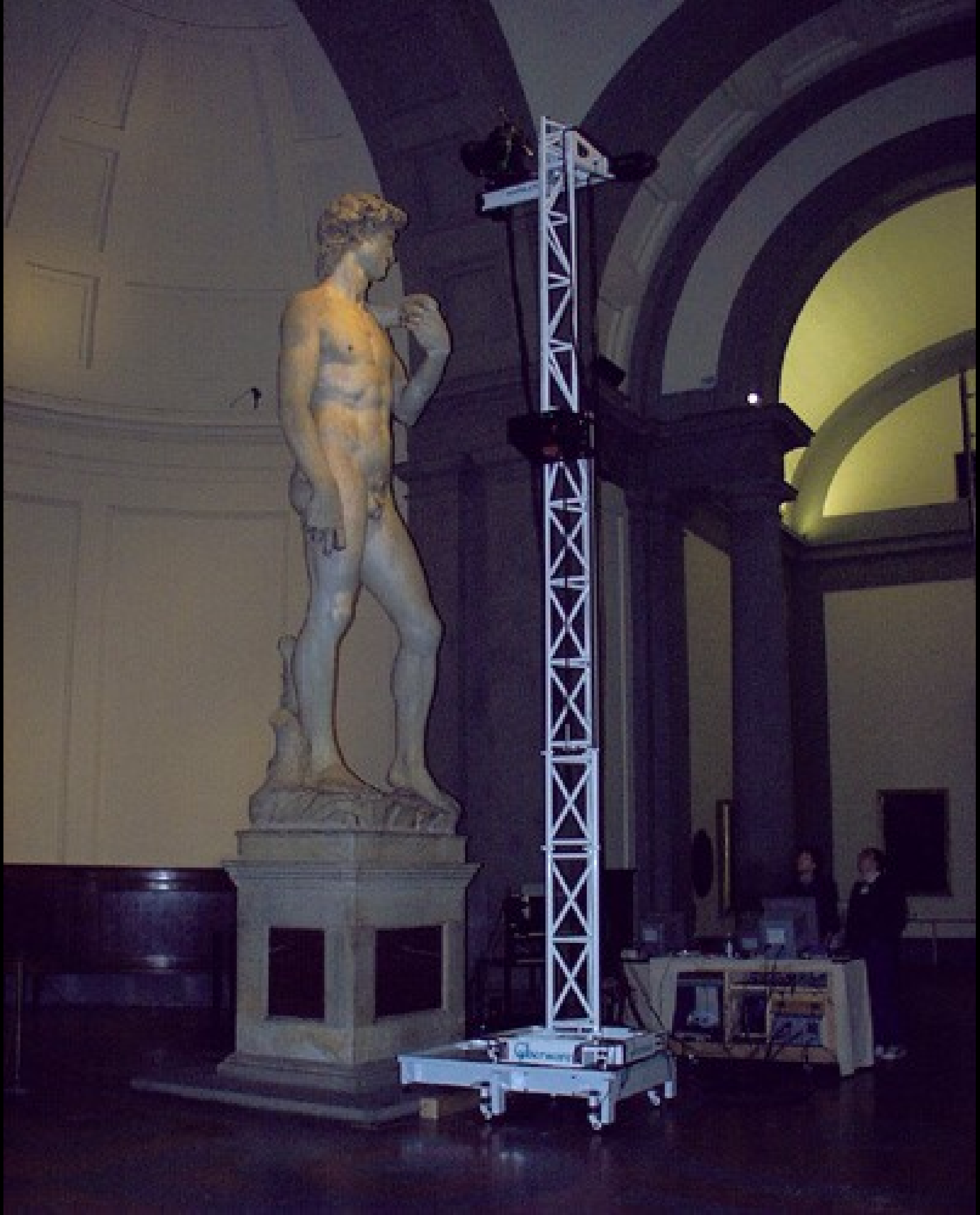
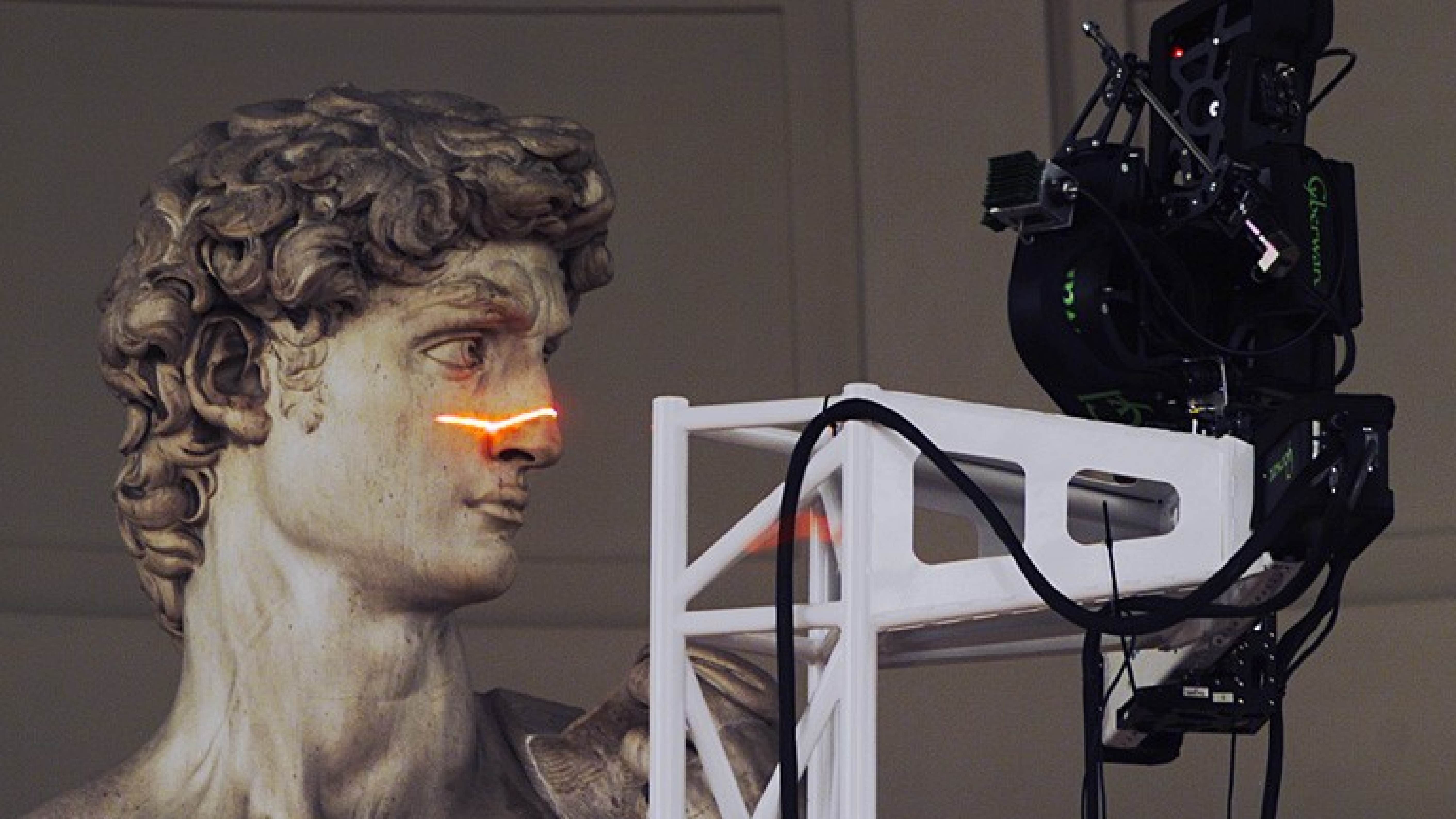
Mesh Simplification

