Attitude Estimation for UAV Using Extended Kalman Filter

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Abstract: A novel attitude estimation algorithm is proposed for unmanned aerial vehicles(UAV) in this paper. It uses attitude quaternion to represent the attitude of UAV, and uses extended Kalman filter(EKF) to fuse the merits of magnetic, angular rate, and gravity(MARG) sensors. First, attitude quaternion and drift bias of gyroscope are selected to construct the state vector, and the state equation is established based on the kinematics model of gyroscope. Then, an orthogonalization method is utilized to obtain the unit attitude quaternion from the outputs of accelerometer and magnetometer, it makes the magnetic field vector perpendicular to the measured gravity vector, which avoids the geomagnetic disturbance. And the unit attitude quaternion is used for the measurements for the EKF. Finally, the EKF update equation is used to determine the attitude of UAV. Experiments are provided on a real-world data set and the results show that the algorithm can precisely represents the orientation of UAV in both static and dynamic situation.

Key Words: Attitude Estimation, UAV, EKF, MARG, Attitude Quaternion

1 INTRODUCTION

In recent years, the researches of unmanned aerial vehicles(UAV) have received much attention all over the world [1]. UAV is widely used in many fields such as military affairs, agriculture, aerial photography, power transmission patrolling, and so on. Attitude estimation is essential for UAV, only if we know accurate orientation of UAV in real-time could we make a good feedback for the attitude controller [2].

Magnetic, angular rate, and gravity(MARG) sensors often consist of a tri-axis micro-electro-mechanical system(MEMS) gyroscope, accelerometer and magnetometer, are widely used in many applications [3,4] of attitude determination because of their low weight, low cost, low power-consumption property and relatively high performance in the real-time [5], and they're critical components of UAV [6].

The gyroscope can be used to measure the angular rate of the vehicle, and the time integration of angular rate derives the rotated angle, but the accumulation error would gradually increased as integral time increase. The accelerometer measures the gravity components along the body frame axes, but the working principle decides it is easily affected by motion acceleration. The magnetometer measures the component of the earth's magnetic field strength along the body frame axes, which can be used to determine the yaw angle of the vehicle, but the magnetometer suffers from heavy magnetic interference since the vehicle is always surrounded by a lot of electromagnetic noise signals [7]. By comparing the merits and drawbacks of MEMS sensors, obviously we can't precisely determine the orientation of an UAV using a single sensor, therefore, it is necessary to fuse the information of sensors, so that their respective advantages and disadvantages are complemented to solve the accurate attitude [8].

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Studying among the existing attitude estimate methods hitherto, we can find that there are mainly two sorts of them: complementary filters(CF) [9∼11] and the EKF [12∼15]. The other methods are usually the variants about these two [16, 17]. Madgwick's gradient descent algorithm(GNA) [18] shows good performance on attitude estimation, and it belongs to CF in a certain degree. EKF needs only the prior and current data from sensors and the procedure is very explicit. CF is quite simple but might not get the desired results if we do not set the parameters properly. Hence in this paper we focus on EKF for attitude estimation.

Among so many nonlinear filters, EKF is the most widely used, the simplest filter, it uses Taylor series expansion method to linearize the nonlinear system, and then uses the basic Kalman filter steps for state estimation update [19]. The selection of system model is vital for EKF. A good model could greatly reduce the computation complexity and speed up the convergence rate, some integrated system algorithms are highly nonlinear so that the linearization process either very complicate or result in divergence [20]. Wu [13] describes a Quaternion EKF method which uses strapdown attitude as state update, and treats the attitude estimated by the bi-vector method as measurement update, Sabatini [14] uses the same state vector, and the outputs of MARG sensors are chosen as observations in his case. Yun [21] gave some good models in his works for posture tracking.

