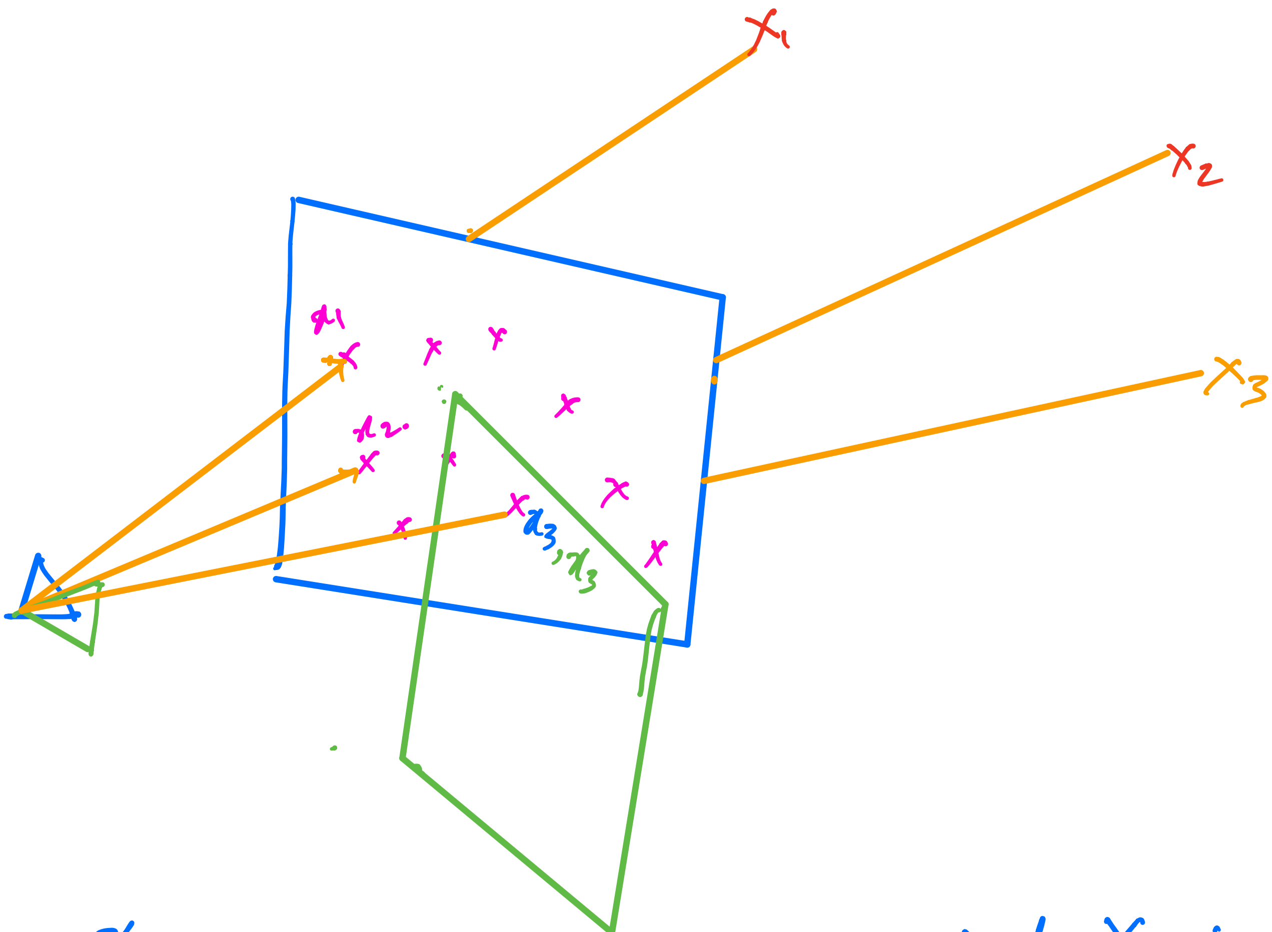


x_3 is the image of the point X_3 in the Blue camera and corresponds to x_3 in the image plane of the green camera that is rotated.