COMP 557

Paul Kry





Level of Detail

- LOD (level of detail)
  - “loosely” the size of the polygons, e.g., length of shortest edge
- Resolution at which a model is displayed could be too coarse or too fine
  - aliasing problems
- David model has 1 billion polygons
- Another important example: terrain

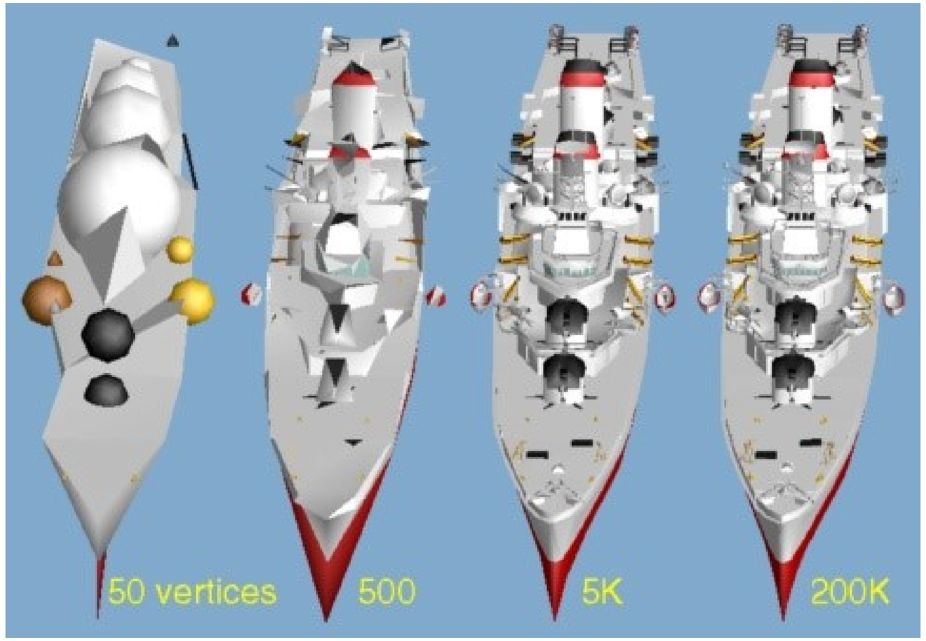
Level of Detail

- Solution: compute several different coarse approximations
- How to choose which model to use?
  - # pixels use in screen space?
  - Continuous levels of detail (progressive meshes) to reduce popping when switching models
  - Viewpoint dependence?
  - Might need multiple levels simultaneously
    - e.g., terrain, coarse geometry for far, fine geometry for near.

Mesh Simplification

- Reduce number of polygons
  - Faster rendering
  - Less storage
  - Simpler manipulation
- Find "good" approximation
  - Visual approximation
  - Geometric approximation
  - Data approximation
- Other desirable qualities
  - Applicability (works on all meshes?)
  - Efficiency, Scalability,
  - Preservation of attributes
    (texture coordinates, normals, etc.)

[Garland and Heckbert 1997]



[Hoppe 1996]

