SigSys - PS10
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Problem 1

$\dot{y} + y = x(t)$

$sY(s) + Y(s) = X(s)$

$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s+1}$

step response $= \frac{1}{s} H(s)$

$\longrightarrow \frac{1}{s} \cdot \frac{1}{s+1} = \frac{A}{s} + \frac{B}{s+1}$

$1 = \cancel{s}(s+1)\frac{A}{\cancel{s}} + s\cancel{(s+1)}\frac{B}{\cancel{s+1}}$

$1 = (s+1)A + sB$

$\underline{s=-1}$: $1 = -B \longrightarrow B = -1$

$\underline{s=0}$: $1 = A$

$Y(s) = \frac{1}{s} - \frac{1}{s+1}$

$\mathcal{L}^{-1}\left\{\frac{1}{s} - \frac{1}{s+1}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\}$

$= u(t) - e^{-t}u(t)$

$\boxed{y(t) = (1-e^{-t})u(t)}$

Problem 2

A. DC gain $= \lim_{s\to 0} H(s)$

$$H(s) = \frac{1/\tau}{s + 1/\tau}$$

$$\frac{Y(s)}{Y_{sp}(s)} = \frac{KH}{1+KH} = \frac{(K_1/s)H}{1+(K_1/s)H} = \frac{K_1 H}{\cancel{s}} \cdot \frac{\cancel{s}}{s + K_1 H} = \frac{K_1 H}{s + K_1 H}$$

$$\lim_{s\to 0} \frac{K_1 H}{s + K_1 H} = \boxed{1}$$

The DC gain is 1 for any H(s), and does not depend on the value of K_1, because that gets cancelled out.

B. $$\frac{Y(s)}{Y_{sp}(s)} = \frac{K_1 H}{s + K_1 H} = \frac{K_1\left(\frac{1/\tau}{s+1/\tau}\right)}{s + K_1\left(\frac{1/\tau}{s+1/\tau}\right)} = \frac{K_1/\tau}{\cancel{s+1/\tau}} \cdot \frac{\cancel{s+1/\tau}}{s^2 + s/\tau + K_1/\tau}$$

$$= \boxed{\frac{K_1/\tau}{s^2 + s/\tau + K_1/\tau}}$$

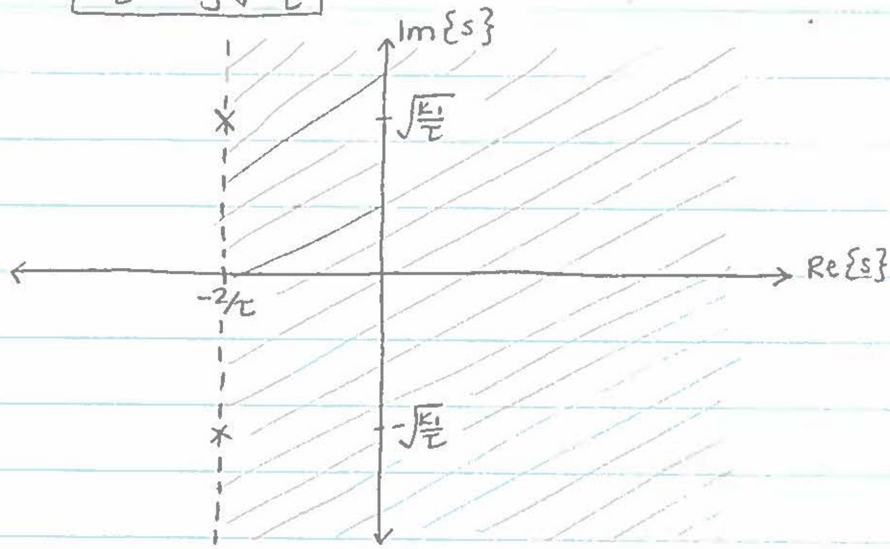
pole:

