GPU Acceleration of the Material Point Method

Fabian Meyer

University of Koblenz

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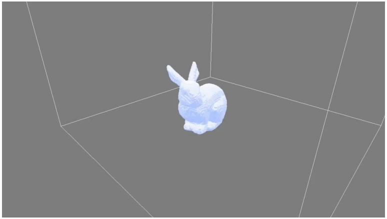
A Brief MPM Overview: Do You Want to Build a Snowman?

A short historical summary of MPM:

- Belongs to family of particle-in-cell(PIC) techniques [EHB57].
- Initial application to solids [SZS95] → MPM
- From research to production in *Disney's* animation film *Frozen* [Sto+13].
- Avalanche research [Gau+18]



Video result of my bachelor thesis on the simulation of snow [Mey15].

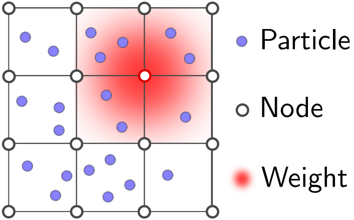
PIC ideas:

- Combine Lagrangian particles & Eulerian grid
- Particles store all information

Typical PIC/MPM roundtrip:

1. Particle-to-grid(P2G) transfer to an unmoving grid
2. Solve discretized governing equations on grid
3. Grid-to-particle(G2P) transfer back to particles & move them

$\Rightarrow$ **meshfree, non-empirical**



Transfers: Interpolation functions are defined over grid nodes.

GPGPU for performance enthusiasts

Why would('nt) you?

Drawbacks:

- Interactivity much easier on CPU, but slow PCI-Bus communication
- Code is mostly written against GPU architecture
- A lot of strain on the programmer

Benefits:

- Data is already on the GPU for rendering
- **Higher parallelization acceleration**

Governing Equations: Conservation of Mass & Momentum

Conservation of mass, continuum assumption holds.
Lagrangian (moving with a particle ${}_0\boldsymbol{x}$):

$$ {}^{t}_{0}J\rho({}_0\boldsymbol{x}, t) = \rho({}_0\boldsymbol{x}, 0). \tag{1} $$

Eulerian (outside observer ${}_t\boldsymbol{x}$):

$$ \frac{\partial}{\partial t}\rho({}_t\boldsymbol{x}, t) = -\vec{\nabla} \cdot (\rho({}_t\boldsymbol{x}, t)\boldsymbol{v}({}_t\boldsymbol{x}, t)). \tag{2} $$

Lagrangian and Eulerian view measure differently but give same results. Equations are given in the strong form! [Jia+16][Abe12]

Conservation of momentum:
Lagrangian (moving with a particle ${}_0\boldsymbol{x}$):

$$\rho({}_0\boldsymbol{x}, 0)\boldsymbol{a}({}_0\boldsymbol{x}, t) = \vec{\nabla} \cdot \boldsymbol{P}({}_0\boldsymbol{x}, t) + \boldsymbol{f}^{\text{body}}({}_0\boldsymbol{x}, t){}_0^tJ. \tag{3}$$

Eulerian (outside observer ${}_t\boldsymbol{x}$):

$$\rho({}_t\boldsymbol{x}, t)\boldsymbol{a}({}_t\boldsymbol{x}, t) = \vec{\nabla} \cdot \boldsymbol{\sigma}({}_t\boldsymbol{x}, t) + \boldsymbol{f}^{\text{body}}({}_t\boldsymbol{x}, t) \tag{4}$$

Solving this equation will tell us how the velocity fields $\boldsymbol{v}({}_t\boldsymbol{x}), \boldsymbol{v}({}_0\boldsymbol{x})$ change on the whole domain due to acceleration $\boldsymbol{a}$. This is important to advect particles accounting for all forces. [Jia+16][Abe12]

