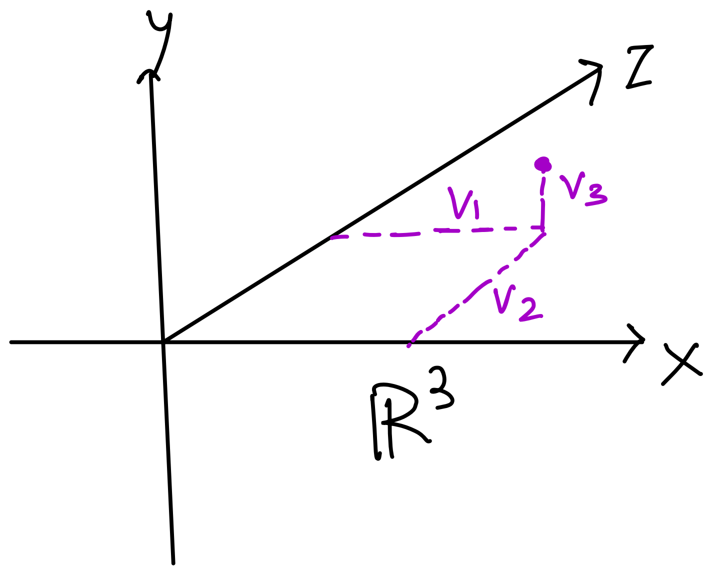
L.A. Review

Notation

Vector $v \in \mathbb{R}^d$



$$v = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_d \end{bmatrix}$$

$$v^T = \begin{bmatrix} v_1 & v_2 & \cdots & v_d \end{bmatrix}$$

[Matrix capital vectors → small]

Matrix A $A \in \mathbb{R}^{m \times n}$

$$A = \begin{bmatrix} a_{11} & \cdots & a_{n1} \\ \vdots & & \vdots \\ a_{m1} & & a_{mn} \end{bmatrix}$$

$$I: \begin{bmatrix} 1 & & & 0 \\ & 1 & & \\ & & 1 & \\ 0 & & & 1 \end{bmatrix}$$

$$\text{Diagonal} = \begin{bmatrix} \ddots & 0 \end{bmatrix}$$

$\begin{bmatrix} 0 & \searrow \end{bmatrix}$

Vector-Vector

Inner / Dot Product

(Same in this course, generally different)