Based on the ideas from above references, we studied a novel quaternion-based extended Kalman filter algorithm for attitude estimation. The quaternion is used in the algorithm since it has relatively less variable and could avoid singularity problem [22]. The paper has the following arrangement of contents: Section II gives the attitude representation method that we use. Section III describes the principle of EKF and its implementation. Section IV presents the experiments and results. Concluding remarks are given in Section V.

2 Attitude Representation Methods

In the field of aviation, the attitude is defined as the orientation of an UAV in the body frame(B) with respect to the navigation frame(N), usually represented with eular angles $roll(\phi$, rotate about X-axis), $pitch(\theta$, rotate about Y-axis) and $yaw(\psi$, rotate about Z-axis).

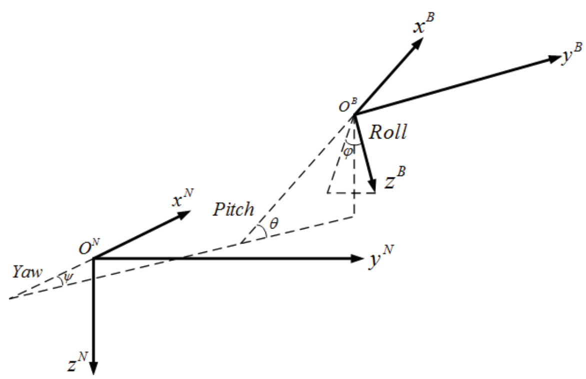

Figure 1: Eular Angles

We define the body frame with X-axis forward, Y points along the right direction, and Z is down to complete the right-hand frame. The navigation frame is defined to be a NED(North East Down) frame. Quaternion can be used to describe the motion of a rigid body [23]. A quaternion itself contains all the information about rotation and can be defined with the following form

$$q = \begin{bmatrix} \cos(\frac{\alpha}{2}) \\ u_x^N \sin(\frac{\alpha}{2}) \\ u_y^N \sin(\frac{\alpha}{2}) \\ u_z^N \sin(\frac{\alpha}{2}) \end{bmatrix}, \tag{1}$$

where u^N stands for the instantaneous axis with direction of rotation, α is the angle of rotation.

To simplify the expression, a quaternion can be rewritten as

$$q = \begin{bmatrix} q_0 \\ q_v \end{bmatrix} = \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix}, \tag{2}$$

where q_0 is a scalar and q_v indicates an 3D vector. The orientation of UAV in the navigation frame after a sequence of rotations can be easily found by rotation matrix multiplication

$$v^N = R(q_N^B)v^B, \tag{3}$$

The superscript and subscript indicates the coordinate frame, v^B denotes that the vector is in the body frame, q_N^B stands for the the quaternion from navigation frame to the body frame, which could be used to compose the rotation matrix from body frame to the navigation frame

$$R = R_B^N = R(q_N^B) = \begin{bmatrix} q_0^2+q_1^2-q_2^2-q_3^2 & 2(q_1q_2-q_0q_3) & 2(q_1q_3+q_0q_2) \\ 2(q_1q_2+q_0q_3) & q_0^2-q_1^2+q_2^2-q_3^2 & 2(q_2q_3-q_0q_1) \\ 2(q_1q_3-q_0q_2) & 2(q_2q_3+q_0q_1) & q_0^2-q_1^2-q_2^2+q_3^2 \end{bmatrix}. \tag{4}$$

To make the expression simpler,the matrix is rewritten as

$$R = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix}. \tag{5}$$

At the same time, attitude quaternion can also be converted from the rotation matrix

$$\begin{cases} q_1 = \dfrac{1}{2}\left(1+R_{11}+R_{22}+R_{33}\right) \\ q_2 = \dfrac{1}{4q_1}\left(R_{32}-R_{23}\right) \\ q_3 = \dfrac{1}{4q_1}\left(R_{13}-R_{31}\right) \\ q_4 = \dfrac{1}{4q_1}\left(R_{21}-R_{12}\right) \end{cases}. \tag{6}$$

