Triangulation & PnP (3D-2D)

Dates Taught	October 23, 2020
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Lecture No.	L19
Module	SLAM: Vision
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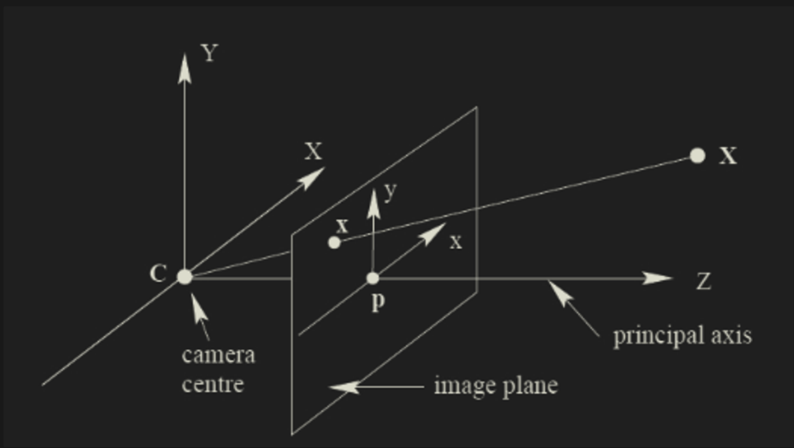
Oct 23rd agenda

1. Triangulation
2. PnP

Next class: revisiting epipolar geometry and then, computation of F.

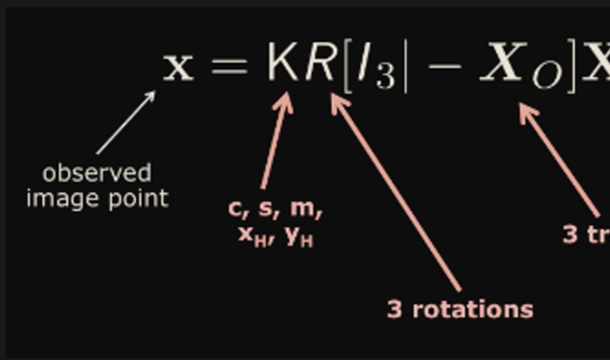
0. Revisiting single view geometry

0.1 Difference between the ray and the image coordinates



Pin hole camera

Few simplications:



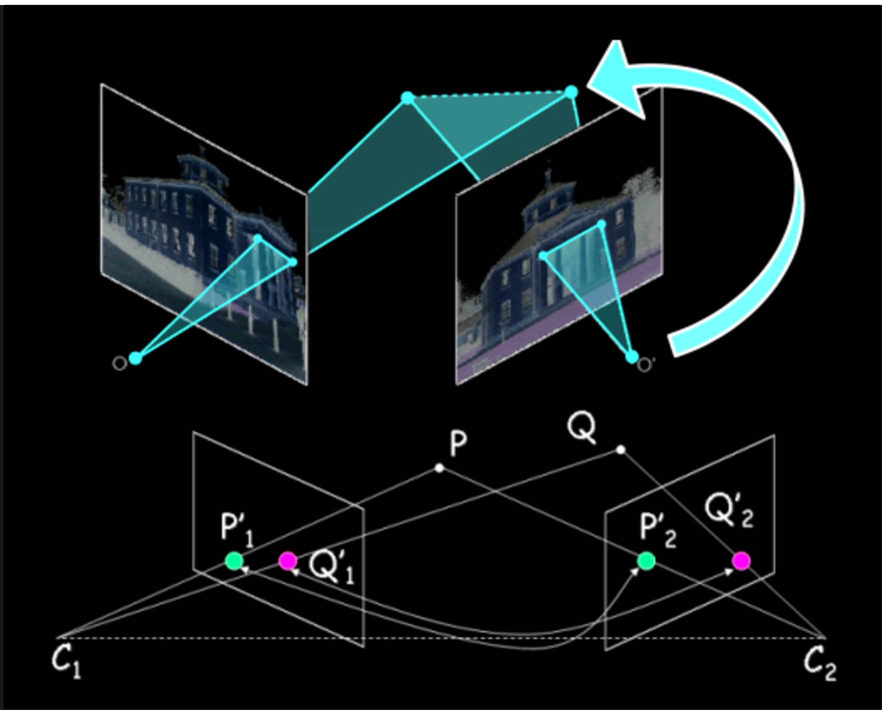
$$\begin{bmatrix} c & 0 & 0 \\ 0 & c & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Coordinates of the image point in the camera coordinate system? Let's call it **normalized image coordinates**.
- What is the relation between **normalized image coordinates** and **3D object point** $\mathbf{X}$?
- How to arrive at the above vector from homogenous **image coordinates** $[x, y, 1]$? Forget about the scaling factor. Clue ⇒