The Pretty Strong but Mathematically Weak Formulation

Weak Formulation (or Principle of Virtual Work):
Dot product equations with arbitrarily 'test functions' $\boldsymbol{q}$ and apply divergence theorem:

$$\int_{\Omega^0} {}_0\boldsymbol{q} \cdot \left[({}_0\rho_0)({}_0\boldsymbol{a}) - {}_0\boldsymbol{f}^{\text{body}}\,{}_0^tJ\right] d_0\boldsymbol{x} = \int_{\partial\Omega^{t^n}} {}_t\boldsymbol{q} \cdot \boldsymbol{\sigma}\, d_t\boldsymbol{A} - \int_{\Omega^{t^n}} \nabla_t \boldsymbol{q} : \boldsymbol{\sigma}\, d_t\boldsymbol{x}. \tag{5}$$

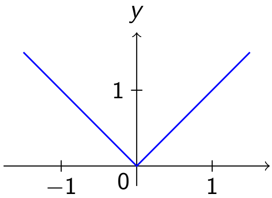
A strong solution is also a solution to the weak formulation. Leave out body forces(like gravity) and boundary condition (e.g. collisions) for now:

$$\int_{\Omega^0} {}_0\boldsymbol{q} \cdot ({}_0\rho_0)({}_0\boldsymbol{a}) d_0\boldsymbol{x} = \int_{\Omega^{t^n}} \nabla_t \boldsymbol{q} : \boldsymbol{\sigma}\, d_t\boldsymbol{x}. \tag{6}$$

Weak Derivative:

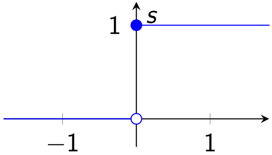
$y = |t|$ has weak derivative:

$$v = \begin{cases} -1, & \text{if } t < 0 \\ c, & \text{if } t = 0 \\ 1, & \text{if } t > 0 \end{cases}$$



Heaviside step function has no weak derivate:

$$s = \begin{cases} 0, & \text{if } t < 0 \\ 1, & \text{if } t \geq 0 \end{cases}$$



Allows for point loads, material discontinuities and more. [Bat06]

Discretization of Space and Time

Time discretization with implicit midpoint scheme:

$$\frac{y^{n+1} - y^{n}}{\Delta t} = f^{n+\frac{1}{2}} = f\left(t^{n} + \frac{\Delta t}{2}, \frac{1}{2}y^{n} + \frac{1}{2}y^{n+1}\right) \qquad (7)$$

- **implicit** requires linear system solve $\Rightarrow$ more stable, larger time steps
- **midpoint** as it conserves gov. equations

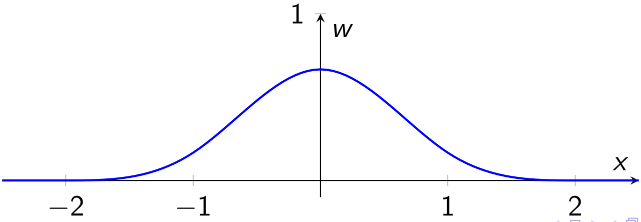
$$\Rightarrow \int_{\Omega^{t}} {}_{t}\boldsymbol{q} \cdot {}_{t}\rho({}_{t}\boldsymbol{v}^{n+1} - {}_{t}\boldsymbol{v}^{n}) d_{t}\boldsymbol{x} = \int_{\Omega^{t^{n}}} \nabla_{t}\boldsymbol{q} : \boldsymbol{\sigma}^{n+\frac{1}{2}} d_{t}\boldsymbol{x}. \qquad (8)$$

Space Discretization is done in a Galerkin/FEM fashion with grid based interpolants w_i with limited support. Here dyadic products

$$w_i(\boldsymbol{x}) = w(\boldsymbol{x} - \boldsymbol{x}_i) = w(\frac{1}{h}(x - x_i))w(\frac{1}{h}(y - y_i)w(\frac{1}{h}(z - z_i)) \quad (9)$$

of cubic b-splines suffice:

$$w(x) = \begin{cases} \frac{1}{2}|x|^3 - |x|^2 + \frac{2}{3} & 0 \leqslant |x| < 1 \\ \frac{1}{6}(2 - |x|)^3 & 1 \leqslant |x| < 2 \\ 0 & 2 \leqslant |x| \end{cases} . \quad (10)$$