$$0 = s^2 + s/\tau + K_1/\tau$$

negligible, as $K \gg 1/\tau$

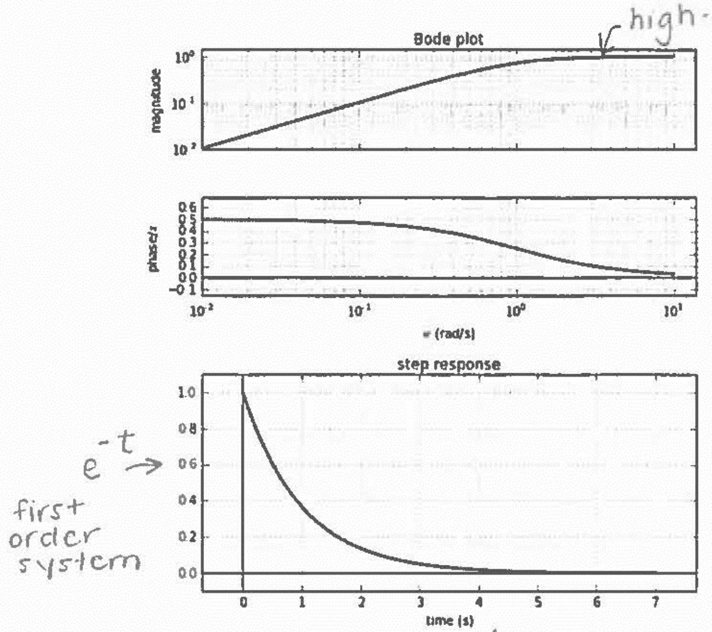
$$\text{poles @ } s = \frac{-\frac{1}{\tau} \pm \sqrt{(1/\tau)^2 - 4K_1/\tau}}{2} = \frac{-2}{\tau} \pm \frac{1}{2}\sqrt{\cancel{\left(\frac{1}{\tau}\right)^2} - 4K_1/\tau}$$

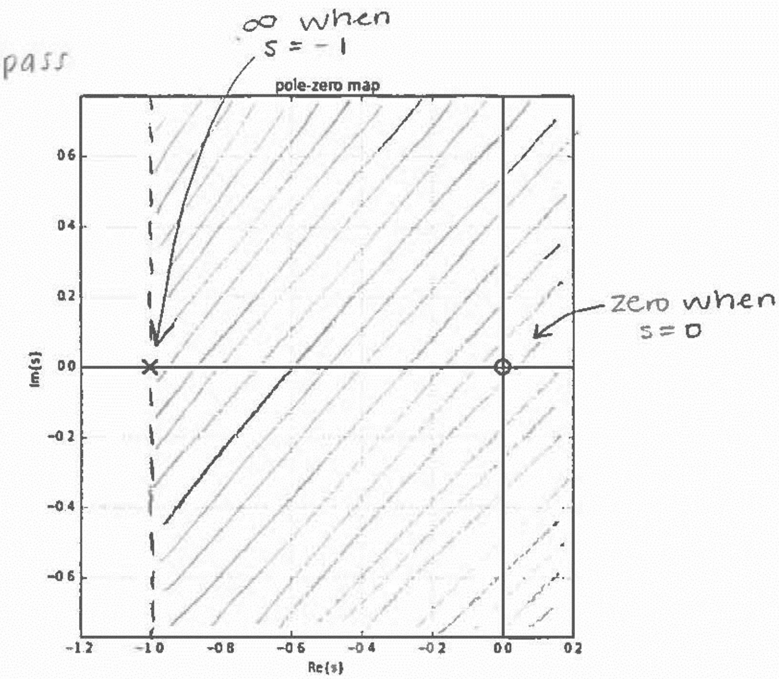
$$= \boxed{\frac{-2}{\tau} \pm j\sqrt{\frac{K_1}{\tau}}}$$



