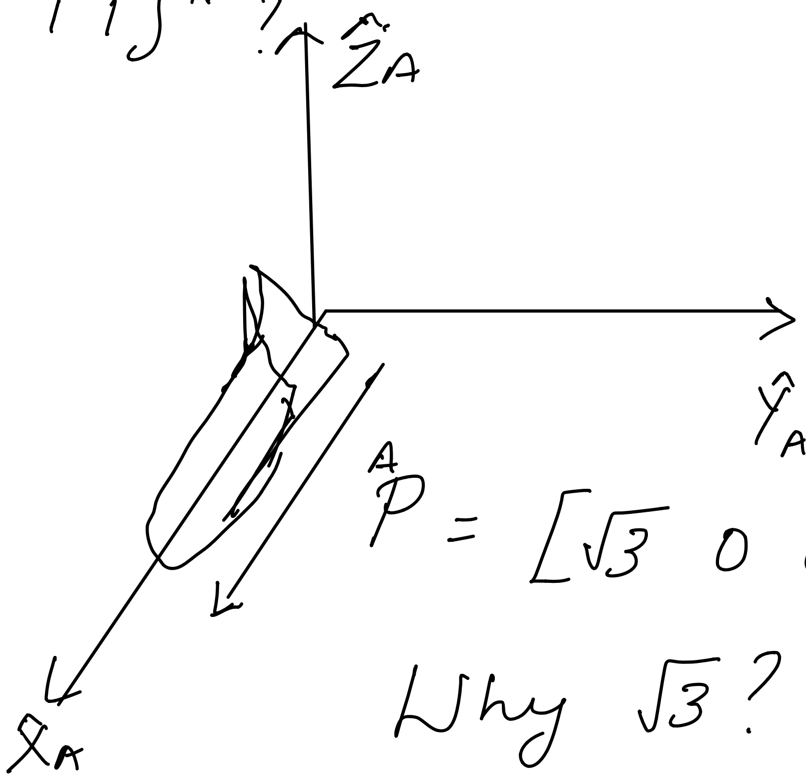
Lecture 4:

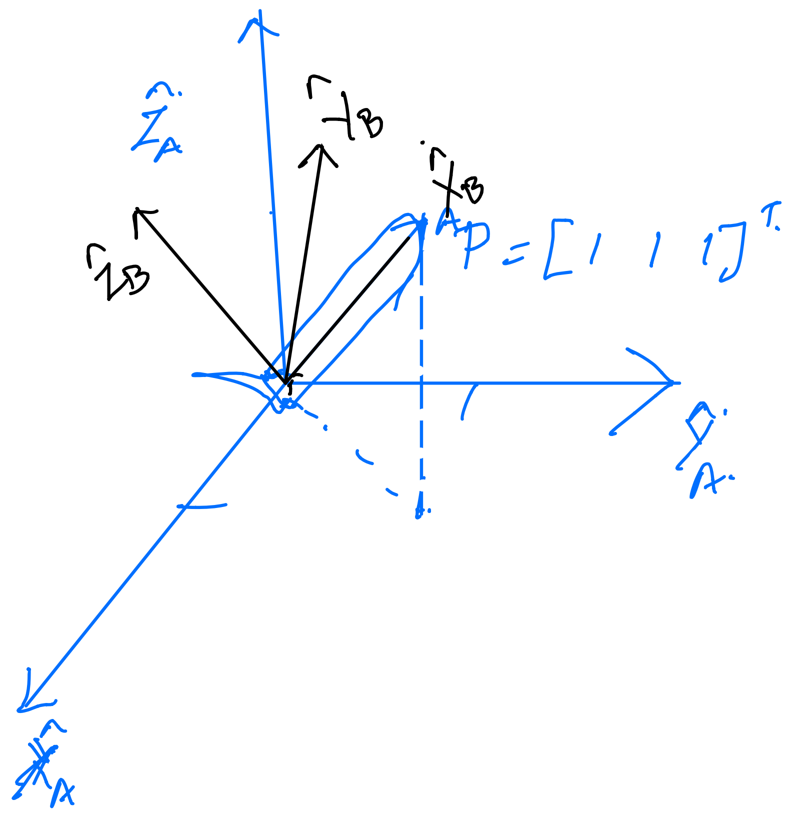
1). The Euler angle expression in Lecture 2 notes is correct.

