqrt{-\left(a_{xN} m_{xN} + a_{yN} m_{yN} + a_{zN} m_{zN}\right)^2 + 1}}$$

$$+ \frac{\left(a_{yN} m_{zN} - a_{zN} m_{yN}\right) \left(a_{xN} m_{xN} + a_{yN} m_{yN} + a_{zN} m_{zN}\right) m_{yN}}{\left(-\left(a_{xN} m_{xN} + a_{yN} m_{yN} + a_{zN} m_{zN}\right)^2 + 1\right)^{3/2}},$$

$$-\frac{m_{yN}}{\sqrt{-\left(a_{xN} m_{xN} + a_{yN} m_{yN} + a_{zN} m_{zN}\right)^2 + 1}}$$

$$\left.+ \frac{\left(a_{yN} m_{zN} - a_{zN} m_{yN}\right) \left(a_{xN} m_{xN} + a_{yN} m_{yN} + a_{zN} m_{zN}\right) m_{zN}}{\left(-\left(a_{xN} m_{xN} + a_{yN} m_{yN} + a_{zN} m_{zN}\right)^2 + 1\right)^{3/2}}\right]\Big]$$

$$J_{ZM} := \left[\left[0, 0, 0\right],\right.$$

$$\left[0, 0, 0\right],$$

$$\left[0, 0, 0\right],$$

$$\left[\frac{\left(a_{yN} m_{zN} - a_{zN} m_{yN}\right) \left(a_{xN} m_{xN} + a_{yN} m_{yN} + a_{zN} m_{zN}\right) a_{xN}}{\left(-\left(a_{xN} m_{xN} + a_{yN} m_{yN} + a_{zN} m_{zN}\right)^2 + 1\right)^{3/2}},\right.$$

$$-\frac{a_{zN}}{\sqrt{-\left(a_{xN}m_{xN}+a_{yN}m_{yN}+a_{zN}m_{zN}\right)^2+1}}+\frac{\left(a_{yN}m_{zN}-a_{zN}m_{yN}\right)\left(a_{xN}m_{xN}+a_{yN}m_{yN}+a_{zN}m_{zN}\right)a_{yN}}{\left(-\left(a_{xN}m_{xN}+a_{yN}m_{yN}+a_{zN}m_{zN}\right)^2+1\right)^{3/2}},$$

$$\frac{a_{yN}}{\sqrt{-\left(a_{xN}m_{xN}+a_{yN}m_{yN}+a_{zN}m_{zN}\right)^2+1}}+\frac{\left(a_{yN}m_{zN}-a_{zN}m_{yN}\right)\left(a_{xN}m_{xN}+a_{yN}m_{yN}+a_{zN}m_{zN}\right)a_{zN}}{\left(-\left(a_{xN}m_{xN}+a_{yN}m_{yN}+a_{zN}m_{zN}\right)^2+1\right)^{3/2}}\Bigg]\Bigg]$$

$$J_{Zsimp} := \Big[\big[1, 0, 0, 0, 0, 0\big], \big[0, 1, 0, 0, 0, 0\big], \big[0, 0, 1, 0, 0, 0\big], \Big[\frac{nZM\,mD\,m_{xN}}{mN^3}, \frac{m_{zN}}{mN}+\frac{nZM\,mD\,m_{yN}}{mN^3}, -\frac{m_{yN}}{mN}+\frac{nZM\,mD\,m_{zN}}{mN^3}, \frac{nZM\,mD\,a_{xN}}{mN^3}, -\frac{a_{zN}}{mN}+\frac{nZM\,mD\,a_{yN}}{mN^3}, \frac{a_{yN}}{mN}+\frac{nZM\,mD\,a_{zN}}{mN^3}\Big]\Big] \quad \textbf{(1.2.2.3)}$$

md=... , mn=... , Zm = ...
Z = [An,Zm]
Rz = Jza.Ran.Jza^T + Jzm.Rmn.Jzm^T
NB: Z_ = H(X) + Rz

Mapping function H

```
> H := Vector[column](4,[ -2*q[3]*q[1]+2*q[4]*q[2],2*q[2]*q[1]+2*q
  [4]*q[3],q[1]^2-q[2]^2-q[3]^2+q[4]^2,2*q[4]*q[1]+2*q[3]*q[2] ]);
```

$$H := \begin{bmatrix} -2\,q_0 q_2 + 2\,q_1 q_3 \\ 2\,q_0 q_1 + 2\,q_2 q_3 \\ q_0^2 - q_1^2 - q_2^2 + q_3^2 \\ 2\,q_0 q_3 + 2\,q_1 q_2 \end{bmatrix} \quad \textbf{(1.2.3.1)}$$

```
> J__H := VectorCalculus:-Jacobian( convert(H,list),convert(X,list)
  );
```

(1.2.3.2)

$$J_H := \begin{bmatrix} -2\,q_2 & 2\,q_3 & -2\,q_0 & 2\,q_1 & 0 & 0 & 0 \\ 2\,q_1 & 2\,q_0 & 2\,q_3 & 2\,q_2 & 0 & 0 & 0 \\ 2\,q_0 & -2\,q_1 & -2\,q_2 & 2\,q_3 & 0 & 0 & 0 \\ 2\,q_3 & 2\,q_2 & 2\,q_1 & 2\,q_0 & 0 & 0 & 0 \end{bmatrix} \tag{1.2.3.2}$$

S(k+1) = Jh.P^(k+1).Jh^T + Rz
W(k+1) = P^(k+1).Jh^t.S(k+1)^-1
X(k+1) = X^(k+1) + W(k+1).(Z(k)-H(X^(k+1)))
P(k+1) = (I-W(k+1).Jh)P^(k+1)

```
>
```

UAV attitude+position with bias on gyro+acc

Variables

State

```
> Xatt := Vector[column](4,[q__0,q__1,q__2,q__3]):#<%>;
  Xbias := Vector[column](6,[b__wx, b__wy, b__wz, b__ax, b__ay,
  b__az]):#<%>;
  Xpos := Vector[column](4,[x, y, v__x, v__y]):#<%>;
  q := Vector[column](4,[q__0,q__1,q__2,q__3]):#<%>;
  pos := Vector[column](2,[x, y]):#<%>;
  v := Vector[column](2,[v__x, v__y]):#<%>;
  pos_v := Vector[column](4,[pos,v]):#<%>;
  b__w := Vector[column](3,[b__wx,b__wy,b__wz]):#<%>;
  b__a := Vector[column](3,[b__ax, b__ay, b__az]):#<%>;
```

Sensors

```
> gyro := Vector[column](3,[w__x,w__y,w__z]):#<%>;
  acc := Vector[column](3,[a__x,a__y,a__z]):#<%>;
  mag := Vector[column](3,[m__x,m__y,m__z]):#<%>;
  uwb := Vector[column](4,[uwb__x,uwb__y,uwb__vx,uwb__vy]):#<%>;
```

Noise

```
> Epsilon__w := Vector[column](3,[epsilon__wx,epsilon__wy,
  epsilon__wz]):#<%>;
  Epsilon__bw := Vector[column](3,[epsilon__wbx,epsilon__wby,
  epsilon__wbz]):#<%>;
> Epsilon__a := Vector[column](3,[epsilon__ax,epsilon__ay,
  epsilon__az]):#<%>;
  Epsilon__ba := Vector[column](3,[epsilon__abx,epsilon__aby,
  epsilon__abz]):#<%>;
> Epsilon__m := Vector[column](3,[epsilon__mx,epsilon__my,
  epsilon__mz]):#<%>;
> Epsilon__uwb := Vector[column](3,[epsilon__uwbx,epsilon__uwby,
  epsilon__uwbz]):#<%>;
```

Considering drone's model

```
> sub_g := g = 9.807;
  acc_nobias := acc-b__a;
```

```
model_eq := [-sin(theta)*g-acc_nobias[1],cos(theta)*sin(phi)*g-
acc_nobias[2],cos(theta)*cos(phi)*g-a__M-acc_nobias[3]]:<%>;
```

$$sub\_g := g = 9.807$$

$$acc\_nobias := \begin{bmatrix} a_x - b_{ax} \\ a_y - b_{ay} \\ a_z - b_{az} \end{bmatrix}$$

$$\begin{bmatrix} -\sin(\theta)\, g - a_x + b_{ax} \\ \cos(\theta)\, \sin(\phi)\, g - a_y + b_{ay} \\ \cos(\theta)\, \cos(\phi)\, g - a_M - a_z + b_{az} \end{bmatrix} \quad \textbf{(2.2.1)}$$

```
> aZg := sqrt(g^2-acc_nobias[1]^2-acc_nobias[2]^2);
  aG := <acc_nobias[1..2]; aZg>/g;

  J_aG_acc := simplify(VectorCalculus:-Jacobian(convert(aG,list),
  convert(acc,list)));
  J_aG_ba := simplify(VectorCalculus:-Jacobian(convert(aG,list),
  convert(b__a,list)));
```

$$aZg := \sqrt{g^2 - (a_x - b_{ax})^2 - (a_y - b_{ay})^2}$$

$$aG := \begin{bmatrix} \dfrac{a_x - b_{ax}}{g} \\ \dfrac{a_y - b_{ay}}{g} \\ \dfrac{\sqrt{g^2 - (a_x - b_{ax})^2 - (a_y - b_{ay})^2}}{g} \end{bmatrix}$$

$$J\_aG\_acc := \left[\left[\frac{1}{g}, 0, 0\right], \left[0, \frac{1}{g}, 0\right], \left[\frac{-a_x + b_{ax}}{\sqrt{-a_x^2 + 2\, a_x b_{ax} - a_y^2 + 2\, a_y b_{ay} - b_{ax}^2 - b_{ay}^2 + g^2}\; g}, \frac{-a_y + b_{ay}}{\sqrt{-a_x^2 + 2\, a_x b_{ax} - a_y^2 + 2\, a_y b_{ay} - b_{ax}^2 - b_{ay}^2 + g^2}\; g}, 0\right]\right]$$