Problem 3

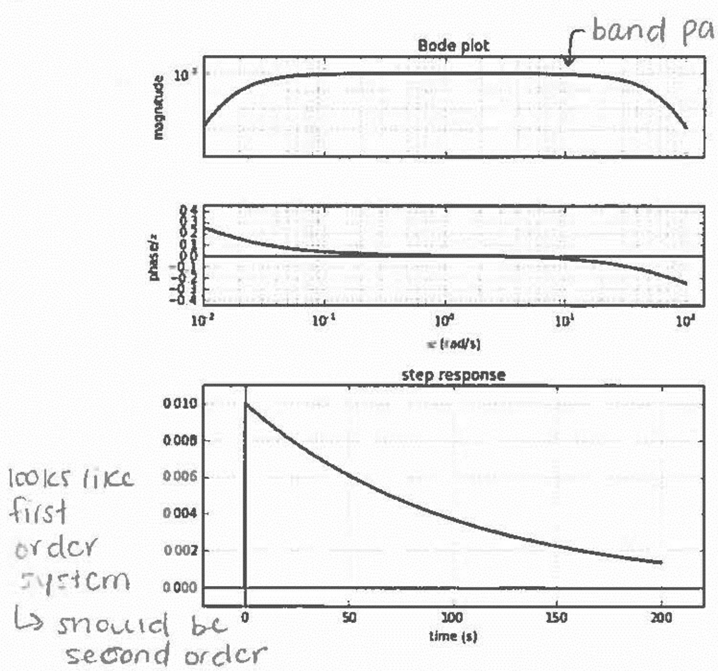
A. $\frac{s}{s+1}$

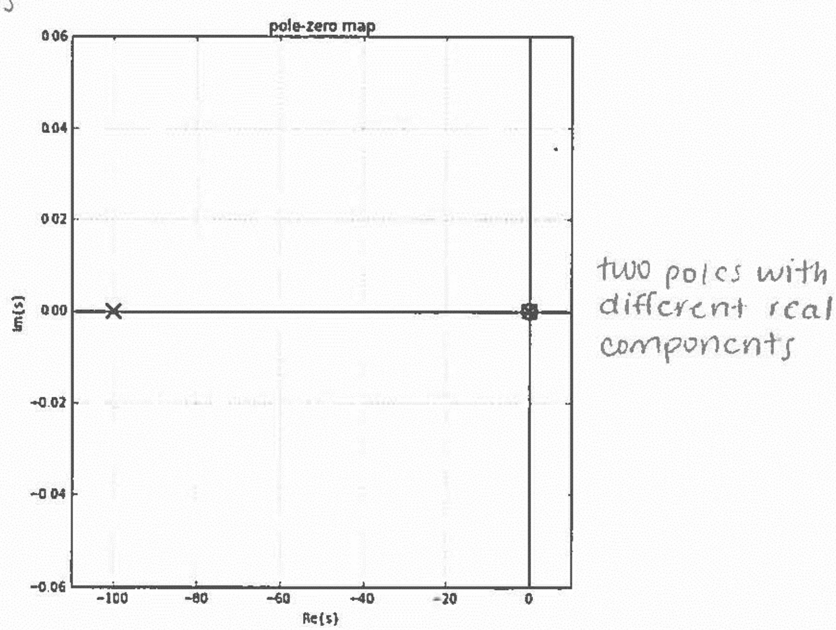




The Bode plot of this system shows a high-pass behavior, and the step response agrees that the system is first-order. The pole-zero map has one real pole and a zero at (0,0).