What is the relation between x_3 and x_3?

$$x_3 = K[I \; 0] X_3^B \quad (1)$$

, where X_3^B is the point X_3 in Blue camera's frame.

$$X_3^C = R X_3^B \longrightarrow (2)$$

$$x_3 = K[I \;\; 0] R X_3^B \longrightarrow (3)$$

$$= K R X_3^B = K R K^{-1} x_3 \longrightarrow (4)$$

(from (1))

or $\boxed{x' = K R K^{-1} x}$ is the pixel to pixel correspondence of an image pixel x in camera C_1's image to the image pixel x' in camera C_1' where $\boxed{C_1 \text{ is rotated by } R \text{ wrt } C_1'}$

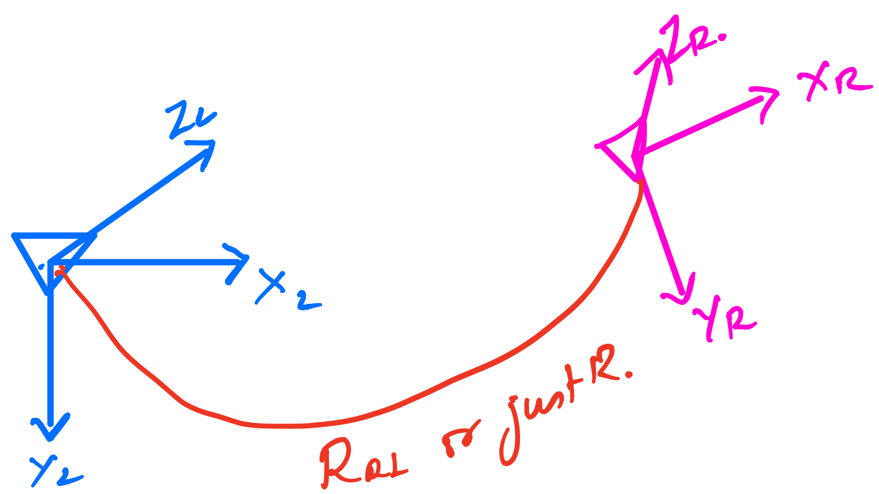
or $x' = K R K^{-1} x$ is the rotational homography that relates two cameras that suffer a pure rotation wrt each other

$$\boxed{H = K R K^{-1}} \longrightarrow \text{Rotational Homography}$$

Other kinds of homography exists.
(see page 46 of the stereo Extensive pdf file).

What happens in Stereo Rectification?

The stereo rectification consists of two rotations for the right camera and one rotation for the left camera.

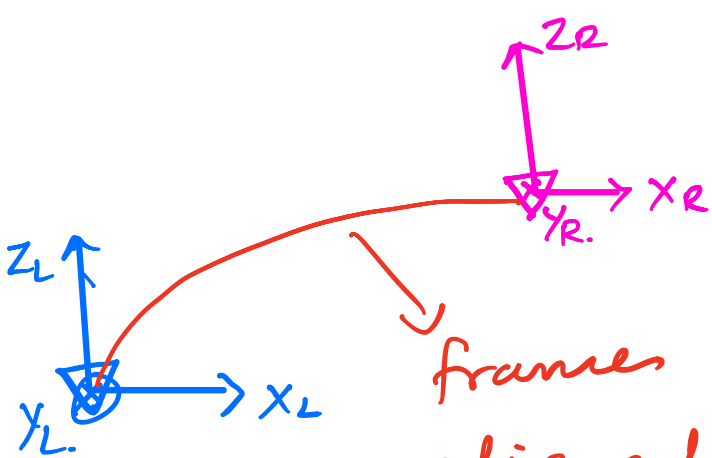


1) Synthesize the image for the right camera whose frame is aligned with the left

$\Rightarrow$ Synthesize the image for the right camera that is now rotated by $R_{RL} = R = R_{LR}^T$

