# Pipeline and Rasterization



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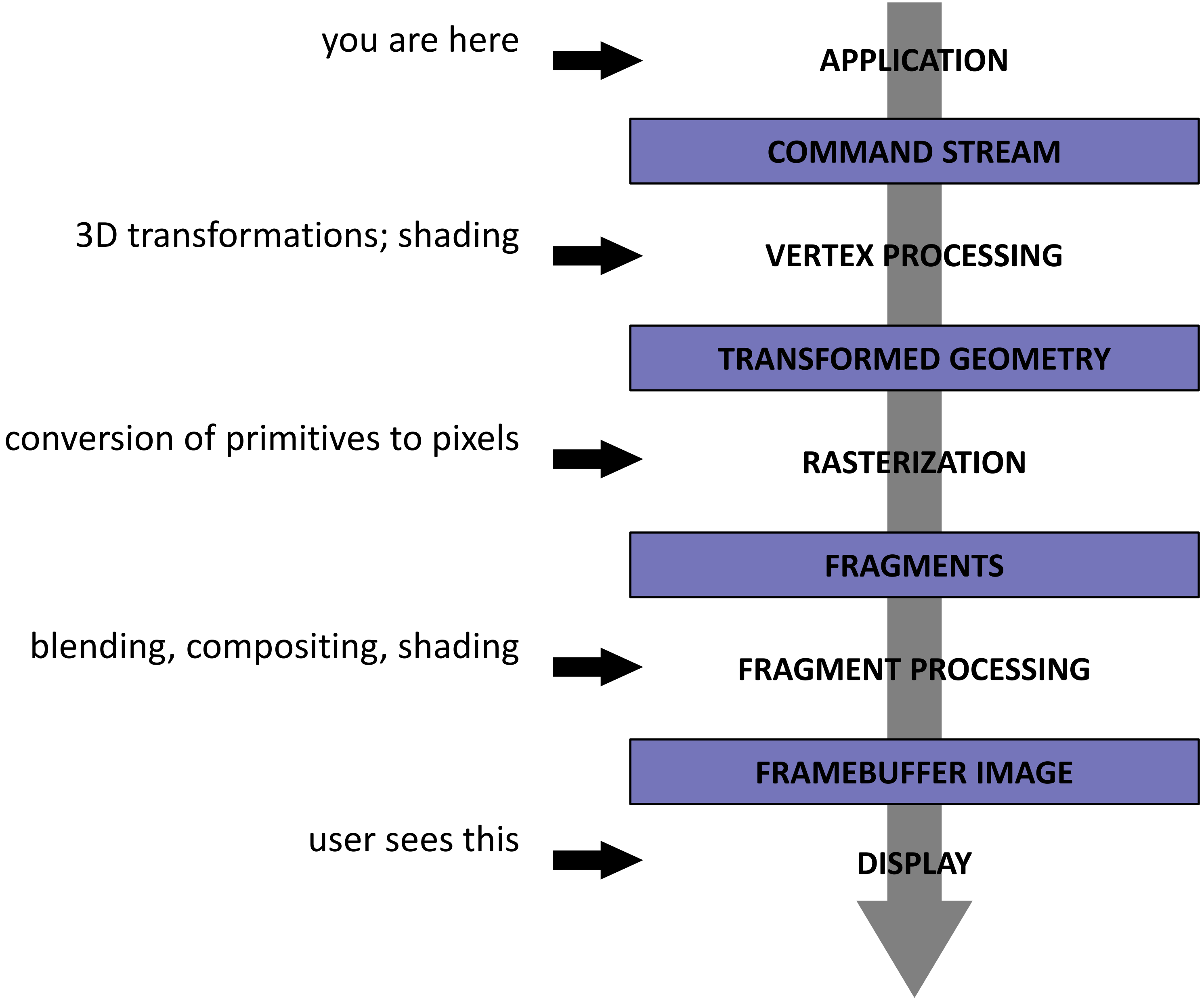
# Pipeline overview



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# Primitives

- Points
- Line segments (and chains, loops of line segments)
- Triangles (and strips and fans of adjacent triangles)
- And that’s all!
  - Curves? Approximate them with chains of line segments
  - Polygons? Break them up into triangles
  - Curved regions? Approximate them with triangles
- Trend has been toward minimal primitives
  - simple, uniform, repetitive: good for parallelism

# Rasterization

- First job: enumerate the pixels covered by a primitive
  - simple, aliased definition: pixels whose centers fall inside
- Second job: interpolate values across the primitive
  - e.g., colors computed at vertices
  - e.g., normals at vertices



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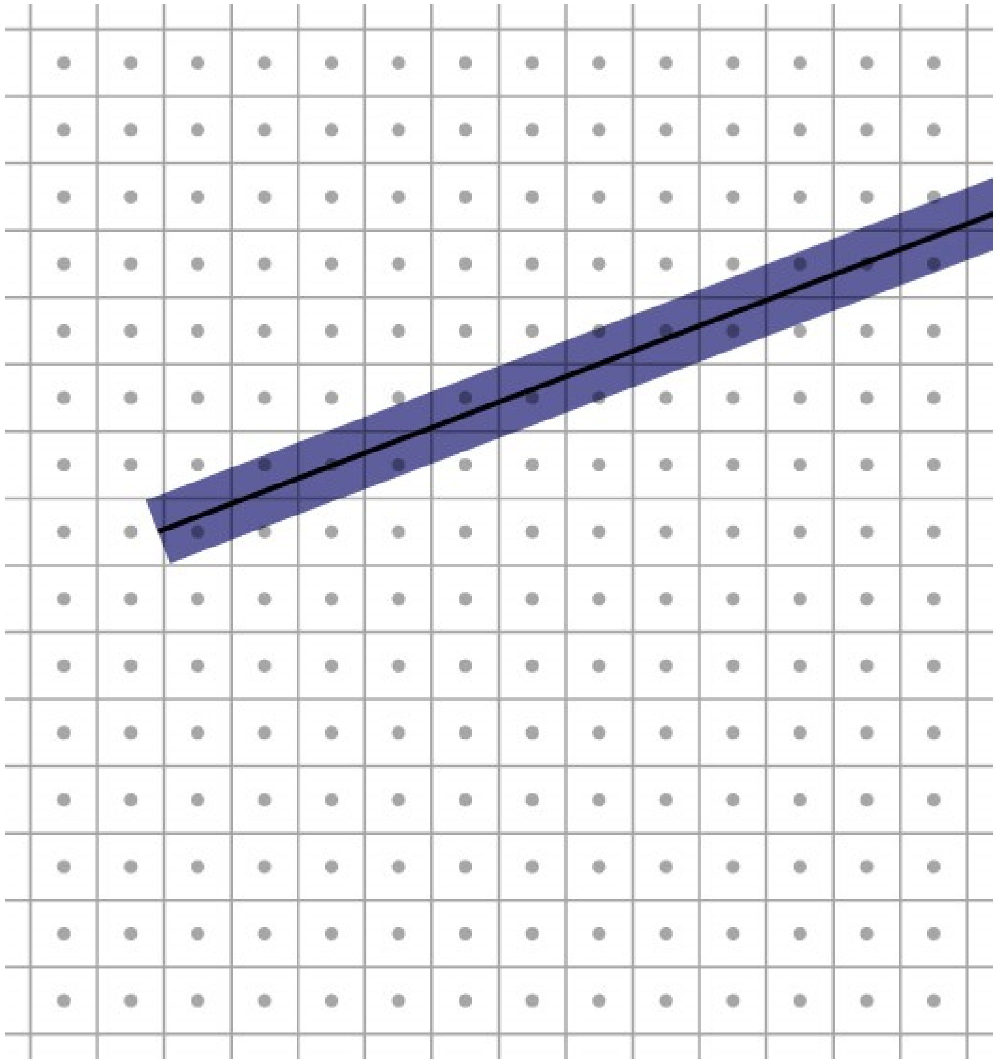
# Rasterizing lines

- Define line as a rectangle
- Specify by two endpoints
- Ideal image: black inside, white outside





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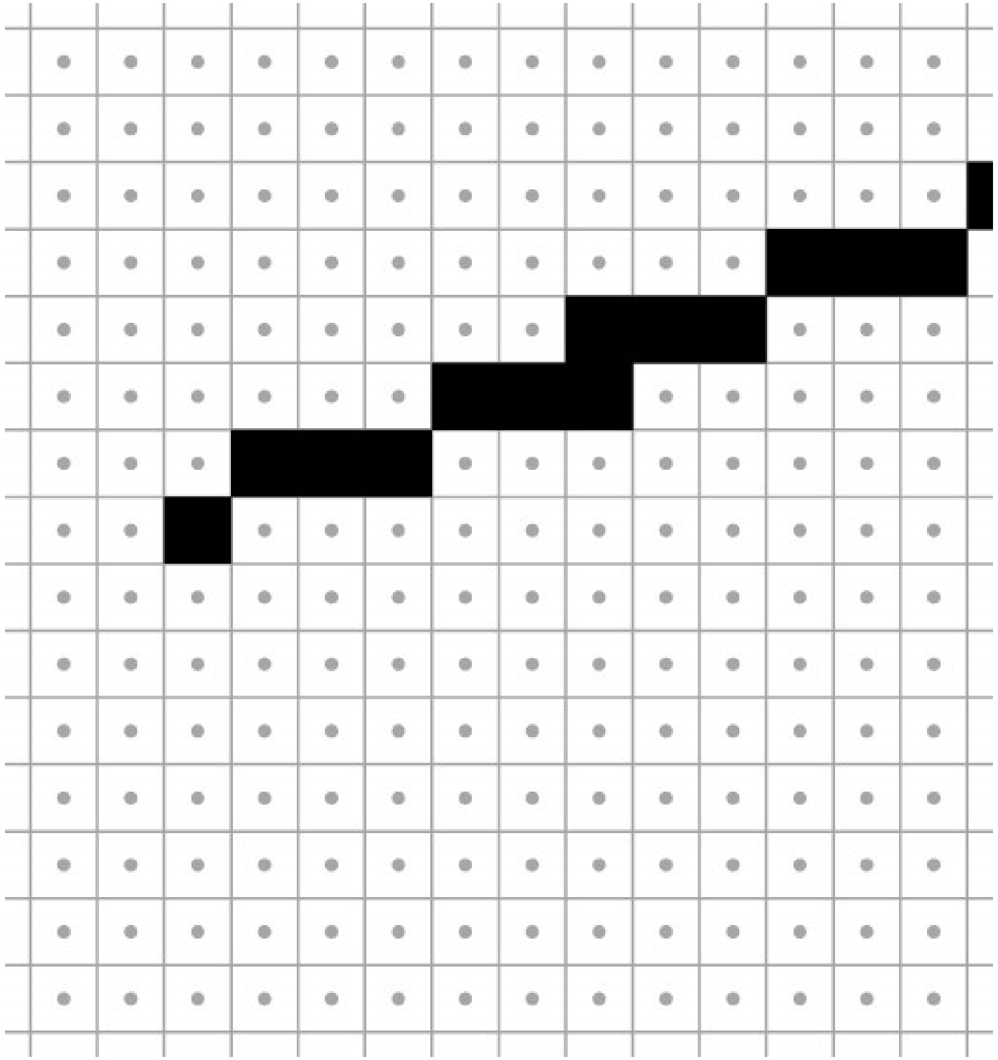
# Point sampling

- Approximate rectangle by drawing all pixels whose centers fall within the line
- Problem: sometimes turns on adjacent pixels