It is well known that the angular rate w and the attitude quaternion derivative are related by the following identity

$$\dot{q} = \frac{1}{2}\Omega(\omega)q = \frac{1}{2}\Xi(q)\begin{bmatrix} 0 \\ \omega \end{bmatrix}, \tag{7}$$

where ω can be measured with a tri-aixs exteroceptive sensor gyroscope in the B frame

$$\omega = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}. \tag{8}$$

$\Omega(\omega)$ and $\Xi(q)$ is composed of angular rates and attitude quaternion respectively.

$$\Omega(w) = \begin{bmatrix} 0 & -\omega_x & -\omega_y & -\omega_z \\ \omega_x & 0 & \omega_y & \omega_z \\ \omega_y & -\omega_z & 0 & \omega_x \\ \omega_z & \omega_y & -\omega_x & 0 \end{bmatrix} = \begin{bmatrix} 0 & -\omega^T \\ \omega & -\lfloor\omega\times\rfloor \end{bmatrix}, \tag{9}$$

$$\Xi(q) = \begin{bmatrix} -q_1 & -q_2 & -q_3 \\ q_0 & -q_3 & q_2 \\ q_3 & q_0 & -q_1 \\ -q_2 & q_1 & q_0 \end{bmatrix} = \begin{bmatrix} -q_v \\ q_0 I_{3\times3} + \lfloor q_v\times\rfloor \end{bmatrix}. \tag{10}$$

where the skew-symmetric matrix operator $\lfloor q_v\times\rfloor$ is defined as

$$\lfloor q_v\times\rfloor = \begin{bmatrix} 0 & -q_3 & q_2 \\ q_3 & 0 & -q_1 \\ -q_2 & q_1 & 0 \end{bmatrix}. \tag{11}$$

The normalized form of a vector is defined as

$$\bar{v} = \frac{v}{|v|}. \tag{12}$$

3 Extended Kalman Filter

Kalman filter is a form of predictor-corrector algorithm used extensively in control systems engineering for estimating unmeasured states of a process. The estimated states may then be used as part of a strategy for control law design [20]. The EKF was developed for non-linear discrete-time processes. Just like the original Kalman filter, the EKF uses a two step predictor-corrector algorithm. The first step involves projecting both the most recent state estimate and an estimate of the error covariance(from the previous time period) forwards in time to compute a predicted(or $apriori$) estimate of the states at the current time. The second step involves correcting the predicted state estimate calculated in the first step by incorporating the most recent process measurement to generate an updated(or $aposteriori$) state estimate.

3.1 Discretization

Like any other typical nonlinear continuous system, the system state equation of an UAV could be described with the following form

$$\begin{cases} \dot{X}(t) = f[X(t), t] + W(t) \\ Z(t) = h[X(t), t] + V(t) \end{cases}, \tag{13}$$

where $f[\cdot]$ is generic multiple-dimensional vector function, it's nonlinear in terms of its independent variables; $h[\cdot]$ is also a nonlinear vector function too. $W(t)$ and $V(t)$ represent the process and observation noise respectively.

The key part of the EKF algorithm is to linearize the state equation and observation equation, and then discretize the system. To linear the state variant, $X(t+\Delta t)$ can be expended about $X(t)$ using its Taylor series expansion. Considering the actual situation, here we express the $X(t+\Delta t)$ with its second-order approximation about $X(t)$

$$\begin{aligned} X(t+\Delta t) &= X(t) + \dot{X}(t) + \frac{1}{2!}\ddot{X}(t)(\Delta t)^2 + \dots \\ &= X(t) + f(X)\Delta t + \frac{\delta f}{\delta X} f(X) \frac{(\Delta t)^2}{2!} + \dots \end{aligned} \tag{14}$$