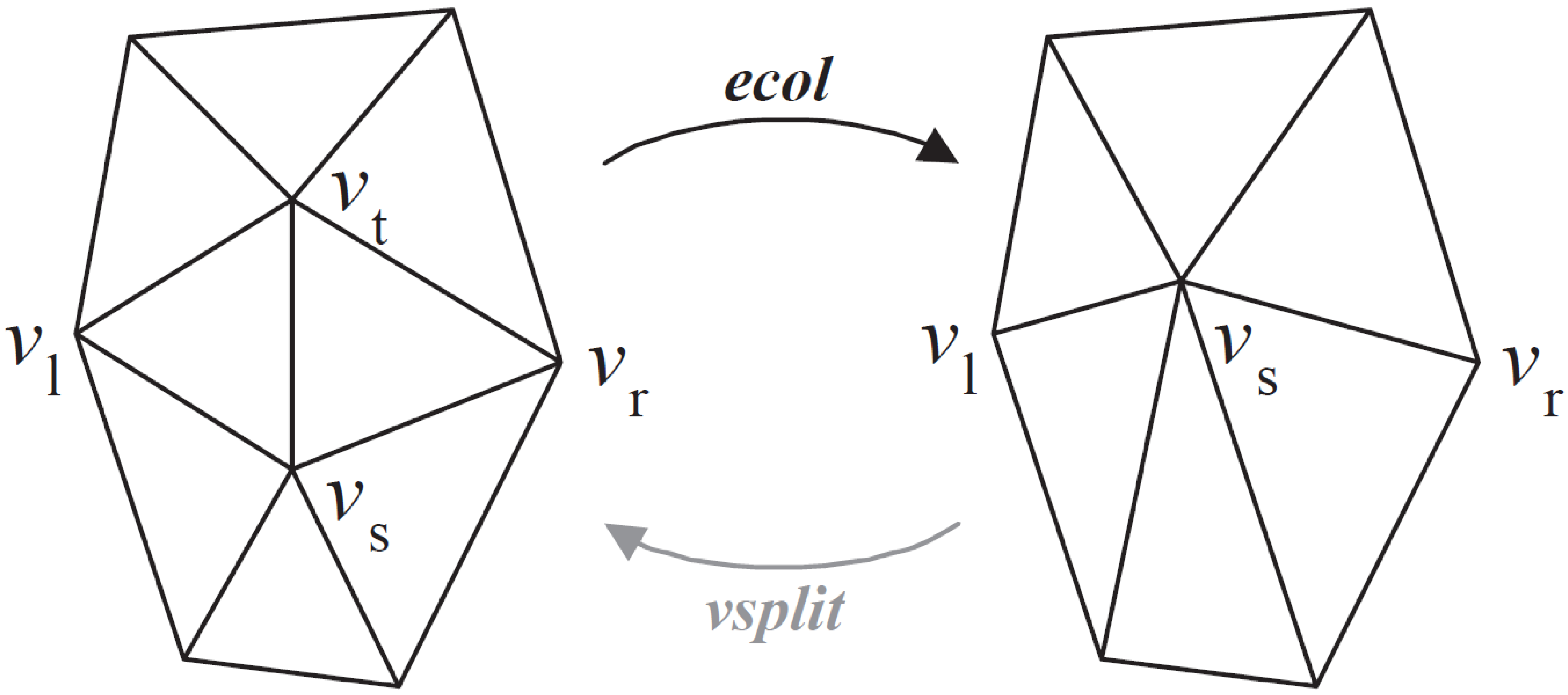
Data Sources

- Measurement
  - Models from laser range finder
  - Iso-surface generation from 3D MRI or CT
  - Terrain from Satellite, Radar, Sonar

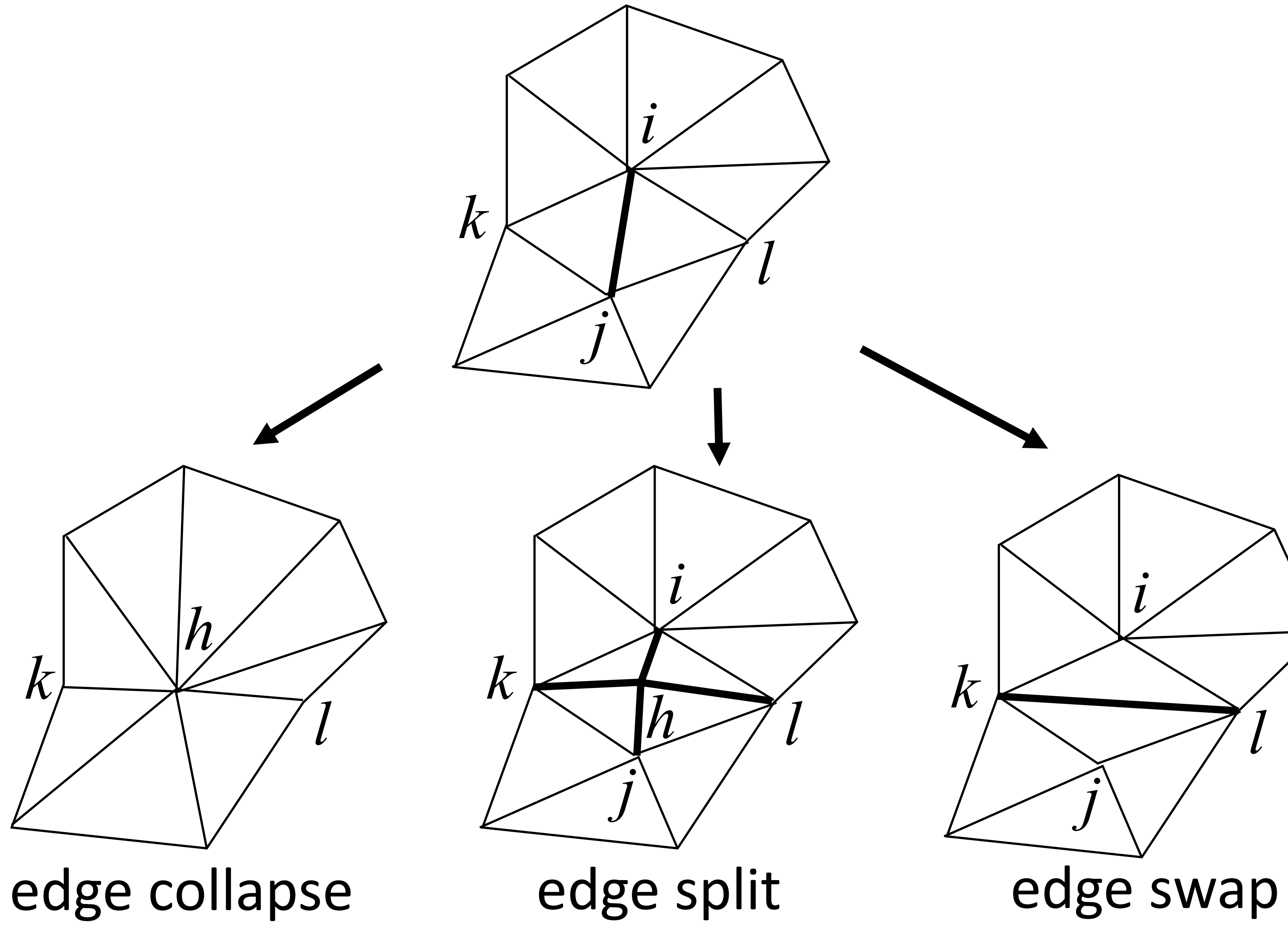
Simplification Approaches

- Geometry refinement
  - Adaptive subdivision
- Geometry resampling
  - Mesh re-tiling
  - Variational Shape Approximation
    - Find a set of geometric proxies that fit the data
- Geometry decimation
  - Vertex decimation
    - remove vertices in planar regions and fill hole with triangles
  - Edge contraction
  - Vertex merging

Local Modification (invertible)



Local Modification (invertible)