Thus set in:

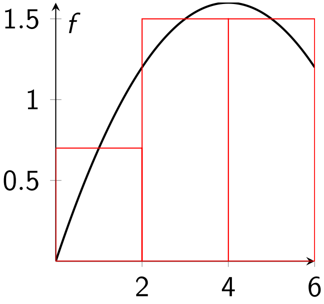
$$ {}_t\boldsymbol{q}(\boldsymbol{x}, t^n) = \sum_i \boldsymbol{e}_i w_i(\boldsymbol{x}), {}_t\boldsymbol{v}^{n(+1)}(\boldsymbol{x}) = \sum_j \boldsymbol{v}_j^{n(+1)} w_j(\boldsymbol{x}). $$

Combine it with numerical integration where the particles function as quadrature points [SKB08]:

$$ g_i = \int_\Omega g(\boldsymbol{x}) w_i(\boldsymbol{x}) d\boldsymbol{x} \approx \sum_p g_p w_i(\boldsymbol{x}_p) V_p. \tag{11} $$

due to integration by midpoint rule:

$$ \int_\Omega f(x)dx \approx \sum_{i=1}^{N} f(x_i)\, h_i. \tag{12} $$



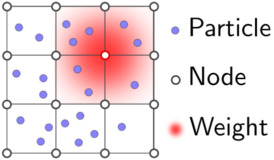
Velocity Fields: APIC-Transfers

PIC-transfer

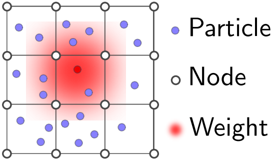
1. $(m\boldsymbol{v})_i^n = \sum_p w_{ip}^n m_p \boldsymbol{v}_p^n$
2. $\boldsymbol{v}_i^n = \frac{(m\boldsymbol{v})_i^n}{m_i^n}$
3. $\boldsymbol{v}_{p,PIC}^{n+1} = \sum_i w_{ip}^n \boldsymbol{v}_i^{n+1}$

APIC-transfers add a local velocity field $\boldsymbol{C}_p^n$ around $\boldsymbol{v}_p^n$:

$$(m\boldsymbol{v})_i^n = \sum_p w_{ip}^n m_p \left(\boldsymbol{v}_p^n + \boldsymbol{C}_p^n(\boldsymbol{x}_i^n - \boldsymbol{x}_p^n)\right)$$



P2G-transfer $\sum_p$



G2P-transfer $\sum_i$

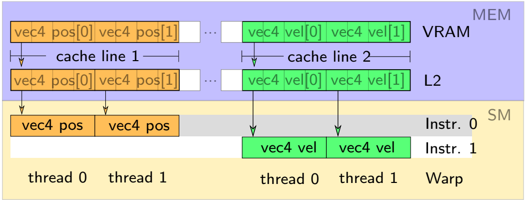
Layout of the data: SoA vs. AoS

SoA-Layout

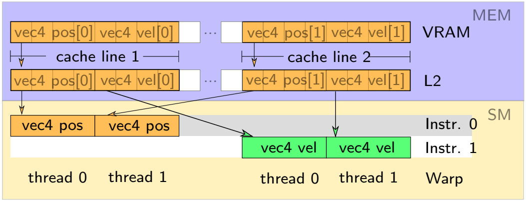
```
struct Container{
  vec4 positions[n];
  vec4 velocities[n];
} Particles;
```

AoS-Layout

```
struct Particle {
  vec4 position;
  vec4 velocity;
} Particles[n];
```



Structures of arrays(SoA)



Arrays of structures(AoS)

Layout	$\Delta t_c (\mu s)$	Speedup	VRAM	SM	L2	SM Issue Ut
AoS	243	-	77.7%	7.3%	30.3%	6.8%
SoA	**120**	2.26x	75.4%	**14.3**%	29.4%	**14.0%**

hi

1. [Jia+16]

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