COMP 557	**MIDTERM**	Prof. Paul Kry
Mike Gao		October 11, 2020
260915701		# 1 – Coordinate Frame

```
// normal vector is n, n dot p is the point, let s, t be coordinates
Matrix4d coordFrame( const Vec3f &n, const Vec3f &p)
{
  Vec3f s,t;
  // if n is near x axis
  if(n.x > 0.9f) {
    s = Vec3f (0.0 f, 1.0 f, 0.0 f );
  } else {
    s = Vec3f (1.0 f, 0.0 f, 0.0 f);
  }
  s -= n* dot(s, n); // make s orthogonal to n
  s *= rsqrt(dot(s, s)); // normalize s
  t = cross(n, s);
  return (new double[] {
    t.x, s.x, n.x, p.x,
    t.y, s.y, n.y, p.y,
    t.z, s.z, n.z, p.z
    0, 0, 0, 1
  })
}
```

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260915701		# 2 – Rotation Using Similarity Transform

Assume the axis pass through two points, $P_1 = (x_1, y_1, z_1)$ and $P_2 = (x_2, y_2, z_2)$

(1) Create the axis passing through origin by translating space by $-P_1$ for example

$$T = \begin{pmatrix} 1 & 0 & 0 & -x_1 \\ 0 & 1 & 0 & -y_1 \\ 0 & 0 & 1 & -z_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(2) Rotate space about the x axis so that the rotation axis lies in the xz plane.

Let u be a unit vector (p, q, r) along the rotation axis.

Project u onto yz-plane, let $s = \sqrt{q^2 + r^2}$ be the length of the projection. Rotate by α in order to get u in xz-plane

$$cos\alpha = \frac{r}{s}\ sin\alpha = \frac{q}{s}\ Rx = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{r}{s} & -\frac{q}{s} & 0 \\ 0 & \frac{q}{s} & \frac{r}{s} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(3) We then rotate by β so the axis overlap the z-axis:

$$cos\beta = \frac{s}{||u||} = s\ sin\beta = \frac{-p}{||u||} = -p$$

$$Ry = \begin{pmatrix} s & 0 & -p & 0 \\ 0 & 1 & 0 & 0 \\ p & 0 & s & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(4) We then rotate around z-axis by the given angle θ

$$Rz = \begin{pmatrix} cos\theta & -sin\theta & 0 & 0 \\ sin\theta & cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Finally, $p' = T^{-1}R_x^{-1}R_y^{-1}R_zR_yR_xTp$

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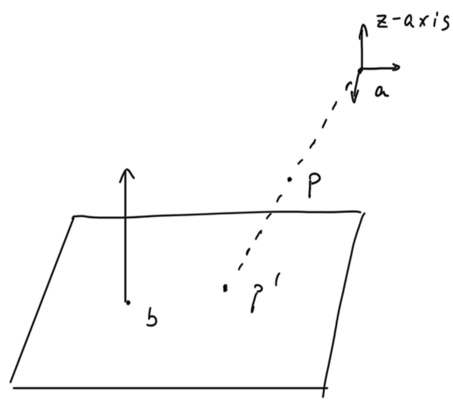
260915701		# 3 – Perspective Projection Matrix

$$P = \begin{pmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & n & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

Where this will take in point $(x, y, z, 1)^T$ to $(nx, ny, nz, -z)^T$, After dividing by the z coordinate we have $(-nx/z, -ny/z, -n, 1)$ which is the desired point of the near plane



As shown above, a point light project $a \in \mathbb{R}^3$ point $p \in \mathbb{R}^3$ onto the plane with normal $\vec{n}$ at p'. We let b be the origin of the canonical basis and $\vec{n}$ be the z-axis

1. Using coordFrame() from Q1, we can transform a, b, p from world frame to camera frame: $F = coordFrame(\vec{n}, a)^{-1}$
2. We let $\vec{n}$ be our z axis. We then find the near plane's position with b: $n = b.z$
3. Since b is in camera frame and we need its coordinate in world frame, we then project onto the ground plane using the projection matrix P from Q3
$$P = \begin{pmatrix} b.z & 0 & 0 & 0 \\ 0 & b.z & 0 & 0 \\ p & 0 & b.z & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$
4. $p' = F^{-1}PFp$

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260915701		# 5 – What is the closest near plane you can set?