B. $\frac{s}{s^2+100s+1}$

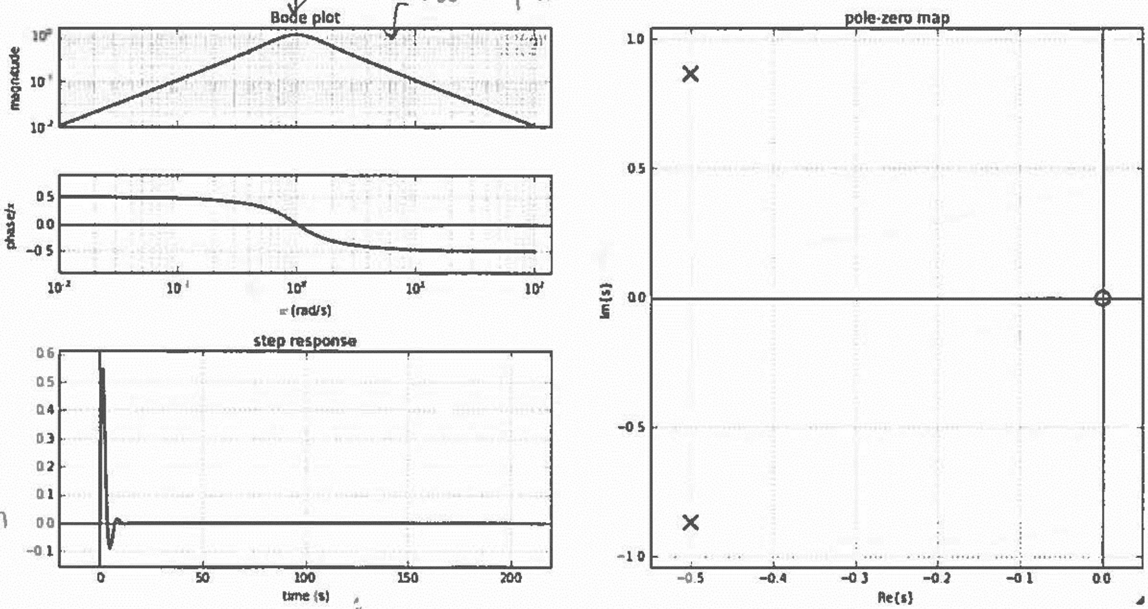




The Bode plot of this system shows a band-pass behavior, and the step response shows a first-order shape, although the system is second-order. The pole-zero map has two real poles and a zero at (0,0).

C. $\frac{s}{s^2+s+1}$

peak forming
band pass



two poles with different imaginary components

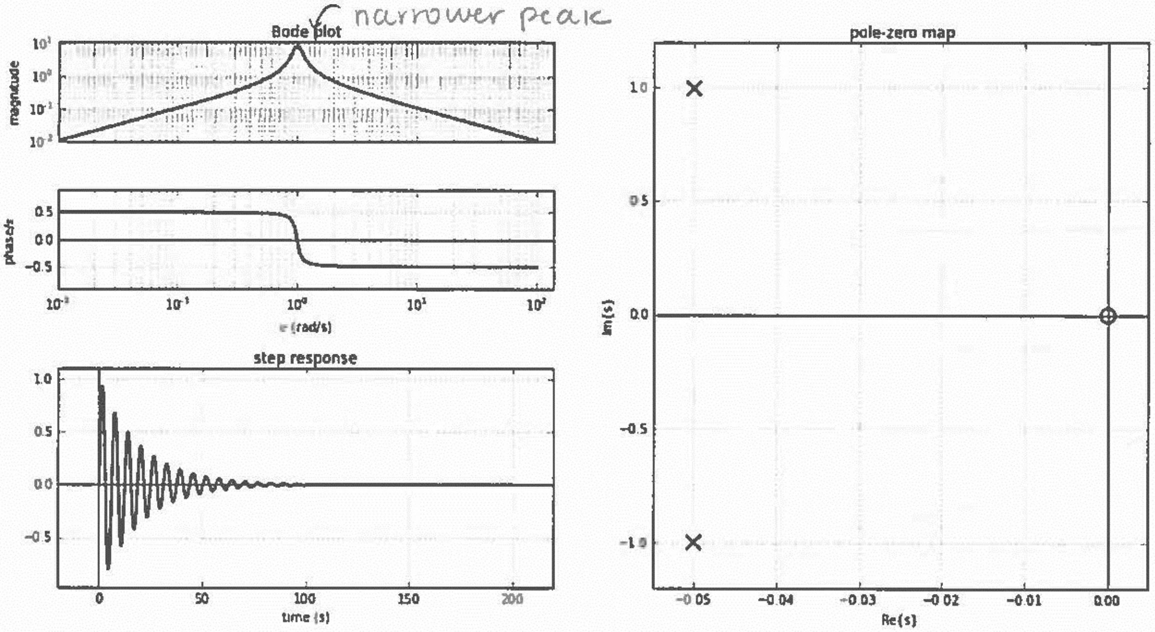
second order system

The Bode plot of this system has a band-pass behavior with a peak in the middle, and the step response appropriately shows a second-order shape. The pole-zero map has two poles with real components and opposite imaginary components. It also has a zero at (0,0).