Consider an aircraft pointing along the X axis. What is the rotation operator or the equivalent Rotation Transform that makes it point towards $[1\ 1\ 1]^T$?



$^{A}P = [\sqrt{3}\ \ 0\ \ 0]^T$

Why $\sqrt{3}$?



$${}^{A}P = [1 \quad 1 \quad 1]^T \; ; \; {}^{B}P = [\sqrt{3} \quad 0 \quad 0]^T$$

$${}^{A}P = R^{A}_{B} \, {}^{B}\vec{P} \longrightarrow (1)$$

Using Z-Y-X Euler angle convention.

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} c\alpha & -s\alpha & 0 \\ s\alpha & c\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\beta & 0 & s\beta \\ 0 & 1 & 0 \\ -s\beta & 0 & c\beta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\gamma & -s\gamma \\ 0 & s\gamma & c\gamma \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Three transcendental equations!!
in 3 unknowns.

How do you solve?

- Newton Raphson!!

→ Use Fsolve in MATLAB

→ Pose it as a non-linear opt.

→ fsolve highly sensitive to initialization.

→ Use a compatible solver in Python

The ECEF-ENU conundrum:

→ Once again: The derivations seem correct to me.

→ Here we indulge again!!

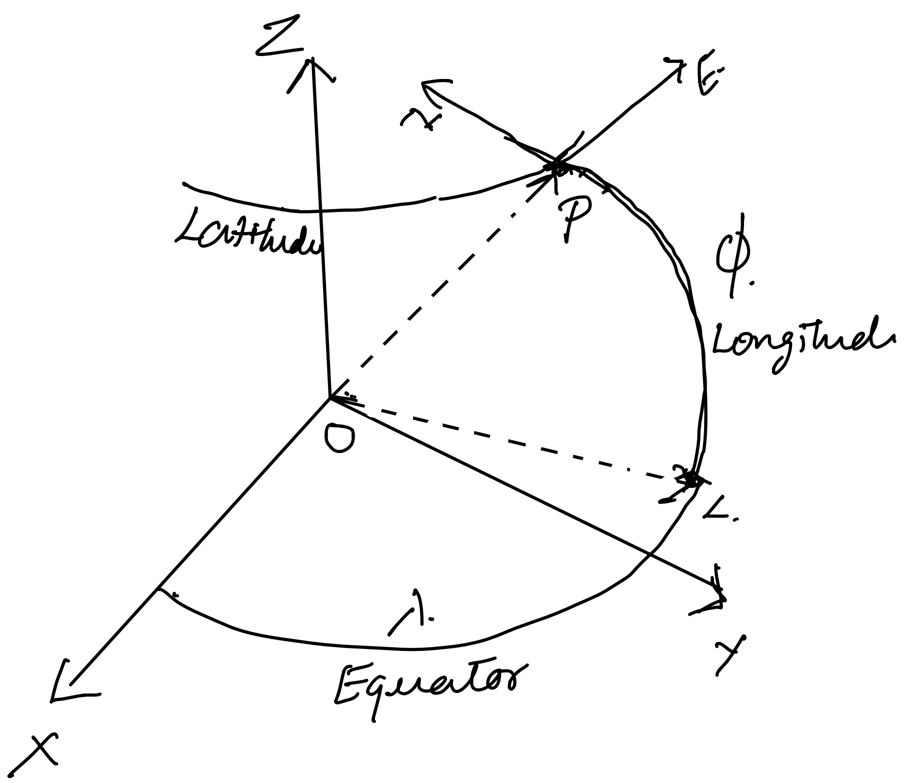


Fig-1

From the top view get the equatorial circle or equivalently the Latitude circle.

