| | | |
|---|---|---|
| COMP 557 | **MIDTERM** | Prof. Paul Kry |
| Mike Gao | | October 11, 2020 |
| 260915701 | | # 1 – Coordinate Frame |

```
// normal vector is n, n dot p is the point, let s, t be coordinates
Matrix4d coordFrame( const Vec3f &n, const Vec3f &p)
{
  // if it is near x axis
  Vec3f s,t;
  if(n.x > 0.9f) {
    s = Vec3f (0.0 f, 1.0 f, 0.0 f );
  } else {
    s = Vec3f (1.0 f, 0.0 f, 0.0 f);
  }
  s -= n* dot(s, n); // make s orthogonal to n
  s *= rsqrt(dot(s, s)); // normalize s
  t = cross(n, s);
  return (new double[] {
    t.x, s.x, n.x, p.x,
    t.y, s.y, n.y, p.y,
    t.z, s.z, n.z, p.z
    0, 0, 0, 1
  })
}
```

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| 260915701 | | # 2 – Rotation Using Similarity Transform |

Assume the axis pass through $(x_1, y_1, z_1)$ and $(x_2, y_2, z_2)$

(1) Create the axis passing through origin by translating space by $-P_1$ for example

$$T = \begin{pmatrix} 1 & 0 & 0 & -x_1 \\ 0 & 1 & 0 & -y_1 \\ 0 & 0 & 1 & -z_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(2) Rotate axis into one of the original axis, Fix z, let u be a unit vector $(p, q, r)$

Project $u$ onto yz-plane to get $u'$ and rotate by $\alpha$ in order to get $u$ in xz-plane

$u' = \sqrt{q^2 + r^2}$, $cos\alpha = r/u'$ $sin\alpha = q/u'$ $Rx = \begin{pmatrix} 1 & 0 & 0 & -x_1 \\ 0 & \frac{r}{u'} & -\frac{q}{u'} & 0 \\ 0 & \frac{q}{u'} & \frac{r}{u'} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

(3) We then rotate $u'$ to overlap z-axis, $cos\beta = u'/||u|| = u'$ $sin\beta = p/||u|| = p$ $Ry = \begin{pmatrix} u' & 0 & -p & 0 \\ 0 & 1 & 0 & 0 \\ p & 0 & u & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

(4) We then rotate around z-axis by $\theta$ $Rz = \begin{pmatrix} cos\theta & -sin\theta & 0 & 0 \\ sin\theta & cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

Finally, $M = T^{-1}R_x^{-1}R_y^{-1}R_zR_yR_xT$

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$$P = \begin{pmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & n & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

Where this will take in point $(x, y, z, -1)^T$ to $(nx, ny, nz, -z)^T$, After dividing by the $z$ coordinate we have $(-nx/z, -ny/z, n, -1)$ which is the desired point of the near plane

Let $s = \begin{pmatrix} 0 \\ 0 \\ -1 \\ 1 \end{pmatrix}$ be a point on the quadrilateral, and $t = \begin{pmatrix} 0 \\ 0 \\ -2 \\ 1 \end{pmatrix}$ be a point on the wall.

$$P = \begin{pmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & n+10 & 10n \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

$$P_s = \begin{pmatrix} 0 \\ 0 \\ -n-10+10n \\ 1 \end{pmatrix}$$

$$P_t = \begin{pmatrix} 0 \\ 0 \\ -2n-20+10n \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -n-10+5n \\ 1 \end{pmatrix}$$

