

Questions

1. Consider the following perspective projection,

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & 0 \end{bmatrix}.$$

What is the projection plane?

2. Suppose the eye is at $(0, 0, -f)$ where $f < 0$ and the projection plane is $z = 0$. Also suppose the camera's y and z axes are the same as those axes for world coordinates. What is the projection matrix?
3. For the previous question, what happens when $f \rightarrow -\infty$?
4. Define a projection matrix that projects onto the the $x = f$ plane. Assume the center of projection (eye) is $(0,0,0)$.

Answers

1.

$$(x, y, z, 1) \rightarrow (x, y, z, 2y) \equiv \left(\frac{x}{2y}, \frac{1}{2}, \frac{z}{2y}, 1\right)$$

and so we are mapping to the $y = \frac{1}{2}$ plane.

2. First we translate the scene by $(0, 0, f)$. This effectively maps from world to camera coordinates. In particular, it moves the viewer to the origin in world coordinates and reduces to the previous case where we projected to the plane $(0, 0, f)$, where $f < 0$ i.e. the projection plane is at a negative value of z . Then do this projection. Finally, we need to translate back from camera to world coordinates so that all projected points lie on the $z = 0$ plane in world coordinates. So we translate by $(0, 0, -f)$, where again we note $f < 0$ so this is a translation in the positive z direction. This gives:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -f \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & f & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & f \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & f \end{bmatrix},$$

3. It is easier to see what happens as $f \rightarrow \infty$ if we rewrite the projection matrix:

$$\begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & f \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{f} & 1 \end{bmatrix}$$

Letting $f \rightarrow \infty$ on the right side gives $\frac{1}{f} \rightarrow 0$ which gives an orthographic projection to the z axis.

4.

$$\begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & f & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}.$$