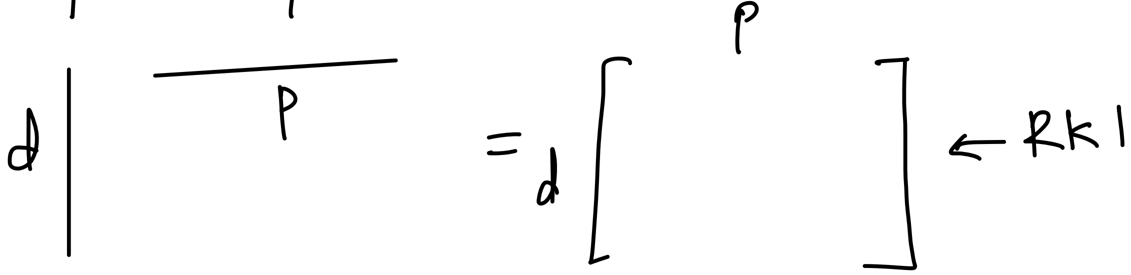
$x, y \in \mathbb{R}^d$

$$\sum_{i=1}^{d} x_i y_i = x^T y = y^T x$$

Outer Product

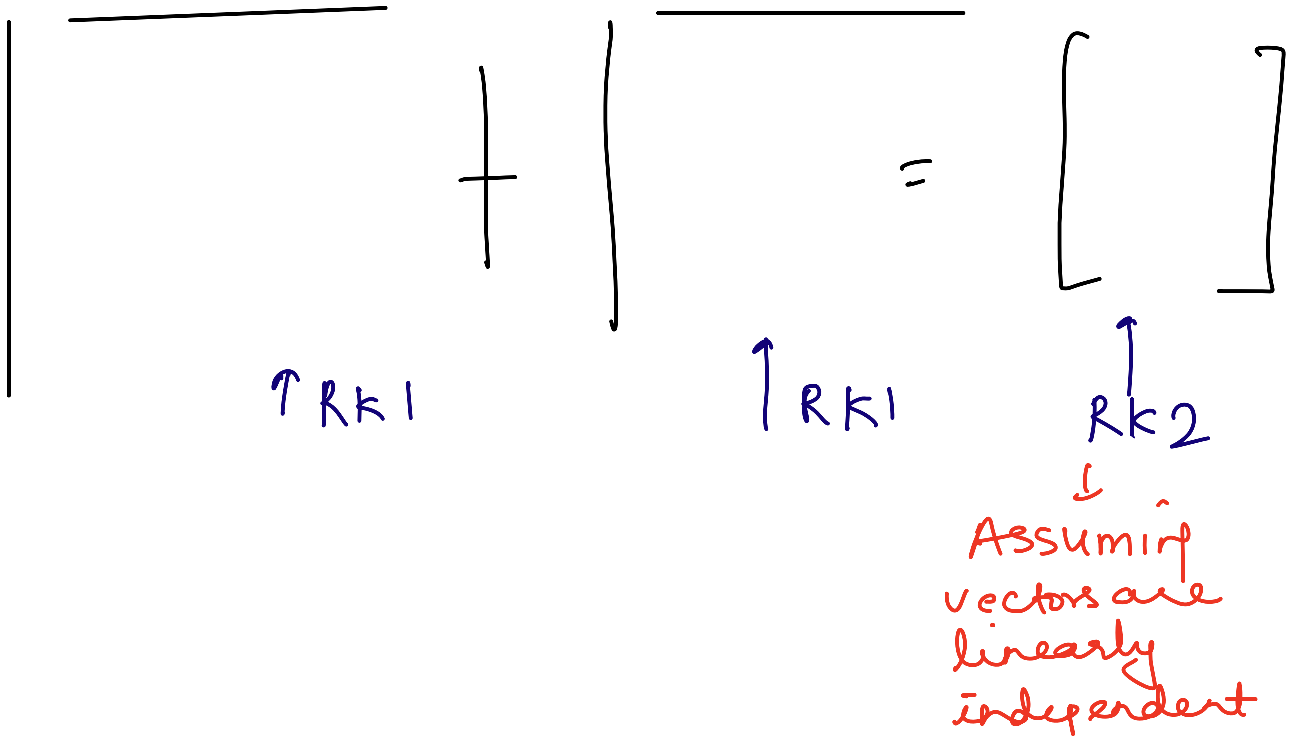
$x \in \mathbb{R}^d \quad y \in \mathbb{R}^p$

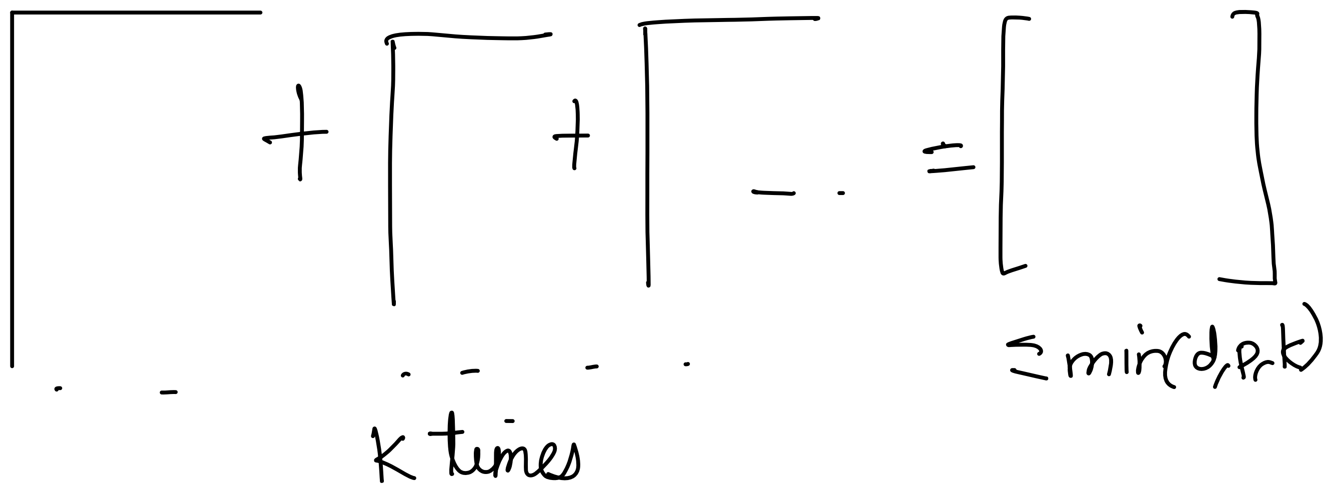
xy^T or yx^T (Both not same here!!)