where

$$\begin{aligned} \ddot{X}(t) &= \begin{bmatrix} \ddot{X}_1(t) \\ \ddot{X}_2(t) \\ \vdots \\ \ddot{X}_n(t) \end{bmatrix} = \begin{bmatrix} \frac{\delta f_1}{\delta X}\frac{\delta X}{dt} \\ \frac{\delta f_2}{\delta X}\frac{\delta X}{dt} \\ \vdots \\ \frac{\delta f_n}{\delta X}\frac{\delta X}{dt} \end{bmatrix} \\ &= \begin{bmatrix} \frac{\delta f_1}{\delta X_1} & \frac{\delta f_1}{\delta X_2} & \cdots & \frac{\delta f_1}{\delta X_n} \\ \frac{\delta f_2}{\delta X_1} & \frac{\delta f_2}{\delta X_2} & \cdots & \frac{\delta f_2}{\delta X_n} \\ & \vdots & & \\ \frac{\delta f_n}{\delta X_1} & \frac{\delta f_n}{\delta X_2} & \cdots & \frac{\delta f_n}{\delta X_n} \end{bmatrix} \begin{bmatrix} \dot{X}_1(t) \\ \dot{X}_2(t) \\ \vdots \\ \dot{X}_n(t) \end{bmatrix} = \frac{\delta f}{\delta X} f(X). \end{aligned} \tag{15}$$

Neglecting the high order terms, the discrete forms of the system state equation could be put as follows with the standard state-space form

$$\begin{cases} X(k+1) = & X(k) + f[X(k)]\Delta t \\ & +A[X(k)]f[X(k)]\frac{(\Delta t)^2}{2!} + W(k) \\ Z(k) = h[X(k)] + V(k) \end{cases}, \tag{16}$$

where $W(k)$ and $V(k)$ are assumed to be zero-mean white Gaussian noise with covariance Q_k and R_k respectively. And $A[X(k)]$ can be described as follows according to the derivation in Equation (15)

$$A[X(k)] = \left.\frac{\delta f}{\delta X}\right|_{X=X(k)}. \tag{17}$$

3.2 Establish Descrete System Process Equation

Considering the gyros bias drift, the attitude quaternion and three gyroscope biases are chosen as state variables, it can be written as

$$X(k) = \begin{bmatrix} q_0(k) & q_1(k) & q_2(k) & q_3(k) & b_x(k) & b_y(k) & b_z(k) \end{bmatrix}^T. \tag{18}$$

A tri-axis gyroscope provides measurements of the rotational velocity, and the gyroscope is known to be subject to different error terms, such as a rate noise error and a null drift. Here we use a simple but realistic model for gyro operation developed by Farrenkopf [24]

$$\omega = \widetilde{\omega} - b - n_r, \tag{19}$$

where $\widetilde{\omega}$ indicates the measured value of gyroscope, ω is the true angular rate of body, b denotes the gyro bias and n_r is the rate noise, which assumed to be zero-mean Gaussian white noise

$$\begin{cases} E[n_r(t)] = 0, \\ E[n_r(t)n_r^T(t')] = Q_r(t)\delta(t-t'). \end{cases} \tag{20}$$

The drift-rate bias itself is not a static quantity but is driven by a second Gaussian white-noise process, the gyro drift-rate ramp noise is described as

$$\dot{b} = n_w, \tag{21}$$

with

$$\begin{cases} E[n_w(t)] = 0, \\ E[n_w(t)n_w^T(t')] = Q_w(t)\delta(t-t'). \end{cases} \tag{22}$$

From Equation (7) and (21), we can find the following system of differential equations governing the state

$$\begin{bmatrix} \dot{q}(t) \\ \dot{b}(t) \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\Omega(\omega)q(t) \\ n_w \end{bmatrix} = f[X(t)]. \tag{23}$$