D. $\frac{s}{s^2+0.1s+1}$

similar to C, but with more oscillations in step response

narrower peak



second order system

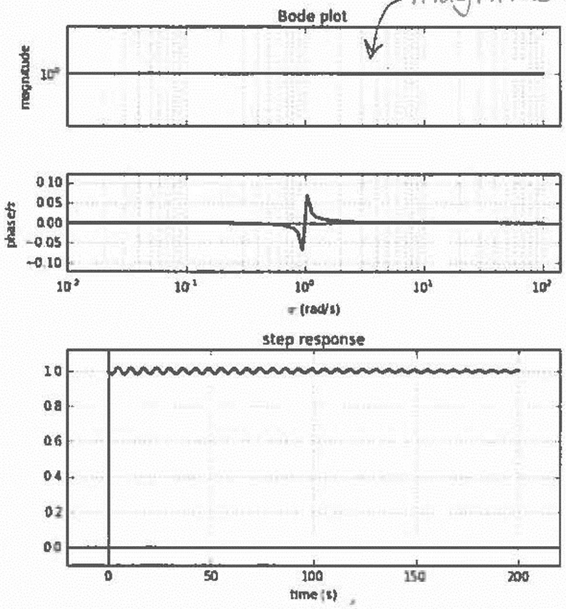
The Bode plot of this system has a band-pass behavior with an even narrower peak than C. The step response shows a second-order shape with more oscillations than C. The pole-zero map looks similar to C, with two poles that have a real component and opposite imaginary components, and a zero at (0,0).

E. $\dfrac{s^2-0.01s+1}{s^2+0.01s+1}$

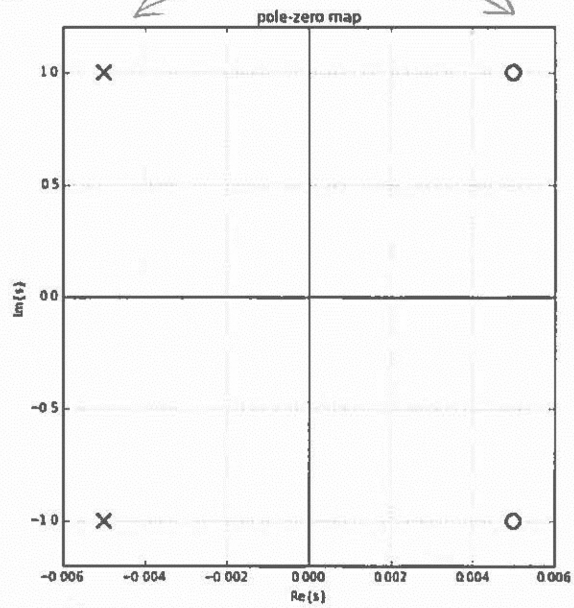
doesn't seem to amplify frequencies

magnitude ~1

poles and zeros very close (~.009 apart)



second order system



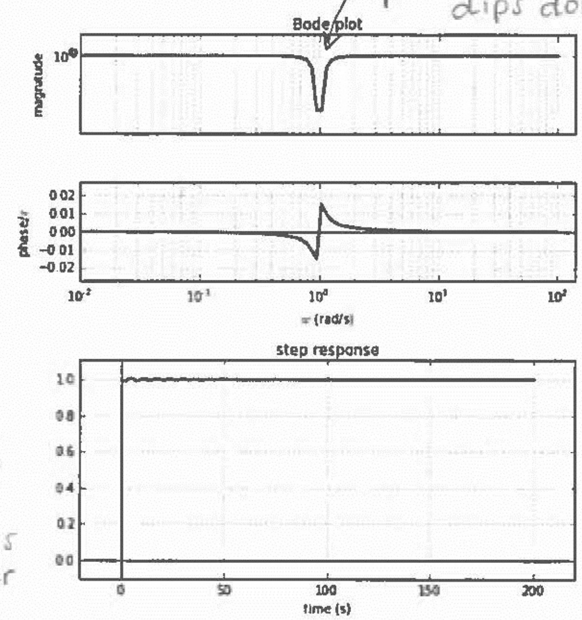
two poles with different imaginary components, and

two zeros with different imag. components.

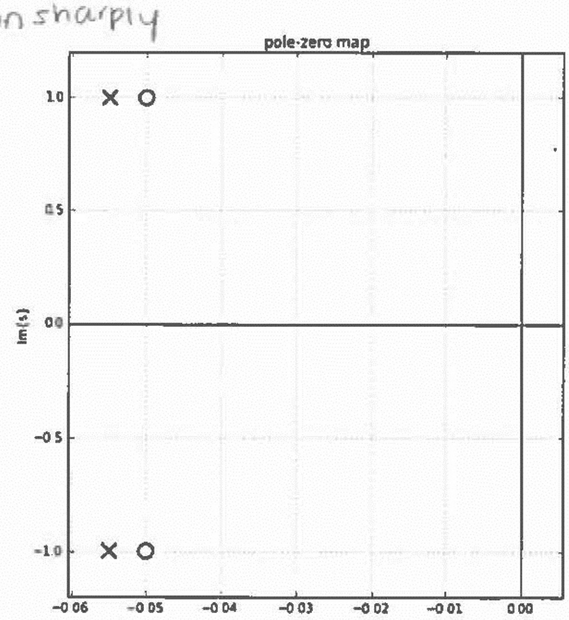
The Bode plot of this system appears flat at 1, implying that the system doesn't amplify the frequencies. The step response shows a second-order system and oscillates for a long time. The pole-zero map has two poles with a ⊖ real component and opposite imaginary components, as well as two zeros with a ⊕ real component and opposite imaginary components.