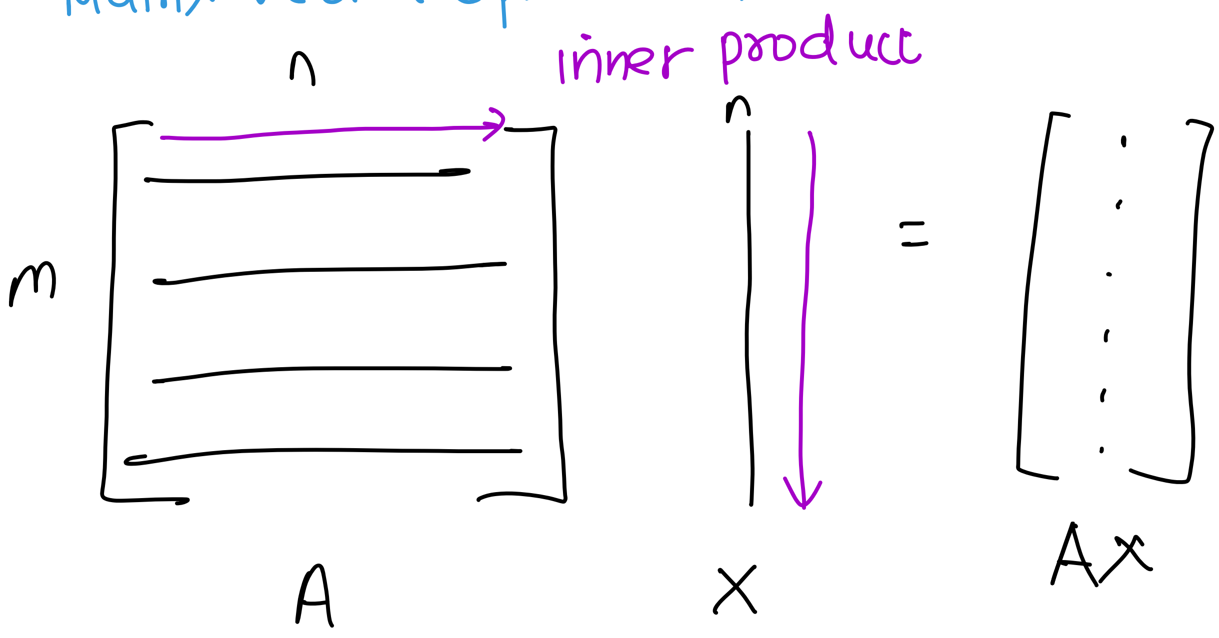
Rank 1 Matrix

Matrix constructed from 1 row vector
1 column vector

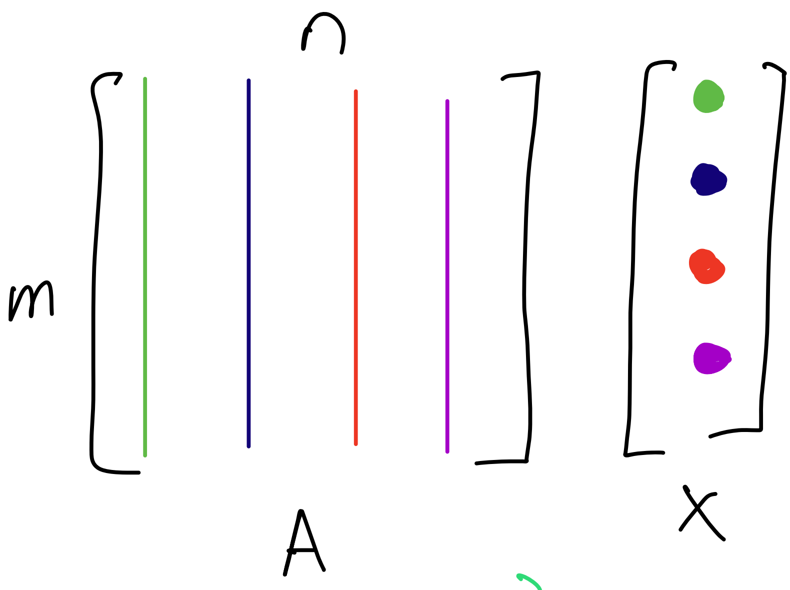




Matrix Vector Operation



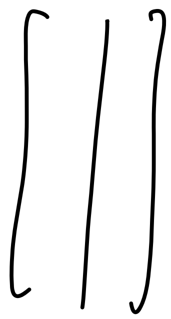
$A \in \mathbb{R}^{m \times n}$ $x \in \mathbb{R}^{n}$ $Ax \in \mathbb{R}^{m}$