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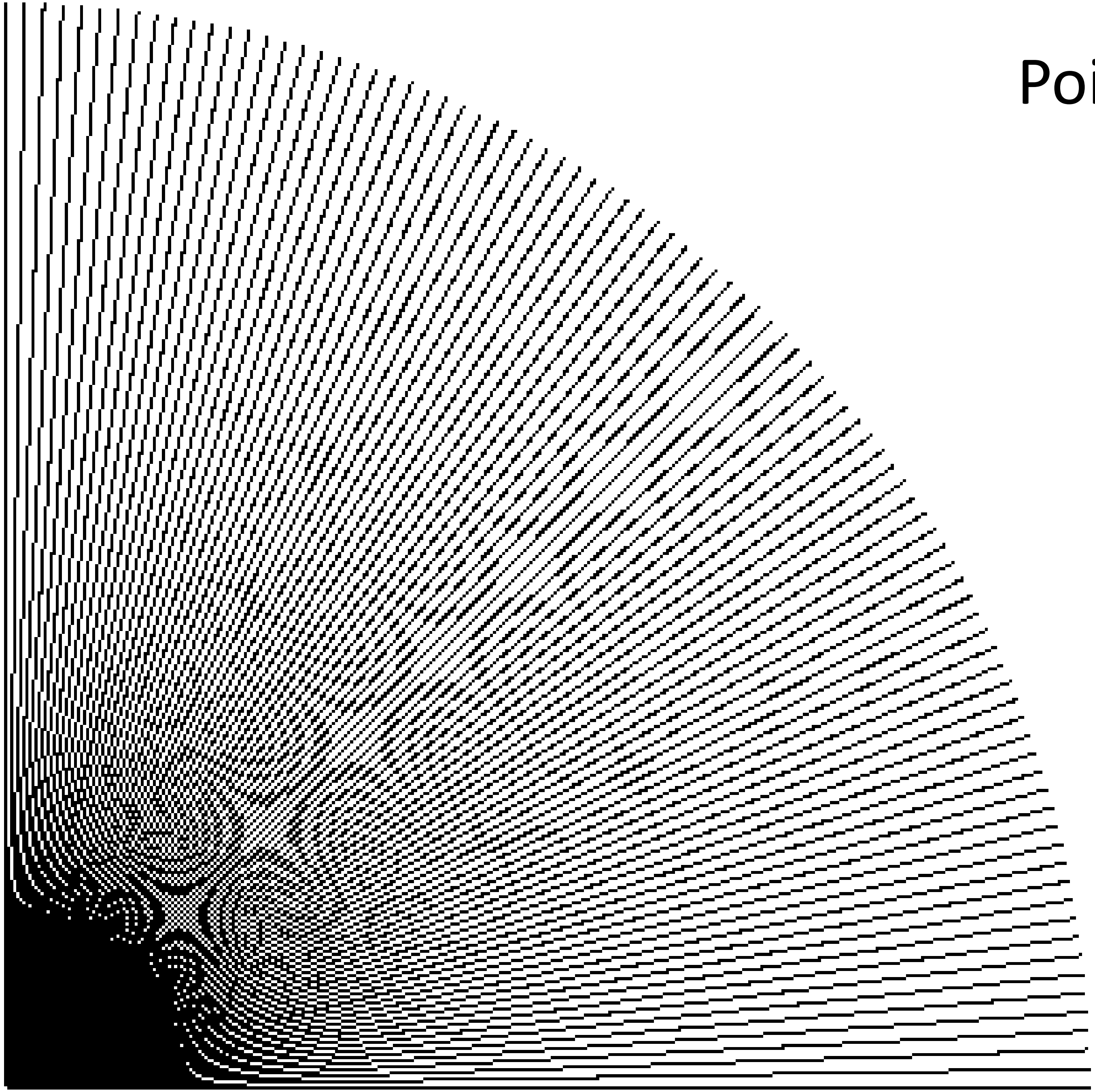

# Point sampling in action



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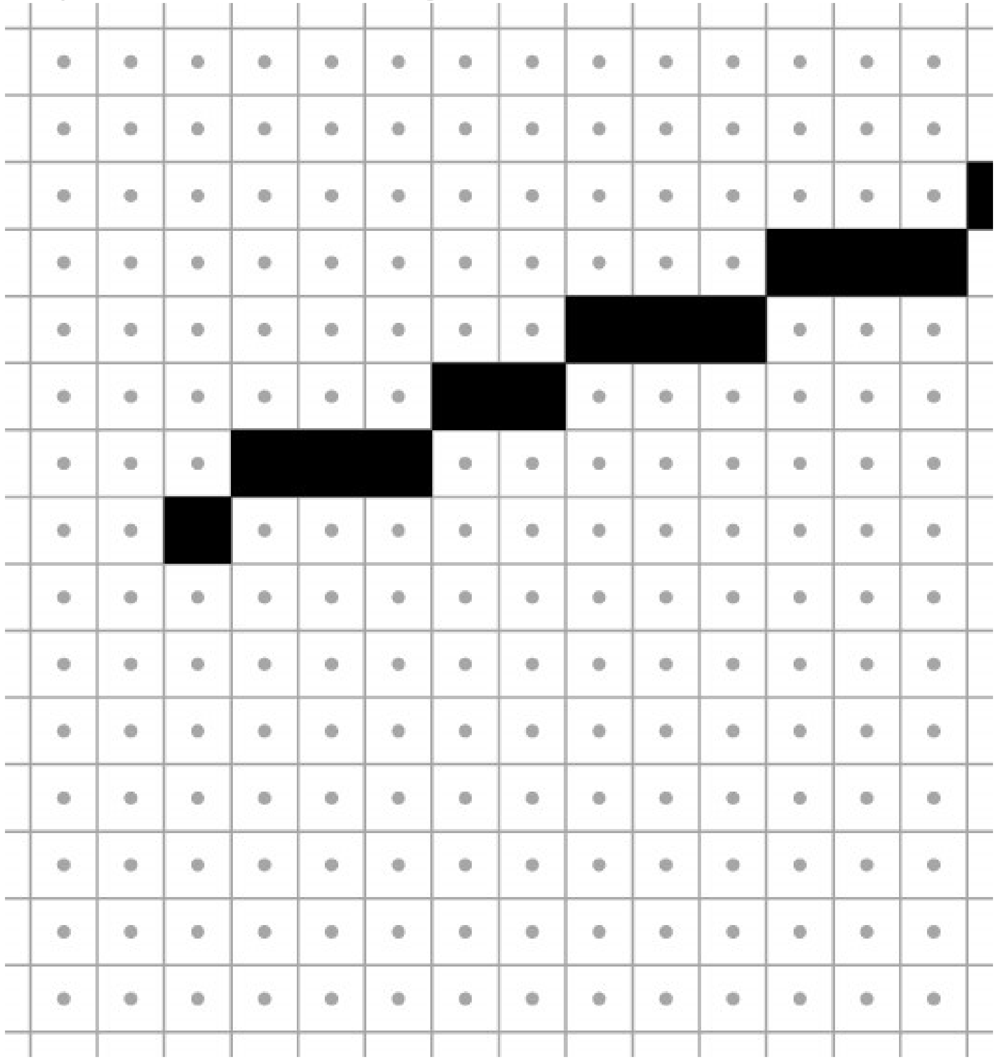
# Bresenham lines (midpoint alg.)

- Point sampling unit width rectangle leads to uneven line width
- Define line width parallel to pixel grid
- That is, turn on the single nearest pixel in each column
- Note that 45 degree lines are now thinner





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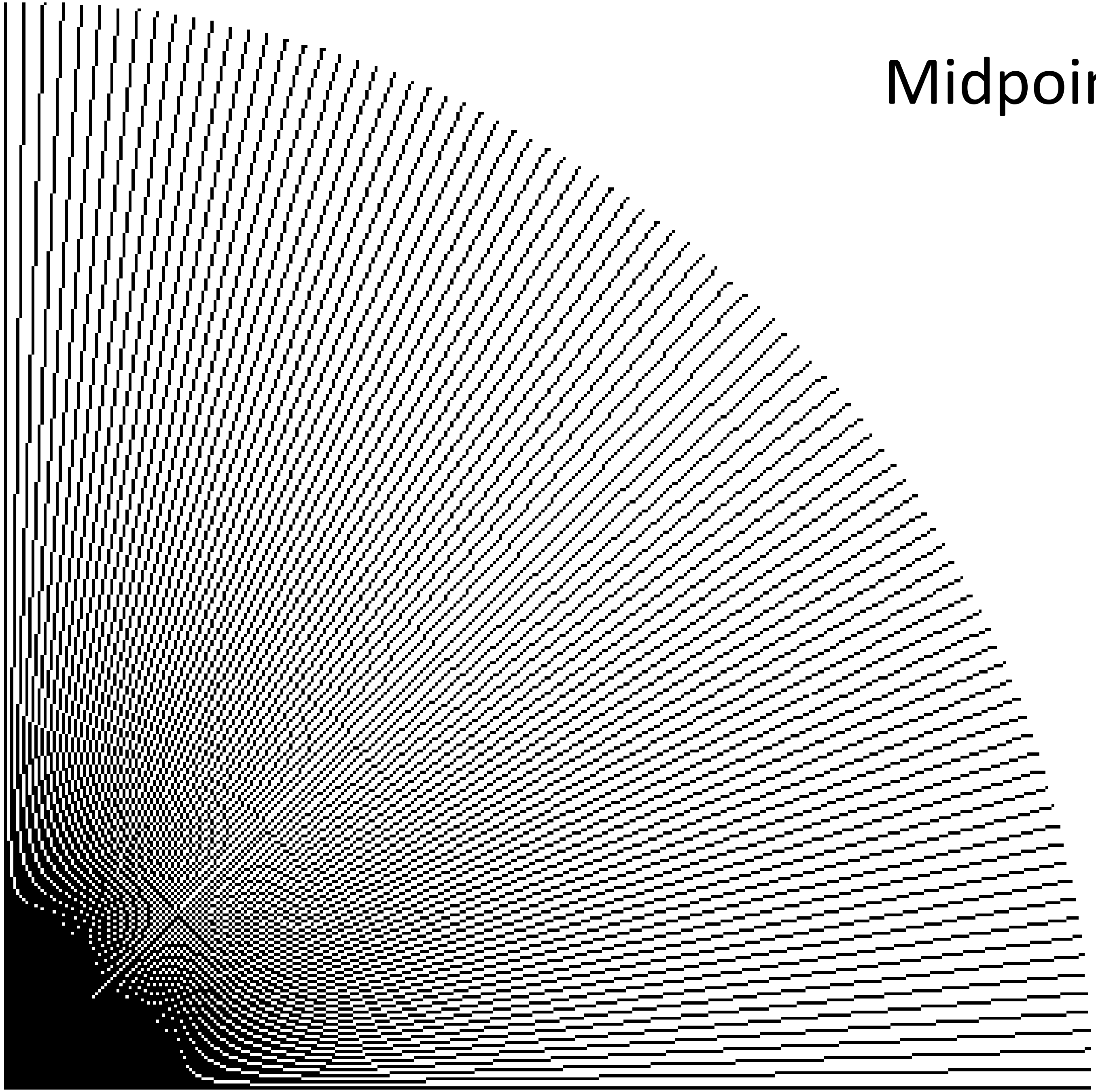

# Midpoint algorithm in action



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# Algorithms for drawing lines