Global Modifications

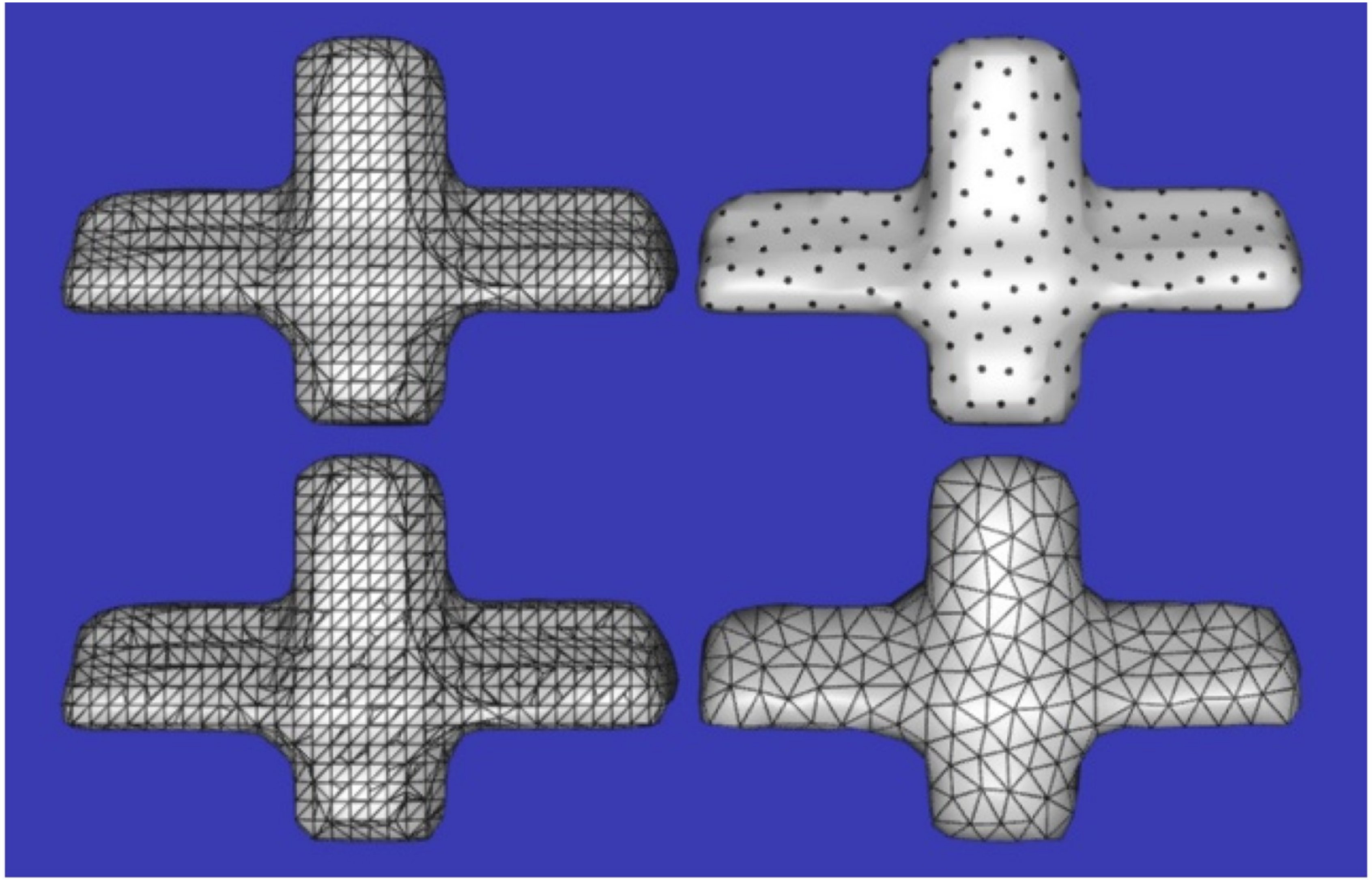


Figure 1: Re-tiling of a radiation iso-dose surface. Upper left: Original surface. Upper right: Candidate vertices after point-repulsion. Lower left: Mutual tessellation. Lower right: Final tessellation.

Global Modifications

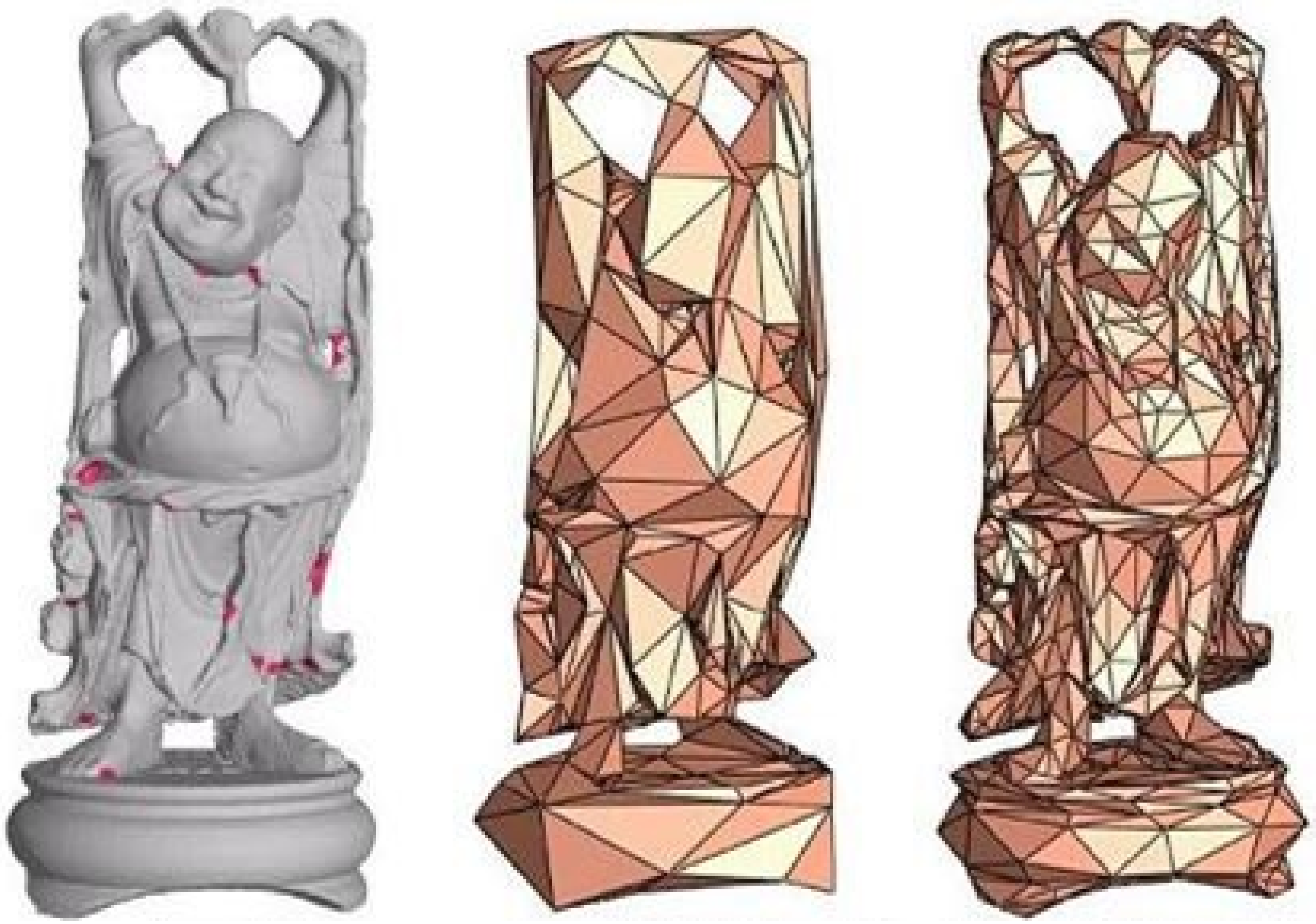
Variational shape approximation from [Cohen-Steiner et al. 2004].