Let $s = \begin{pmatrix} 0 \\ 0 \\ -1 \\ 1 \end{pmatrix}$ be a point on the quadrilateral, and $t = \begin{pmatrix} 0 \\ 0 \\ -2 \\ 1 \end{pmatrix}$ be a point on the wall.

$$P = \begin{pmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & n+10 & 10n \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

$$P_s = \begin{pmatrix} 0 \\ 0 \\ -n-10+10n \\ 1 \end{pmatrix}$$

$$P_t = \begin{pmatrix} 0 \\ 0 \\ -2n-20+10n \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -n-10+5n \\ 1 \end{pmatrix}$$

We are only concerned about the z coordinates of those points

$z_{P_s} = -n - 10 + 10n$

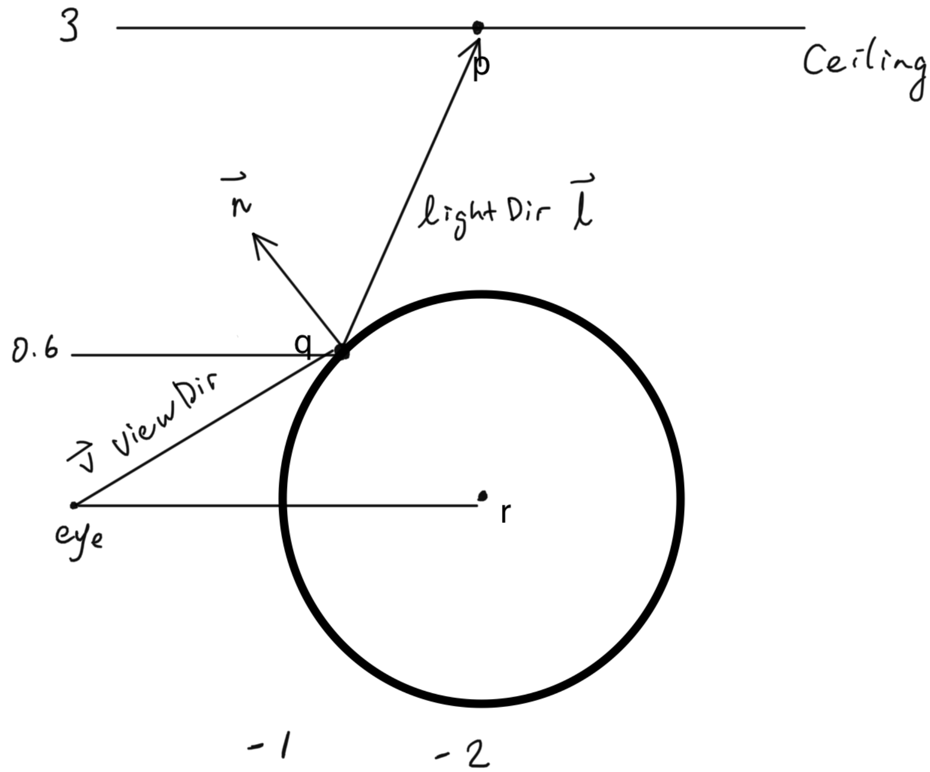
$z_{P_t} = -n - 10 + 5n$

We want to minimize, i.e $|z_{P_s} - z_{P_t}| = \epsilon$ so:

$|-n-10+10n-(-n-10+5n)| = \epsilon$

$|5n| = \epsilon$

$n = \frac{\epsilon}{5}$



Where p is the light position (unknown), r is the center of the circle, q is the brightest spot of a Blinn-Phong specular highlight.