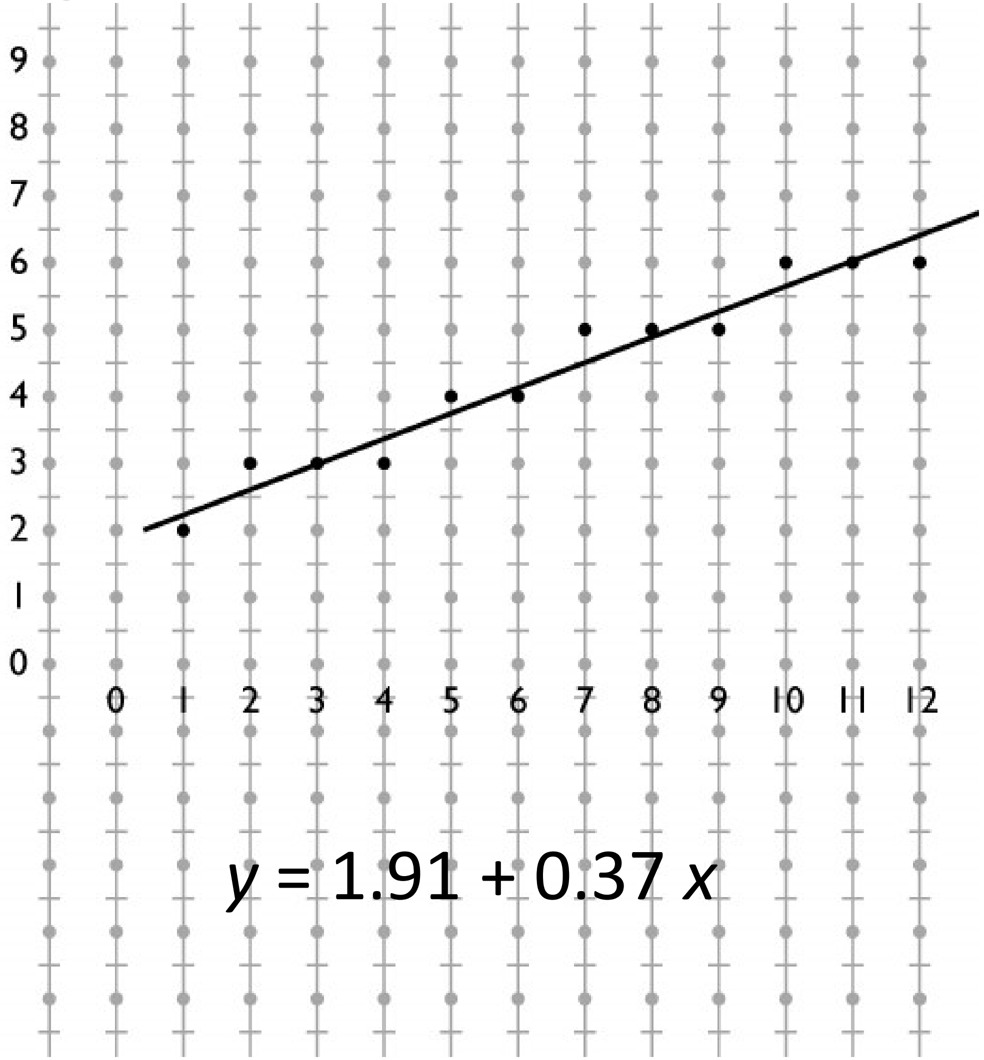
- Line equation:

$$y = b + mx$$

- **Simple algorithm:** evaluate line equation per column
- Without loss of generality, assume,

$$x_0 < x_1; 0 \leq m \leq 1$$

```
for x = ceil(x0) to floor(x1)
     y = b + m*x
     output(x, round(y))
```



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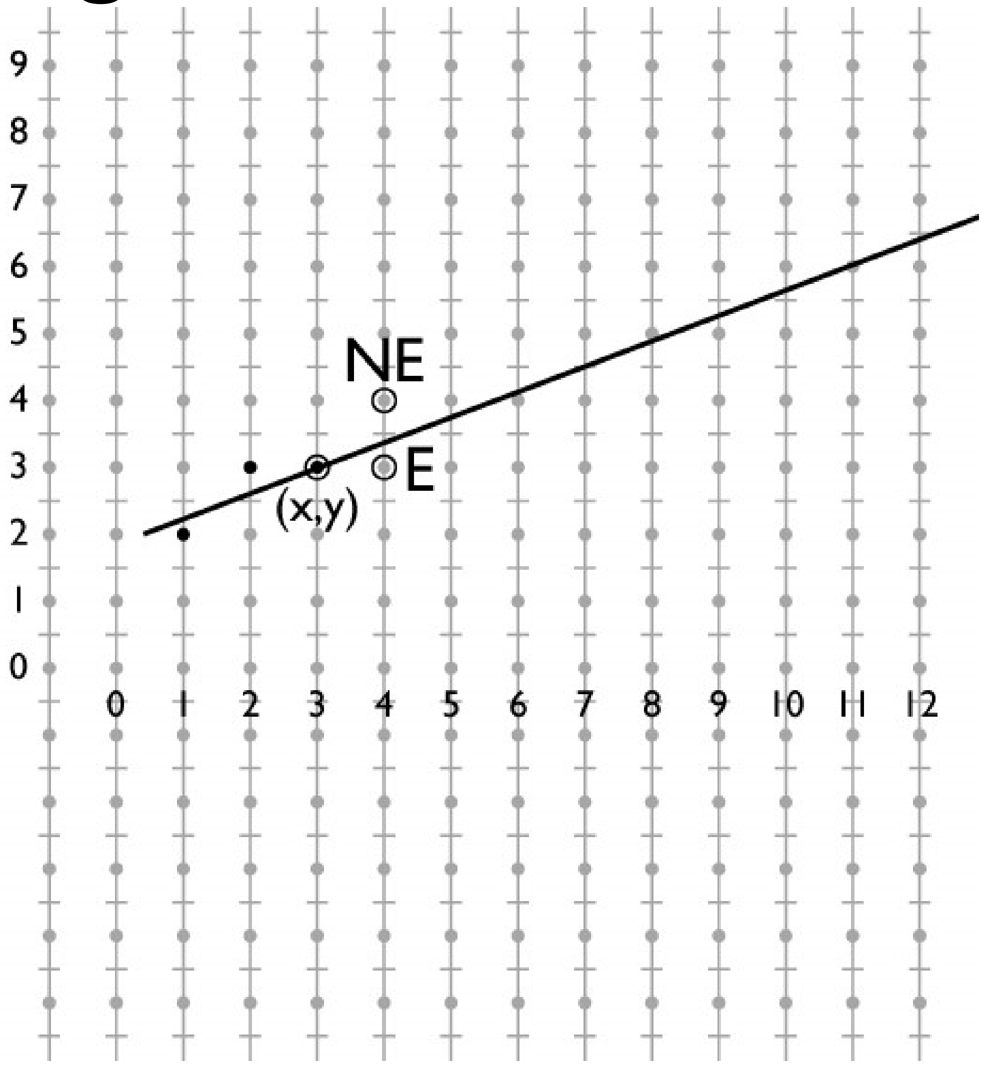
# Optimizing line drawing

- Multiplying and rounding is **slow**
- At each pixel the only options are E and NE
- $d = m(x+1) + b - y$
- $d > 0.5$ **decides** between E and NE



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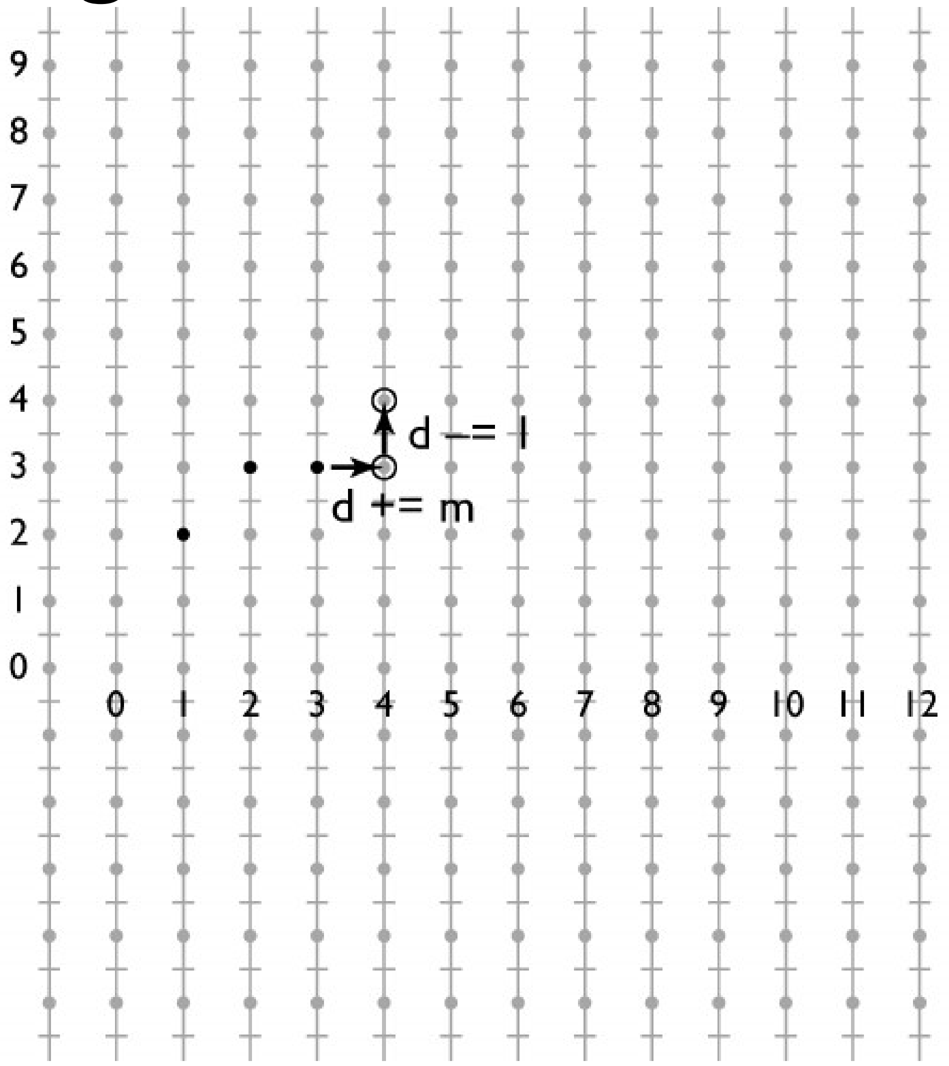
# Optimizing line drawing

- $d = m(x+1) + b - y$
- Only need to update d for integer steps in $x$ and $y$
- Do that with addition
- Known as "DDA" (digital differential analyzer)