We are only concerned about the z coordinates of those points

$z_{P_s} = -n - 10 + 10n$

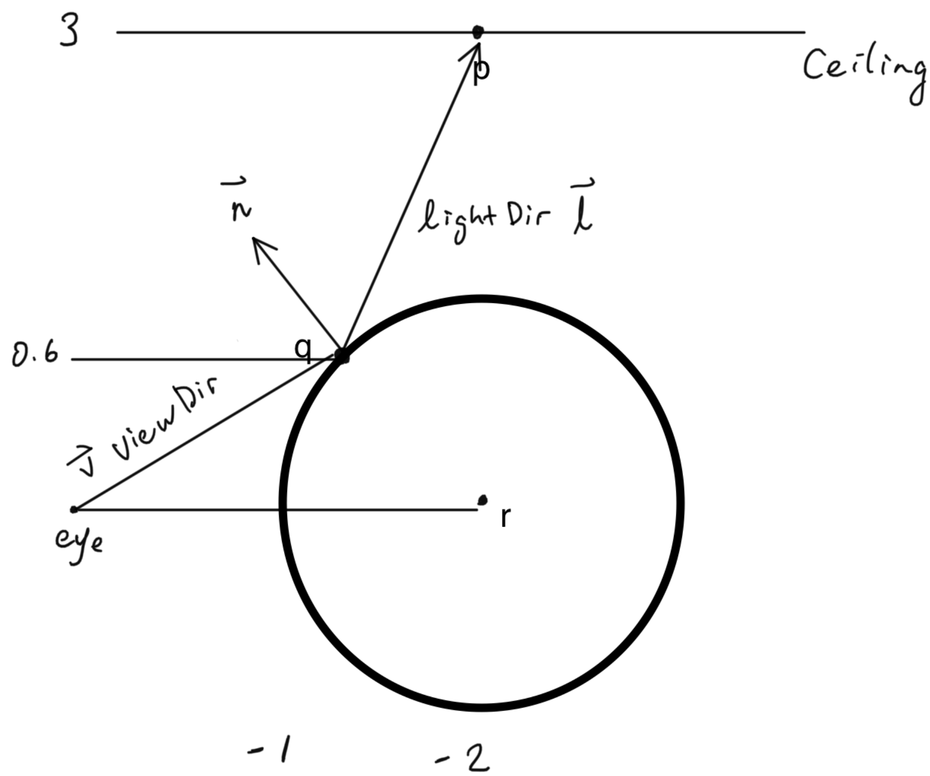
$z_{P_t} = -n - 10 + 5n$

We want to minimize, i.e $|z_{P_s} - z_{P_t}| = \epsilon$ so:

$|-n-10+10n-(-n-10+5n)| = \epsilon$

$|5n| = \epsilon$

$n = \frac{\epsilon}{5}$




Where p is the light position (unknown), r is the center of the circle, q is the brightest spot of a Blinn-Phong specular highlight.

We need to find the position of $q$ first. Since we know $y_q = 0.6$, we get:

$x_q^2 + 0.6^2 = 1$

$x_q = 0.8$

Thus, $z_q = -2 + 0.8 = 1.2$

Now we can find the view direction $v$ and the normal $n$.

$$n = q - r = \begin{pmatrix} 0 \\ 0.6 \\ 0.8 \\ 0 \end{pmatrix}$$

$$v = eye - q = \begin{pmatrix} 0 \\ -0.6 \\ 1.2 \\ 0 \end{pmatrix}$$

Now let's find $l$. Suppose $u$ is a vector such that $v - 2u = l$

$$u = v - proj_n v = \begin{pmatrix} 0 \\ -0.6 \\ 1.2 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 0.36 \\ 0.48 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -0.96 \\ 0.72 \\ 0 \end{pmatrix}$$

$$l = v - 2u = \begin{pmatrix} 0 \\ -0.6 \\ 1.2 \\ 0 \end{pmatrix} - 2\begin{pmatrix} 0 \\ -0.96 \\ 0.72 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1.32 \\ -0.24 \\ 0 \end{pmatrix}$$

Now let's find out the location of the Light

Parametric equations of the direction: $x = 0 \quad y = 0.6 + 1.32t \quad z = -1.2 - 0.24t$

Equation of the ceiling:$y = 3$

$3 = 0.6 + 1.32t$

$t = \frac{20}{11}$

$z = -1.2 - 0.24t$

$z = -\frac{18}{11}$

So the light should be at $\begin{pmatrix} 0 \\ 3 \\ -\frac{18}{11} \\ 1 \end{pmatrix}$

Vertex Shader

```
#version 400 core
uniform mat4 M;
// modeling matrix
uniform mat4 V;
// view matrix
uniform mat4 P;
// projection matrix
uniform mat3 MinvT; // inverse transpose of linear part of M
uniform mat3 VinvT; // inverse transpose of linear part of V

in vec3 VertexNormal;
in vec4 VertexPosition;

out vec4 PositionForFP; // camera coordinates
out vec3 NormalForFP;
// interpolate the normalized surface normal

void main() {

NormalForFP = MinvT * VinvT * VertexNormal; // Should change to NormalForFP= normalize(V*MinvT* vec4(VertexNormal,0))

PositionForFP = V * M * VertexPosition;

gl_Position = P * V * M * VertexPosition;
}
```

Fragment Shader

```
#version 400 core
uniform vec3 LightColor;
// rgb intensity of light
uniform vec3 LightPosition; // light position in camera coordiantes
uniform float Shininess;
// exponent for Blinn-Phong model
uniform vec3 kd;
// diffuse material property

in vec4 PositionForFP; // fragment position in camera coordinates
in vec3 NormalForFP;
// normal at each fragment in camera coordinates

out vec4 FragColor;

void main() {

vec3 LightDirection = PositionForFP - LightPosition; // Should change to LightPosition - PositionForFP

float diffuse = dot( NormalForFP, LightDirection ); // Should change to max( dot(NormalForFP, LightDirection), 0)

vec3 ViewDirection = vec3(0,0,0) - PositionForFP;

HalfVector = (LightDirection + ViewDirection) / 2; // Should change to normalize (LightDirection + ViewDirection)

float specular = max(0.0, dot(NormalForFP, HalfVector));

if (diffuse == 0.0) {

specular = 0.0;

} else {

specular = pow( specular, Shininess );

}

vec3 scatteredLight = kd * LightColor * diffuse;

vec3 reflectedLight = LightColor * specular;

vec3 rgb = min( scatteredLight + reflectedLight, vec3(1,1,1) );

FragColor = vec4( rgb, 1 );
}
```

Yes, I actually looked into his work of OpenVDB, apparently its very widely used as a library of manipulating sparse volumetric data.