$$\lambda \mathbf{x} = \mathbf{K}\mathbf{X}$$

4. Triangulation

> How to compute the position of a point in 3-space given its image in two views and the camera matrices of those views?



We know C_1, C_2 and P'_1, P'_2 and camera matrix K. We want to find P, the world point.

Equations of Lines:

▸ Revisiting what vector and position vectors are.

Here, I have 2 lines and want to find its intersection.

Every line is determined by a point (*position vector*) and a direction (*vector*).

1. Is there a point?
2. Is there a direction?

$$\boldsymbol{f} = \boldsymbol{P} + \lambda \boldsymbol{r}$$
$$\boldsymbol{g} = \boldsymbol{Q} + \mu \boldsymbol{s}$$

▸ **Knowns:**

Unknown:

- scalars μ and λ. Which will in turn give us the world points.

Rays from the camera to the 3D point in the world:

$$\boldsymbol{r} = K_1^{-1} \boldsymbol{x_1}$$
$$\boldsymbol{s} = R_2^1 K_2^{-1} \boldsymbol{x_2}$$

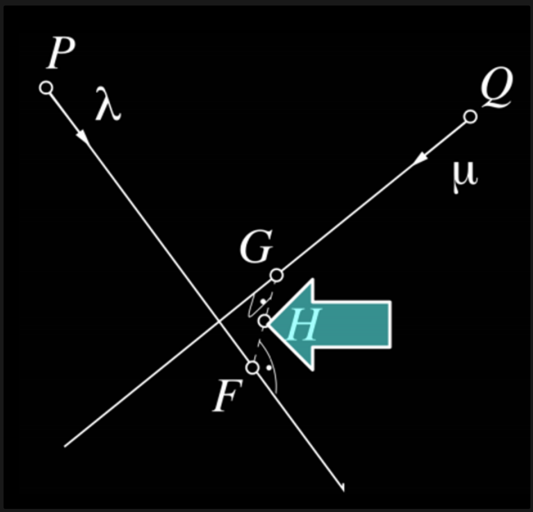
▸ How s?

- $\boldsymbol{x_1}, \boldsymbol{x_2}$: image pixel coordinate (homogeneous) in camera 1 & 2.
- K_1 and K_2 are intrinsic matrices for camera 1 and 2.
- R_2^1 is the relative orientation of camera 2 with respect to camera 1.

If they intersected

$$1. \quad \vec{f} = \vec{g} \iff ||f|| = ||g||$$

$$2. \quad \frac{\vec{f} \cdot \vec{g}}{||f||||g||} = 1$$



- **Knowns:**

Unknown:

F and G are the world points.

- scalars μ and λ.