In the predict stage, we use the simplified expression of the above equation

$$\dot{\hat{q}}(t) = \frac{1}{2}\Omega(\hat{\omega})\hat{q}(t). \tag{24}$$

The bias is considered to be constant over the integration interval since the sampling period os short enough, so it can be found that

$$\begin{cases} \dot{\hat{b}}(t) = 0_{3\times1}, \\ \hat{b}(k+1) = \hat{b}(k). \end{cases} \tag{25}$$

and

$$\hat{\omega} = \widetilde{\omega} - \hat{b}. \tag{26}$$

From the detailed derivation in Equation (15), the system state process equation can be established as below

$$\begin{aligned}\hat{X}(k+1) =& \hat{X}(k) + f[\hat{X}(k)]\Delta t \\ &+ A[\hat{X}(k)]f[\hat{X}(k)]\frac{(\Delta t)^2}{2}.\end{aligned} \tag{27}$$

where the state vector X contains both attitude quaternion and gyro bias.

3.3 Establish Descrete System Measurement Equation

To make measurement more intuitive from the state vector, namely making the measurement matrix more simple, the attitude quaternion is chosen as measurement variable. So the measurement vector is

$$Z(k) = \left[\begin{array}{cccc} q_0(k) & q_1(k) & q_2(k) & q_3(k) \end{array}\right]^T. \tag{28}$$

The outputs of magnetometer and accelerometers can be directly converted into quaternion with GNA [21,25], Wahba's loss function [26] or algebraic solutions [27] and etc. Considering that the geomagnetic constant is difficult to determine and easily perturbed by ferromagnetic objects, an orthogonalization method is proposed, which uses only accelerometer and magnetometer measurements to obtain an orthogonal matrix and then derives an unit quaternion. The rotation matrix is composed of three 3-D column vectors i, j, k where i represents the X-axis of body frame, and j, k represent the Y and Z-axis respectively. We arrive at

$$k = \widetilde{acc}, \tag{29}$$

where $\widetilde{acc}$ indicates the raw outputs of accelerometer. We define $\bar{k}$ to be the normalized form of vector k, which can be calculated with Equation (12).

Comparing with magnetometer, we trust the acclerometer measurements more, since the magnetic field around the UAV is prone to be effected by the environment. So the outputs from acclerometer are used to form the basis axis(Z-axis in body frame), and X-axis is forced to be orthogonal with Z-axis. Then i can be calculated with the following formulation

$$i = \widetilde{mag} - \bar{k} \cdot (\widetilde{mag} \cdot \bar{k}), \tag{30}$$

where $\widetilde{mag}$ represents the outputs of magnetometer and $\cdot$ stands for the dot product between two vectors.

Finally, $\bar{j}$ can be directly calculated with the cross product of $\bar{k}$ and $\bar{i}$, that is

$$\bar{j} = \bar{k} \times \bar{i}. \tag{31}$$

So the rotation matrix from B to N can be written as

$$R = \begin{bmatrix} \bar{i}(1) & \bar{i}(2) & \bar{i}(3) \\ \bar{j}(1) & \bar{j}(2) & \bar{j}(3) \\ \bar{k}(1) & \bar{k}(2) & \bar{k}(3) \end{bmatrix}. \tag{32}$$

The unit attitude quaternion from rotation matrix is given in Equation(6). So the measurement equation is

$$Z(k) = h[X(k)] = \hat{q}_{k+1}. \tag{33}$$

3.4 EKF Implementation

The filtering algorithm can be illustrated with Fig.2.

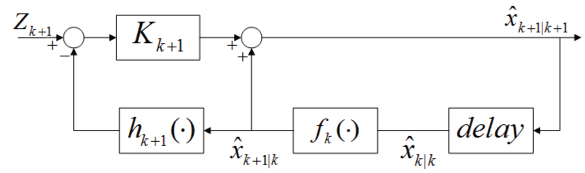

Figure 2: The EKF procedure

According to the derived result about state vector and measurement vector in Equation (27) and Equation (33) respectively, and standard Kalman procedure [19], we can express the quatenion-based extended Kalman filter algorithm process as follows.