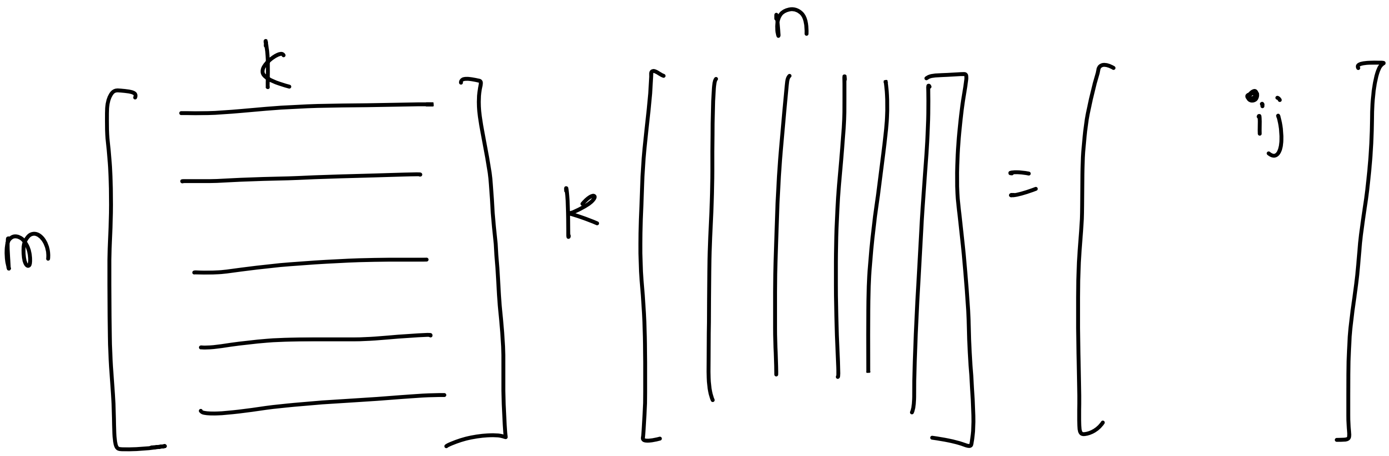
Scale it acc. to colour code

add after scaling

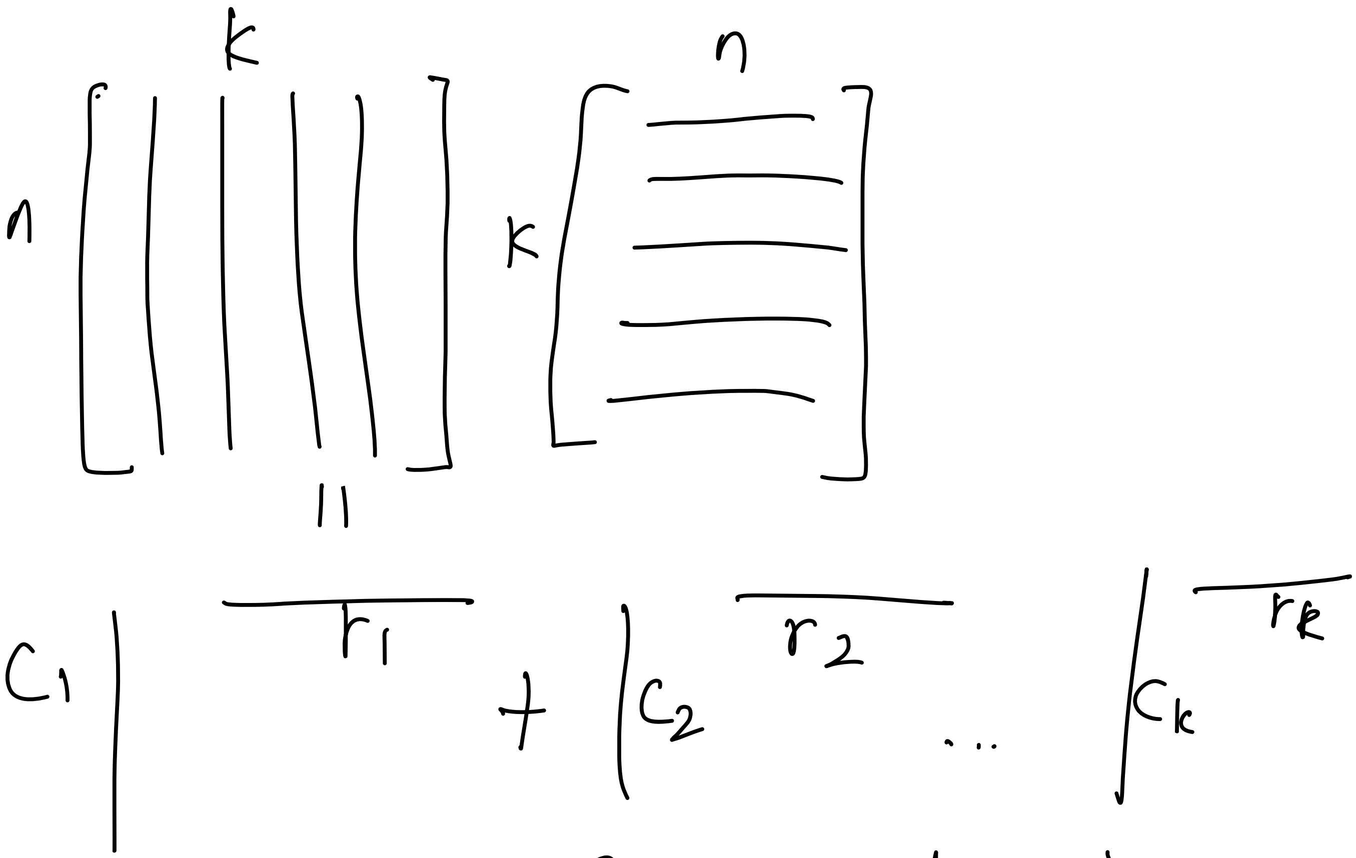
$Ax =$



Matrix-Matrix