(2.2.2)

$$J\_aG\_ba := \left[\left[-\frac{1}{g}, 0, 0\right], \left[0, -\frac{1}{g}, 0\right], \left[\frac{a_x - b_{ax}}{\sqrt{-a_x^2 + 2\,a_x b_{ax} - a_y^2 + 2\,a_y b_{ay} - b_{ax}^2 - b_{ay}^2 + g^2}\; g}, \frac{a_y - b_{ay}}{\sqrt{-a_x^2 + 2\,a_x b_{ax} - a_y^2 + 2\,a_y b_{ay} - b_{ax}^2 - b_{ay}^2 + g^2}\; g}, 0\right]\right] \quad \mathbf{(2.2.2)}$$

```
> wRb := <<q(1)^2 + q(2)^2 - q(3)^2 - q(4)^2, 2*q(4)*q(1) + 2*q(3)*
  q(2), -2*q(3)*q(1) + 2*q(4)*q(2)>|
  < 2*q(3)*q(2) - 2*q(4)*q(1), q(1)^2 - q(2)^2 + q(3)^2 - q(4)^2,
  2*q(2)*q(1) + 2*q(4)*q(3)>|
  < 2*q(3)*q(1) + 2*q(4)*q(2), -2*q(2)*q(1) + 2*q(4)*q(3), q(1)^2 -
  q(2)^2 - q(3)^2 + q(4)^2>>;
  aTrust := aZg - acc_nobias[3];
  aM := <0,0,-aTrust>;
  aW := wRb.aM;
  J_aW_acc := simplify(VectorCalculus:-Jacobian(convert(aW,list),
  convert(acc,list)));
  J_aW_ba := simplify(VectorCalculus:-Jacobian(convert(aW,list),
  convert(b__a,list)));
  J_aW_q := simplify(VectorCalculus:-Jacobian(convert(aW,list),
  convert(q,list)));
  simplify(VectorCalculus:-Jacobian(convert(< 2*q(3)*q(1) + 2*q(4)*
  q(2), -2*q(2)*q(1) + 2*q(4)*q(3), q(1)^2 - q(2)^2 - q(3)^2 + q(4)
  ^2>,list),convert(q,list)));
```

$$wRb := \begin{bmatrix} q_0^2 + q_1^2 - q_2^2 - q_3^2 & -2\,q_3 q_0 + 2\,q_2 q_1 & 2\,q_2 q_0 + 2\,q_3 q_1 \\ 2\,q_3 q_0 + 2\,q_2 q_1 & q_0^2 - q_1^2 + q_2^2 - q_3^2 & -2\,q_1 q_0 + 2\,q_3 q_2 \\ -2\,q_2 q_0 + 2\,q_3 q_1 & 2\,q_1 q_0 + 2\,q_3 q_2 & q_0^2 - q_1^2 - q_2^2 + q_3^2 \end{bmatrix}$$

$$aTrust := \sqrt{g^2 - (a_x - b_{ax})^2 - (a_y - b_{ay})^2} - a_z + b_{az}$$

$$aM := \begin{bmatrix} 0 \\ 0 \\ -\sqrt{g^2 - (a_x - b_{ax})^2 - (a_y - b_{ay})^2} + a_z - b_{az} \end{bmatrix}$$

$$aW := \begin{bmatrix} (2q_2q_0 + 2q_3q_1)\left(-\sqrt{g^2 - (a_x - b_{ax})^2 - (a_y - b_{ay})^2} + a_z - b_{az}\right) \\ (-2q_1q_0 + 2q_3q_2)\left(-\sqrt{g^2 - (a_x - b_{ax})^2 - (a_y - b_{ay})^2} + a_z - b_{az}\right) \\ (q_0^2 - q_1^2 - q_2^2 + q_3^2)\left(-\sqrt{g^2 - (a_x - b_{ax})^2 - (a_y - b_{ay})^2} + a_z - b_{az}\right) \end{bmatrix}$$

$$J\_aW\_acc := \Bigg[\Bigg[\frac{2(q_2q_0 + q_3q_1)(a_x - b_{ax})}{\sqrt{-a_x^2 + 2a_xb_{ax} - a_y^2 + 2a_yb_{ay} - b_{ax}^2 - b_{ay}^2 + g^2}},$$

$$\frac{2(q_2q_0 + q_3q_1)(a_y - b_{ay})}{\sqrt{-a_x^2 + 2a_xb_{ax} - a_y^2 + 2a_yb_{ay} - b_{ax}^2 - b_{ay}^2 + g^2}}, 2q_2q_0 + 2q_3q_1\Bigg],$$

$$\Bigg[-\frac{2(q_1q_0 - q_3q_2)(a_x - b_{ax})}{\sqrt{-a_x^2 + 2a_xb_{ax} - a_y^2 + 2a_yb_{ay} - b_{ax}^2 - b_{ay}^2 + g^2}},$$

$$-\frac{2(q_1q_0 - q_3q_2)(a_y - b_{ay})}{\sqrt{-a_x^2 + 2a_xb_{ax} - a_y^2 + 2a_yb_{ay} - b_{ax}^2 - b_{ay}^2 + g^2}}, -2q_1q_0 + 2q_3q_2\Bigg],$$

$$\Bigg[\frac{(q_0^2 - q_1^2 - q_2^2 + q_3^2)(a_x - b_{ax})}{\sqrt{-a_x^2 + 2a_xb_{ax} - a_y^2 + 2a_yb_{ay} - b_{ax}^2 - b_{ay}^2 + g^2}},$$

$$\frac{(q_0^2 - q_1^2 - q_2^2 + q_3^2)(a_y - b_{ay})}{\sqrt{-a_x^2 + 2a_xb_{ax} - a_y^2 + 2a_yb_{ay} - b_{ax}^2 - b_{ay}^2 + g^2}}, q_0^2 - q_1^2 - q_2^2 + q_3^2\Bigg]\Bigg]$$

$$J\_aW\_ba := \Bigg[\Bigg[-\frac{2(q_2q_0 + q_3q_1)(a_x - b_{ax})}{\sqrt{-a_x^2 + 2a_xb_{ax} - a_y^2 + 2a_yb_{ay} - b_{ax}^2 - b_{ay}^2 + g^2}},$$

$$-\frac{2(q_2q_0 + q_3q_1)(a_y - b_{ay})}{\sqrt{-a_x^2 + 2a_xb_{ax} - a_y^2 + 2a_yb_{ay} - b_{ax}^2 - b_{ay}^2 + g^2}}, -2q_2q_0 - 2q_3q_1\Bigg],$$

$$\Bigg[\frac{2(q_1q_0 - q_3q_2)(a_x - b_{ax})}{\sqrt{-a_x^2 + 2a_xb_{ax} - a_y^2 + 2a_yb_{ay} - b_{ax}^2 - b_{ay}^2 + g^2}},$$

$$\frac{2(q_1q_0 - q_3q_2)(a_y - b_{ay})}{\sqrt{-a_x^2 + 2a_xb_{ax} - a_y^2 + 2a_yb_{ay} - b_{ax}^2 - b_{ay}^2 + g^2}}, 2q_1q_0 - 2q_3q_2\Bigg],$$

$$\Bigg[-\frac{(q_0^2 - q_1^2 - q_2^2 + q_3^2)(a_x - b_{ax})}{\sqrt{-a_x^2 + 2a_xb_{ax} - a_y^2 + 2a_yb_{ay} - b_{ax}^2 - b_{ay}^2 + g^2}},$$

$$-\frac{\left(q_0^2-q_1^2-q_2^2+q_3^2\right)\left(a_y-b_{ay}\right)}{\sqrt{-a_x^2+2\,a_x\,b_{ax}-a_y^2+2\,a_y\,b_{ay}-b_{ax}^2-b_{ay}^2+g^2}},\ -q_0^2+q_1^2+q_2^2-q_3^2\Big]\Big]$$