$Step$ 1 The $apriori$ state estimate

$$\begin{aligned}\hat{X}(k+1|k) &= \hat{X}(k|k) + f[\hat{X}(k|k)]\Delta t \\ &+ A[\hat{X}(k|k)]f[\hat{X}(k|k)]\frac{(\Delta t)^2}{2}.\end{aligned}$$

$Step$ 2 The $apriori$ error covariance matrix

$$P(k+1|k) = \Phi(k)P(k|k)\Phi^T(k) + Q_k.$$

where

$$\begin{aligned}\Phi(k) &= I_{7\times7} + A[\hat{X}(k|k)] \cdot \Delta t \\ &= I_{7\times7} + \frac{\Delta t}{2}\begin{bmatrix} \Omega(\widetilde{\omega} - b) & \Xi(q) \\ 0_{4\times3} & 0_{3\times3} \end{bmatrix},\end{aligned}$$

$$Q(k) = E[W(k), W(k)^T].$$

$Step$ 3 The Kalman gain

$$\begin{aligned}K(k+1) =& P(k+1|k)H^T(k+1) \cdot \\ & [H(k+1)P(k+1|k)H^T(k+1) + R_{k+1}]^{-1},\end{aligned}$$

where

$$H(k+1) = \left[\begin{array}{cc} I_{4\times4} & 0_{4\times3} \end{array}\right],$$

$$R(k) = E[V(k), V(k)^T].$$

$Step$ 4 The $aposteriori$ state estimate

$$\begin{aligned}\hat{X}(k+1|k+1) =& \hat{X}(k+1|k) + K(k+1)\{Z(k+1) \\ &- h[\hat{X}(k+1|k)]\}.\end{aligned}$$

$Step$ 5 The $aposteriori$ error covariance matrix

$$P(k+1|k+1) = [I - K(k+1)H(k+1)]P(k+1|k).$$

4 EXPERIMENTS AND RESULTS

In this section, the performance of EKF is tested and analyzed. To prove the validity of the EKF attitude estimation algorithm, some simulations are revealed using real sensors data sampled from the UAV during a dynamic state. The sampling frequency we adopt is 50Hz. In order to get the real orientation of the vehicle, we use a two-axis turntable whose closed-loop control system is composed of stepping motor and an absolute encoder, and the rotation precision can reach 0.1 degrees.

Figure 3: The Two-Axis Turntable

Fig.4 to Fig.7 show the results of the experiments, where Fig.4 and Fig.5 shows the orientation of UAV in pitch motion, and Fig.6 and Fig.7 in yaw motion. The orientations are represented with eular angles estimated with the EKF algorithm we proposed, one another EKF by Wu [14], Madgwick's GDA [18], and the two-axis turntable provides the ground truth data. The two algorithms are chosen is because that Wu implemented his method on the helicopter and Madgwick's shows great performance in body tracking and UAV attitude estimation, their applications are similar to ours to a certain extent.