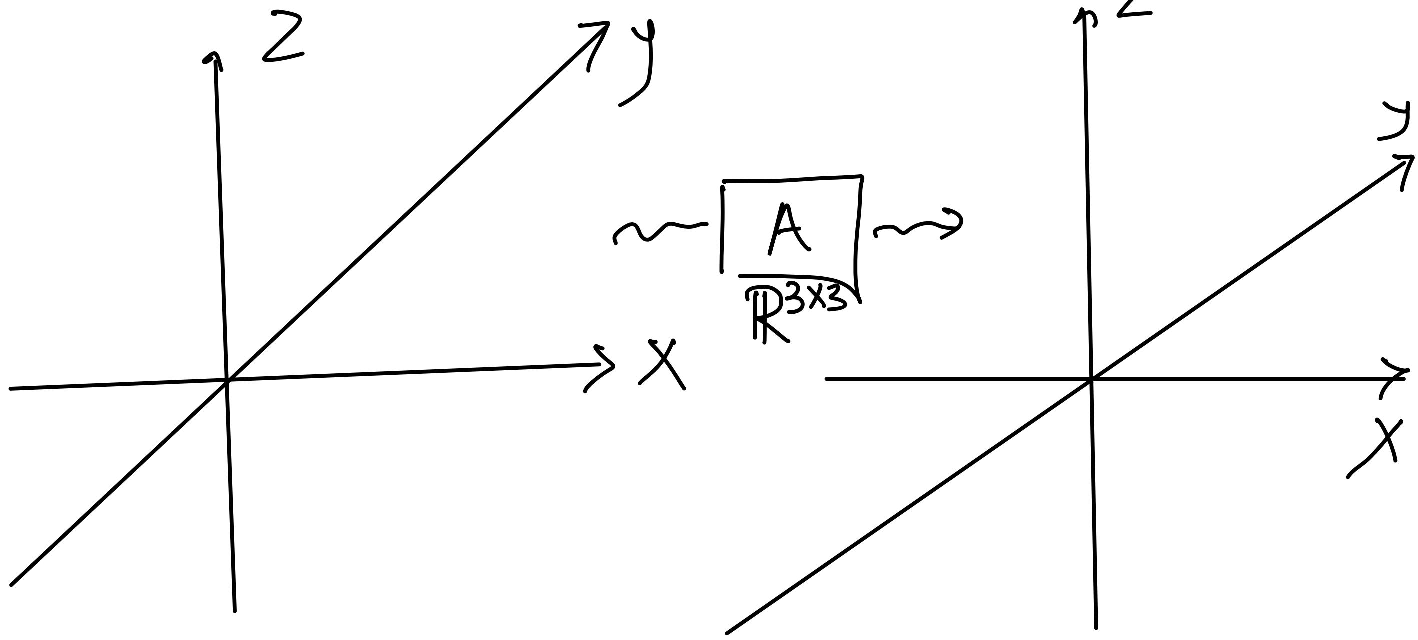
Take all possible R-C pairs, dot product of them & ij^{th} element is $R_i \cdot C_j$



(outer product)