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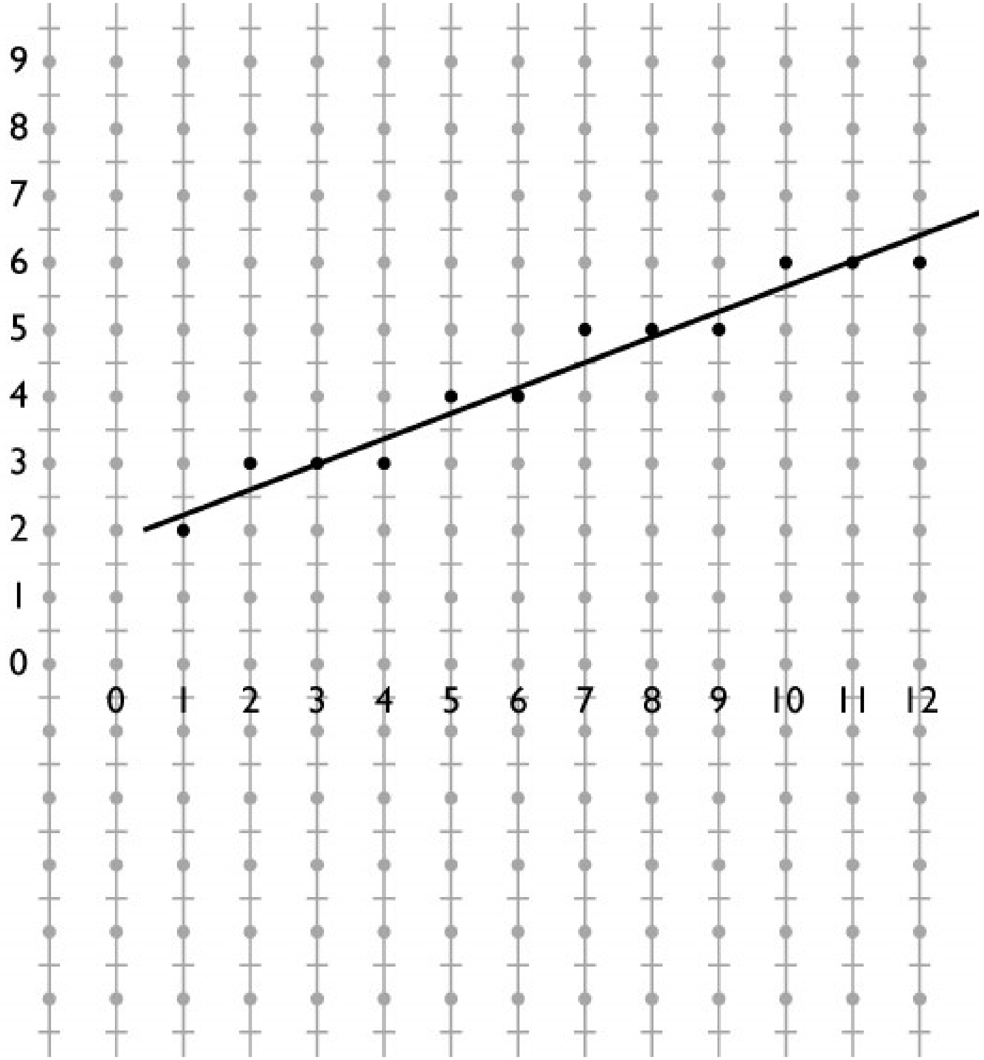
# Midpoint line algorithm

```
x = ceil(x0)
y = round(m*x + b)
d = m*(x + 1) + b – y
while x < floor(x1)
      if d > 0.5
            y += 1
            d –= 1
      x += 1
      d += m
      output(x, y)
```




Bresenham's algorithm for drawing lines does this using only integer operations in the inner loop.



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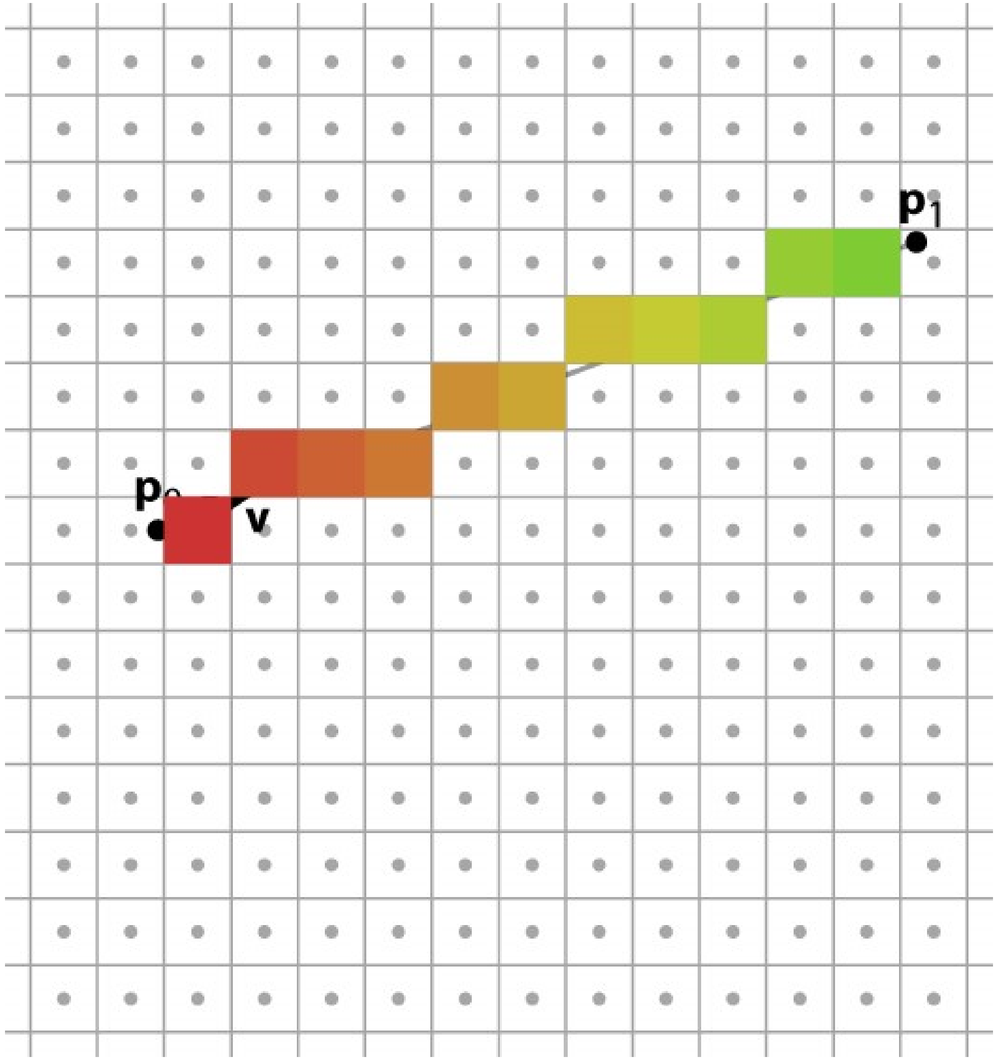
# Linear interpolation

- We often attach attributes to vertices
  - e.g., computed diffuse color of a hair being drawn using lines
  - want color to vary smoothly along a chain of line segments
- Recall basic definition
  - 1D: $f(x) = (1 - \alpha)y_0 + \alpha y_1$ where $\alpha = \frac{x - x_0}{x_1 - x_0}$
- In the 2D case of a line segment, alpha is just the fraction of the distance from $(x_0, y_0)$ to $(x_1, y_1)$

# Linear interpolation

- Pixels are not exactly on the line
- Define 2D function by projection on line
  - this is linear in 2D
  - therefore can use DDA to interpolate



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# Alternate interpretation

- We update $d$ and $\alpha$ as we step from pixel to pixel
    - Scalar $d$ tells us how far from the line we are
    - Scalar $\alpha$ tells us how far along the line we are
- Thus, $d$ and $\alpha$ are coordinates in a coordinate system oriented to the line
    - What are the axes of this coordinate system?
    - Is it orthogonal coordinate system?



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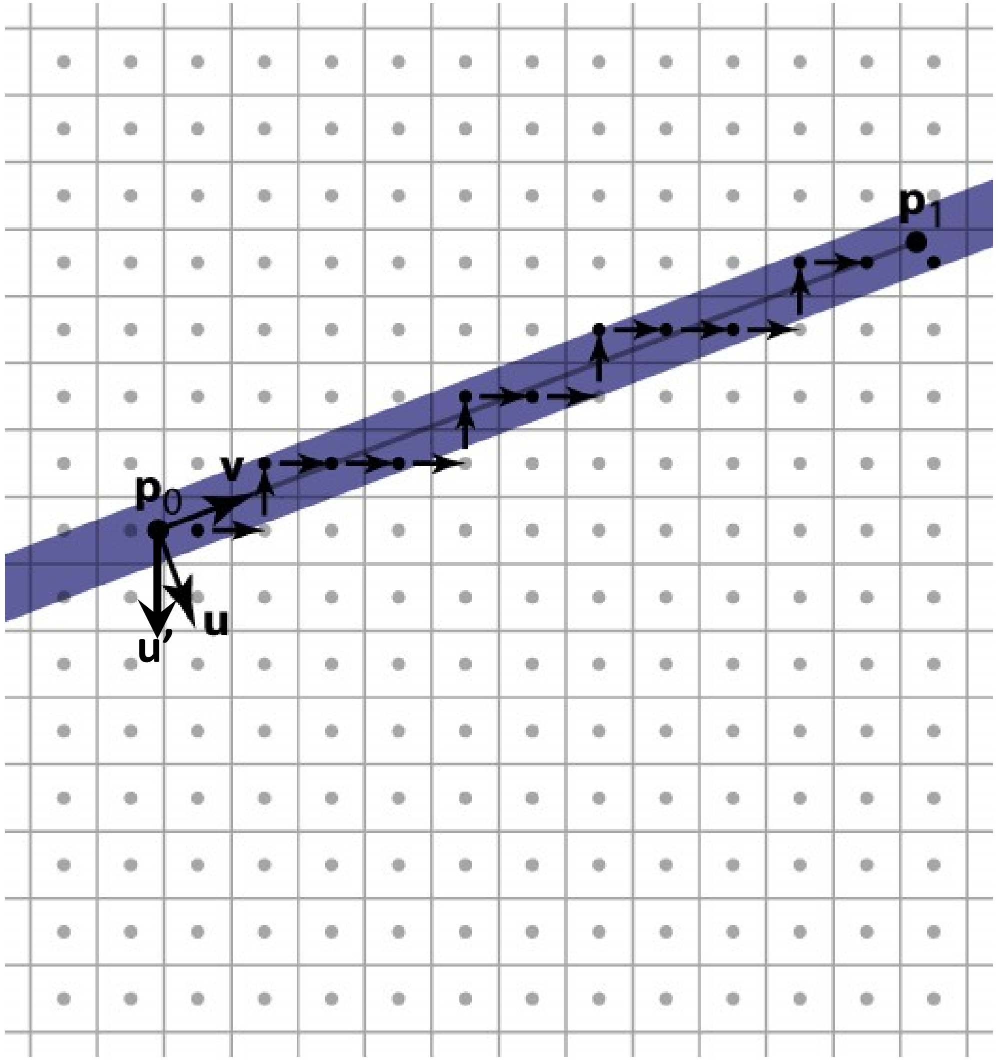
# Alternate interpretation

- View loop as visiting all pixels the line passes through
  - Interpolate $d$ and $\alpha$ for each pixel
  - Only output fragment if the pixel is in band
- This makes linear interpolation the primary operation