$$J\_aW\_q := \Big[\Big[-2\,q_2\left(\sqrt{-a_x^2+2\,a_x\,b_{ax}-a_y^2+2\,a_y\,b_{ay}-b_{ax}^2-b_{ay}^2+g^2}-a_z+b_{az}\right),$$
$$-2\,q_3\left(\sqrt{-a_x^2+2\,a_x\,b_{ax}-a_y^2+2\,a_y\,b_{ay}-b_{ax}^2-b_{ay}^2+g^2}-a_z+b_{az}\right),$$
$$-2\,q_0\left(\sqrt{-a_x^2+2\,a_x\,b_{ax}-a_y^2+2\,a_y\,b_{ay}-b_{ax}^2-b_{ay}^2+g^2}-a_z+b_{az}\right),$$
$$-2\,q_1\left(\sqrt{-a_x^2+2\,a_x\,b_{ax}-a_y^2+2\,a_y\,b_{ay}-b_{ax}^2-b_{ay}^2+g^2}-a_z+b_{az}\right)\Big],$$
$$\Big[2\,q_1\left(\sqrt{-a_x^2+2\,a_x\,b_{ax}-a_y^2+2\,a_y\,b_{ay}-b_{ax}^2-b_{ay}^2+g^2}-a_z+b_{az}\right),$$
$$2\,q_0\left(\sqrt{-a_x^2+2\,a_x\,b_{ax}-a_y^2+2\,a_y\,b_{ay}-b_{ax}^2-b_{ay}^2+g^2}-a_z+b_{az}\right),$$
$$-2\,q_3\left(\sqrt{-a_x^2+2\,a_x\,b_{ax}-a_y^2+2\,a_y\,b_{ay}-b_{ax}^2-b_{ay}^2+g^2}-a_z+b_{az}\right),$$
$$-2\,q_2\left(\sqrt{-a_x^2+2\,a_x\,b_{ax}-a_y^2+2\,a_y\,b_{ay}-b_{ax}^2-b_{ay}^2+g^2}-a_z+b_{az}\right)\Big],$$
$$\Big[-2\,q_0\left(\sqrt{-a_x^2+2\,a_x\,b_{ax}-a_y^2+2\,a_y\,b_{ay}-b_{ax}^2-b_{ay}^2+g^2}-a_z+b_{az}\right),$$
$$2\,q_1\left(\sqrt{-a_x^2+2\,a_x\,b_{ax}-a_y^2+2\,a_y\,b_{ay}-b_{ax}^2-b_{ay}^2+g^2}-a_z+b_{az}\right),$$
$$2\,q_2\left(\sqrt{-a_x^2+2\,a_x\,b_{ax}-a_y^2+2\,a_y\,b_{ay}-b_{ax}^2-b_{ay}^2+g^2}-a_z+b_{az}\right),$$
$$-2\,q_3\left(\sqrt{-a_x^2+2\,a_x\,b_{ax}-a_y^2+2\,a_y\,b_{ay}-b_{ax}^2-b_{ay}^2+g^2}-a_z+b_{az}\right)\Big]\Big]$$

$$\begin{bmatrix} 2\,q_2 & 2\,q_3 & 2\,q_0 & 2\,q_1 \\ -2\,q_1 & -2\,q_0 & 2\,q_3 & 2\,q_2 \\ 2\,q_0 & -2\,q_1 & -2\,q_2 & 2\,q_3 \end{bmatrix} \tag{2.2.3}$$

Attitude

Prediction

```
> S := w -> <<0,w[1],w[2],w[3]>|<-w[1],0,-w[3],w[2]>|<-w[2],w[3],0,
  -w[1]>|<-w[3],-w[2],w[1],0>>;
  W := gyro-b__w+Epsilon__w-Epsilon__bw;
  Sw := S(W);
  Emean := {seq(Epsilon__w[i] = 0,i=1..3),seq(Epsilon__a[i] = 0,i=
  1..3),seq(Epsilon__bw[i] = 0,i=1..3),seq(Epsilon__ba[i] = 0,i=1.
  .3),seq(Epsilon__m[i] = 0,i=1..3),seq(Epsilon__uwb[i] = 0,i=1..3)
  };
  F := Vector[column](7,[(Ts/2*Sw+ IdentityMatrix(4)).q,b__w+
  Epsilon__bw]);
  subs(Emean_W,F);
```

$$S := w \mapsto \left\langle \left\langle 0, w_1, w_2, w_3\right\rangle \middle| \left\langle -w_1, 0, -w_3, w_2\right\rangle \middle| \left\langle -w_2, w_3, 0, -w_1\right\rangle \middle| \left\langle -w_3, -w_2, w_1, 0\right\rangle \right\rangle$$

$$W := \begin{bmatrix} w_x - b_{wx} + \epsilon_{wx} - \epsilon_{wbx} \\ w_y - b_{wy} + \epsilon_{wy} - \epsilon_{wby} \\ w_z - b_{wz} + \epsilon_{wz} - \epsilon_{wbz} \end{bmatrix}$$

$$Sw := [[0, -w_x + b_{wx} - \epsilon_{wx} + \epsilon_{wbx}, -w_y + b_{wy} - \epsilon_{wy} + \epsilon_{wby}, -w_z + b_{wz} - \epsilon_{wz} + \epsilon_{wbz}],$$
$$[w_x - b_{wx} + \epsilon_{wx} - \epsilon_{wbx}, 0, w_z - b_{wz} + \epsilon_{wz} - \epsilon_{wbz}, -w_y + b_{wy} - \epsilon_{wy} + \epsilon_{wby}],$$
$$[w_y - b_{wy} + \epsilon_{wy} - \epsilon_{wby}, -w_z + b_{wz} - \epsilon_{wz} + \epsilon_{wbz}, 0, w_x - b_{wx} + \epsilon_{wx} - \epsilon_{wbx}],$$
$$[w_z - b_{wz} + \epsilon_{wz} - \epsilon_{wbz}, w_y - b_{wy} + \epsilon_{wy} - \epsilon_{wby}, -w_x + b_{wx} - \epsilon_{wx} + \epsilon_{wbx}, 0]]$$

$$Emean := \{\epsilon_{abx} = 0, \epsilon_{aby} = 0, \epsilon_{abz} = 0, \epsilon_{ax} = 0, \epsilon_{ay} = 0, \epsilon_{az} = 0, \epsilon_{mx} = 0, \epsilon_{my} = 0, \epsilon_{mz} = 0, \epsilon_{uwbx}$$
$$= 0, \epsilon_{uwby} = 0, \epsilon_{uwbz} = 0, \epsilon_{wbx} = 0, \epsilon_{wby} = 0, \epsilon_{wbz} = 0, \epsilon_{wx} = 0, \epsilon_{wy} = 0, \epsilon_{wz} = 0\}$$

$$F := \left[\left[q_0 + \frac{Ts\,(-w_x + b_{wx} - \epsilon_{wx} + \epsilon_{wbx})\, q_1}{2} + \frac{Ts\,(-w_y + b_{wy} - \epsilon_{wy} + \epsilon_{wby})\, q_2}{2}\right.\right.$$
$$\left. + \frac{Ts\,(-w_z + b_{wz} - \epsilon_{wz} + \epsilon_{wbz})\, q_3}{2}\right],$$
$$\left[\frac{Ts\,(w_x - b_{wx} + \epsilon_{wx} - \epsilon_{wbx})\, q_0}{2} + q_1 + \frac{Ts\,(w_z - b_{wz} + \epsilon_{wz} - \epsilon_{wbz})\, q_2}{2}\right.$$
$$\left. + \frac{Ts\,(-w_y + b_{wy} - \epsilon_{wy} + \epsilon_{wby})\, q_3}{2}\right],$$
$$\left[\frac{Ts\,(w_y - b_{wy} + \epsilon_{wy} - \epsilon_{wby})\, q_0}{2} + \frac{Ts\,(-w_z + b_{wz} - \epsilon_{wz} + \epsilon_{wbz})\, q_1}{2} + q_2\right.$$
$$\left. + \frac{Ts\,(w_x - b_{wx} + \epsilon_{wx} - \epsilon_{wbx})\, q_3}{2}\right],$$
$$\left[\frac{Ts\,(w_z - b_{wz} + \epsilon_{wz} - \epsilon_{wbz})\, q_0}{2} + \frac{Ts\,(w_y - b_{wy} + \epsilon_{wy} - \epsilon_{wby})\, q_1}{2}\right.$$
$$\left. + \frac{Ts\,(-w_x + b_{wx} - \epsilon_{wx} + \epsilon_{wbx})\, q_2}{2} + q_3\right],$$
$$\left[b_{wx} + \epsilon_{wbx}\right],$$
$$\left[b_{wy} + \epsilon_{wby}\right],$$
$$\left.\left[b_{wz} + \epsilon_{wbz}\right]\right]$$

$$
\left[\begin{array}{c}
q_0 + \frac{Ts\left(-w_x + b_{wx}\right) q_1}{2} + \frac{Ts\left(-w_y + b_{wy}\right) q_2}{2} + \frac{Ts\left(-w_z + b_{wz}\right) q_3}{2} \\
\frac{Ts\left(w_x - b_{wx}\right) q_0}{2} + q_1 + \frac{Ts\left(w_z - b_{wz}\right) q_2}{2} + \frac{Ts\left(-w_y + b_{wy}\right) q_3}{2} \\
\frac{Ts\left(w_y - b_{wy}\right) q_0}{2} + \frac{Ts\left(-w_z + b_{wz}\right) q_1}{2} + q_2 + \frac{Ts\left(w_x - b_{wx}\right) q_3}{2} \\
\frac{Ts\left(w_z - b_{wz}\right) q_0}{2} + \frac{Ts\left(w_y - b_{wy}\right) q_1}{2} + \frac{Ts\left(-w_x + b_{wx}\right) q_2}{2} + q_3 \\
b_{wx} \\
b_{wy} \\
b_{wz}
\end{array}\right] \tag{2.3.1.1}
$$

```
> J_Xatt := subs(Emean,VectorCalculus:-Jacobian(convert(F,list),
  convert(ArrayTools:-Concatenate(1,Xatt,b__w),list)));
  #J__Epsilon__b := VectorCalculus:-Jacobian(convert(F,list),
  convert(Epsilon__bw,list) );
  J_Uatt := VectorCalculus:-Jacobian(convert(F,list),convert
  (Epsilon__w,list) );
```