Tangent drawn to the equatorial circle is the local East

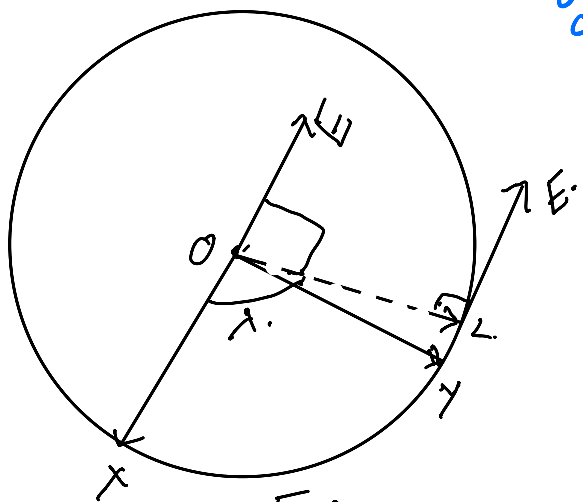


Fig 2

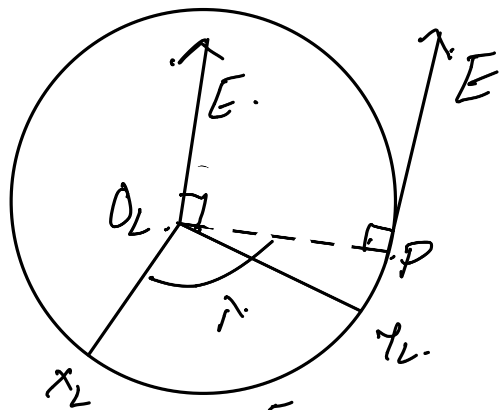


Fig 3

Equatorial Circle.

$\vec{OE} \perp_r \vec{LE}$.

$\angle EOX = 90 + \lambda$.

(Looks like a straight line)

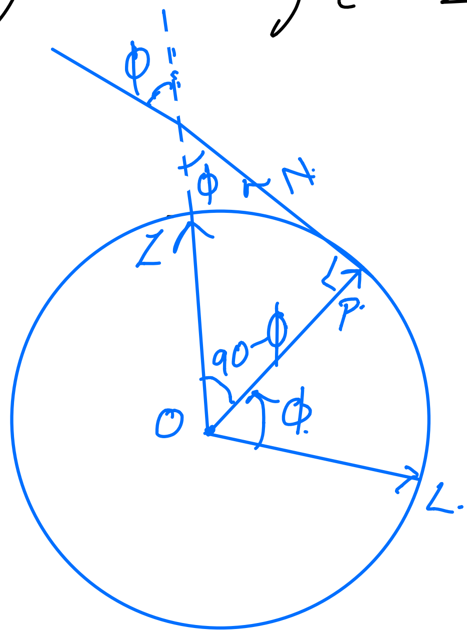
Latitude Circle

$\vec{O_L P} \perp \vec{PE}$.

$\angle E O_L X_L = 90 + \lambda$.