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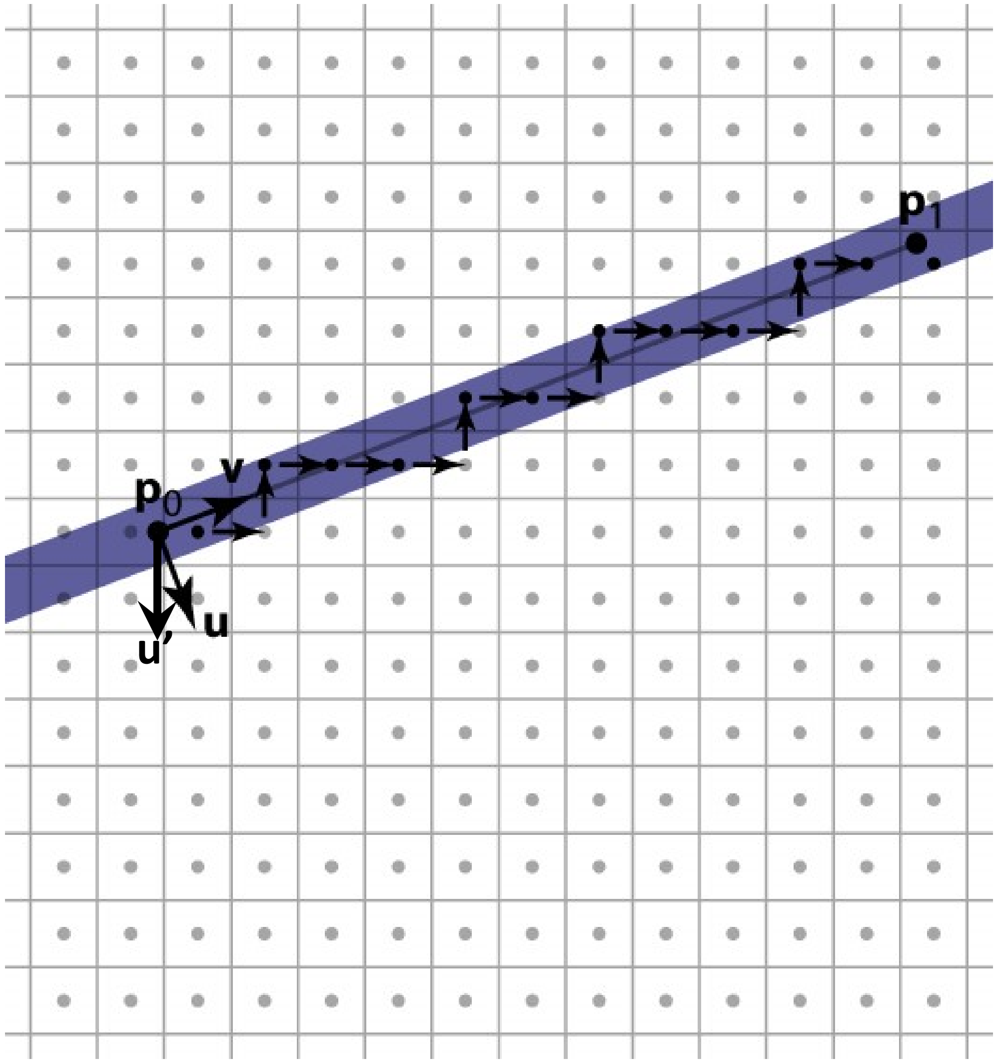
# Pixel-walk line rasterization

```
x = ceil(x0)
y = round(m*x + b)
d = m*x + b – y
while x < floor(x1)
  if d > 0.5
    y += 1; d –= 1;
  else
    x += 1; d += m;
  if –0.5 < d ≤ 0.5
    output(x, y)
```



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# Rasterizing triangles

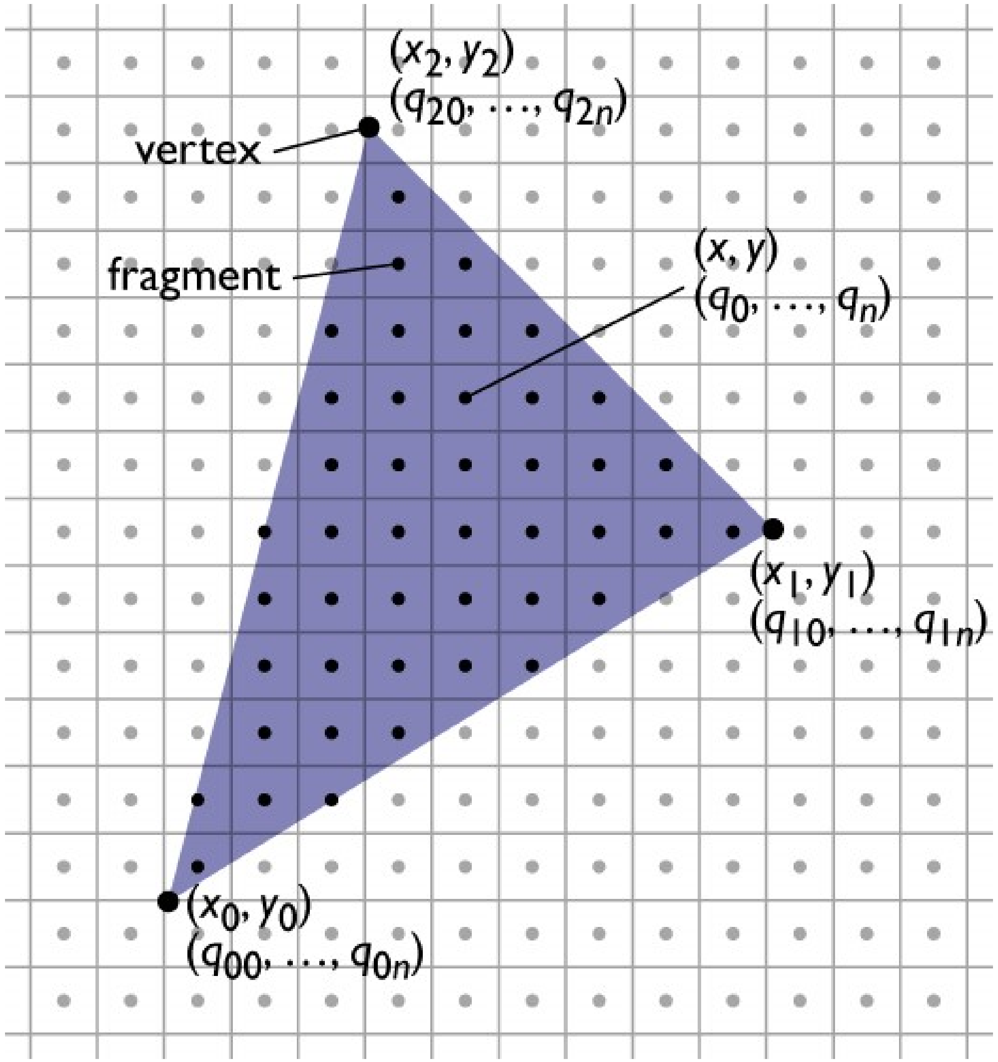
- The most common case in most applications
  - with good antialiasing can be the only case
  - some systems render a line as two skinny triangles
- Triangle represented by three vertices
- Simple way to think of algorithm follows the pixel-walk interpretation of line rasterization
  - walk from pixel to pixel over (at least) the polygon’s area
  - evaluate linear functions as you go
  - use those functions to decide which pixels are inside

# Rasterizing triangles

- Input:
  - Three 2D points (the triangle's vertices in pixel space)
    $$(x_0, y_0); (x_1, y_1); (x_2, y_2)$$
  - Parameter values at each vertex
    $$q_{00}, \dots, q_{0n}; q_{10}, \dots, q_{1n}; q_{20}, \dots, q_{2n}$$
- Output: a list of *fragments*, each with
  - The integer pixel coordinates $(x, y)$
  - Interpolated parameter values $q_0, \dots, q_n$

# Rasterizing triangles

- Summary
  1. evaluation of linear functions on pixel grid
  2. functions defined by parameter values at vertices
  3. using extra parameters to determine fragment set



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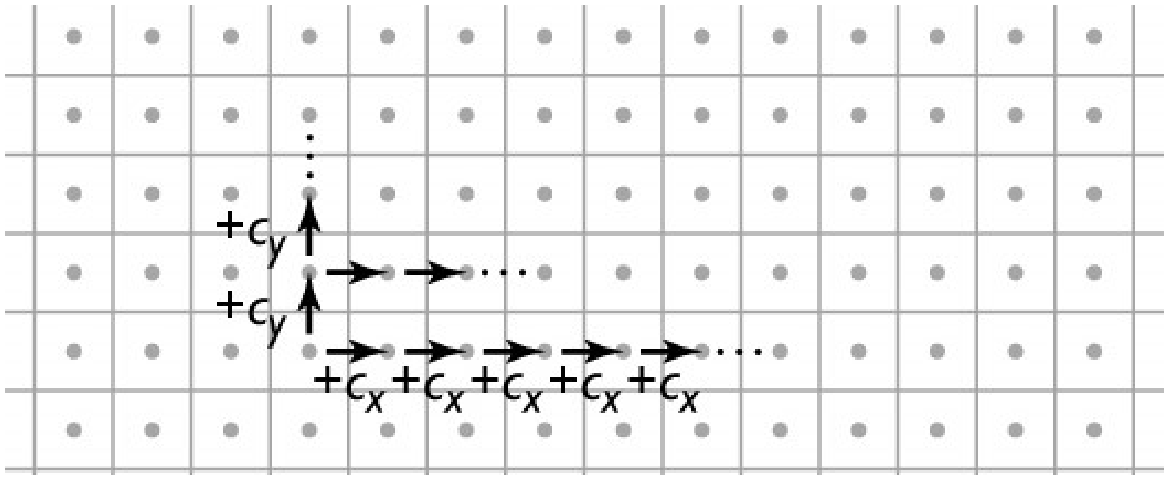
# Consider… Incremental linear evaluation

- A linear (***affine***, really) function on the plane is
$$q(x, y) = c_x x + c_y y + c_k$$
- Linear functions are efficient to evaluate on a grid:
$$q(x+1, y) = c_x(x+1) + c_y y + c_k = q(x, y) + c_x$$
$$q(x, y+1) = c_x x + c_y(y+1) + c_k = q(x, y) + c_y$$



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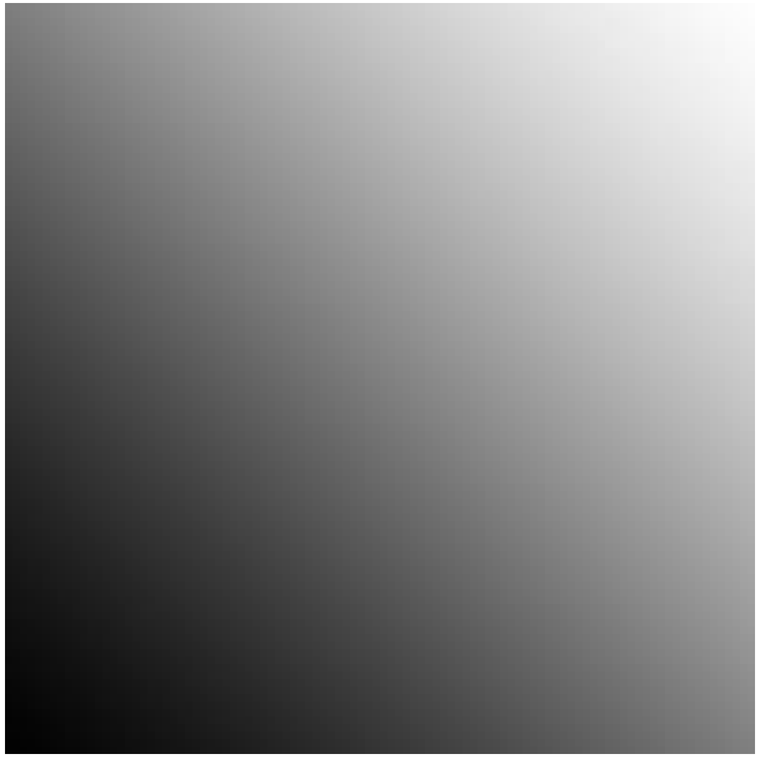
# Incremental linear evaluation

```
linEval(xl, xh, yl, yh, cx, cy, ck) {

    // setup
    qRow = cx*xl + cy*yl + ck;

    // traversal
    for y = yl to yh {
        qPix = qRow;
        for x = xl to xh {
            output(x, y, qPix);
            qPix += cx;
        }
        qRow += cy;
    }
}
```