$$
J\_Xatt := \left[\left[1, \frac{Ts\left(-w_x + b_{wx}\right)}{2}, \frac{Ts\left(-w_y + b_{wy}\right)}{2}, \frac{Ts\left(-w_z + b_{wz}\right)}{2}, \frac{Ts\, q_1}{2}, \frac{Ts\, q_2}{2}, \frac{Ts\, q_3}{2}\right],\right.
$$

$$
\left[\frac{Ts\left(w_x - b_{wx}\right)}{2}, 1, \frac{Ts\left(w_z - b_{wz}\right)}{2}, \frac{Ts\left(-w_y + b_{wy}\right)}{2}, -\frac{Ts\, q_0}{2}, \frac{Ts\, q_3}{2}, -\frac{Ts\, q_2}{2}\right],
$$

$$
\left[\frac{Ts\left(w_y - b_{wy}\right)}{2}, \frac{Ts\left(-w_z + b_{wz}\right)}{2}, 1, \frac{Ts\left(w_x - b_{wx}\right)}{2}, -\frac{Ts\, q_3}{2}, -\frac{Ts\, q_0}{2}, \frac{Ts\, q_1}{2}\right],
$$

$$
\left[\frac{Ts\left(w_z - b_{wz}\right)}{2}, \frac{Ts\left(w_y - b_{wy}\right)}{2}, \frac{Ts\left(-w_x + b_{wx}\right)}{2}, 1, \frac{Ts\, q_2}{2}, -\frac{Ts\, q_1}{2}, -\frac{Ts\, q_0}{2}\right],
$$

$$
\left[0, 0, 0, 0, 1, 0, 0\right],
$$

$$
\left[0, 0, 0, 0, 0, 1, 0\right],
$$

$$
\left.\left[0, 0, 0, 0, 0, 0, 1\right]\right]
$$

$$J\_Uatt := \begin{bmatrix} -\frac{Ts\,q_1}{2} & -\frac{Ts\,q_2}{2} & -\frac{Ts\,q_3}{2} \\ \frac{Ts\,q_0}{2} & -\frac{Ts\,q_3}{2} & \frac{Ts\,q_2}{2} \\ \frac{Ts\,q_3}{2} & \frac{Ts\,q_0}{2} & -\frac{Ts\,q_1}{2} \\ -\frac{Ts\,q_2}{2} & \frac{Ts\,q_1}{2} & \frac{Ts\,q_0}{2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{2.3.1.2}$$

Update

Accelerometer

```
> aG;
  R__a := DiagonalMatrix(Epsilon__a);
  R__ba := DiagonalMatrix(Epsilon__ba);
```

$$\begin{bmatrix} \frac{a_x - b_{ax}}{g} \\ \frac{a_y - b_{ay}}{g} \\ \frac{\sqrt{g^2 - (a_x - b_{ax})^2 - (a_y - b_{ay})^2}}{g} \end{bmatrix}$$

$$R_a := \begin{bmatrix} \epsilon_{ax} & 0 & 0 \\ 0 & \epsilon_{ay} & 0 \\ 0 & 0 & \epsilon_{az} \end{bmatrix}$$

$$R_{ba} := \begin{bmatrix} \epsilon_{abx} & 0 & 0 \\ 0 & \epsilon_{aby} & 0 \\ 0 & 0 & \epsilon_{abz} \end{bmatrix} \tag{2.3.2.1}$$

Magnetometer

```
> M := mag;
  R__m := DiagonalMatrix(Epsilon__m);
```

$$M := \begin{bmatrix} m_x \\ m_y \\ m_z \end{bmatrix}$$

$$R_m := \begin{bmatrix} \epsilon_{mx} & 0 & 0 \\ 0 & \epsilon_{my} & 0 \\ 0 & 0 & \epsilon_{mz} \end{bmatrix} \tag{2.3.2.2}$$

Normalization

```
> modM := sqrt(M[1]^2+M[2]^2+M[3]^2);
  m__N := map(x->x/modM,M);
  J__N := simplify(subs(Emean,VectorCalculus:-Jacobian( convert
  (m__N,list),convert(M,list) )));

  M__N := Vector[column](3,[m__xN,m__yN,m__zN]);
  Epsilon__MN := Vector[column](3,[epsilon__mxN,epsilon__myN,
  epsilon__mzN]);
  Emean_MN := {seq(Epsilon__MN[i] = 0,i=1..3)};
  R__MN := DiagonalMatrix(Epsilon__MN);
```

$$modM := \sqrt{m_x^2 + m_y^2 + m_z^2}$$

$$m_N := \begin{bmatrix} \dfrac{m_x}{\sqrt{m_x^2 + m_y^2 + m_z^2}} \\ \dfrac{m_y}{\sqrt{m_x^2 + m_y^2 + m_z^2}} \\ \dfrac{m_z}{\sqrt{m_x^2 + m_y^2 + m_z^2}} \end{bmatrix}$$

$$J_N := \begin{bmatrix} \dfrac{m_y^2 + m_z^2}{\left(m_x^2 + m_y^2 + m_z^2\right)^{3/2}} & -\dfrac{m_x m_y}{\left(m_x^2 + m_y^2 + m_z^2\right)^{3/2}} & -\dfrac{m_x m_z}{\left(m_x^2 + m_y^2 + m_z^2\right)^{3/2}} \\ -\dfrac{m_x m_y}{\left(m_x^2 + m_y^2 + m_z^2\right)^{3/2}} & \dfrac{m_x^2 + m_z^2}{\left(m_x^2 + m_y^2 + m_z^2\right)^{3/2}} & -\dfrac{m_y m_z}{\left(m_x^2 + m_y^2 + m_z^2\right)^{3/2}} \\ -\dfrac{m_x m_z}{\left(m_x^2 + m_y^2 + m_z^2\right)^{3/2}} & -\dfrac{m_y m_z}{\left(m_x^2 + m_y^2 + m_z^2\right)^{3/2}} & \dfrac{m_x^2 + m_y^2}{\left(m_x^2 + m_y^2 + m_z^2\right)^{3/2}} \end{bmatrix}$$

$$M_N := \begin{bmatrix} m_{xN} \\ m_{yN} \\ m_{zN} \end{bmatrix}$$

$$\mathrm{E}_{MN} := \begin{bmatrix} \epsilon_{mxN} \\ \epsilon_{myN} \\ \epsilon_{mzN} \end{bmatrix}$$

$$Emean\_MN := \{\epsilon_{mxN} = 0, \epsilon_{myN} = 0, \epsilon_{mzN} = 0\}$$

$$R_{MN} := \begin{bmatrix} \epsilon_{mxN} & 0 & 0 \\ 0 & \epsilon_{myN} & 0 \\ 0 & 0 & \epsilon_{mzN} \end{bmatrix} \tag{2.3.2.3}$$

TRIAD algorithm

```
> a__G := [aG__x, aG__y, aG__z];
  m__D := VdotV(a__G,M__N);
  m__N := sqrt(1-m__D^2);
  N__ZM := a__G[2]*M__N[3]-a__G[3]*M__N[2];
  Z__M := (N__ZM/m__N);
  Z := Vector[column](4,[a__G,Z__M]);
```

$$a_G := [aG_x, aG_y, aG_z]$$

$$m_{\mathrm{D}} := aG_x m_{xN} + aG_y m_{yN} + aG_z m_{zN}$$

$$m_N := \sqrt{-\left(aG_x m_{xN} + aG_y m_{yN} + aG_z m_{zN}\right)^2 + 1}$$

$$N_{ZM} := aG_y m_{zN} - aG_z m_{yN}$$

$$Z_M := \frac{aG_y m_{zN} - aG_z m_{yN}}{\sqrt{-\left(aG_x m_{xN} + aG_y m_{yN} + aG_z m_{zN}\right)^2 + 1}}$$

$$Z := \begin{bmatrix} aG_x \\ aG_y \\ aG_z \\ \dfrac{aG_y m_{zN} - aG_z m_{yN}}{\sqrt{-\left(aG_x m_{xN} + aG_y m_{yN} + aG_z m_{zN}\right)^2 + 1}} \end{bmatrix} \tag{2.3.2.4}$$

```
> J__ZA := VectorCalculus:-Jacobian( convert(Z,list),convert(a__G,
  list) );
  J__ZM := VectorCalculus:-Jacobian( convert(Z,list),convert(M__N,
  list) );
  J__Zsimp := <<1,0,0,nZM*mD*m__xN/mN^3>|<0,1,0,m__zN/mN+nZM*mD*
  m__yN/mN^3>|<0,0,1,-m__yN/mN+nZM*mD*m__zN/mN^3>|<0,0,0,nZM*mD*
```

```
a__xN/mN^3>|<0,0,0,-a__zN/mN+nZM*mD*a__yN/mN^3>|<0,0,0,a__yN/mN+
nZM*mD*a__zN/mN^3>>;
```

$$J_{ZA} := \left[\left[\begin{bmatrix}1, 0, 0\end{bmatrix}\right.,\right.$$

$$\begin{bmatrix}0, 1, 0\end{bmatrix},$$

$$\begin{bmatrix}0, 0, 1\end{bmatrix},$$

$$\left[\frac{\left(aG_y m_{zN} - aG_z m_{yN}\right)\left(aG_x m_{xN} + aG_y m_{yN} + aG_z m_{zN}\right) m_{xN}}{\left(-\left(aG_x m_{xN} + aG_y m_{yN} + aG_z m_{zN}\right)^2 + 1\right)^{3/2}},\right.$$