F. $\dfrac{s^2+0.1s+1}{s^2+0.11s+1}$

peak that dips down sharply



second order system ↓ dampens quicker than E



about the same as above

The Bode plot of this system has a band-stop behavior with a fairly narrow peak, and the step response has a second-order shape with a quicker damping behavior (less oscillations) than E. The pole-zero map also is very similar to E, with two poles and two zeros containing real and imaginary components.

Problem 4

$$H(s) = \frac{1}{s^2 - 0.01s + 1}$$

A. step response $= \frac{1}{s} H(s) = \frac{1}{s} \cdot \frac{1}{s^2 - 0.01s + 1}$

$$\frac{1}{s} \cdot \frac{1}{s^2 - 0.01s + 1} = \frac{A}{s} + \frac{B}{s^2 - 0.01s + 1}$$

$$1 = \cancel{s}(s^2 - 0.01s + 1)\frac{A}{\cancel{s}} + s\cancel{(s^2 - 0.01s + 1)}\frac{B}{\cancel{s^2 - 0.01s + 1}}$$

$$1 = A(s^2 - 0.01s + 1) + Bs$$

$s=0$: $1 = A$

$$1 = s^2 - 0.01s + 1 + Bs$$

$s=1$: $\cancel{1} = \cancel{1} - 0.01 + 1 + B$

$$B = -.99$$

partial fraction expansion again

step response $= \frac{1}{s} - \frac{.99}{s^2 - .01s + 1}$

$$\frac{.99}{s - .005 + .999987i} \cdot \frac{1}{s - .005 - .999987i} = \frac{A}{s - .005 + .999987i} + \frac{B}{s - .005 - .999987i}$$

$$.99 = (s - .005 \pm .999987i)\frac{A}{s - .05 + .999987i} + (s - .005 \pm .999987i)\frac{B}{s - .005 - .999987i}$$

$$.99 = (s - (.005 + .999987i))A + (s - (.005 - .999987i))B$$

$s = .005 + .99987i$: $.99 = (\cancel{.005} + .999987i - \cancel{.005} + .999987i)B$

$.99 = 1.999974iB \rightarrow \underline{B = -0.495006435i}$

$s = .005 - .9999i$: $.99 = (\cancel{.005} - .999987i - \cancel{.005} - .999987i)A$

$.99 = -1.9999874iA \rightarrow \underline{A = 0.495006435i}$

see python plot of step response plot

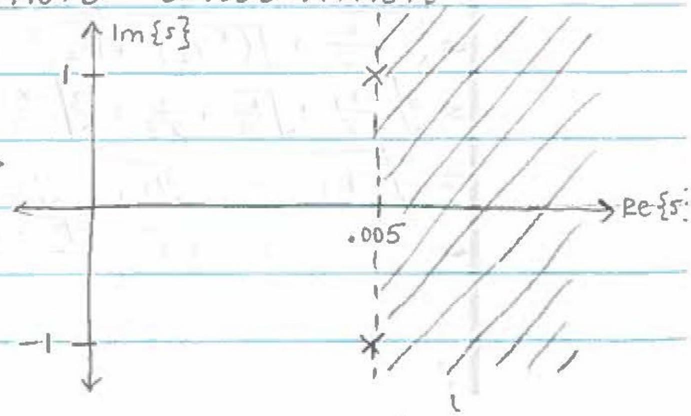
step response $= \frac{1}{s} - \frac{0.495006435i}{s - .005 + .999987i} - \frac{0.495006435i}{s - .005 - .999987i}$