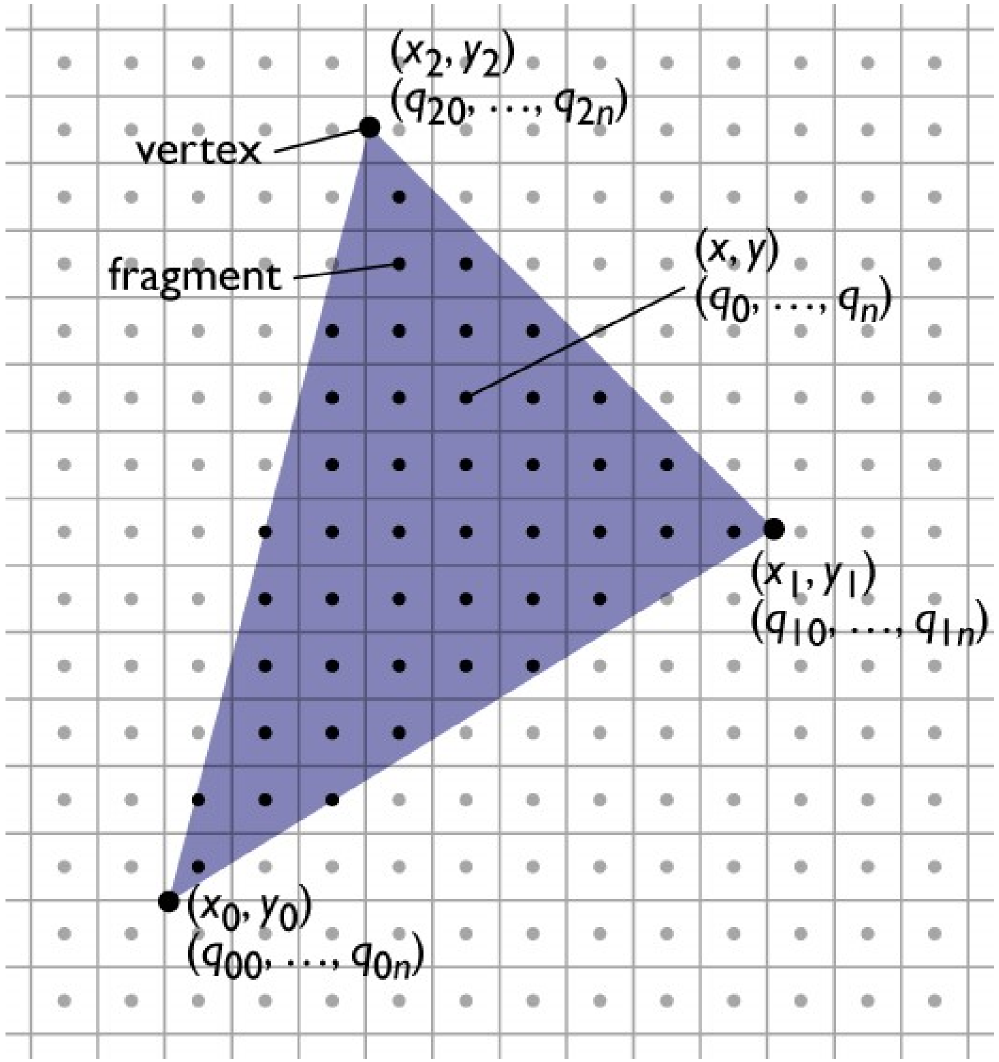


$c_x = 0.005; c_y = 0.005; c_k = 0$

(image size 100x100)

# Rasterizing triangles

- Summary
  1. evaluation of linear functions on pixel grid
  2. functions defined by parameter values at vertices
  3. using extra parameters to determine fragment set



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# Defining parameter functions

- To interpolate parameters across a triangle we need to find the $c_x$, $c_y$, and $c_k$ that define the (unique) linear function that matches the given values at all 3 vertices
  - This is 3 constraints on 3 unknown coefficients:

$$\begin{aligned} c_x x_0 + c_y y_0 + c_k &= q_0 \\ c_x x_1 + c_y y_1 + c_k &= q_1 \\ c_x x_2 + c_y y_2 + c_k &= q_2 \end{aligned}$$

(each states that the function agrees with the given value at one vertex)

  - Leads us to a 3x3 matrix equation for the coefficients:

$$\begin{pmatrix} x_0 & y_0 & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{pmatrix} \begin{pmatrix} c_x \\ c_y \\ c_k \end{pmatrix} = \begin{pmatrix} q_0 \\ q_1 \\ q_2 \end{pmatrix}$$

(singular iff triangle is degenerate)



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# Defining parameter functions

- More efficient version: shift origin to $(x_0, y_0)$

$$q(x,y) = c_x(x - x_0) + c_y(y - y_0) + q_0$$
$$q(x_1,y_1) = c_x(x_1 - x_0) + c_y(y_1 - y_0) + q_0 = q_1$$
$$q(x_2,y_2) = c_x(x_2 - x_0) + c_y(y_2 - y_0) + q_0 = q_2$$

  - Now this is a 2x2 linear system (since $q_0$ falls out):

$$\begin{pmatrix} (x_1 - x_0) & (y_1 - y_0) \\ (x_2 - x_0) & (y_2 - y_0) \end{pmatrix} \begin{pmatrix} c_x \\ c_y \end{pmatrix} = \begin{pmatrix} q_1 - q_0 \\ q_2 - q_0 \end{pmatrix}$$

  - Solve using Cramer's rule (see Shirley):

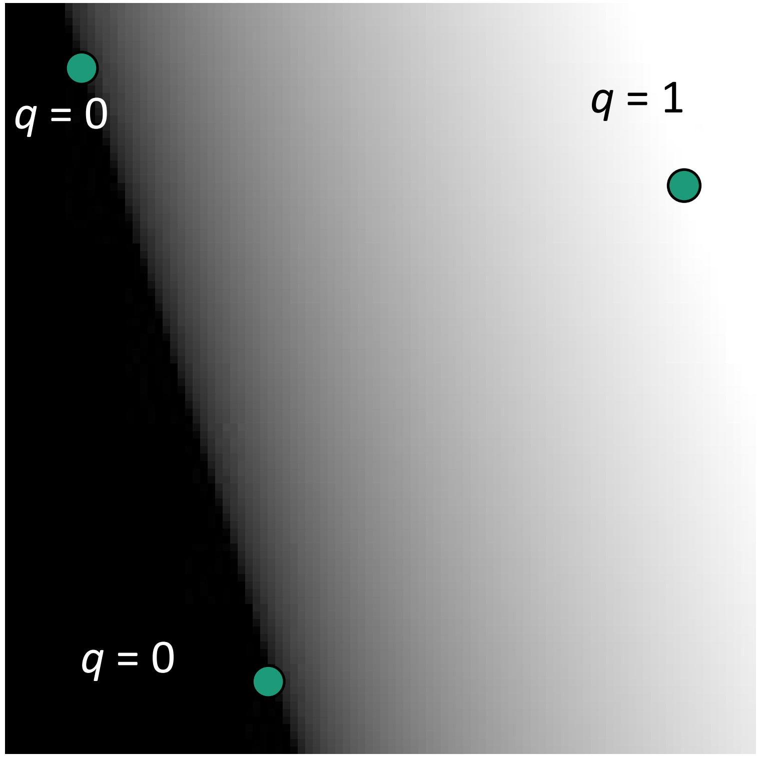
$$c_x = \frac{\Delta q_1 \Delta y_2 - \Delta q_2 \Delta y_1}{\Delta x_1 \Delta y_2 - \Delta x_2 \Delta y_1} \quad c_y = \frac{\Delta q_2 \Delta x_1 - \Delta q_1 \Delta x_2}{\Delta x_1 \Delta y_2 - \Delta x_2 \Delta y_1}$$

```
linInterp(xl, xh, yl, yh, x0, y0, q0, x1, y1, q1, x2, y2, q2) {

  // setup
  det = (x1-x0)*(y2-y0) - (x2-x0)*(y1-y0);
  cx = ((q1-q0)*(y2-y0) - (q2-q0)*(y1-y0)) / det;
  cy = ((q2-q0)*(x1-x0) - (q1-q0)*(x2-x0)) / det;
  qRow = cx*(xl-x0) + cy*(yl-y0) + q0;

  // traversal (same as before)
  for y = yl to yh {
    qPix = qRow;
    for x = xl to xh {
      output(x, y, qPix);
      qPix += cx;
    }
    qRow += cy;
  }
}
```

# Defining parameter functions



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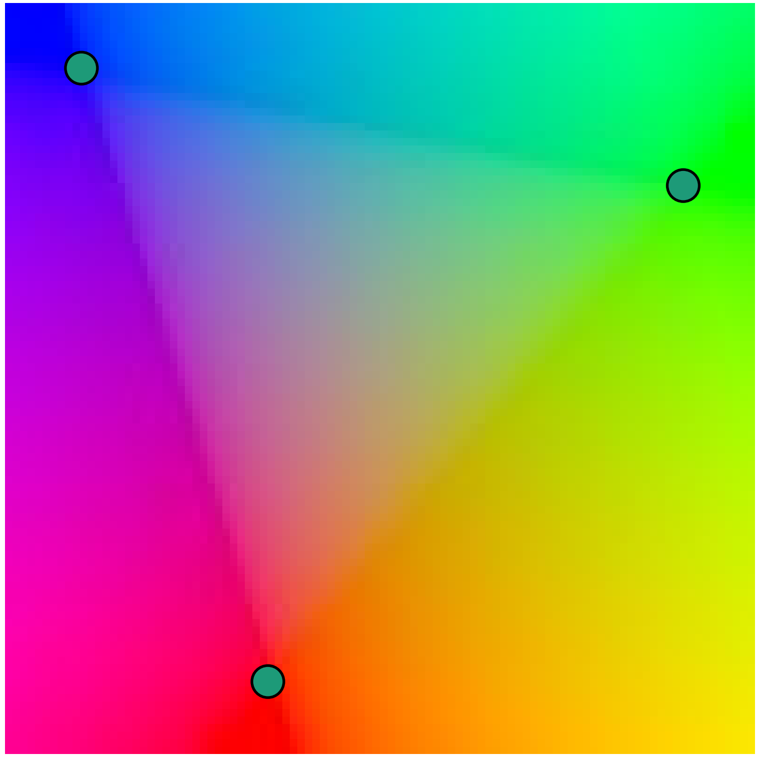
# Interpolating several parameters

```
linInterp(xl, xh, yl, yh, n, x0, y0, q0[], x1, y1, q1[], x2, y2, q2[]) {

  // setup
  for k = 0 to n-1
    // compute cx[k], cy[k], qRow[k]
    // from q0[k], q1[k], q2[k]

  // traversal
  for y = yl to yh {
    for k = 1 to n, qPix[k] = qRow[k];
    for x = xl to xh {
      output(x, y, qPix);
      for k = 1 to n, qPix[k] += cx[k];
    }
    for k = 1 to n, qRow[k] += cy[k];
  }
}
```