$$\frac{m_{zN}}{\sqrt{-\left(aG_x m_{xN} + aG_y m_{yN} + aG_z m_{zN}\right)^2 + 1}}$$

$$+ \frac{\left(aG_y m_{zN} - aG_z m_{yN}\right)\left(aG_x m_{xN} + aG_y m_{yN} + aG_z m_{zN}\right) m_{yN}}{\left(-\left(aG_x m_{xN} + aG_y m_{yN} + aG_z m_{zN}\right)^2 + 1\right)^{3/2}},$$

$$-\frac{m_{yN}}{\sqrt{-\left(aG_x m_{xN} + aG_y m_{yN} + aG_z m_{zN}\right)^2 + 1}}$$

$$\left.\left. + \frac{\left(aG_y m_{zN} - aG_z m_{yN}\right)\left(aG_x m_{xN} + aG_y m_{yN} + aG_z m_{zN}\right) m_{zN}}{\left(-\left(aG_x m_{xN} + aG_y m_{yN} + aG_z m_{zN}\right)^2 + 1\right)^{3/2}}\right]\right]$$

$$J_{ZM} := \left[\left[\begin{bmatrix}0, 0, 0\end{bmatrix}\right.,\right.$$

$$\begin{bmatrix}0, 0, 0\end{bmatrix},$$

$$\begin{bmatrix}0, 0, 0\end{bmatrix},$$

$$\left[\frac{\left(aG_y m_{zN} - aG_z m_{yN}\right)\left(aG_x m_{xN} + aG_y m_{yN} + aG_z m_{zN}\right) aG_x}{\left(-\left(aG_x m_{xN} + aG_y m_{yN} + aG_z m_{zN}\right)^2 + 1\right)^{3/2}},\right.$$

$$-\frac{aG_z}{\sqrt{-\left(aG_x m_{xN} + aG_y m_{yN} + aG_z m_{zN}\right)^2 + 1}}$$

$$+\frac{\left(aG_y m_{zN}-aG_z m_{yN}\right)\left(aG_x m_{xN}+aG_y m_{yN}+aG_z m_{zN}\right) aG_y}{\left(-\left(aG_x m_{xN}+aG_y m_{yN}+aG_z m_{zN}\right)^2+1\right)^{3/2}},$$

$$\frac{aG_y}{\sqrt{-\left(aG_x m_{xN}+aG_y m_{yN}+aG_z m_{zN}\right)^2+1}} + \frac{\left(aG_y m_{zN}-aG_z m_{yN}\right)\left(aG_x m_{xN}+aG_y m_{yN}+aG_z m_{zN}\right) aG_z}{\left(-\left(aG_x m_{xN}+aG_y m_{yN}+aG_z m_{zN}\right)^2+1\right)^{3/2}}\Bigg]\Bigg]$$

$$J_{Zsimp} := \Bigg[\Big[1, 0, 0, 0, 0, 0\Big], \Big[0, 1, 0, 0, 0, 0\Big], \Big[0, 0, 1, 0, 0, 0\Big], \Big[\frac{nZM\, mD\, m_{xN}}{mN^3}, \frac{m_{zN}}{mN}+\frac{nZM\, mD\, m_{yN}}{mN^3}, -\frac{m_{yN}}{mN}+\frac{nZM\, mD\, m_{zN}}{mN^3}, \frac{nZM\, mD\, a_{xN}}{mN^3}, -\frac{a_{zN}}{mN}+\frac{nZM\, mD\, a_{yN}}{mN^3}, \frac{a_{yN}}{mN}+\frac{nZM\, mD\, a_{zN}}{mN^3}\Big]\Bigg] \quad \textbf{(2.3.2.5)}$$

md=... , mn=... , Zm = ...
Z = [An,Zm]
Rz = Jza.Ran.Jza^T + Jzm.Rmn.Jzm^T
NB: Z_ = H(X) + Rz

Position

Prediction

```
> Aposition := <<1,0,0,0>|<0,1,0,0>|<dt,0,1,0>|<0,dt,0,1>>:
  Bposition := <<dt^2/2,0,dt,0>|<0,dt^2/2,0,dt>>:
  F_pos := [op(convert( Aposition.Xpos + Bposition.aW[1..2] ,list)
  ), op(convert(Xbias[4..6],list))]:<%>;
```

$$\Bigg[\Bigg[dt\, v_x + x + \frac{dt^2\left(2\, q_2 q_0 + 2\, q_3 q_1\right)\left(-\sqrt{g^2-\left(a_x-b_{ax}\right)^2-\left(a_y-b_{ay}\right)^2}+a_z-b_{az}\right)}{2}\Bigg], \quad \textbf{(2.4.1.1)}$$

$$\Bigg[dt\, v_y + y$$

$$+\frac{dt^2\left(-2q_1q_0+2q_3q_2\right)\left(-\sqrt{g^2-\left(a_x-b_{ax}\right)^2-\left(a_y-b_{ay}\right)^2}+a_z-b_{az}\right)}{2}\Bigg],$$

$$\left[v_x+dt\left(2q_2q_0+2q_3q_1\right)\left(-\sqrt{g^2-\left(a_x-b_{ax}\right)^2-\left(a_y-b_{ay}\right)^2}+a_z-b_{az}\right)\right],$$

$$\left[v_y+dt\left(-2q_1q_0+2q_3q_2\right)\left(-\sqrt{g^2-\left(a_x-b_{ax}\right)^2-\left(a_y-b_{ay}\right)^2}+a_z-b_{az}\right)\right],$$

$$\left[b_{ax}\right],$$

$$\left[b_{ay}\right],$$

$$\left[b_{az}\right]\Big]$$

```
> wRb
```

$$\begin{bmatrix} q_0^2+q_1^2-q_2^2-q_3^2 & -2q_0q_3+2q_1q_2 & 2q_2q_0+2q_3q_1 \\ 2q_0q_3+2q_1q_2 & q_0^2-q_1^2+q_2^2-q_3^2 & -2q_1q_0+2q_3q_2 \\ -2q_2q_0+2q_3q_1 & 2q_1q_0+2q_3q_2 & q_0^2-q_1^2-q_2^2+q_3^2 \end{bmatrix} \quad \textbf{(2.4.1.2)}$$

```
> state := [op(convert(Xpos,list)),op(convert(Xbias[4..6],list))];
  input := [op(convert(acc,list)),op(convert(q,list))];
  J_Xpos := (subs(Emean,VectorCalculus:-Jacobian(F_pos,state)));
  J_Upos := subs(Emean,VectorCalculus:-Jacobian(F_pos,input));
  Matrix(3, 4, [[2*q__2, 2*q__3, 2*q__0, 2*q__1], [-2*q__1, -2*
  q__0, 2*q__3, 2*q__2], [2*q__0, -2*q__1, -2*q__2, 2*q__3]])*(-1);
```

$$state := \left[x, y, v_x, v_y, b_{ax}, b_{ay}, b_{az}\right]$$

$$input := \left[a_x, a_y, a_z, q_0, q_1, q_2, q_3\right]$$

$$J\_Xpos := \Bigg[\Bigg[1, 0, dt, 0, -\frac{dt^2\left(2q_2q_0+2q_3q_1\right)\left(2a_x-2b_{ax}\right)}{4\sqrt{g^2-\left(a_x-b_{ax}\right)^2-\left(a_y-b_{ay}\right)^2}},$$

$$-\frac{dt^2\left(2q_2q_0+2q_3q_1\right)\left(2a_y-2b_{ay}\right)}{4\sqrt{g^2-\left(a_x-b_{ax}\right)^2-\left(a_y-b_{ay}\right)^2}}, -\frac{dt^2\left(2q_2q_0+2q_3q_1\right)}{2}\Bigg],$$

$$\Bigg[0, 1, 0, dt, -\frac{dt^2\left(-2q_1q_0+2q_3q_2\right)\left(2a_x-2b_{ax}\right)}{4\sqrt{g^2-\left(a_x-b_{ax}\right)^2-\left(a_y-b_{ay}\right)^2}},$$

$$-\frac{dt^2\left(-2q_1q_0+2q_3q_2\right)\left(2a_y-2b_{ay}\right)}{4\sqrt{g^2-\left(a_x-b_{ax}\right)^2-\left(a_y-b_{ay}\right)^2}},\ -\frac{dt^2\left(-2q_1q_0+2q_3q_2\right)}{2}\Bigg],$$

$$\Bigg[0,0,1,0,-\frac{dt\left(2q_2q_0+2q_3q_1\right)\left(2a_x-2b_{ax}\right)}{2\sqrt{g^2-\left(a_x-b_{ax}\right)^2-\left(a_y-b_{ay}\right)^2}},$$

$$-\frac{dt\left(2q_2q_0+2q_3q_1\right)\left(2a_y-2b_{ay}\right)}{2\sqrt{g^2-\left(a_x-b_{ax}\right)^2-\left(a_y-b_{ay}\right)^2}},\ -dt\left(2q_2q_0+2q_3q_1\right)\Bigg],$$

$$\Bigg[0,0,0,1,-\frac{dt\left(-2q_1q_0+2q_3q_2\right)\left(2a_x-2b_{ax}\right)}{2\sqrt{g^2-\left(a_x-b_{ax}\right)^2-\left(a_y-b_{ay}\right)^2}},$$

$$-\frac{dt\left(-2q_1q_0+2q_3q_2\right)\left(2a_y-2b_{ay}\right)}{2\sqrt{g^2-\left(a_x-b_{ax}\right)^2-\left(a_y-b_{ay}\right)^2}},\ -dt\left(-2q_1q_0+2q_3q_2\right)\Bigg],$$

