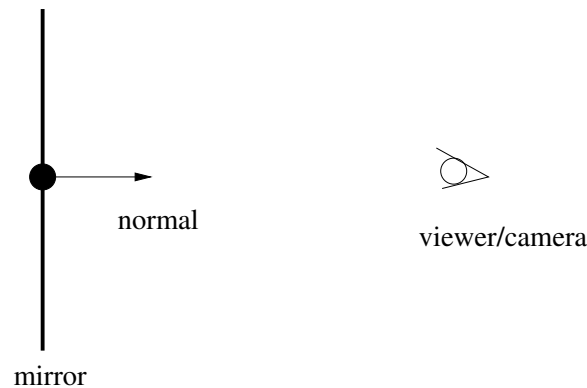


Questions

1. Consider the figure below. Indicate the scene volume that is visible to the viewer, as a reflection in the mirror. The viewer position lies in this volume, for example.

Compare two cases:

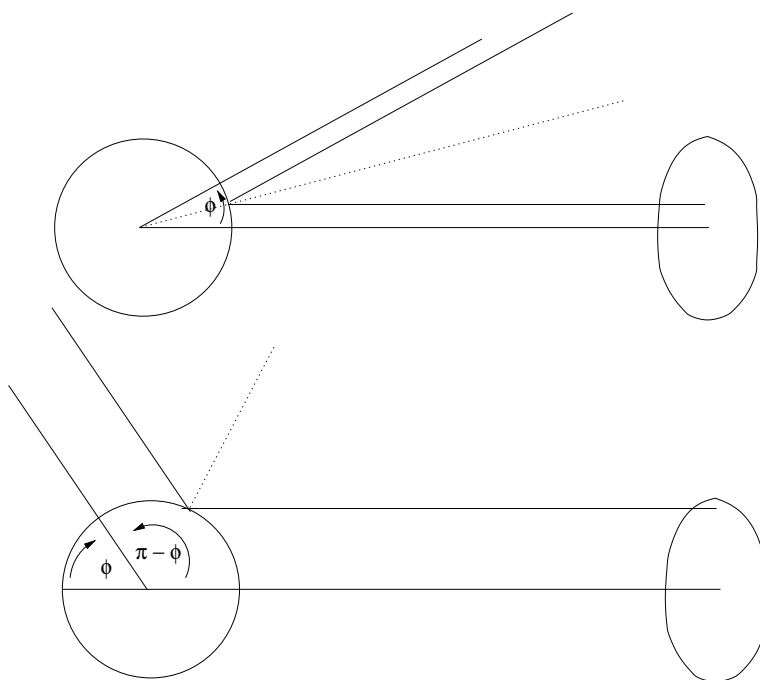
- (a) if ray tracing were used;
- (b) if environment mapping were used. (Assume that the environment map is computed from the black dot at the base of the surface normal in the figure.)



2. Clipping should not be used by ray tracing algorithms. Why not?
3. Cube maps were introduced in the environment mapping lecture. They have been used in many clever ways in computer graphics. For example, they are sometimes used as a fast way to lookup an approximation of a unit vector in the direction of some given 3D vector. How could that work?
4. The environment mapping model that I presented in the lecture uses orthographic projection of the mirror sphere onto an image plane. I argued in class that it covers nearly all directions visible from the center of the (imaginary) sphere. Now ask yourself, does it use more or fewer pixels to represent directions in the environment behind the sphere versus in front of the sphere?