If they don't intersect

- Ensure distance is minimum.
- Line $(\mathrm{F} - \mathrm{G})$ perpendicular to both lines r and s.

$$\begin{aligned}(\boldsymbol{F} - \boldsymbol{G}) \cdot \boldsymbol{r} = 0 \\ (\boldsymbol{F} - \boldsymbol{G}) \cdot \boldsymbol{s} = 0\end{aligned}$$

$$\begin{aligned}\boldsymbol{f} = \boldsymbol{P} + \lambda \boldsymbol{r} \\ \boldsymbol{g} = \boldsymbol{Q} + \mu \boldsymbol{s}\end{aligned}$$

We have two equations and two unknowns λ and μ.

$$\begin{aligned}(\boldsymbol{P} + \lambda \boldsymbol{r} - (\boldsymbol{Q} + \mu \boldsymbol{s})) \cdot \boldsymbol{r} = 0 \\ (\boldsymbol{P} + \lambda \boldsymbol{r} - (\boldsymbol{Q} + \mu \boldsymbol{s})) \cdot \boldsymbol{s} = 0\end{aligned}$$

$$\begin{bmatrix} r \cdot r & -s \cdot r \\ r \cdot s & -s \cdot s \end{bmatrix} \begin{bmatrix} \lambda \\ \mu \end{bmatrix} = \begin{bmatrix} (\boldsymbol{Q} - \boldsymbol{P}) \cdot r \\ (\boldsymbol{Q} - \boldsymbol{P}) \cdot s \end{bmatrix}$$

$$\begin{bmatrix} \lambda \\ \mu \end{bmatrix} = \begin{bmatrix} r \cdot r & -s \cdot r \\ r \cdot s & -s \cdot s \end{bmatrix}^{-1} \begin{bmatrix} (\boldsymbol{Q} - \boldsymbol{P}) \cdot r \\ (\boldsymbol{Q} - \boldsymbol{P}) \cdot s \end{bmatrix}$$

- λ and μ found.
- Obtain $\boldsymbol{F}$ and $\boldsymbol{G}$ from right equation.
- Mid-point of this line segment $\boldsymbol{H} = \boldsymbol{F} - \boldsymbol{G}$ is the final estimate for the $3\mathrm{D}$ triangulated world point.

$$\boldsymbol{f} = \boldsymbol{P} + \lambda \boldsymbol{r}$$
$$\boldsymbol{g} = \boldsymbol{Q} + \mu \boldsymbol{s}$$

$$\vec{h} = \vec{f} + \frac{||FG||}{2}\widehat{fg}$$

5. PnP

5.0 Introduction

5.0.1 What is the Perspective n Points (PnP) problem?

💡 **Given:** known 3D landmarks positions in the **world frame** and given their 2D image correspondences in the **camera frame**.

💡 **Determine:** 6DOF pose of the camera (or camera motion) in the world frame (including the intrinsic parameters if uncalibrated).

- However, if the 3D position of the feature points is known, then at least 3 point pairs (and at least one additional point verification result) are needed to estimate camera motion. (This is P3P)

▸ The 2D−2D epipolar geometry method

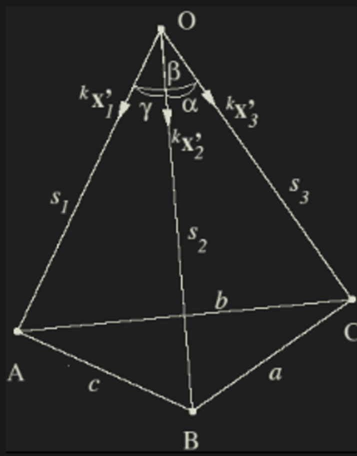
5.0.2 The P3P/Spatial Resection Problem

Given:

- $3D$ coordinates of object points X_i
- 2D image coordinates x_i of corresponding object points
- K matrix, it is a **calibrated camera.**

Find:

- Extrinsic parameters R, X_O of the **calibrated** camera (unlike DLT)