$$\Big[0,0,0,0,1,0,0\Big],$$

$$\Big[0,0,0,0,0,1,0\Big],$$

$$\Big[0,0,0,0,0,0,1\Big]\Big]$$

$$J\_Upos := \Bigg[\Bigg[-\frac{dt^2\left(2q_2q_0+2q_3q_1\right)\left(-2a_x+2b_{ax}\right)}{4\sqrt{g^2-\left(a_x-b_{ax}\right)^2-\left(a_y-b_{ay}\right)^2}},$$

$$-\frac{dt^2\left(2q_2q_0+2q_3q_1\right)\left(-2a_y+2b_{ay}\right)}{4\sqrt{g^2-\left(a_x-b_{ax}\right)^2-\left(a_y-b_{ay}\right)^2}},\ \frac{dt^2\left(2q_2q_0+2q_3q_1\right)}{2},\ dt^2q_2\Big($$

$$-\sqrt{g^2-\left(a_x-b_{ax}\right)^2-\left(a_y-b_{ay}\right)^2}+a_z-b_{az}\Big),\ dt^2q_3\Big($$

$$-\sqrt{g^2-\left(a_x-b_{ax}\right)^2-\left(a_y-b_{ay}\right)^2}+a_z-b_{az}\Big),\ dt^2q_0\Big($$

$$-\sqrt{g^2-\left(a_x-b_{ax}\right)^2-\left(a_y-b_{ay}\right)^2}+a_z-b_{az}\Big),\ dt^2q_1\Big($$

$$-\sqrt{g^2-\left(a_x-b_{ax}\right)^2-\left(a_y-b_{ay}\right)^2}+a_z-b_{az}\Big)\Bigg],$$

$$\Bigg[-\frac{dt^2\left(-2q_1q_0+2q_3q_2\right)\left(-2a_x+2b_{ax}\right)}{4\sqrt{g^2-\left(a_x-b_{ax}\right)^2-\left(a_y-b_{ay}\right)^2}},$$

$$-\frac{dt^2\left(-2 q_1 q_0+2 q_3 q_2\right)\left(-2 a_y+2 b_{ay}\right)}{4\sqrt{g^2-\left(a_x-b_{ax}\right)^2-\left(a_y-b_{ay}\right)^2}}, \frac{dt^2\left(-2 q_1 q_0+2 q_3 q_2\right)}{2}, -dt^2 q_1\left(\right.$$

$$\left.-\sqrt{g^2-\left(a_x-b_{ax}\right)^2-\left(a_y-b_{ay}\right)^2}+a_z-b_{az}\right), -dt^2 q_0\left(\right.$$

$$\left.-\sqrt{g^2-\left(a_x-b_{ax}\right)^2-\left(a_y-b_{ay}\right)^2}+a_z-b_{az}\right), dt^2 q_3\left(\right.$$

$$\left.-\sqrt{g^2-\left(a_x-b_{ax}\right)^2-\left(a_y-b_{ay}\right)^2}+a_z-b_{az}\right), dt^2 q_2\left(\right.$$

$$\left.\left.-\sqrt{g^2-\left(a_x-b_{ax}\right)^2-\left(a_y-b_{ay}\right)^2}+a_z-b_{az}\right)\right],$$

$$\left[-\frac{dt\left(2 q_2 q_0+2 q_3 q_1\right)\left(-2 a_x+2 b_{ax}\right)}{2\sqrt{g^2-\left(a_x-b_{ax}\right)^2-\left(a_y-b_{ay}\right)^2}}, -\frac{dt\left(2 q_2 q_0+2 q_3 q_1\right)\left(-2 a_y+2 b_{ay}\right)}{2\sqrt{g^2-\left(a_x-b_{ax}\right)^2-\left(a_y-b_{ay}\right)^2}},\right.$$

$$dt\left(2 q_2 q_0+2 q_3 q_1\right), 2\,dt\,q_2\left(-\sqrt{g^2-\left(a_x-b_{ax}\right)^2-\left(a_y-b_{ay}\right)^2}+a_z-b_{az}\right),$$

$$2\,dt\,q_3\left(-\sqrt{g^2-\left(a_x-b_{ax}\right)^2-\left(a_y-b_{ay}\right)^2}+a_z-b_{az}\right), 2\,dt\,q_0\left(\right.$$

$$\left.-\sqrt{g^2-\left(a_x-b_{ax}\right)^2-\left(a_y-b_{ay}\right)^2}+a_z-b_{az}\right), 2\,dt\,q_1\left(\right.$$

$$\left.\left.-\sqrt{g^2-\left(a_x-b_{ax}\right)^2-\left(a_y-b_{ay}\right)^2}+a_z-b_{az}\right)\right],$$

$$\left[-\frac{dt\left(-2 q_1 q_0+2 q_3 q_2\right)\left(-2 a_x+2 b_{ax}\right)}{2\sqrt{g^2-\left(a_x-b_{ax}\right)^2-\left(a_y-b_{ay}\right)^2}},\right.$$

$$-\frac{dt\left(-2 q_1 q_0+2 q_3 q_2\right)\left(-2 a_y+2 b_{ay}\right)}{2\sqrt{g^2-\left(a_x-b_{ax}\right)^2-\left(a_y-b_{ay}\right)^2}}, dt\left(-2 q_1 q_0+2 q_3 q_2\right), -2\,dt\,q_1\left(\right.$$

$$\left.-\sqrt{g^2-\left(a_x-b_{ax}\right)^2-\left(a_y-b_{ay}\right)^2}+a_z-b_{az}\right), -2\,dt\,q_0\left(\right.$$

$$\left.-\sqrt{g^2-\left(a_x-b_{ax}\right)^2-\left(a_y-b_{ay}\right)^2}+a_z-b_{az}\right), 2\,dt\,q_3\left(\right.$$

$$\left.-\sqrt{g^2-\left(a_x-b_{ax}\right)^2-\left(a_y-b_{ay}\right)^2}+a_z-b_{az}\right), 2\,dt\,q_2\left(\right.$$

$$\left.\left.-\sqrt{g^2-\left(a_x-b_{ax}\right)^2-\left(a_y-b_{ay}\right)^2}+a_z-b_{az}\right)\right],$$

$$\left[0, 0, 0, 0, 0, 0, 0\right],$$

$$\left[0, 0, 0, 0, 0, 0, 0\right],$$

$$\left[\left[0, 0, 0, 0, 0, 0, 0\right]\right]$$

$$\begin{bmatrix} -2\,q_2 & -2\,q_3 & -2\,q_0 & -2\,q_1 \\ 2\,q_1 & 2\,q_0 & -2\,q_3 & -2\,q_2 \\ -2\,q_0 & 2\,q_1 & 2\,q_2 & -2\,q_3 \end{bmatrix} \tag{2.4.1.3}$$

>

Update

```
> uwb;
  H_v := state[1..4];
  J_H := VectorCalculus:-Jacobian(H_v,state);
  H_p := state[1..2];
  J_H := VectorCalculus:-Jacobian(H_p,state);
```

$$\begin{bmatrix} uwb_x \\ uwb_y \\ uwb_{vx} \\ uwb_{vy} \end{bmatrix}$$

$$H\_v := \left[x, y, v_x, v_y\right]$$

$$J\_H := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$H\_p := [x, y]$$

$$J\_H := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{2.4.2.1}$$

Update

Measurement model for update

Accelerometer

```
> A := Vector[column](3,[a__x+epsilon__ax,a__y+epsilon__ay,a__z+
  epsilon__az]);
  Epsilon__A := Vector[column](3,[epsilon__ax,epsilon__ay,
  epsilon__az]);
  R__A := DiagonalMatrix(Epsilon__A);
```

$$A := \begin{bmatrix} a_x + \epsilon_{ax} \\ a_y + \epsilon_{ay} \\ a_z + \epsilon_{az} \end{bmatrix}$$

$$\mathrm{E}_A := \begin{bmatrix} \epsilon_{ax} \\ \epsilon_{ay} \\ \epsilon_{az} \end{bmatrix}$$

$$R_A := \begin{bmatrix} \epsilon_{ax} & 0 & 0 \\ 0 & \epsilon_{ay} & 0 \\ 0 & 0 & \epsilon_{az} \end{bmatrix} \tag{2.5.1.1.1}$$