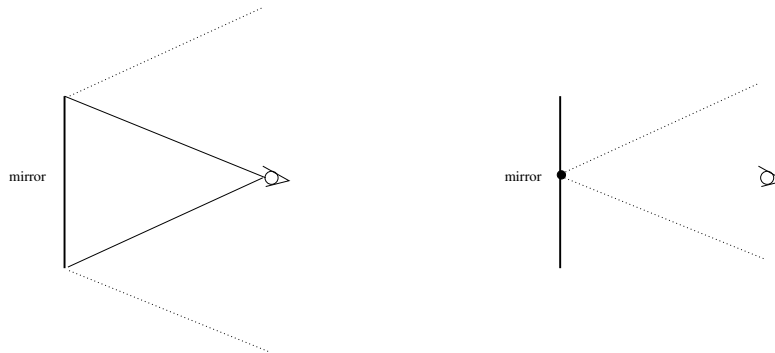
To pose this question more generally, consider any cone of radius say $\phi < \frac{\pi}{2}$ centered in the direction of the camera (z). What is the area on the disk map occupied by the directions that are bounded by this cone? Give a similar expression for the case the $\phi > \frac{\pi}{2}$.

Hint: I don't expect most you to take the time to work this one out. But do think about it, since it will help you understand environment maps. Examine the geometry in the two sketches below. In each sketch, an environment ray arriving at the sphere is reflected parallel to the z axis and arrives (orthographic projection) at the disk map.

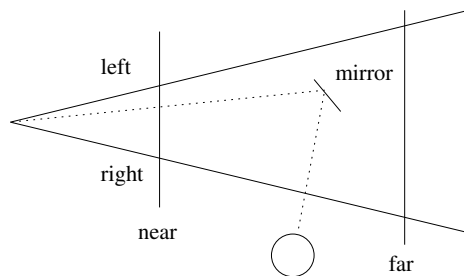


Answers

1. The view volume that would be visible in the two cases lies between the dotted lines. The case on the left is ray tracing. The case on the right is environment mapping (with environment map computed from the black dot).



2. Ray tracing was motivated by mirror surfaces. To trace rays back into the scene, we cannot restrict ourselves to points that lie in the view volume. Surfaces may be visible in the reflection off the mirror surface which do not lie in the view volume. For example, in the figure below, the disk is outside the view volume, but would be visible in the mirror. Such surfaces would need to be considered by any ray tracing method. Thus, we would need to consider the whole scene in solving for visibility.



3. Given a vector \mathbf{v} , computing a unit vector in the direction of \mathbf{v} requires that we compute a square root i.e. $\mathbf{v} / \sqrt{v_x^2 + v_y^2 + v_z^2}$. This is relatively expensive.

Suppose that instead of dividing \mathbf{v} by its norm, we divide by the maximum absolute value of its components:

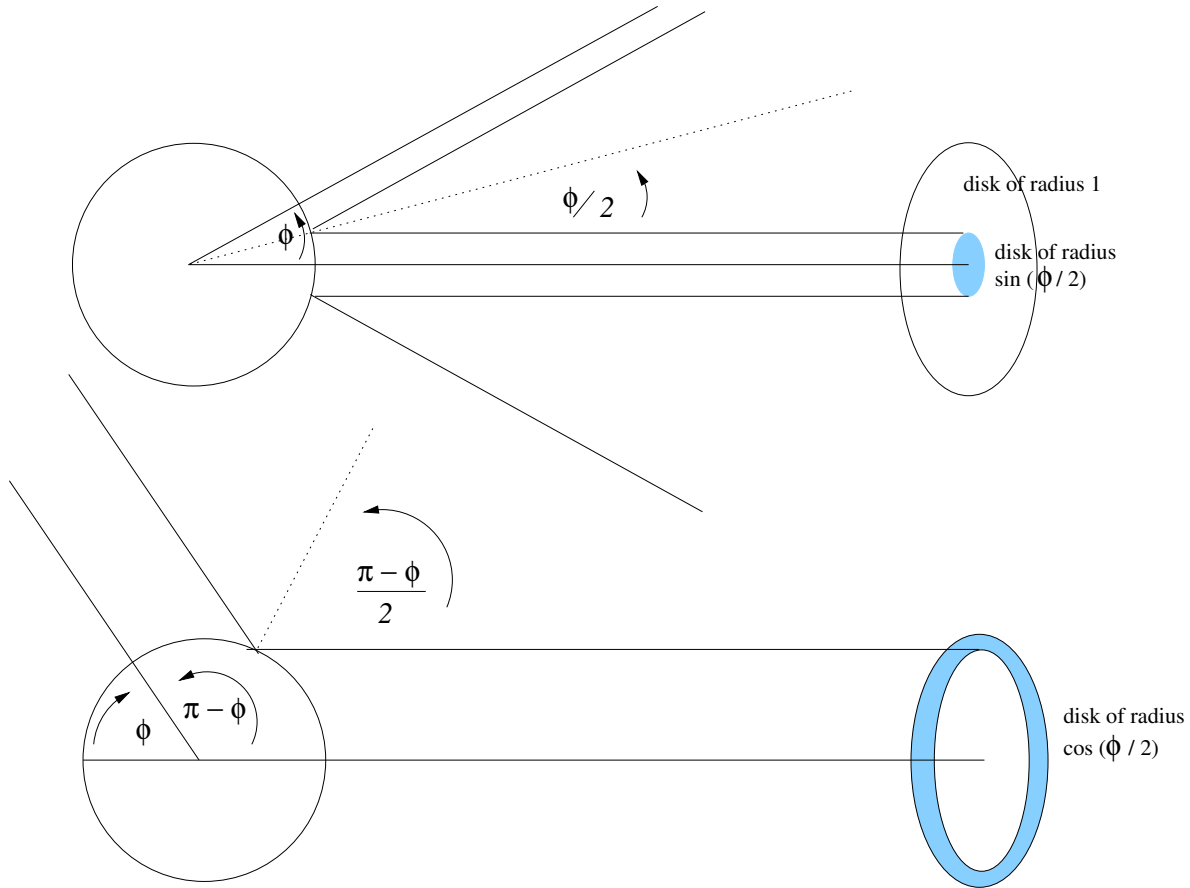
$$\mathbf{v} = (v_x, v_y, v_z) / \max(|v_x|, |v_y|, |v_z|)$$

Typically one of the components will be 1 or -1 and this can be used as an index into a cube map. What to store in the cube map?

Each pixel in the cube map can store an RGB value, namely the (precomputed) normalized vector in that direction on the cube. Typically a given vector \mathbf{v} will not project exactly to a pixel on the cube map, so one could take the closest pixel and get a good approximation (if

the cube map is large). This seems like madness the first time you hear it, but now imagine you need to normalize billions of vectors $\mathbf{v} \in \mathbb{R}^3$. It might be better to have such a lookup table.

- Consider a cone of radius ϕ with apex at the center of the sphere and directions centered in the viewing direction. For a ray whose direction is parallel to this cone (see figure) to be reflected in the direction of the z axis, this ray must strike the mirror sphere at a point that is $\frac{\phi}{2}$ degrees from the z axis direction. Thus the set of directions in the scene that are bounded by this cone and that reflect in the z axis direction will be represented on the disk map by a disk of radius $\sin(\frac{\phi}{2})$. This disk has area $\pi \sin(\frac{\phi}{2})^2$.



By a similar argument, the environment map directions that are on the far side of the sphere and lie in a cone of radius $\pi - \phi$ that is centered in the opposite direction ($-z$) will be reflected in direction z when they reflect off an annulus of points on the disk map. This annulus is the whole disk of unit radius minus a disk of radius $\sin(\frac{\pi - \phi}{2}) = \cos(\frac{\phi}{2})$. The area of this annulus is $\pi - \pi \cos(\frac{\phi}{2})^2 = \pi \sin(\frac{\phi}{2})^2$ which is the same as the area above.

The angle $\phi = \frac{\pi}{2}$ partitions space into points behind the viewer and points in front of the viewer. It follows from the above calculation that the disk map representation uses as many samples of the environment map for 3D points behind the sphere as it does for 3D points in front of the sphere.