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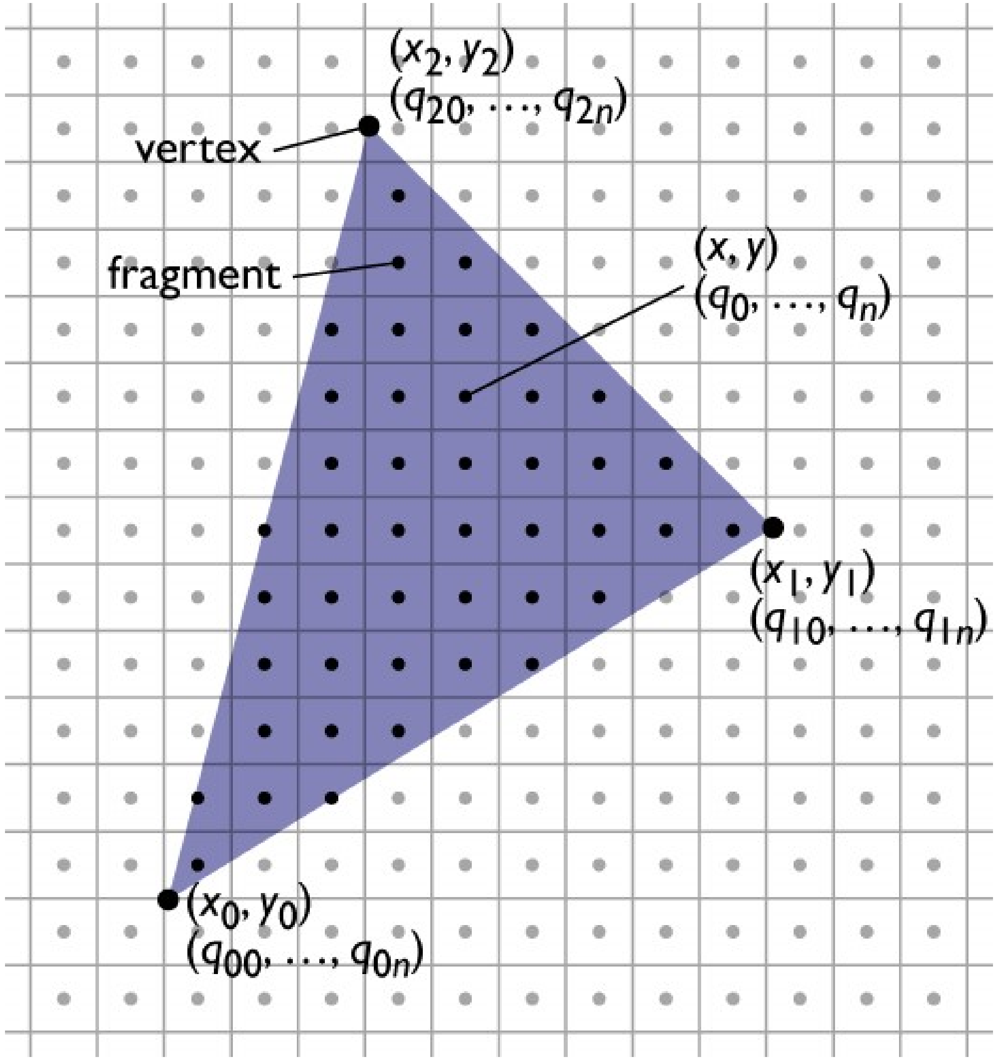
# Rasterizing triangles

- Summary
  1 evaluation of linear functions on pixel grid
  2 functions defined by parameter values at vertices
  3 using extra parameters to determine fragment set



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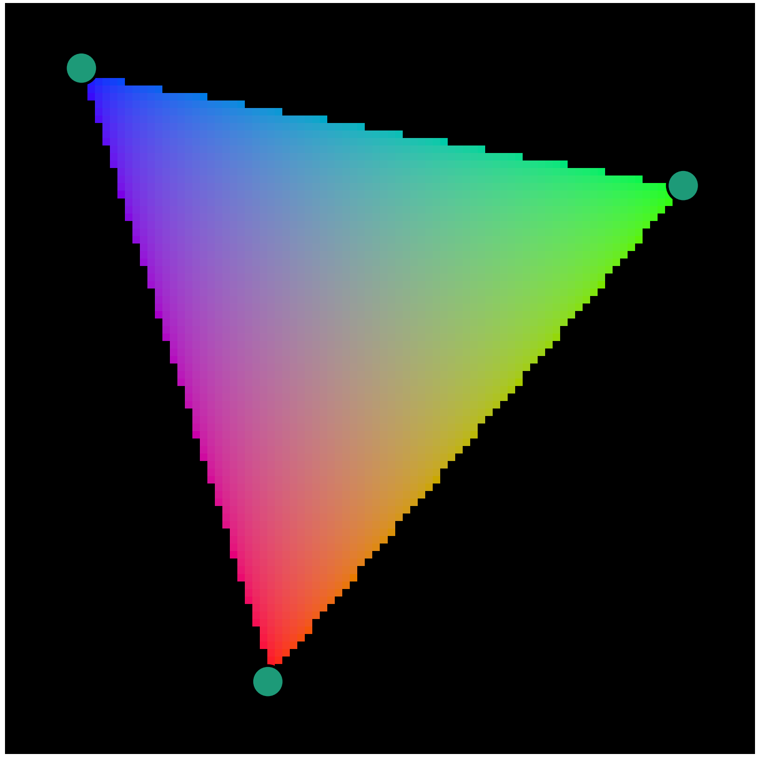
# Clipping to the triangle

- Interpolate three *barycentric coordinates* across the plane
  - each barycentric coord is 1 at one vert. and 0 at the other two
- Output fragments only when all three are > 0.



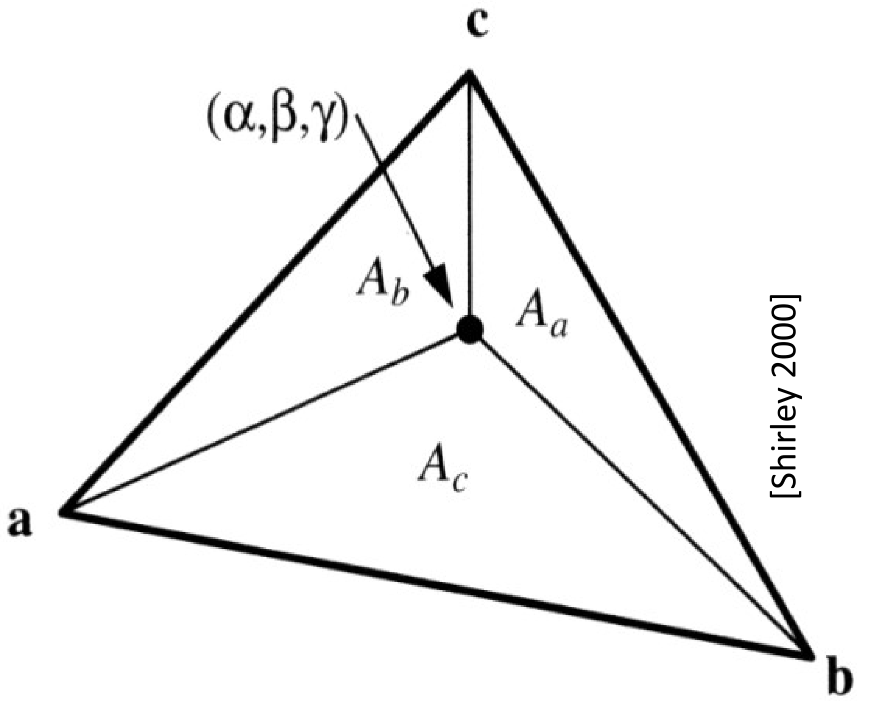
# Barycentric coordinates

- A coordinate system for triangles
  - Algebraic viewpoint:

$$\mathbf{p} = \alpha\mathbf{a} + \beta\mathbf{b} + \gamma\mathbf{c}$$
$$\alpha + \beta + \gamma = 1$$

  - Geometric viewpoint: area ratios
- Triangle interior test:

$$\alpha > 0; \beta > 0;\ \gamma > 0$$




[Shirley 2000]



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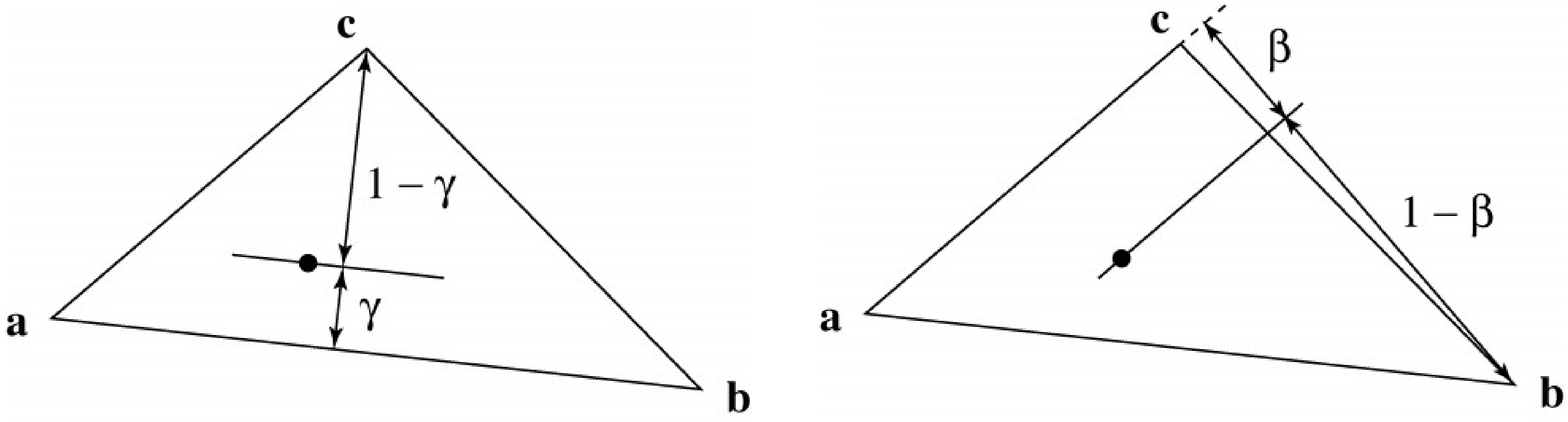
# Barycentric coordinates

- A coordinate system for triangles
  - Another geometric viewpoint: distances ratios



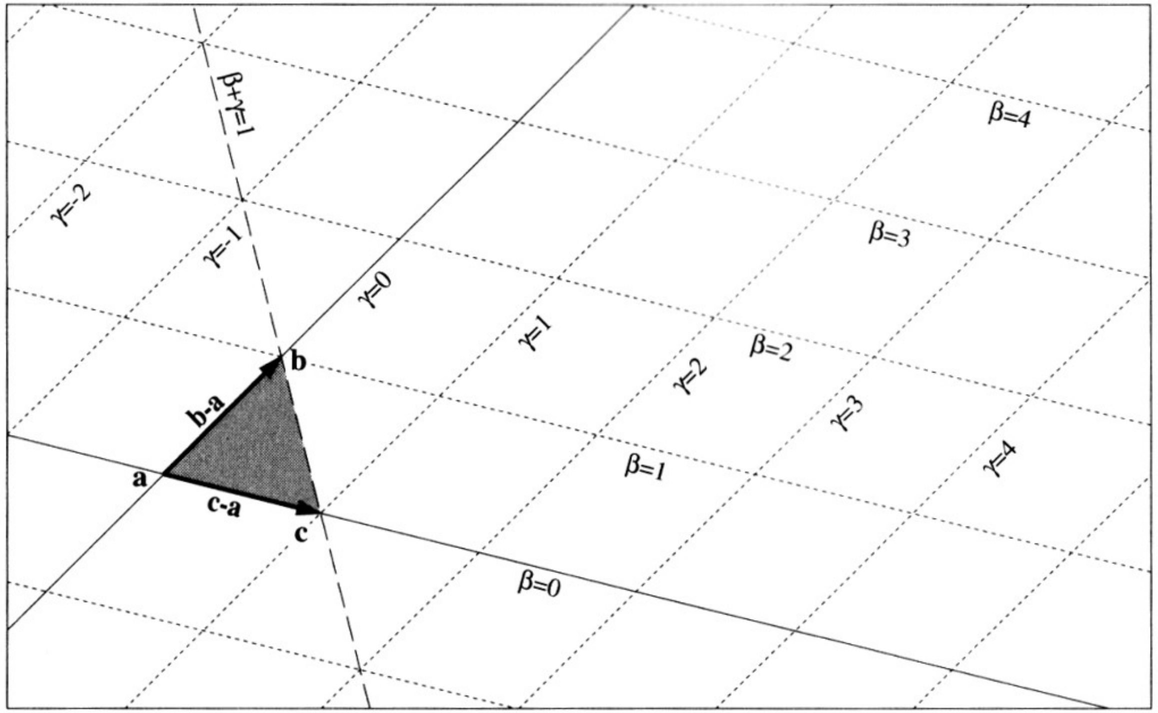

  - linear viewpoint: basis of edges

$$\alpha + \beta + \gamma = 1$$
$$\mathbf{p} = \mathbf{a} + \beta(\mathbf{b} - \mathbf{a}) + \gamma(\mathbf{c} - \mathbf{a})$$