Normalization

```
> Z__A := map(x->x/g,A);
  J__A := VectorCalculus:-Jacobian( convert(Z__A,list),convert
  (Epsilon__A,list) );
  R__ZA := simplify(J__A.R__A.Transpose(J__A));

  A__N := Vector[column](3,[a__xN+epsilon__axN,a__yN+epsilon__ayN,
  a__zN+epsilon__azN]);
  Epsilon__AN := Vector[column](3,[epsilon__axN,epsilon__ayN,
  epsilon__azN]);
  Emean_AN := {seq(Epsilon__AN[i] = 0,i=1..3)};
  R__AN := DiagonalMatrix(Epsilon__AN);
```

$$Z_A := \begin{bmatrix} \frac{a_x + \epsilon_{ax}}{g} \\ \frac{a_y + \epsilon_{ay}}{g} \\ \frac{a_z + \epsilon_{az}}{g} \end{bmatrix}$$

$$J_A := \begin{bmatrix} \frac{1}{g} & 0 & 0 \\ 0 & \frac{1}{g} & 0 \\ 0 & 0 & \frac{1}{g} \end{bmatrix}$$

$$R_{ZA} := \begin{bmatrix} \frac{\epsilon_{ax}}{g^2} & 0 & 0 \\ 0 & \frac{\epsilon_{ay}}{g^2} & 0 \\ 0 & 0 & \frac{\epsilon_{az}}{g^2} \end{bmatrix}$$

$$A_N := \begin{bmatrix} a_{xN} + \epsilon_{axN} \\ a_{yN} + \epsilon_{ayN} \\ a_{zN} + \epsilon_{azN} \end{bmatrix}$$

$$\mathrm{E}_{AN} := \begin{bmatrix} \epsilon_{axN} \\ \epsilon_{ayN} \\ \epsilon_{azN} \end{bmatrix}$$

$$Emean\_AN := \{\epsilon_{axN} = 0, \epsilon_{ayN} = 0, \epsilon_{azN} = 0\}$$

$$R_{AN} := \begin{bmatrix} \epsilon_{axN} & 0 & 0 \\ 0 & \epsilon_{ayN} & 0 \\ 0 & 0 & \epsilon_{azN} \end{bmatrix} \qquad \textbf{(2.5.1.1.2)}$$

an := a/g
Ran := Ra/g^2

Magnetometer

```
> M := Vector[column](3,[m__x+epsilon__mx,m__y+epsilon__my,m__z+
  epsilon__mz]);
  Epsilon__M := Vector[column](3,[epsilon__mx,epsilon__my,
  epsilon__mz]);
  R__M := DiagonalMatrix(Epsilon__M);
```

$$M := \begin{bmatrix} m_x + \epsilon_{mx} \\ m_y + \epsilon_{my} \\ m_z + \epsilon_{mz} \end{bmatrix}$$

$$\mathrm{E}_M := \begin{bmatrix} \epsilon_{mx} \\ \epsilon_{my} \\ \epsilon_{mz} \end{bmatrix}$$

$$R_M := \begin{bmatrix} \epsilon_{mx} & 0 & 0 \\ 0 & \epsilon_{my} & 0 \\ 0 & 0 & \epsilon_{mz} \end{bmatrix} \quad \textbf{(2.5.1.2.1)}$$

Normalization

```
> modM := sqrt(M[1]^2+M[2]^2+M[3]^2);
  m__N := map(x->x/modM,M);
  Emean_M := {seq(Epsilon__M[i] = 0,i=1..3)};
  J__N := simplify(subs(Emean_M,VectorCalculus:-Jacobian( convert
  (m__N,list),convert(Epsilon__M,list) )));
  R__mN := simplify(J__N.R__M.Transpose(J__N));

  M__N := Vector[column](3,[m__xN+epsilon__mxN,m__yN+epsilon__myN,
  m__zN+epsilon__mzN]);
  Epsilon__MN := Vector[column](3,[epsilon__mxN,epsilon__myN,
  epsilon__mzN]);
  Emean_MN := {seq(Epsilon__MN[i] = 0,i=1..3)};
  R__MN := DiagonalMatrix(Epsilon__MN);
```

$$modM := \sqrt{\left(m_x+\epsilon_{mx}\right)^2+\left(m_y+\epsilon_{my}\right)^2+\left(m_z+\epsilon_{mz}\right)^2}$$

$$m_N := \begin{bmatrix} \dfrac{m_x+\epsilon_{mx}}{\sqrt{\left(m_x+\epsilon_{mx}\right)^2+\left(m_y+\epsilon_{my}\right)^2+\left(m_z+\epsilon_{mz}\right)^2}} \\ \dfrac{m_y+\epsilon_{my}}{\sqrt{\left(m_x+\epsilon_{mx}\right)^2+\left(m_y+\epsilon_{my}\right)^2+\left(m_z+\epsilon_{mz}\right)^2}} \\ \dfrac{m_z+\epsilon_{mz}}{\sqrt{\left(m_x+\epsilon_{mx}\right)^2+\left(m_y+\epsilon_{my}\right)^2+\left(m_z+\epsilon_{mz}\right)^2}} \end{bmatrix}$$

$$Emean\_M := \left\{\epsilon_{mx}=0, \epsilon_{my}=0, \epsilon_{mz}=0\right\}$$

$$J_N := \begin{bmatrix} \dfrac{m_y^2+m_z^2}{\left(m_x^2+m_y^2+m_z^2\right)^{3/2}} & -\dfrac{m_x m_y}{\left(m_x^2+m_y^2+m_z^2\right)^{3/2}} & -\dfrac{m_x m_z}{\left(m_x^2+m_y^2+m_z^2\right)^{3/2}} \\ -\dfrac{m_x m_y}{\left(m_x^2+m_y^2+m_z^2\right)^{3/2}} & \dfrac{m_x^2+m_z^2}{\left(m_x^2+m_y^2+m_z^2\right)^{3/2}} & -\dfrac{m_y m_z}{\left(m_x^2+m_y^2+m_z^2\right)^{3/2}} \\ -\dfrac{m_x m_z}{\left(m_x^2+m_y^2+m_z^2\right)^{3/2}} & -\dfrac{m_y m_z}{\left(m_x^2+m_y^2+m_z^2\right)^{3/2}} & \dfrac{m_x^2+m_y^2}{\left(m_x^2+m_y^2+m_z^2\right)^{3/2}} \end{bmatrix}$$

$$R_{mN} := \left[\left[\frac{\left(m_y^2+m_z^2\right)^2 \epsilon_{mx}+m_x^2\left(m_y^2 \epsilon_{my}+m_z^2 \epsilon_{mz}\right)}{\left(m_x^2+m_y^2+m_z^2\right)^3},\right.\right.$$

$$-\frac{m_y m_x \left(\left(\epsilon_{mx}+\epsilon_{my}-\epsilon_{mz}\right) m_z^2+m_x^2 \epsilon_{my}+m_y^2 \epsilon_{mx}\right)}{\left(m_x^2+m_y^2+m_z^2\right)^3},$$

$$-\frac{m_x \left(\left(\epsilon_{mx}-\epsilon_{my}+\epsilon_{mz}\right) m_y^2+m_x^2 \epsilon_{mz}+m_z^2 \epsilon_{mx}\right) m_z}{\left(m_x^2+m_y^2+m_z^2\right)^3}\Bigg],$$

$$\Bigg[-\frac{m_y m_x \left(\left(\epsilon_{mx}+\epsilon_{my}-\epsilon_{mz}\right) m_z^2+m_x^2 \epsilon_{my}+m_y^2 \epsilon_{mx}\right)}{\left(m_x^2+m_y^2+m_z^2\right)^3},$$

$$\frac{\left(m_x^2+m_z^2\right)^2 \epsilon_{my}+m_y^2 \left(m_x^2 \epsilon_{mx}+m_z^2 \epsilon_{mz}\right)}{\left(m_x^2+m_y^2+m_z^2\right)^3},$$

$$\frac{m_y \left(\left(\epsilon_{mx}-\epsilon_{my}-\epsilon_{mz}\right) m_x^2-m_y^2 \epsilon_{mz}-m_z^2 \epsilon_{my}\right) m_z}{\left(m_x^2+m_y^2+m_z^2\right)^3}\Bigg],$$

$$\Bigg[-\frac{m_x \left(\left(\epsilon_{mx}-\epsilon_{my}+\epsilon_{mz}\right) m_y^2+m_x^2 \epsilon_{mz}+m_z^2 \epsilon_{mx}\right) m_z}{\left(m_x^2+m_y^2+m_z^2\right)^3},$$

$$\frac{m_y \left(\left(\epsilon_{mx}-\epsilon_{my}-\epsilon_{mz}\right) m_x^2-m_y^2 \epsilon_{mz}-m_z^2 \epsilon_{my}\right) m_z}{\left(m_x^2+m_y^2+m_z^2\right)^3},$$

$$\frac{\left(m_x^2+m_y^2\right)^2 \epsilon_{mz}+m_z^2 \left(m_x^2 \epsilon_{mx}+m_y^2 \epsilon_{my}\right)}{\left(m_x^2+m_y^2+m_z^2\right)^3}\Bigg]\Bigg]$$

$$M_N := \begin{bmatrix} m_{xN}+\epsilon_{mxN} \\ m_{yN}+\epsilon_{myN} \\ m_{zN}+\epsilon_{mzN} \end{bmatrix}$$

$$\mathrm{E}_{MN} := \begin{bmatrix} \epsilon_{mxN} \\ \epsilon_{myN} \\ \epsilon_{mzN} \end{bmatrix}$$

$$Emean\_MN := \left\{\epsilon_{mxN}=0, \epsilon_{myN}=0, \epsilon_{mzN}=0\right\}$$

$$R_{MN} := \begin{bmatrix} \epsilon_{mxN} & 0 & 0 \\ 0 & \epsilon_{myN} & 0 \\ 0 & 0 & \epsilon_{mzN} \end{bmatrix} \qquad \textbf{(2.5.1.2.2)}$$