Geometrical Interpretation

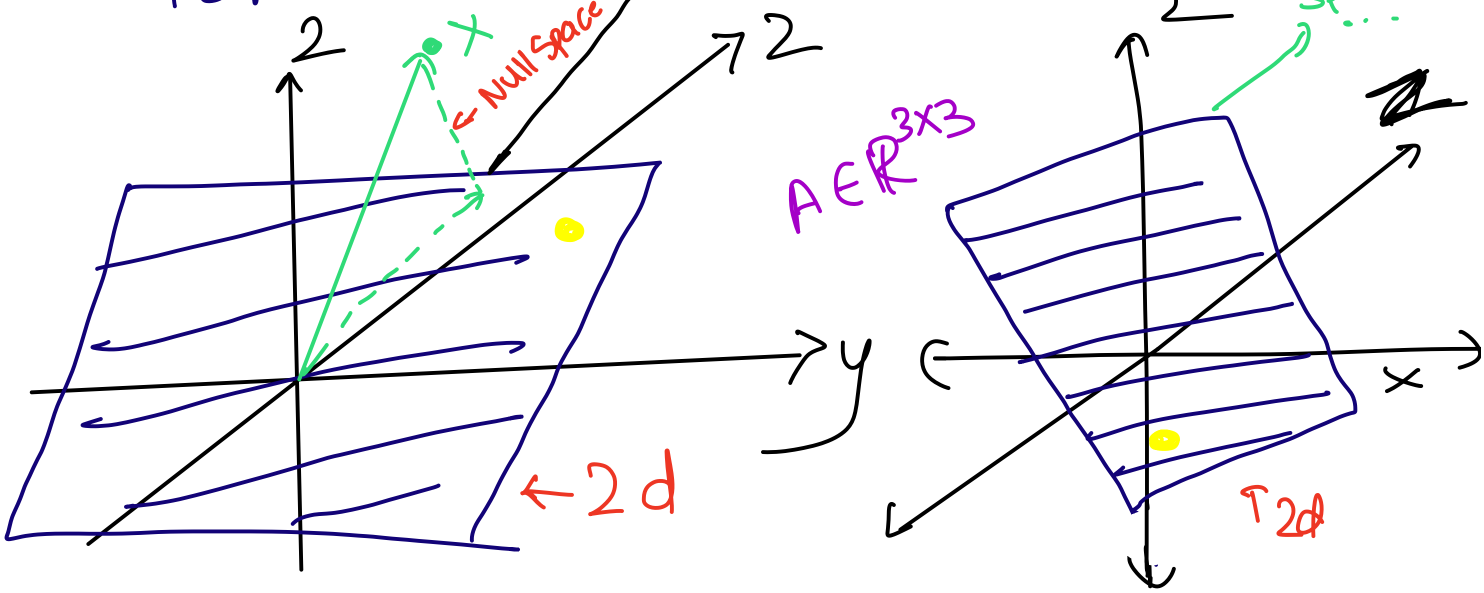
$A \in \mathbb{R}^{m \times n}$ $x \in \mathbb{R}^n$ $Ax \in \mathbb{R}^m$