# Barycentric coordinates

- Linear viewpoint: basis for the plane
  - In this view, the triangle interior test is just

$$\beta > 0;\ \gamma > 0;\ \beta + \gamma < 1$$




[Shirley 2000]



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# Walking edge equations

- We need to update values of the three edge equations with single-pixel steps in $x$ and $y$
- Edge equation already in form of dot product
- components of vector are the increments



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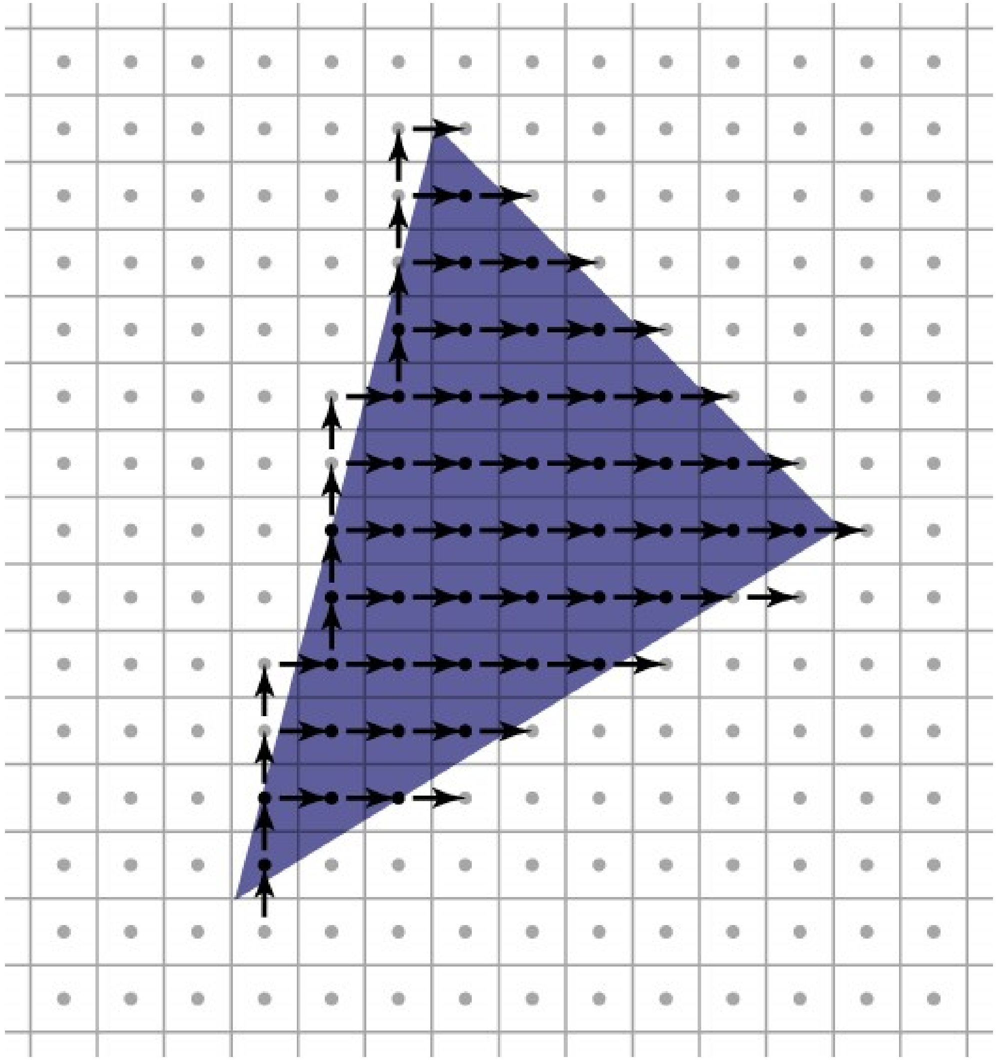
# Pixel-walk (Pineda) rasterization

- Conservatively visit a superset of the pixels you want
- Interpolate linear functions
- Use those functions to determine when to emit a fragment





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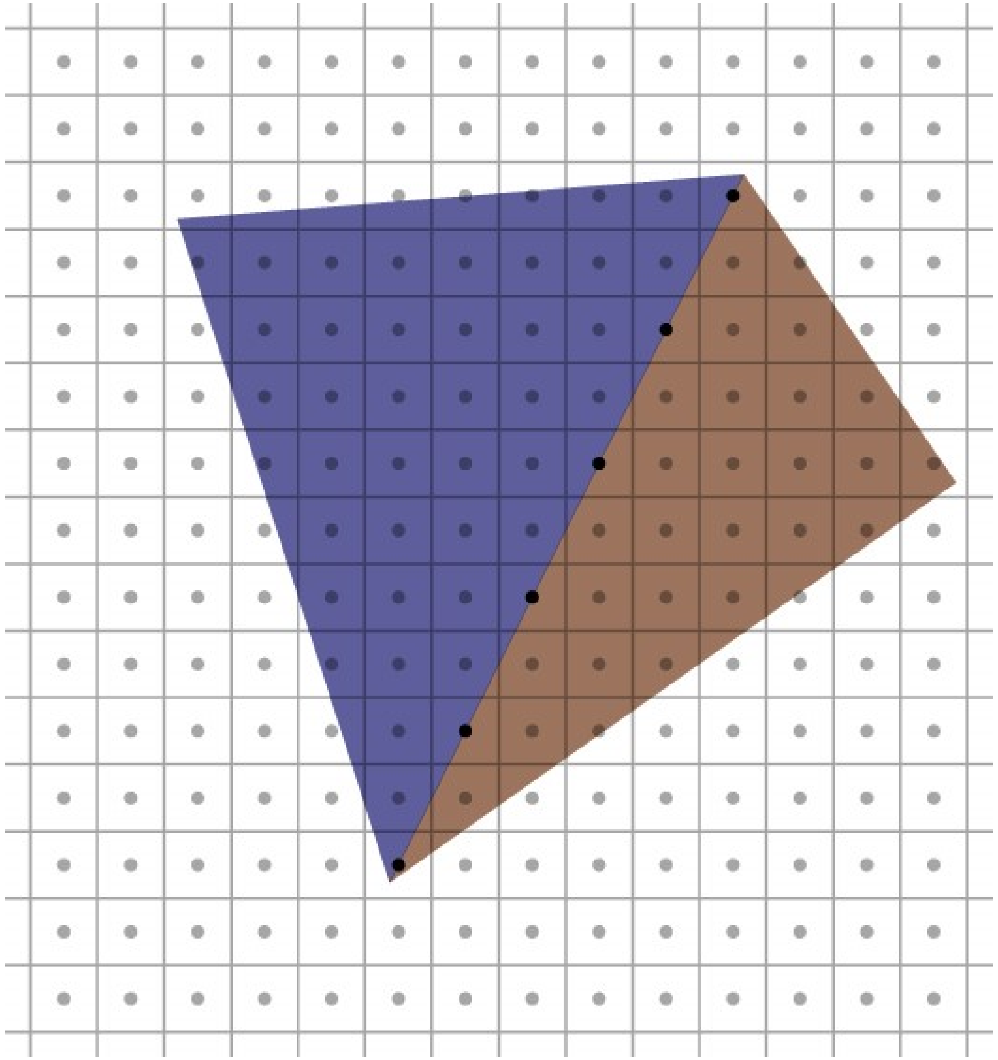
# Rasterizing triangles

- Exercise caution with rounding and arbitrary decisions
    - need to visit these pixels once
    - but it's important not to visit them twice!
      (for instance, when drawing partially transparent surfaces)





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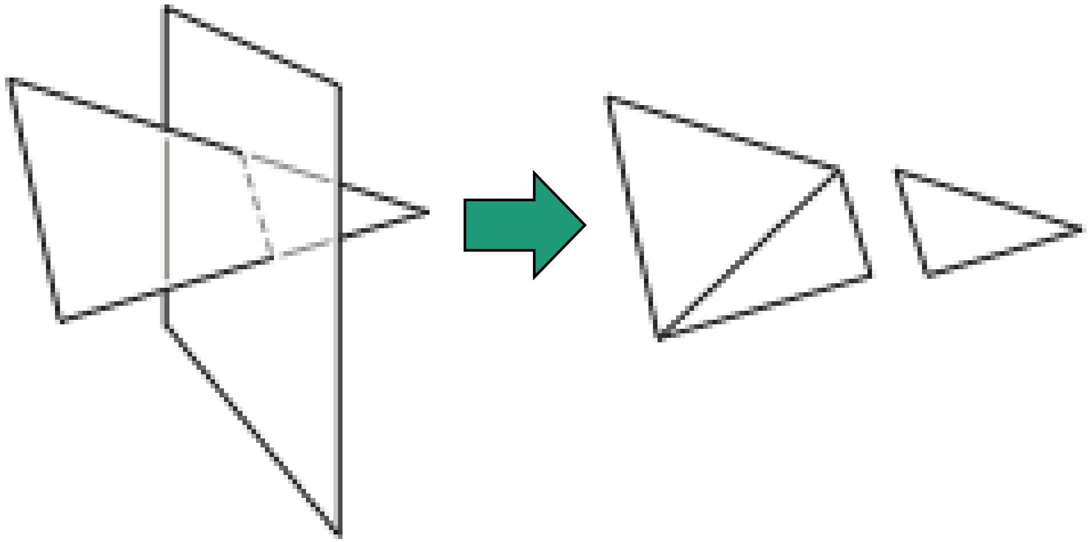
# Clipping

- Rasterizer tends to assume triangles are on screen
  - Particularly problematic to have triangles crossing the plane $z = 0$
- After projection, before perspective divide
  - Clip against the planes $x$, $y$, $z$ = $1, -1$ (6 planes)
  - Primitive operation: clip triangle against axis-aligned plane
  - *Homogeneous clip coordinates*

# Clipping a triangle against a plane

- 4 cases, based on sidedness of vertices
  - all in (keep)
  - all out (discard)
  - one in, two out (one clipped triangle)
  - two in, one out (two clipped triangles)





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# Tessellation

polygons into triangles…

# Review and more information

- FCG Chapter 8 the graphics pipeline
  - 8.1 Rasterization
  - 8.1.3 and 8.1.4 contain more details on clipping, but we haven’t discussed this in any depth
- See also FCG Chapter 2, miscellaneous math
  - 2.7 Triangles and Barycentric coordinates