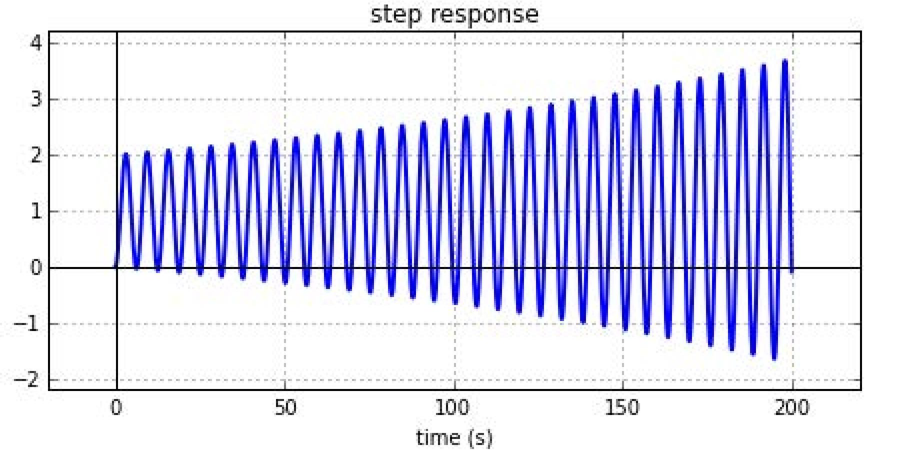
Anyway,

no zeros, poles @:

$$0 = s^2 - .01s + 1$$

$$\frac{.01 \pm \sqrt{.01^2 - 4}}{2} \approx \boxed{.005 \pm j}$$




step response
4
3
2
1
0
−1
−2
0
50
100
150
200
time (s)

B. $\text{gain} = \frac{Y}{Y_{sp}} = \frac{KH}{1+KH}$

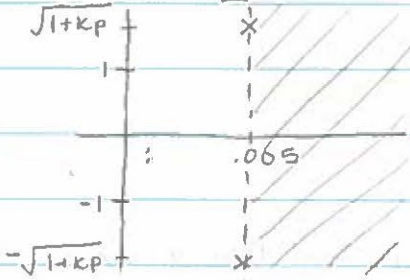
$$= \frac{K_p\left(\frac{1}{s^2-0.01s+1}\right)}{1+K_p\left(\frac{1}{s^2-0.01s+1}\right)} = \frac{K_p}{s^2\cancel{-0.01s}+1}\cdot\frac{s^2\cancel{-0.01s+1}}{s^2-0.01s+1+K_p}$$

$$= \frac{K_p}{s^2-0.01s+1+K_p}$$

no zero, pole @:

$$0 = s^2-0.01s+(1+K_p)$$

$$\frac{.01 \pm \sqrt{.01^2-4(1+K_p)}}{2} = \frac{.01}{2} \pm \frac{\sqrt{\cancel{.0001}-4(1+K_p)}}{2} \approx .005 \pm j\sqrt{1+K_p}$$



The imaginary components of the poles ar[e] shifted out to about $\sqrt{1+K_p}$. So if K_p was 3, the poles would exist at $.005 \pm 2j$, and if K_p was 8, the poles would be $.005 \pm 3j$.

C. $\text{gain} = \frac{Y}{Y_{sp}} = \frac{KH}{1+KH} = \frac{K_1 H}{s+K_1 H}$ ↙ from Problem 2

$$= \frac{K_1\left(\frac{1}{s^2-.01s+1}\right)}{s+K_1\left(\frac{1}{s^2-.01s+1}\right)} = \frac{K_1}{s^2\cancel{-.01s}+1}\cdot\frac{s^2\cancel{-.01s}+1}{s(s^2-.01s+1)+K_1}$$

$$= \frac{K_1}{s(s^2-.01s+1)+K_1}$$

no zero, pole @ Yay, cubic formula...

$$0 = s^3-.01s^2+s+K_1$$

$$\sqrt[3]{\left(\cancel{\frac{.01^3}{27}}-\cancel{\frac{.01}{6}}-\frac{K_1}{2}\right)+\sqrt{\left(\frac{-.01^3}{27}-\frac{.01}{6}-\frac{K_1}{2}\right)^2+\left(\frac{1}{3}-\cancel{\frac{.01^2}{9}}\right)^3}} + \sqrt[3]{〃 - \sqrt{〃}} + \cancel{\frac{.01}{3}}$$ ← wtf?