For one-to-one mapping, A → full rank

Rowspace: linear combⁿ of columns of A

For rank-2



In this 2d subspace, one to-one mapping

x → not in our subspace, so decompose it.
Projection on subspace + ⊥ to it

If A has 3 rows

$$A = \begin{bmatrix} \text{———} \\ \text{———} \\ \text{———} \end{bmatrix} \begin{array}{l} 1 \leftarrow \text{coefficient} \\ 0 \\ 0 \end{array}$$

, so first row lies inside our subspace and all other combinations

$$\bar{X} = \text{Proj}(\bar{X}; \text{Row Space}) + \text{Proj}(\bar{X}; \text{Null space})$$

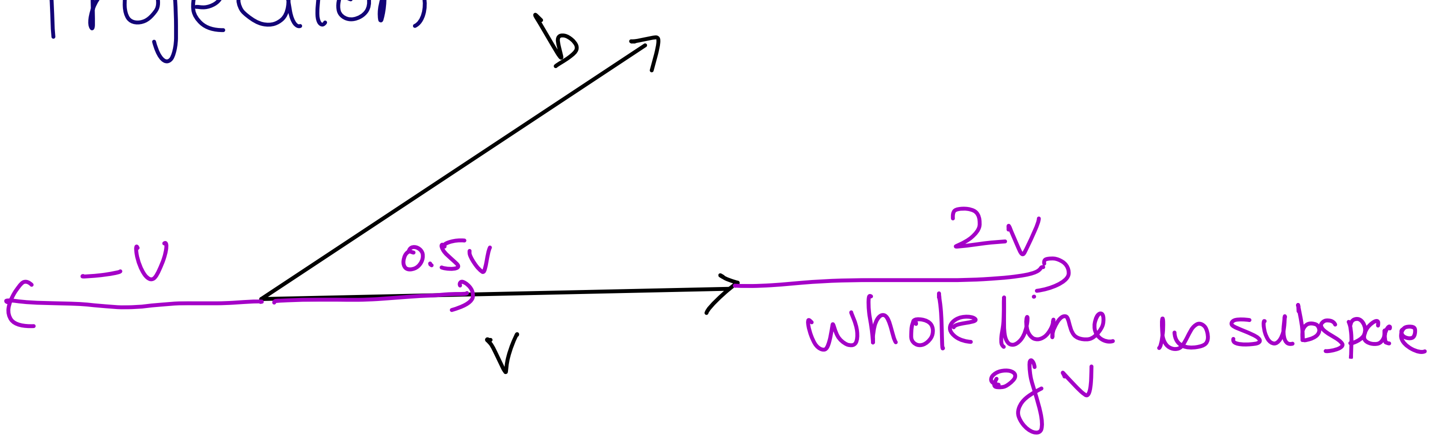
$$A(X) = A(X_R + X_N)$$ Linear Function

$$= A(X_R) + A(\cancel{X_N})$$

$$\swarrow 0$$

$$= A(X_R)$$

Projection



Project b on subspace of v : (2v, 1.5v)