Characteristics

- Speed vs quality?
- Type of Mesh
  - Height field or parametric
  - Manifold
  - Polygon soup
- Modifies topology?
- Continuous LOD?
- View-Dependent refinement?
- Simplify topology?
  - David head with genus 340?

Fixing Topology

- Preserving topology during simplification is not always a good idea

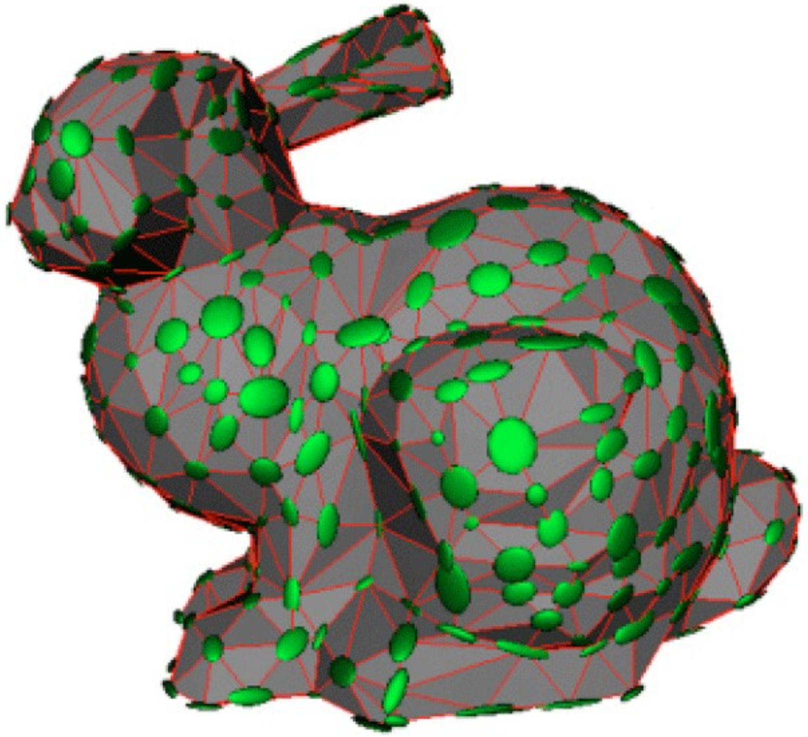


Genus 104 | **Genus 104 2K triangles** | **Genus 6 2K triangles topologically simplified**

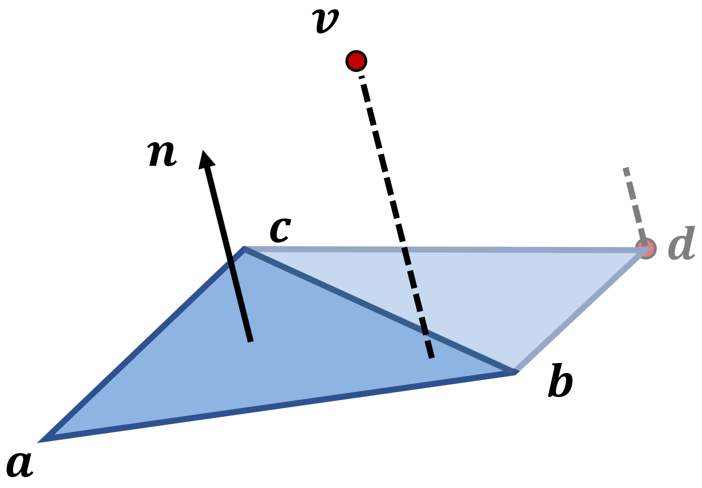
Quadric error metric for mesh simplification

- Let us look more closely at a specific method
- Only use edge collapses
- Choose a good location for collapsed edges
- Always collapse edge with minimal error

SIGGRAPH 1997, Garland, Heckbert
Surface simplification using quadric error metrics



Signed distance to triangle's plane



$$\boldsymbol{n} = \frac{(\boldsymbol{b}-\boldsymbol{a}) \times (\boldsymbol{c}-\boldsymbol{a})}{\|(\boldsymbol{b}-\boldsymbol{a}) \times (\boldsymbol{c}-\boldsymbol{a})\|}$$

$$D = -\boldsymbol{n}_x\boldsymbol{a}_x - \boldsymbol{n}_y\boldsymbol{a}_y - \boldsymbol{n}_z\boldsymbol{a}_z$$