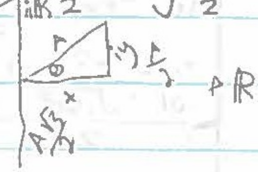
lots of estimating...

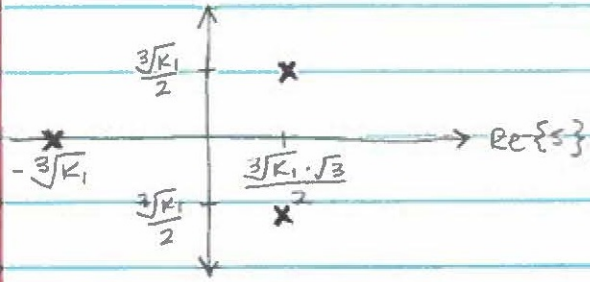
$$\approx \sqrt[3]{\frac{-K_1}{2}+\sqrt{\left(\frac{K_1}{2}\right)^2+\left(\frac{1}{3}\right)^3}} + \sqrt[3]{\frac{-K_1}{2}+\sqrt{\left(\frac{K_1}{2}\right)^2+\frac{1}{3}^3}}$$

$$\approx \sqrt[3]{\frac{-K_1}{2}+\sqrt{\frac{K_1^2}{4}+\cancel{\frac{1}{27}}}} + \sqrt[3]{\frac{-K_1}{2}-\sqrt{\frac{K_1^2}{4}+\cancel{\frac{1}{27}}}} \approx \sqrt[3]{\cancel{\frac{-K_1}{2}+\frac{K_1}{2}}} + \sqrt[3]{\frac{-K_1}{2}-\frac{K_1}{2}}$$

$$\approx \sqrt[3]{-K_1} = -\sqrt[3]{K_1},\ \sqrt[3]{K_1}\cdot e^{\pi/3},\ \sqrt[3]{K_1}\cdot e^{-\pi/3}$$

$$= -\sqrt[3]{K_1},\ \frac{\sqrt[3]{K_1}\cdot\sqrt{3}}{2} \pm \frac{\sqrt[3]{K_1}}{2}$$





There are now three poles - one real pole at $-\sqrt[3]{K_1}$, and the poles $\frac{\sqrt[3]{K_1}\cdot\sqrt{3}}{2} \pm \frac{\sqrt[3]{K_1}}{2}$. I'm not sure why this happens conceptually

D. $$\text{gain} = \frac{Y}{Y_{SP}} = \frac{KH}{1+KH} = \frac{sK_dH}{1+sK_dH}$$

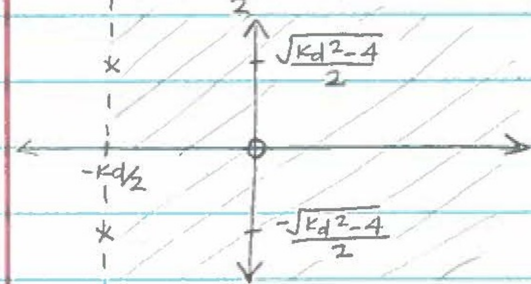
$$= \frac{sK_d\left(\frac{1}{s^2-.01s+1}\right)}{1+sK_d\left(\frac{1}{s^2-.01s+1}\right)} = \frac{sK_d}{\cancel{s^2-.01s+1}} \cdot \frac{\cancel{s^2-.01s+1}}{s^2-.01s+1+sK_d}$$

$$= \frac{sK_d}{s^2+(K_d-.01)s+1}$$

zero @ $s=0$, pole @:

$$0 = s^2+(K_d-.01)s+1$$

$$\frac{(.01-K_d)\pm\sqrt{(K_d-.01)^2-4}}{2} = \frac{.01-K_d}{2} \pm \frac{\sqrt{K_d^2-4}}{2} \approx \frac{-K_d}{2} \pm \frac{\sqrt{K_d^2-4}}{2}$$



The poles now exist at $\frac{-K_d}{2} \pm \frac{\sqrt{K_d^2-4}}{2}$. So if Kd was 2, there would be one pole at -1, and if Kd was 1, the poles would be at $\frac{-1}{2} \pm j\frac{\sqrt{3}}{2}$. Again, don't understand this on a conceptual level.