$$\text{Projection matrix}(v) = \left(\frac{vv^T}{v^Tv}\right)$$

$$\text{So b onto } v = \frac{vv^T b}{v^T v}$$

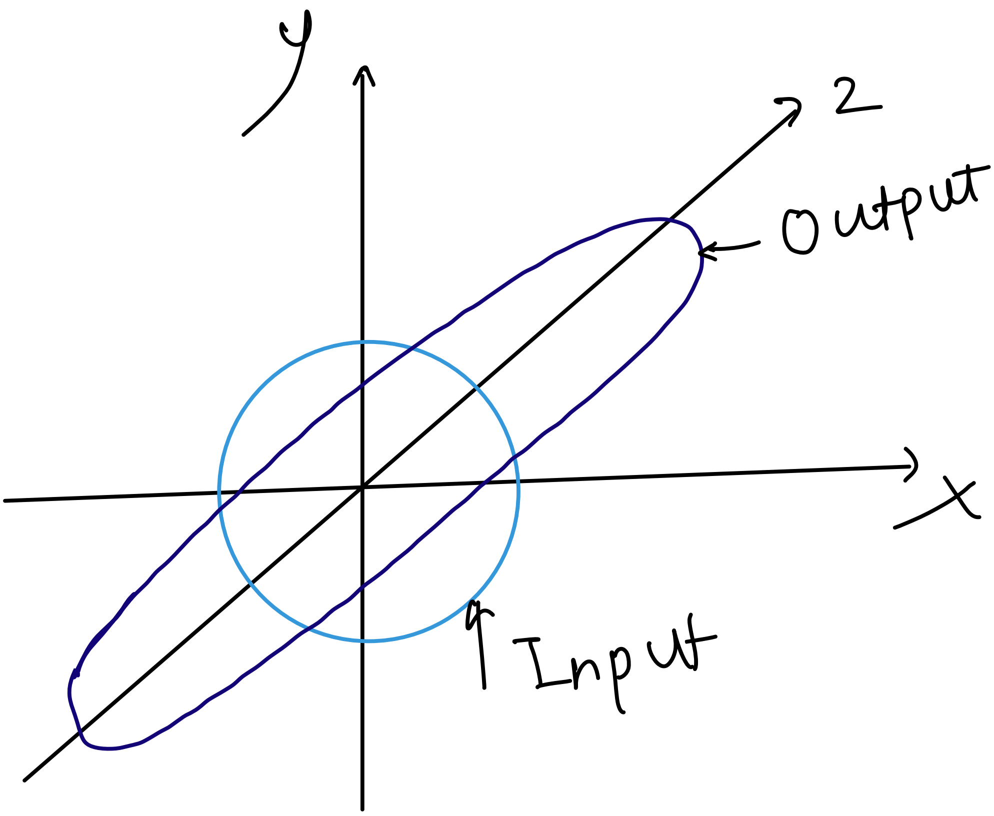
$$\left(\frac{v}{\|v\|}\right)\left(\frac{v^T}{\|v\|}\right) b = \tilde{v}\left(\tilde{v}^T b\right)$$

Dot product
(length of projection of b on v)

For a matrix X,

$$X(X^TX)^{-1}X^T$$

So for rank deficient A, we can't reach input from a given output as A^{-1} doesn't exist (Because multiple inputs map to the per past → nullspace)



$A \in \mathbb{R}^{3\times3}$
so input & output space are overlaid

Assume $A^{3\times3}$ ∈ symmetric

For full rank 3×3 matrix, 3 eigen

vectors and if symmetric, eigen vectors are $\perp$

Rank = no. of non-zero eigen values

Determinant = Product of eigen values

$$= \frac{\text{Volume of output shape}}{\text{Volume of input shape}}$$

Spectrum → collection of eigen values in descending order

Spectral Theorem: For every matrix $A \in \mathbb{R}^{d \times d}$, $A = A^T$ has real valued eigenvalues & ortho normal eigenvectors

eg. Hessians, Covariance, Kernel