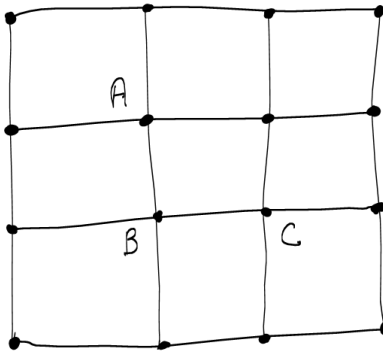


Questions

1. Suppose you are given three vertices $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3 \in \mathbb{R}^3$ which define a triangle. Suppose you are also given a point \mathbf{v} on the plane containing that triangle. How would you find a, b, c such that $\mathbf{v} = a\mathbf{v}_1 + b\mathbf{v}_2 + c\mathbf{v}_3$ and $a + b + c = 1$?
2. Suppose you are given three vertices $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ and \mathbf{v} as in the previous question. How would you decide if point \mathbf{v} lies in the triangle defined by $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$?
3. Suppose we have several properties that are defined on the vertices of a triangle, e.g. surface normals, RGB values, and possibly other values. What might be the advantage of parameterizing the points within the triangle using an a, b, c as above?
4. Given the following mesh, show what happens when you collapse edge AB to vertex B, and then you collapse edge BC to C. How many vertices, edges, and faces are lost in each case?



Answers

- Without the constraint that \mathbf{v} lies on the plane containing the triangle, we would just have a linear system of 3 equations and 3 unknowns, that is,

$$\begin{bmatrix} \mathbf{v}_x \\ \mathbf{v}_y \\ \mathbf{v}_z \end{bmatrix} = \begin{bmatrix} \mathbf{v}_{1,x} & \mathbf{v}_{2,x} & \mathbf{v}_{3,x} \\ \mathbf{v}_{1,y} & \mathbf{v}_{2,y} & \mathbf{v}_{3,y} \\ \mathbf{v}_{1,z} & \mathbf{v}_{2,z} & \mathbf{v}_{3,z} \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

The constraint that \mathbf{v} lies on the plane means that we can be sure there is a solution such that $a + b + c = 1$. To find that solution, we could just solve the original system, of course. But to take advantage of the stronger constraint we do the following:

$$\mathbf{v} = (1 - b - c)\mathbf{v}_1 + b\mathbf{v}_2 + c\mathbf{v}_3$$

and so

$$\mathbf{v} - \mathbf{v}_1 = b(\mathbf{v}_2 - \mathbf{v}_1) + c(\mathbf{v}_3 - \mathbf{v}_1)$$

which is now three equations (i.e. x, y, z) with two unknown b, c . So we can use just two of the equations to solve for b, c .

$$\begin{bmatrix} \mathbf{v}_x - \mathbf{v}_{1,x} \\ \mathbf{v}_y - \mathbf{v}_{1,y} \end{bmatrix} = \begin{bmatrix} \mathbf{v}_{2,x} - \mathbf{v}_{1,x} & \mathbf{v}_{3,x} - \mathbf{v}_{1,x} \\ \mathbf{v}_{2,y} - \mathbf{v}_{1,y} & \mathbf{v}_{3,y} - \mathbf{v}_{1,y} \end{bmatrix} \begin{bmatrix} b \\ c \end{bmatrix}$$

- Solve for b and c as in the previous question, and check that both are between 0 and 1, and that $1 - b - c$ is between 0 and 1. If any of those conditions fails to hold, then the point \mathbf{v} is not in the triangle.
- Once you know the a, b, c values for a point \mathbf{v} , you can interpolate any values that are defined on the vertices. Interpolating the normals and RGB values comes up next lecture, for example, when we discuss shading.
- In the collapse from AB to B, one vertex is lost, one edge is lost, and no faces are lost.
In the collapse from BC to C, one vertex is lost, two edges are lost, and one face is lost.