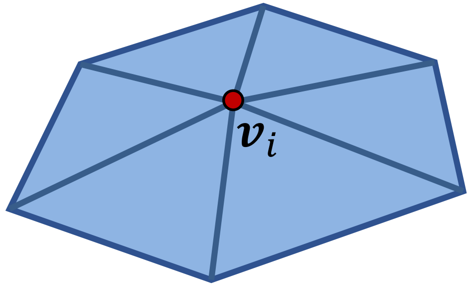
- Implicit plane equation
  $$Ax + By + Cz + D = 0$$
  - Plane normal $\boldsymbol{n} = (A, B, C)^T$
  - With unit length $\boldsymbol{n}$, equation gives the signed distance of point $\boldsymbol{v} = (x, y, z, 1)^T$ from the plane
- Letting $\boldsymbol{p} = (A, B, C, D)^T$, can compute signed distance in homogenous coordinates as
  $$\boldsymbol{p}^T\boldsymbol{v}$$

Squared distance to plane

$$K_{\boldsymbol{p}} = \begin{pmatrix} A^2 & \mathrm{AB} & \mathrm{AC} & AD \\ \mathrm{BA} & B^2 & \mathrm{BC} & BD \\ \mathrm{CA} & \mathrm{CB} & C^2 & CD \\ DA & DB & DC & D^2 \end{pmatrix}$$

$$\begin{aligned} \|\boldsymbol{p}^T\boldsymbol{v}\|^2 &= (\boldsymbol{p}^T\boldsymbol{v}) \cdot (\boldsymbol{p}^T\boldsymbol{v}) \\ &= \boldsymbol{v}^T\boldsymbol{p}\boldsymbol{p}^T\boldsymbol{v} \\ &= \boldsymbol{v}^T K_{\boldsymbol{p}}\boldsymbol{v} \qquad \text{where } K_{\boldsymbol{p}} = \boldsymbol{p}\boldsymbol{p}^T \end{aligned}$$

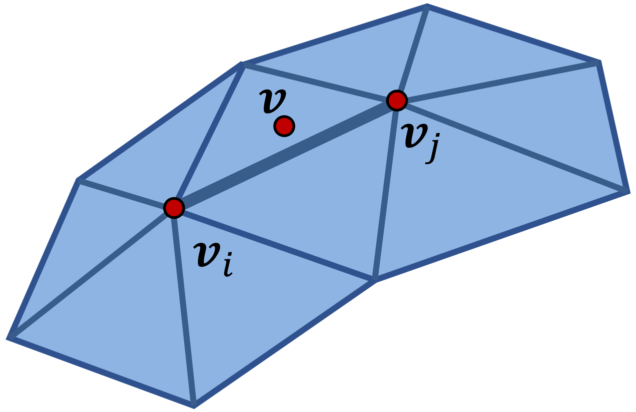
- Now, consider the sum of squared distances to the planes defined by the triangles adjacent to a vertex



$$\sum_{\boldsymbol{p}\in\mathrm{adj}(\boldsymbol{v}_i)} \boldsymbol{v}^T K_{\boldsymbol{p}}\boldsymbol{v} = \boldsymbol{v}^T Q_i \boldsymbol{v}$$

Where $Q_i = \sum_{\boldsymbol{p}\in\mathrm{adj}(\boldsymbol{v}_i)} K_{\boldsymbol{p}}$

Quadric Error Metric



- Each face defines a plane
- Each vertex lies in the planes of all its adjacent faces
- Consider moving a vertex to a new position $\boldsymbol{v}$
  - How well does vertex $\boldsymbol{v}$ lie in a set of planes?
  - Sum of squared distances to adjacent planes, $\min_{\boldsymbol{v}} \boldsymbol{v}^T Q_i \boldsymbol{v}$
- To collapse an edge, we ask this question for both vertices on either side of the edge, specifically, minimizing for best choice of position v for both ends simultaneously (we will want to regularize this)

$$e = \min_{v} v^T \left(Q_i + Q_j\right) v$$

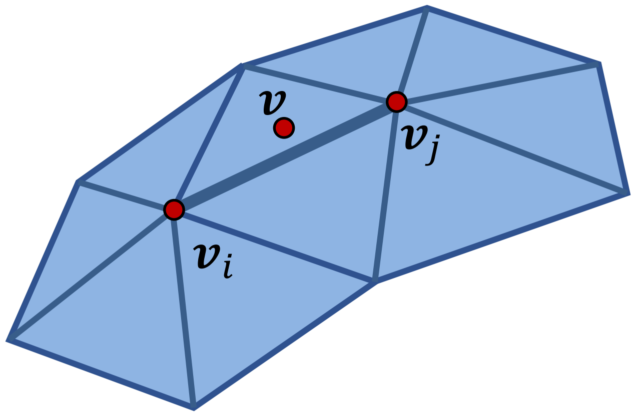
Solving $e = \min_{\boldsymbol{v}} v^T (Q_i + Q_j) v$

- Take care in minimizing the quadratic function because $\boldsymbol{v}$ is in homogeneous coordinates
- Can rewrite as (nonhomogeneous)

$$\min_{\boldsymbol{v}} \boldsymbol{v}^T A \boldsymbol{v} + 2\boldsymbol{b}^T \boldsymbol{v} + c$$

- Here, A, $\boldsymbol{b}$, and c come from

$$(Q_i + Q_j) = \begin{pmatrix} A_{xx} & A_{xy} & A_{xz} & b_x \\ A_{yx} & A_{yy} & A_{yz} & b_y \\ A_{zx} & A_{zy} & A_{zz} & b_z \\ b_x & b_y & b_z & c \end{pmatrix}$$



Matrix A is symmetric, minimum occurs where gradient is zero

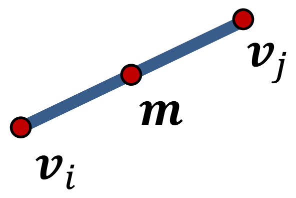
$$2A\boldsymbol{v} + 2\boldsymbol{b} = 0$$

That is, solve for v in

$$A\boldsymbol{v} = -\boldsymbol{b}$$

Regularization

$$Q_{reg} = \begin{pmatrix} 1 & 0 & 0 & -\boldsymbol{m}_x \\ 0 & 1 & 0 & -\boldsymbol{m}_y \\ 0 & 0 & 1 & -\boldsymbol{m}_z \\ -\boldsymbol{m}_x & -\boldsymbol{m}_y & -\boldsymbol{m}_z & \boldsymbol{m}^T\boldsymbol{m} \end{pmatrix}$$