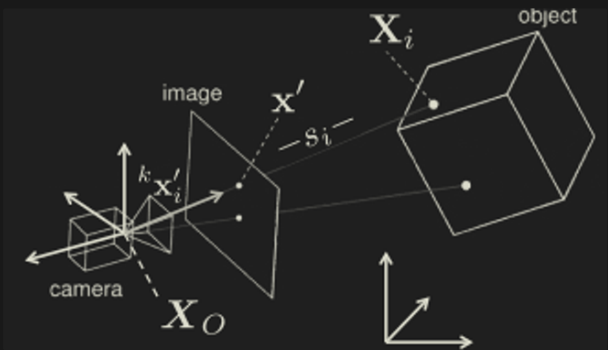


5.0.3 Difference between P3P and DLT

- P3P/Spatial Resection for calibrated cameras
    - 6 unknowns, so at least 3 points are needed
- DLT for uncalibrated cameras (seen)
    - 11 unknowns, so at least 6 points are needed

5.1 Solution to P3P

5.1.1 Revisiting normalized coordinates

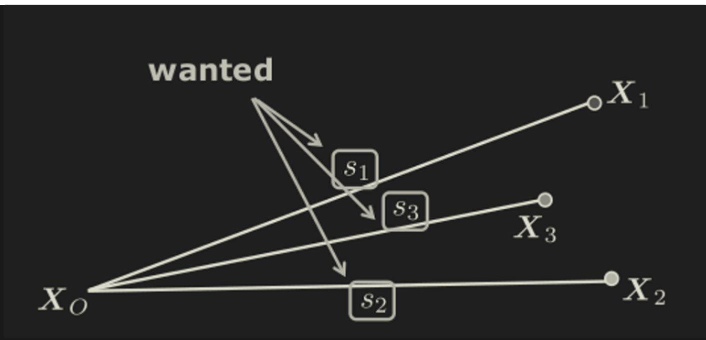


$$\mathrm{x} = \mathrm{K} R \left[I_3 | - X_O \right] \mathrm{X}$$

$$^k\mathbf{x}'_i = \mathrm{K}^{-1} \mathbf{x}'_i$$

5.1.2 Two step process

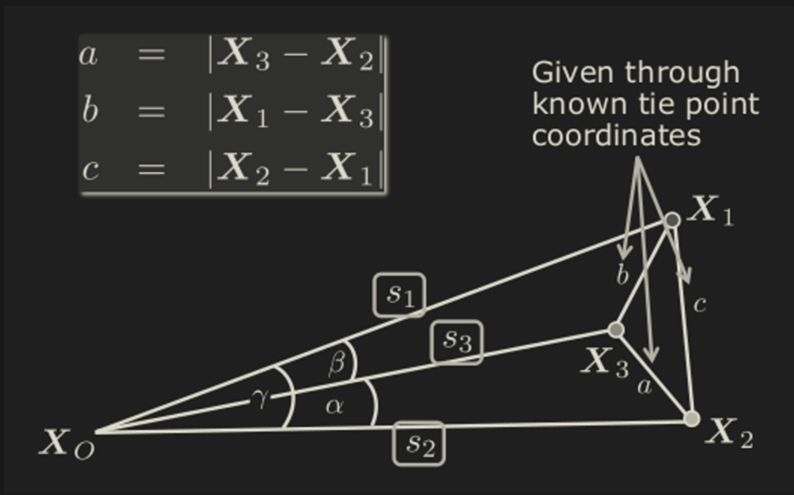
1. Length of projection rays
2. Orientation



💡 Clarity about camera frame and world frame: angles and distances between points