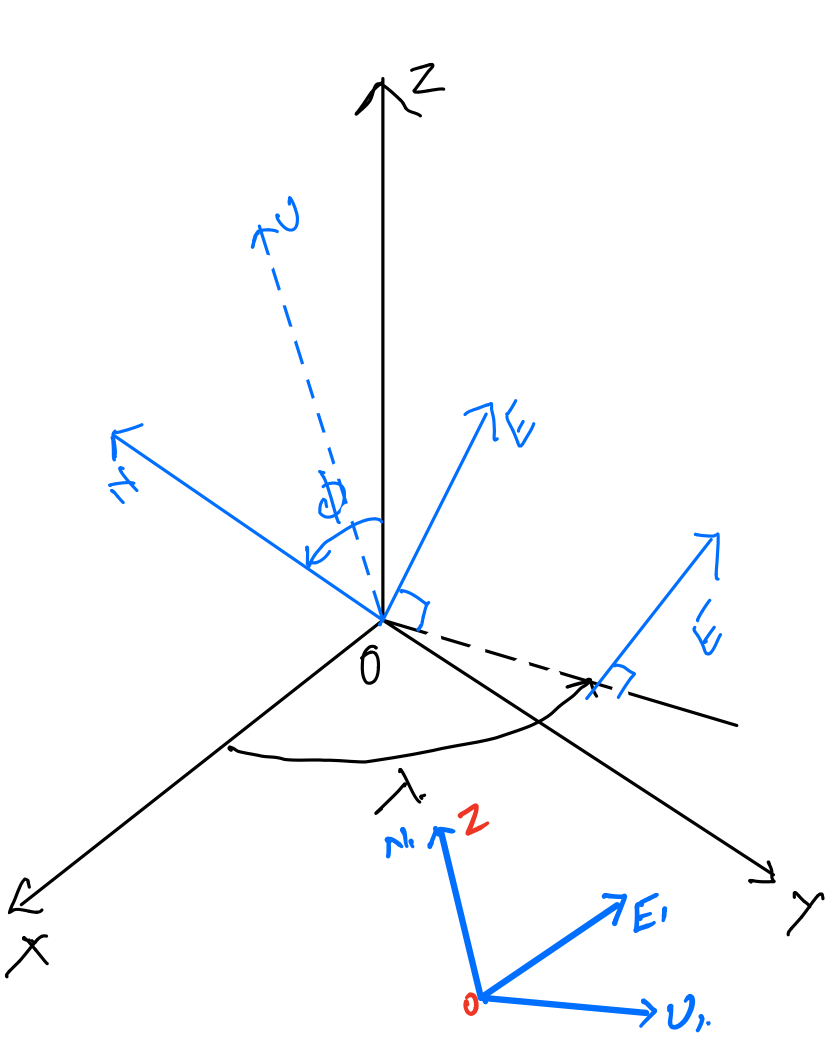
Longitudional Circle:

Consider the ZOL plane in figure 1 and the O'le with O as the centre and passing through Z L and P



Note $\angle ZOL$ is 90 as L lies on the equator.
(Tangent drawn to the longitudinal circle is the local North)

∴ What do we have?



1) $\vec{OE}$ lies in the equatorial plane or O'le.

2) Rotate $\hat{N}$ about E by ϕ to align N with Z.

3). Rotate E about N by $-(90+\lambda)$ to align E with X*. (see fig A below)

$$R_{XYZ}^{ENU} = R_{(ENU)_1}^{ENU} \, R_{(ENU)_2}^{(ENU)_1}$$

$$= R_E(\phi) \, . \, R_{N_1}(-(90+\lambda)) \equiv \underline{\underline{R_x(\phi) . R_y(-(90+\lambda))}}$$

=

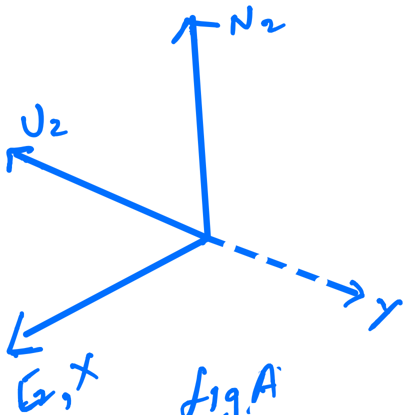


fig. A

* Rotate N_2 about E_2 by $(-\pi/2)$ to align N_2 with Y and U_2 with Z.

Then $R_{XYZ}^{ENU} = R_{(ENU)_2}^{ENU} . R_{(ENU)_3}^{ENU_2}$

$$= \boxed{R_x(\phi) . R_y(-(90+\lambda)) \, R_x(-\pi/2)}$$