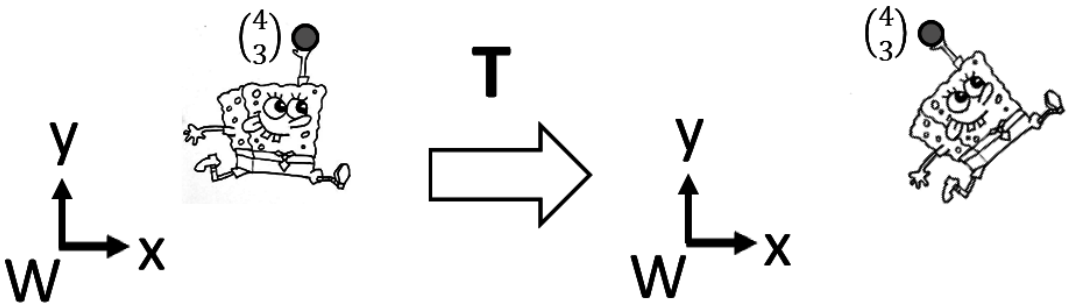
Transformations, Viewing, Projection (7 Marks)

[1] 1. Given four homogeneous points $p_0 = (9, 12, 6, 1.5)$, $p_1 = (12, 16, 8, 2)$, $p_2 = (9, 12, 6, 1)$, and $p_3 = (18, 24, 12, 3)$. All of them except one represent the same 3D point. What is that 3D point, and which of the homogeneous points $p_0, ...p_3$ represents a different 3D point?

[3] 2. Let $\boldsymbol{p}_A$ be a point, $\boldsymbol{v}_A$ a vector, and $\boldsymbol{n}_A$ a normal vector, all expressed in homogeneous coordinates of reference frame A. Let reference frame B be different from reference frame A. Let the 4x4 matrix $\mathsf{M} = \begin{pmatrix} \boldsymbol{x} & \boldsymbol{y} & \boldsymbol{z} & \boldsymbol{c} \end{pmatrix}$, where $\boldsymbol{x}$, $\boldsymbol{y}$, and $\boldsymbol{z}$ are vectors that specify the axes of frame B in homogeneous coordinates of frame A, and $\boldsymbol{c}$ is a point that specifies the origin of frame B in homogeneous coordinates of frame A. Use the matrix M to write the expressions for computing the point, vector, and normal in coordinates of reference frame B, that is, $\boldsymbol{p}_B$, $\boldsymbol{v}_B$, and $\boldsymbol{n}_B$, respectively.

[3] 3. Suppose you are given SpongeBob as a set of points expressed in a 2D coordinate system W. Give a sequence of affine transformations that have the action of rotating the points by 45 degrees counter clockwise while leaving the point at $(x, y) = (4, 3)$ fixed. Use homogeneous representation for your transformations.



Subdivision and Splines (5 Marks)

[1] 4. Consider a subdivision curve using the Chaikin scheme averaging mask $(r_{-1}, r_0, r_1) = (0, 0.5, 0.5)$, which averages the position of a point with its neighbour to the right. In class, we only saw an example of using the Chaikin scheme to subdivide a closed curve (one where the end wraps around to the beginning). What is a reasonable special rule to define for the endpoints of a curve subdivided with the Chaikin scheme?

[1] 5. Whats the difference between even and odd vertices in a subdivision scheme?

[1] 6. How many real scalar values are necessary to describe the shape of a rational Bezier curve? Suppose the curve is in d dimensions and the degree of the Bezier is n. What is your answer when $d = 2$ and $n = 3$.

[2] 7. Show that the Bernstein basis polynomails $B_{i,n}(t)$ for $i = 1 \cdots n - 1$ attains only one maximum on $[0, 1]$ and does so at $t = i/n$. *Hint: take the derivative and set it to zero.*

Triangles and Meshes (10 Marks)

[2] 8. Write a parametric equation for the plane that contains the triangle defined by points $A, B, C \in \mathbb{R}^3$.

[2] 9. Write an implicit equation for the plane that contains the triangle defined by points $A, B, C \in \mathbb{R}^3$.

[2] 10. List the most important advantages and disadvantages of triangles and quadrilaterals.

[1] 11. Describe and sketch a mesh which is an orientable manifold with one boundary and genus one. Note you do not need to draw the mesh showing all the faces. You just need to draw enough to illustrate your answer.

[3] 12. Write pseudocode for method `CollapseEdge( HalfEdge e, Vertex v )`, which performs an edge collapse on a half edge data structure. The method takes as parameter a half edge `e` specifying the edge to collapse, and a new vertex `v` with the new position for the collapsed edge vertices. Assume the mesh is a watertight manifold (i.e., no boundaries). You may also assume that the two faces involved will be removed from the face list by the caller. Draw a diagram with labels on the mesh around the edge being collapsed to help explain variables in your code.

Rasterization, Texture Mapping, Illumination (5 Marks)

[2] 13. Suppose you are rendering an image to a HD screen, 1920 by 1080 pixels, and have a vertical FOV of 27 degrees. You have have an eye point at 10 units on the positive z axis and a look at point set to the origin, and y axis as the up direction. The scene consists of a 1 unit by 1 unit square, centered at the origin or the world, with normal in the z direction. The square has a texture stored in a 512x512 image. Does this scenario need a minificaiton filter or a magnificaiton filter? Show your work to explain your answer.

[2] 14. Suppose you must compute texture coordinates for a leg of an animated character. In its local coordinates, suppose it is centered at the origin and the length of the leg is oriented in the z axis. Write pseudocode to compute a cylinder mapping, that is, for every vertex v compute a texture coordinate in the interval [0,1] using a cylinder mapping.

[1] 15. What kind of projection would you use to create a shadow map texture for a directional light?

Colour (3 Marks)

[3] 16. In the following diagram, the one eyed monster has designed a matrix so that a glowing luminous coloured disk has the same appearance in real life as it does when displayed on the monitor. Suppose that the camera is special in that it records a discretized vector $\boldsymbol{s}$ for spectral power density at each pixel. Assume that the monster has cone receptors on his retina identical to those of a human. Describe how this is possible (to design such a matrix) and under what conditions the colours can actually be made to match. You might start by formalizing how the monitor produces a spectral power density at each pixel given an RGB colour specification.