- If all planes of triangles adjacent to the two vertices of an edge are coplanar, then any point in the plane has zero error, and A will not be full rank
- Good location for this case: the edge midpoint, $\boldsymbol{m}$
- With $\boldsymbol{m} = \frac{1}{2}(\boldsymbol{v}_i + \boldsymbol{v}_j)$, distance squared between $\boldsymbol{v}$ and $\boldsymbol{m}$ is

$$(\boldsymbol{v} - \boldsymbol{m})^{\boldsymbol{T}}(\boldsymbol{v} - \boldsymbol{m}) = \boldsymbol{v}^{\boldsymbol{T}} I \boldsymbol{v} - 2\boldsymbol{m}^T\boldsymbol{v} + \boldsymbol{m}^T\boldsymbol{m}$$

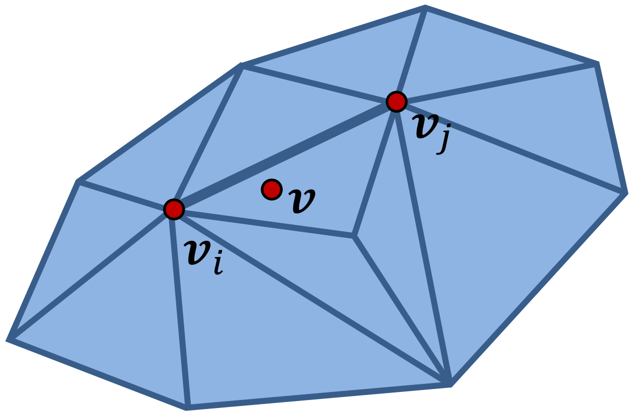
- Use this to regularize the problem with small factor γ
- Instead, solve $e = \min_{\boldsymbol{v}} \boldsymbol{v}^T\left(Q_i + Q_j + \gamma Q_{reg}\right)\boldsymbol{v}$

Quadratic Error Metric Implementation Issues

- Let us discuss the following issues
  - Regularization of the minimization problem
    - Use distance squared to point halfway along an edge
  - Computation of the error for each edge
    - Solving the minimum of the quadratic equation
    - That is, take the derivative and solve for the zero
  - Use a priority queue to keep track of which edge should be collapsed next
    - Each collapse requires adjacent edges to be revisited
    - Removed from queue, recompute error, re-insert in queue

Edge Collapse Problems

- Preserve topology?
- Preserve the manifold?
- Don’t create self-intersections in geometry?
- Don’t create non-manifold topology, use heuristics
  - Number of common adjacent vertices to collapsing edge should be 2
  - If {i} and {j} are both boundary vertices, only collapse if {i,j} is a boundary edge