We need to find the position of q first. Since we know $y_q = 0.6$, we get:

$x_q^2 + 0.6^2 = 1$

$x_q = 0.8$

Thus, $z_q = -2 + 0.8 = 1.2$

Now we can find the view direction v and the normal n.

$$n = q - r = \begin{pmatrix} 0 \\ 0.6 \\ 0.8 \\ 0 \end{pmatrix}$$

$$v = eye - q = \begin{pmatrix} 0 \\ -0.6 \\ 1.2 \\ 0 \end{pmatrix}$$

Now let's find l. Suppose u is a vector such that $v - 2u = l$

$$u = v - proj_n v = \begin{pmatrix} 0 \\ -0.6 \\ 1.2 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 0.36 \\ 0.48 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -0.96 \\ 0.72 \\ 0 \end{pmatrix}$$

$$l = v - 2u = \begin{pmatrix} 0 \\ -0.6 \\ 1.2 \\ 0 \end{pmatrix} - 2\begin{pmatrix} 0 \\ -0.96 \\ 0.72 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1.32 \\ -0.24 \\ 0 \end{pmatrix}$$

Now let's find out the location of the Light

Parametric equations of the direction: $x = 0 \quad y = 0.6 + 1.32t \quad z = -1.2 - 0.24t$

Equation of the ceiling:$y = 3$

$3 = 0.6 + 1.32t$

$t = \frac{20}{11}$

$z = -1.2 - 0.24t$

$z = -\frac{18}{11}$

So the light should be at $\begin{pmatrix} 0 \\ 3 \\ -\frac{18}{11} \\ 1 \end{pmatrix}$

Vertex Shader

```
#version 400 core
uniform mat4 M;
uniform mat4 V;
uniform mat4 P;
uniform mat3 MinvT;
uniform mat3 VinvT;

in vec3 VertexNormal;
in vec4 VertexPosition;

out vec4 PositionForFP;
out vec3 NormalForFP;

void main() {

// FIX: VinvT is used instead of V
// Should change to
// vec4 tmp = V*MinvT* vec4(VertexNormal,0);
// NormalForFP= normalize(tmp.xyz)
NormalForFP = MinvT * VinvT * VertexNormal;

PositionForFP = V * M * VertexPosition;
gl_Position = P * V * M * VertexPosition;

}
```

Fragment Shader

```
#version 400 core
uniform vec3 LightColor;
uniform vec3 LightPosition;
uniform float Shininess;
uniform vec3 kd;

in vec4 PositionForFP;
in vec3 NormalForFP;

out vec4 FragColor;

void main() {

// FIX: Direction should go to the light.
// Should change to normalize(LightPosition - PositionForFP.xyz)
vec3 LightDirection = PositionForFP - LightPosition;

// FIX: Diffuse may go negative in the original implementation.
// Should change to max(dot(NormalForFP, LightDirection), 0)
float diffuse = dot( NormalForFP, LightDirection );

// FIX: Should change to normalize (vec3(0,0,0) - PositionForFP.xyz)
vec3 ViewDirection = vec3(0,0,0) - PositionForFP;

// FIX: HalfVector not unit verctor, also shouldn't divide by 2.
// Should change to normalize (LightDirection + ViewDirection)
HalfVector = (LightDirection + ViewDirection) / 2;

float specular = max(0.0, dot(NormalForFP, HalfVector));

if (diffuse == 0.0) {
specular = 0.0;
} else {
specular = pow( specular, Shininess );
}
vec3 scatteredLight = kd * LightColor * diffuse;
// FIX: ks is 1 here, this may be unintentional
// reflectedLight = ks * LightColor * specular
vec3 reflectedLight = LightColor * specular;
vec3 rgb = min( scatteredLight + reflectedLight, vec3(1,1,1) );
FragColor = vec4( rgb, 1 );

}
```

Yes, I actually looked into his work of OpenVDB, apparently its very widely used as a library of manipulating sparse volumetric data. In addition to Ken Museth's Keynote, I also attended Papers 2 - Waves. I was only familiar with J. Tessendorf's Work (Clemson) prior to coming to SCA 2020, its very refreshing to see new development on procedural generation of ocean waves.