5.1.3 Length of projection rays

1. **Do we know a, b, c?**

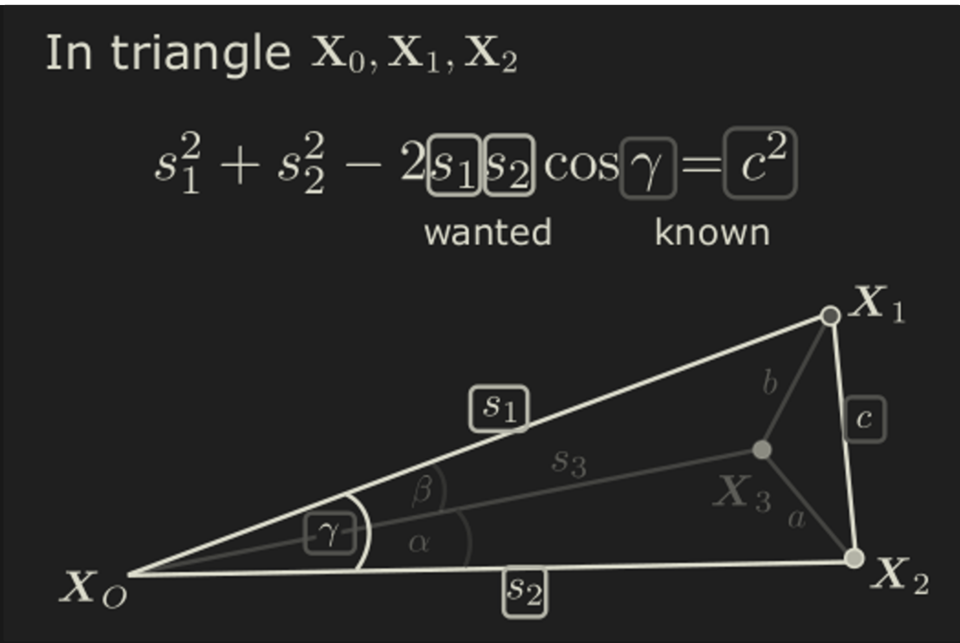


2. Do we know angles?

$$\cos\gamma = \frac{(X_1 - X_0)\cdot(X_2 - X_0)}{\|X_1 - X_0\|\,\|X_2 - X_0\|}$$

$$\text{Clue: Normalized Coords}$$

Cosine rule:



$$
\begin{aligned}
a^2 &= s_2^2 + s_3^2 - 2s_2 s_3 \cos\alpha && -(1)\\
b^2 &= s_1^2 + s_3^2 - 2s_1 s_3 \cos\beta && -(2)\\
c^2 &= s_1^2 + s_2^2 - 2s_1 s_2 \cos\gamma && -(3)
\end{aligned}
$$

We have: $\quad a^2 = s_2^2 + s_3^2 - 2s_2 s_3 \cos\alpha$

Define: $\quad u = \frac{s_2}{s_1} \quad v = \frac{s_3}{s_1} \qquad -(4)$

$$
\implies a^2 = s_1^2\left(u^2 + v^2 - 2uv\cos\alpha\right)
$$

$$
\begin{aligned}
s_1^2 &= \frac{a^2}{u^2 + v^2 - 2uv\cos\alpha}\\
&= \frac{b^2}{1 + v^2 - 2v\cos\beta}\\
&= \frac{c^2}{1 + u^2 - 2u\cos\gamma}
\end{aligned}
\tag{5}
$$

$$
\begin{aligned}
b^2 &= s_1^2 + s_3^2 - 2s_1 s_3 \cos\beta\\
c^2 &= s_1^2 + s_2^2 - 2s_1 s_2 \cos\gamma
\end{aligned}
$$

Substitute ***u*** in other equation — **4th degree polynomial:**

$$A_4 v^4 + A_3 v^3 + A_2 v^2 + A_1 v + A_0 = 0$$

$$\begin{aligned}
A_4 = & \left(\tfrac{a^2-c^2}{b^2} - 1\right)^2 - \tfrac{4c^2}{b^2}\cos^2\alpha \\
A_3 = & 4\left[\frac{a^2-c^2}{b^2}\left(1 - \frac{a^2-c^2}{b^2}\right)\cos\beta \right. \\
& \left. - \left(1 - \frac{a^2+c^2}{b^2}\right)\cos\alpha\cos\gamma + 2\frac{c^2}{b^2}\cos^2\alpha\cos\beta\right]
\end{aligned}$$