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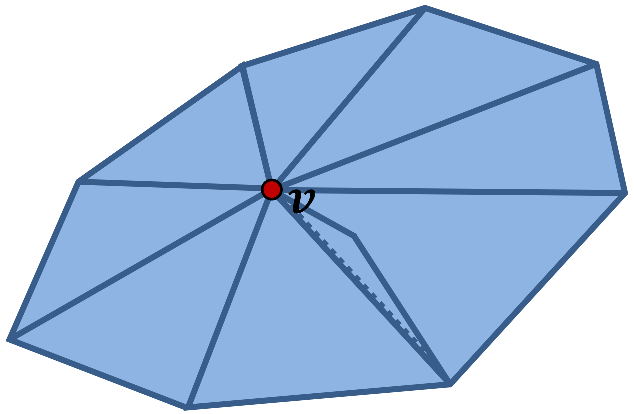
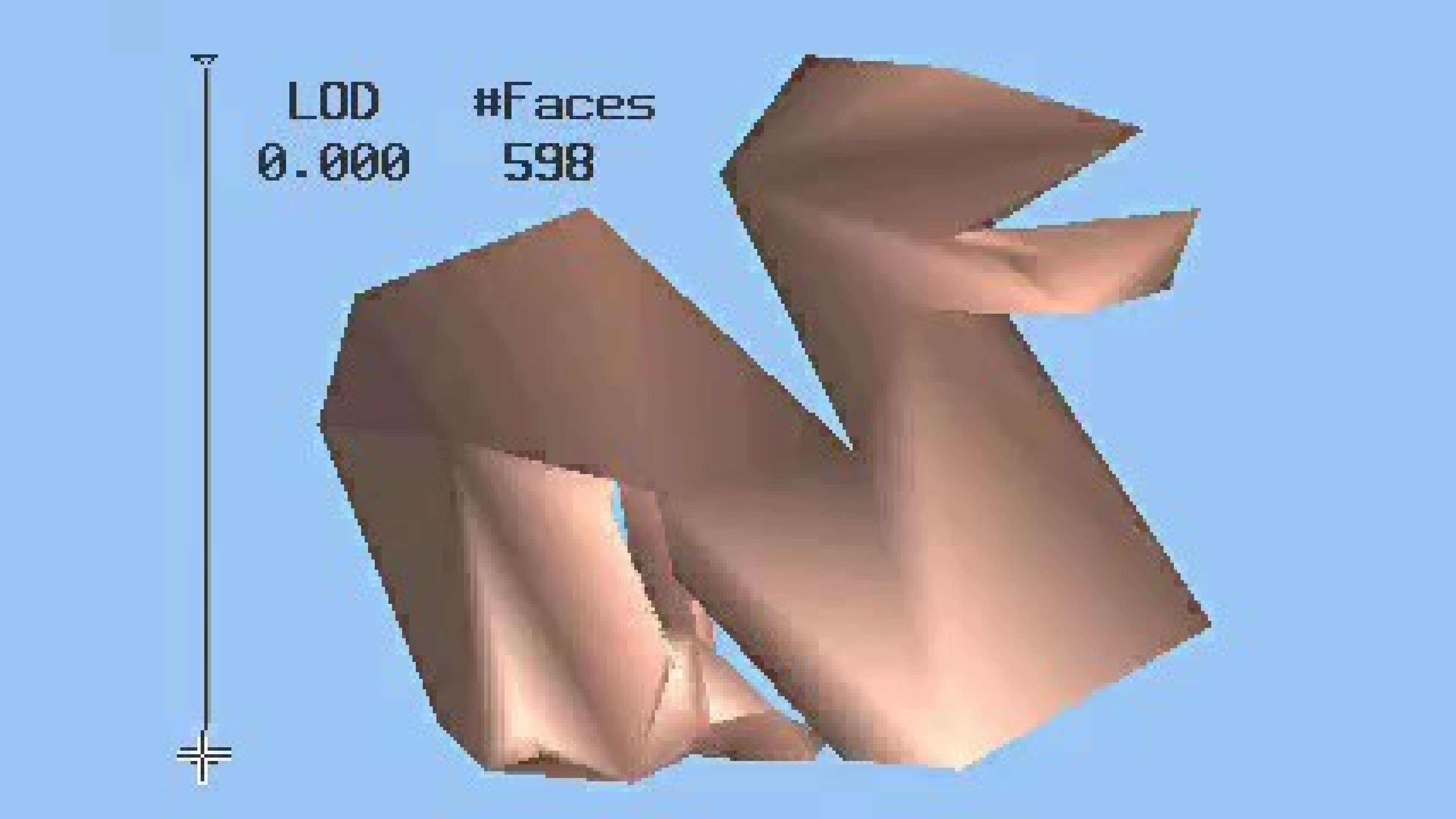
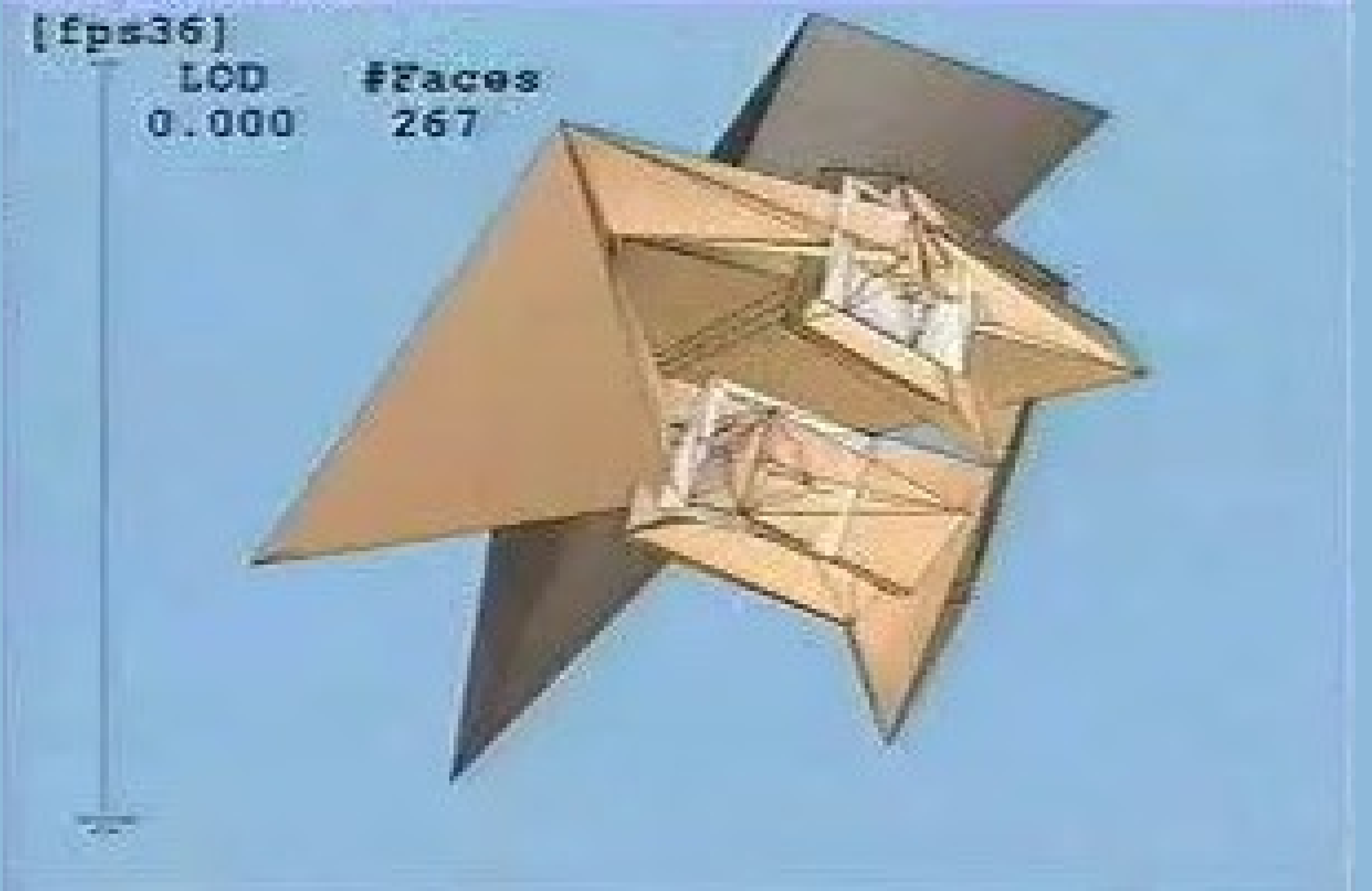


Quadratic Error Metric - Simplification Steps

- Compute planes equation for each face, $\boldsymbol{p}$
- Compute quadratic function coefficient for each face, $\boldsymbol{K}$
- Compute quadratic function coefficient for each vertex, $\boldsymbol{Q}$
- Solve optimal vertex position and error for each edge (i, j) using $\boldsymbol{Q}_i + \boldsymbol{Q}_j$ and possibly including a regularization term.
- Insert all edge errors into a priority queue, with the minimum error at the top of the queue
- Pop top off queue until we find an edge collapse that does not cause problems (avoid bad topology, perhaps check geometry)
- Collapse the edge to optimal vertex position, set quadratic function coefficient of this vertex as $\boldsymbol{Q}_i + \boldsymbol{Q}_j$,
- Remove adjacent edges from queue, re-compute edge collapse error, and reinsert into queue
- Repeat until desired level of detail is reached.


LOD #Faces
0.000 48


LOD #Faces
0.000 598


[fps36]
LOD #Faces
0.000 267

More information

- Surface simplification using quadric error metrics
    - Garland and Heckbert, 1997
    - http://dl.acm.org/citation.cfm?id=258849
- Progressive meshes
    - Hoppe, 1996
    - http://research.microsoft.com/en-us/um/people/hoppe/proj/pm/
- CGPP Chapter 25.4 Level of Detail and Progressive Meshes