- simulation for pitch motion

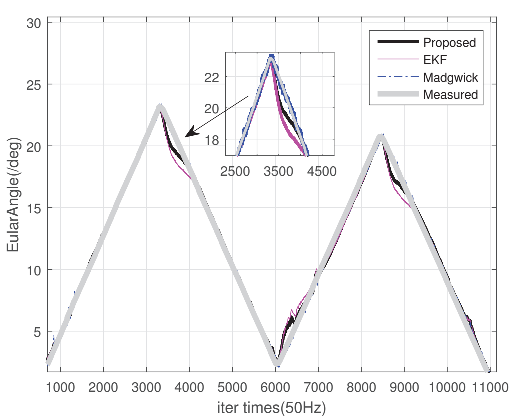

Figure 4: The pitch angular during elevating

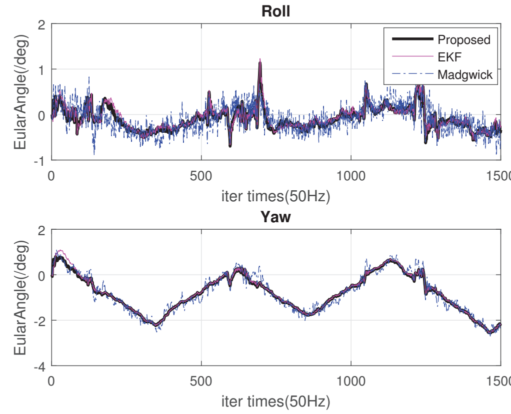

Figure 5: The roll and yaw angular during elevating

From the figures above, it's easy to be shown that all the three algorithms are capable of estimating the true attitude of UAV. While GDA has the advantage of fast convergence when UAV turns but the curve estimated contains relatively more noise. And the EKF we proposed converges faster then Wu's and can still keep smooth.

- simulation for yaw motion

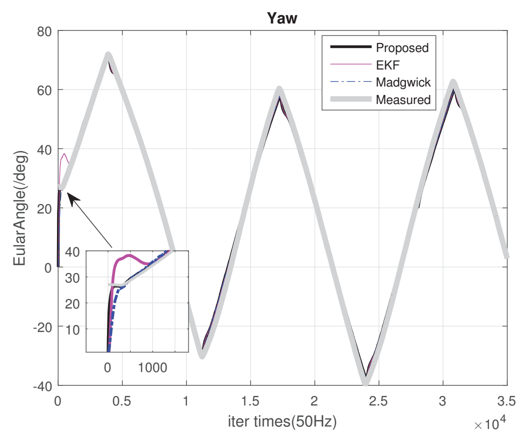

Figure 6: The yaw angular during ruddering

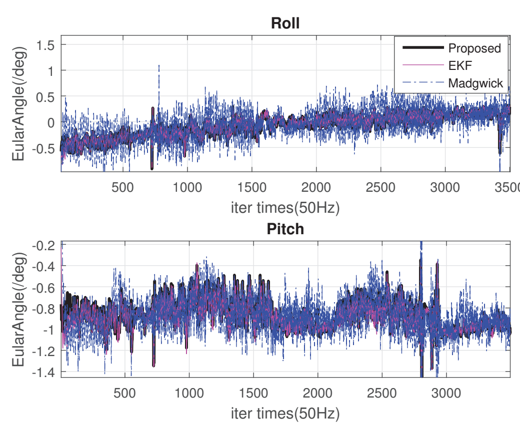

Figure 7: The roll and yaw angular during ruddering

When UAV is in yaw motion, we choose an initial attitude quaternion with value $q = [q_0\ q_1\ q_2\ q_3]^T$, the convergence rate difference among the three algorithms is self-evident. Seeing from Fig.6, it's obvious that Wu's EKF algorithm converges the slowest, and the three algorithms can all well track the true values dynamically.

The turntable can only output one kind of measurement data when it's working. So when it's in pitch motion we just compared the accuracy of the three algorithms within themselves. We can still find that the performances are very close on attitude estimation, and EKF is smoother. The roll angle as well as yaw angle fluctuate in a small range because of the vibration and the installation error. The yaw motion performance of the three algorithms is similar to the pitch motion in converge rate and curve smoothness, which proves the validity of the method we proposed.

5 CONCLUSION

This paper provides an UAV attitude estimation method based on EKF to fuse the advantages of the low-cost MARG sensors. The Taylor series linear expansion process of gyroscope kinematic equation derives the state matrix in the prediction stage. The unit orthogonal rotation matrix is used to get the attitude quaternion through the outputs of earth field exteroceptive sensors, which makes the measurement stage of EKF linear and reduce the computational efforts. The results show that EKF we proposed maintains a balance between the accuracy and time consumption. However, the algorithm has its drawbacks, since the gyroscope model we used is an empirical model, this might result in the estimate error. And we are going to studying on more accurate models in our future works.

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