$$\begin{aligned}
A_2 = & 2\left[\left(\frac{a^2-c^2}{b^2}\right)^2 - 1 + 2\left(\frac{a^2-c^2}{b^2}\right)^2\cos^2\beta \right. \\
& + 2\left(\frac{b^2-c^2}{b^2}\right)\cos^2\alpha \\
& - 4\left(\frac{a^2+c^2}{b^2}\right)\cos\alpha\cos\beta\cos\gamma \\
& \left. + 2\left(\frac{b^2-a^2}{b^2}\right)\cos^2\gamma\right]
\end{aligned}$$

$$\begin{aligned}
A_1 = & 4\left[-\left(\frac{a^2-c^2}{b^2}\right)\left(1 + \frac{a^2-c^2}{b^2}\right)\cos\beta \right. \\
& + \frac{2a^2}{b^2}\cos^2\gamma\cos\beta \\
& \left. - \left(1 - \left(\frac{a^2+c^2}{b^2}\right)\right)\cos\alpha\cos\gamma\right]
\end{aligned}$$

$$A_0 = \left(1 + \frac{a^2-c^2}{b^2}\right)^2 - \frac{4a^2}{b^2}\cos^2\gamma$$

But upto 4 possible solutions possible. So we consider 4th point to confirm the right solution:

So say we know 2D-3D correspondence of (x, X) of 4th point, say (x_4, X_4). Just substitute X of 4th point (we know the K matrix) and the possible solutions of R, t in our camera equation and only one solution will give you the right (x_4).

3

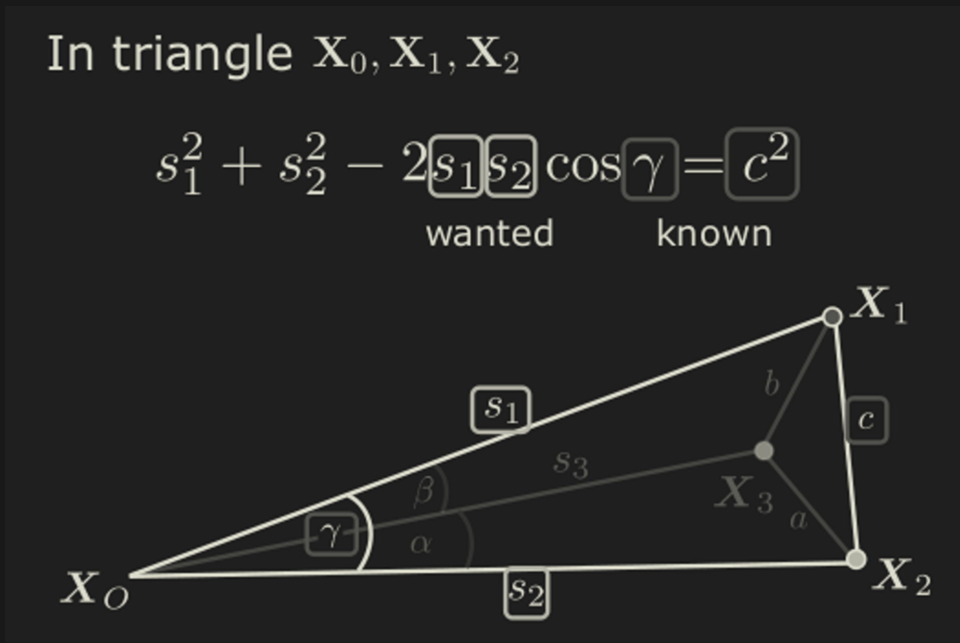
5.1.4 Transformation between camera frame and world frame

KaTeX parse error: Got group of unknown type: 'internal'

$\hat{X}$ are unit (direction) vectors..

$$\lambda \mathbf{x} = \mathbf{K}\mathbf{X}$$

($\mathbf{K^{-1}x_1} / its\ norm$)= √x Invalid equation gives direction in camera's frame. Divide by its norm to get the unit vector.



Now the question becomes: I have 3 points in one frame and same 3 points in another frame: I can use ICP now.