mn = m/norm(m)
Rmn = Jn.Rm.Jn^T

TRIAD algorithm

```
> m__D := VdotV(A__N,M__N);
  m__N := sqrt(1-m__D^2);
  N__ZM := A__N[2]*M__N[3]-A__N[3]*M__N[2];
  Z__M := N__ZM/m__N;
  Z := Vector[column](4,[A__N,Z__M]);
  eqns := subs(Emean_AN,Emean_MN ,{m__D = mD,N__ZM = nZM});
  J__ZA := subs(Emean_AN,Emean_MN ,VectorCalculus:-Jacobian(
  convert(Z,list),convert(Epsilon__AN,list) ));
  J__ZM := subs(Emean_AN,Emean_MN ,VectorCalculus:-Jacobian(
  convert(Z,list),convert(Epsilon__MN,list) ));
  simplify(%, eqns):
  J__Zsimp := <<1,0,0,nZM*mD*m__xN/mN^3>|<0,1,0,m__zN/mN+nZM*mD*
  m__yN/mN^3>|<0,0,1,-m__yN/mN+nZM*mD*m__zN/mN^3>|<0,0,0,nZM*mD*
  a__xN/mN^3>|<0,0,0,-a__zN/mN+nZM*mD*a__yN/mN^3>|<0,0,0,a__yN/mN+
  nZM*mD*a__zN/mN^3>>;
```

$$m_D := (a_{xN}+\epsilon_{axN})(m_{xN}+\epsilon_{mxN})+(a_{yN}+\epsilon_{ayN})(m_{yN}+\epsilon_{myN})+(a_{zN}+\epsilon_{azN})(m_{zN}+\epsilon_{mzN})$$

$$m_N := \left(-\left((a_{xN}+\epsilon_{axN})(m_{xN}+\epsilon_{mxN})+(a_{yN}+\epsilon_{ayN})(m_{yN}+\epsilon_{myN})+(a_{zN}+\epsilon_{azN})(m_{zN}+\epsilon_{mzN})\right)^2+1\right)^{1/2}$$

$$N_{ZM} := (a_{yN}+\epsilon_{ayN})(m_{zN}+\epsilon_{mzN})-(a_{zN}+\epsilon_{azN})(m_{yN}+\epsilon_{myN})$$

$$Z_M := \left((a_{yN}+\epsilon_{ayN})(m_{zN}+\epsilon_{mzN})-(a_{zN}+\epsilon_{azN})(m_{yN}+\epsilon_{myN})\right) \Big/ \left(-\left((a_{xN}+\epsilon_{axN})(m_{xN}+\epsilon_{mxN})+(a_{yN}+\epsilon_{ayN})(m_{yN}+\epsilon_{myN})+(a_{zN}+\epsilon_{azN})(m_{zN}+\epsilon_{mzN})\right)^2+1\right)^{1/2}$$

$$Z := \Big[\left[a_{xN}+\epsilon_{axN}\right],$$

$$\left[a_{yN}+\epsilon_{ayN}\right],$$

$$\left[a_{zN}+\epsilon_{azN}\right],$$

$$\Big[\left((a_{yN}+\epsilon_{ayN})(m_{zN}+\epsilon_{mzN})-(a_{zN}+\epsilon_{azN})(m_{yN}+\epsilon_{myN})\right) \Big/$$

$$\left(-\left((a_{xN}+\epsilon_{axN})(m_{xN}+\epsilon_{mxN})+(a_{yN}+\epsilon_{ayN})(m_{yN}+\epsilon_{myN})+(a_{zN}\right.\right.$$

$$+ \epsilon_{azN}) \left(m_{zN} + \epsilon_{mzN})\right)^2 + 1\right)^{1/2} \Big]\Big]$$

$$eqns := \{a_{yN} m_{zN} - a_{zN} m_{yN} = nZM, a_{xN} m_{xN} + a_{yN} m_{yN} + a_{zN} m_{zN} = mD\}$$

$$J_{ZA} := \left[\left[1, 0, 0 \right],\right.$$

$$\left[0, 1, 0 \right],$$

$$\left[0, 0, 1 \right],$$

$$\left[\frac{(a_{yN} m_{zN} - a_{zN} m_{yN})(a_{xN} m_{xN} + a_{yN} m_{yN} + a_{zN} m_{zN}) m_{xN}}{(-(a_{xN} m_{xN} + a_{yN} m_{yN} + a_{zN} m_{zN})^2 + 1)^{3/2}}, \right.$$

$$\frac{m_{zN}}{\sqrt{-(a_{xN} m_{xN} + a_{yN} m_{yN} + a_{zN} m_{zN})^2 + 1}}$$

$$+ \frac{(a_{yN} m_{zN} - a_{zN} m_{yN})(a_{xN} m_{xN} + a_{yN} m_{yN} + a_{zN} m_{zN}) m_{yN}}{(-(a_{xN} m_{xN} + a_{yN} m_{yN} + a_{zN} m_{zN})^2 + 1)^{3/2}},$$

$$- \frac{m_{yN}}{\sqrt{-(a_{xN} m_{xN} + a_{yN} m_{yN} + a_{zN} m_{zN})^2 + 1}}$$

$$\left. + \frac{(a_{yN} m_{zN} - a_{zN} m_{yN})(a_{xN} m_{xN} + a_{yN} m_{yN} + a_{zN} m_{zN}) m_{zN}}{(-(a_{xN} m_{xN} + a_{yN} m_{yN} + a_{zN} m_{zN})^2 + 1)^{3/2}} \right]\Big]$$

$$J_{ZM} := \left[\left[0, 0, 0 \right],\right.$$

$$\left[0, 0, 0 \right],$$

$$\left[0, 0, 0 \right],$$

$$\left[\frac{(a_{yN} m_{zN} - a_{zN} m_{yN})(a_{xN} m_{xN} + a_{yN} m_{yN} + a_{zN} m_{zN}) a_{xN}}{(-(a_{xN} m_{xN} + a_{yN} m_{yN} + a_{zN} m_{zN})^2 + 1)^{3/2}}, \right.$$

$$- \frac{a_{zN}}{\sqrt{-\left(a_{xN} m_{xN} + a_{yN} m_{yN} + a_{zN} m_{zN}\right)^2 + 1}} + \frac{\left(a_{yN} m_{zN} - a_{zN} m_{yN}\right) \left(a_{xN} m_{xN} + a_{yN} m_{yN} + a_{zN} m_{zN}\right) a_{yN}}{\left(-\left(a_{xN} m_{xN} + a_{yN} m_{yN} + a_{zN} m_{zN}\right)^2 + 1\right)^{3/2}},$$

$$\frac{a_{yN}}{\sqrt{-\left(a_{xN} m_{xN} + a_{yN} m_{yN} + a_{zN} m_{zN}\right)^2 + 1}} + \frac{\left(a_{yN} m_{zN} - a_{zN} m_{yN}\right) \left(a_{xN} m_{xN} + a_{yN} m_{yN} + a_{zN} m_{zN}\right) a_{zN}}{\left(-\left(a_{xN} m_{xN} + a_{yN} m_{yN} + a_{zN} m_{zN}\right)^2 + 1\right)^{3/2}} \Bigg]\Bigg]$$

$$J_{Zsimp} := \Bigg[\Big[1, 0, 0, 0, 0, 0\Big], \Big[0, 1, 0, 0, 0, 0\Big], \Big[0, 0, 1, 0, 0, 0\Big], \Big[\frac{nZM\, mD\, m_{xN}}{mN^3}, \frac{m_{zN}}{mN} + \frac{nZM\, mD\, m_{yN}}{mN^3}, -\frac{m_{yN}}{mN} + \frac{nZM\, mD\, m_{zN}}{mN^3}, \frac{nZM\, mD\, a_{xN}}{mN^3}, -\frac{a_{zN}}{mN} + \frac{nZM\, mD\, a_{yN}}{mN^3}, \frac{a_{yN}}{mN} + \frac{nZM\, mD\, a_{zN}}{mN^3}\Big]\Bigg] \qquad \textbf{(2.5.1.2.3)}$$

md=... , mn=... , Zm = ...
Z = [An,Zm]
Rz = Jza.Ran.Jza^T + Jzm.Rmn.Jzm^T
NB: Z_ = H(X) + Rz

Mapping function H

```
> H := Vector[column](4,[ -2*q[3]*q[1]+2*q[4]*q[2],2*q[2]*q[1]+2*q[4]*q[3],q[1]^2-q[2]^2-q[3]^2+q[4]^2,2*q[4]*q[1]+2*q[3]*q[2] ]);
```

$$H := \begin{bmatrix} -2\, q_2 q_0 + 2\, q_3 q_1 \\ 2\, q_1 q_0 + 2\, q_3 q_2 \\ q_0^2 - q_1^2 - q_2^2 + q_3^2 \\ 2\, q_0 q_3 + 2\, q_1 q_2 \end{bmatrix} \qquad \textbf{(2.5.1.3.1)}$$

```
> J__H := VectorCalculus:-Jacobian( convert(H,list),convert(X,list) );
```

(2.5.1.3.2)

$$J_H := \begin{bmatrix} -2\,q_2 & 2\,q_3 & -2\,q_0 & 2\,q_1 & 0 & 0 & 0 \\ 2\,q_1 & 2\,q_0 & 2\,q_3 & 2\,q_2 & 0 & 0 & 0 \\ 2\,q_0 & -2\,q_1 & -2\,q_2 & 2\,q_3 & 0 & 0 & 0 \\ 2\,q_3 & 2\,q_2 & 2\,q_1 & 2\,q_0 & 0 & 0 & 0 \end{bmatrix} \tag{2.5.1.3.2}$$

S(k+1) = Jh.P^(k+1).Jh^T + Rz
W(k+1) = P^(k+1).Jh^t.S(k+1)^-1
X(k+1) = X^(k+1) + W(k+1).(Z(k)-H(X^(k+1)))
P(k+1) = (I-W(k+1).Jh)P^(k+1)

>

>

>