Uniform Distribution

Definition.

- **Uniform distribution**
- A random variable X is said to be uniform on the interval $[a, b]$ if its probability density function is of the form
- $f(x) = 1/b - a \quad , \quad a \leq x \leq b,$
- where a and b are constants.
- We denote a random variable X with the uniform distribution on the interval $[a, b]$ as $X \sim UNIF(a, b)$.
- An important application of uniform distribution lies in random number generation

Applications

- Useful in life testing and traffic flow experiments
- Waiting time for a facility that goes in cycles eg. Shuttle bus and an elevator. If user comes to a stop at a random time and wait till the facility arrives, the waiting time will be uniformly distributed between a minimum of 0 and a maximum of equal to cycle time.

Mean and variance of Uniform distribution

- If X is uniform on the interval [a, b]

then the mean and variance of X are given by

- <u>E(X)=(b+a)/2 ,V(X)=(b-a)^2 /12</u>

- Mean

=E(X)

= $\int_{-\infty}^{\infty} xf(x)dx$

= $\int_a^b x(1/\{b-a\})dx$

= (b+a)/2

$E(X^2)$

$= \int_{-\infty}^{\infty} x^2 f(x)dx$

$= \int_{a}^{b} x^2 (1/\{b-a\})dx$

$= (1/\{b-a\}) \int_{a}^{b} x^2 dx =$

$=(1/3)\ (a^2+ab+b^2)$

Now V(X)

$= E(X^2) - \{E(x)\}^2$

$= (1/3)\ (a^2+ab+b^2) - ((b+a)/2)^2$

$=(b-a)^2 /12$

Distribution Function and Probabilities for U(a,b)

- $F(x) = P(X \leq x) = \int_a^x f(t)dt = \frac{x-a}{b-a}$
- $P(X \leq x) = \frac{x-a}{b-a}$
- $P(X \geq x) = 1 - \frac{x-a}{b-a}$
- $P(x_1 \leq X \leq x_2) = \frac{x_2-x_1}{b-a}, a \leq x_1 \leq x_2 \leq b$

Exercise 1- Suppose that for a certain company, the conference time X has uniform distribution over interval (0,4)hrs. (a) what is the pdf of X
(b) what is the prob that (i) any conference lasts at least 3 hrs (ii) it lasts for 2 hrs but does not exceed more than 3.5 hrs.

- Given: a=0, b=4, $X \sim U(0,4)$

(a) pdf of X: f(x) =1/b – a , a ≤ x ≤ b therefore

$$f(x) = \begin{cases} 1/4, 0 \leq x \leq 4 \\ 0, \quad otherwise \end{cases}$$

(b) (i) $P(X \geq x) = 1 - \frac{x-a}{b-a}$

$$P(X \geq 3) = 1 - \frac{x-a}{b-a} = 1/4$$

(ii) $P(x_1 \leq X \leq x_2) = \frac{x_2-x_1}{b-a}, a \leq x_1 \leq x_2 \leq b$

$$P(2 \leq X \leq 3.5) = \frac{x_2-x_1}{b-a} = 0.375$$

Ex 2- On a route from Railway station to college, every 20 minutes , there is a bus. A student arrive at bus stop and waits for a bus. His waiting time till bus arrives is uniform over the interval (0,20). On one fine Monday what is the prob that the student waits for (i) less than 5 minutes (ii) between 7 minutes to 15 minutes (iii) more than 10 minutes

Given: a=0, b=20, $X \sim U(0,20)$

$$f(x) = \begin{cases} 1/20, 0 \leq x \leq 4 \\ 0, \quad otherwise \end{cases}$$

(i) 0.25

(ii) 0.4

(iii) 0.5

Ex 3-If $X \sim U(a, b)$ such that mean and variance are 15/2 and 25/12 respectively. Determine the values of a and b. Hence find $P(6 \leq X \leq 8)$

- mean $= \frac{a+b}{2}$ variance $= \frac{(b-a)^2}{12}$
- $a + b = 15, b - a = 5 \ (b > a)$
- $a = 5, b = 10$
- $f(x) = \begin{cases} \frac{1}{5}, & 5 \leq x \leq 10 \\ 0, & otherwise \end{cases}$
- $P(6 \leq X \leq 8) = 0.4$

Ex 4. If X is uniform on the interval from 0 to 3, what is the probability that the quadratic equation $4t^2 + 4tX + X + 2 = 0$ has real solutions?

- Since X~ UNIF(0, 3),
- the probability density function of X is
- $f(x) = 1/3$, $0 \leq x \leq 3$
- 0 ,otherwise.
- The quadratic equation $4t^2 + 4tX + X + 2 = 0$ has real solution if the discriminant of this equation is positive.
- That is $16X^2 - 16(X + 2) \geq 0$,
- i.e. $X^2 - X - 2 \geq 0$.
- i.e. $(X - 2)(X + 1) \geq 0$.
- The probability that the quadratic equation
- $4t^2 + 4tX + X + 2 = 0$ has real roots is equivalent to
- $p((X - 2)(X + 1) \geq 0)$
- $= p(X \leq -1) + p(x \geq 2)$
- $= \int_{-\infty}^{-1} f(x)dx + \int_{2}^{3} f(x)dx$
- $= \int_{2}^{3} (\frac{1}{3})dx =$
- $= 1/3$