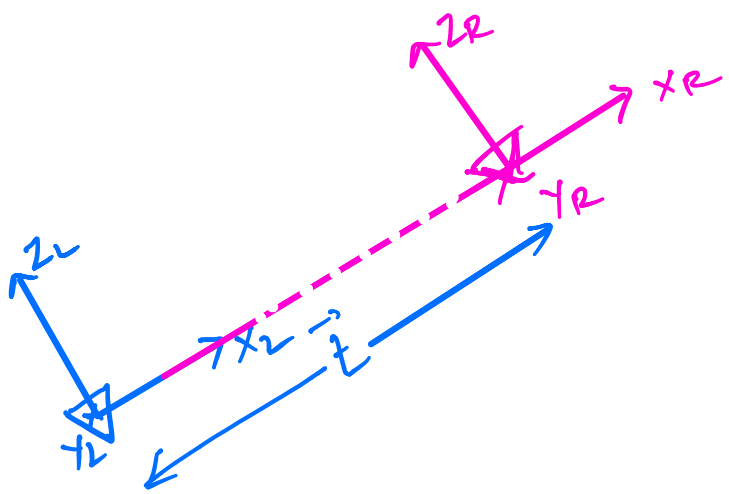
2) How does one get $R \rightarrow$ from the decomposition of the E matrix given 8 point correspondences between L & R views. Through the $F \rightarrow E \rightarrow (R, t)$ route

Now we have the L & R frame rotationally aligned



frames rotationally aligned. or rather we have images of the right camera that is obtained from the point of view of the right camera rotationally aligned with the left.

But we want



In other words we want the cameras to be rotationally aligned as well as the X axis of both the cameras to be coincident/collinear

So we need to subject to one more such rotation R_{rect}.

How do we get R_{rect}?

We know that for the stereo setting the epipoles e_1, e_2 are at infinity.

In the homogenous system of representat of ∞, $e_1 = [e_{x1}\ e_{y1}\ 0]^T$ & $e_2 = [e_{x2}\ e_{y2}\ 0]^T$

— Why?

Moreover the epipolar lines are horizontal, implies the epipole lies at x_∞ or $e_1 = [e_{x1}\ 0\ 0]^T$ & $e_2 = [e_{x2}\ 0\ 0]^T$

→ We want to find R_{rect} s.t the epipole moves to something like $[1\ 0\ 0]^T$

→ How does one compute e?

$F e_2 = F^T e_1 = 0$

or $e_1, e_2 \in N(F)$.

→ One can use SVD to compute

the null vectors that span the null space of F.

→ Or one uses standard methods from Linear Algebra to solve for e_1, e_2

Let us say upon solving we get

$$e_2 = [e_{2x} \;\; e_{2y} \;\; e_{2z}]^T = e_{2z}[e_{2x}' \;\; e_{2y}' \;\; 1]^T$$

$$= [e_{2x} \;\; e_{2y} \;\; 1]^T$$

Then $R_{rect} = \begin{bmatrix} r_1^T \\ r_2^T \\ r_3^T \end{bmatrix}$

By reverse engineering so as to speak, let

$r_1^T = \widehat{K^{-1} e_2} = \dfrac{\vec{t}}{\|t\|} = \hat{t}$ ($\widehat{K^{-1}} e_2$ points to the direction of 2nd camera).

$r_2^T = \hat{t} \times [0 \;\; 0 \;\; 1]^T$ → the cross product with the optical axis

$$= [\hat{t}_x \;\; \hat{t}_y \;\; \hat{t}_z] \times [0 \;\; 0 \;\; 1]$$

$$= \begin{bmatrix} 0 & -\hat{t}_z & \hat{t}_y \\ \hat{t}_z & 0 & -\hat{t}_x \\ -\hat{t}_y & \hat{t}_x & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= [\hat{t}_y \;\; -\hat{t}_x \;\; 0]^T$$

where $\hat{t}_x, \hat{t}_y, \hat{t}_z$ are the component of the unit vector $\hat{t} = [\hat{t}_x \;\; \hat{t}_y \;\; \hat{t}_z]^T$

$$r_3^T = r_1^T \times r_2^T$$

Where $R_{rect} = \begin{bmatrix} r_1^T \\ r_2^T \\ r_3^T \end{bmatrix}$ forms an orthonormal coordinate system.

Then $R_{rect} \hat{K^{-1}e_2} = \begin{bmatrix} r_1^T \\ r_2^T \\ r_3^T \end{bmatrix} K^{-1}e_2$

$$= \begin{bmatrix} \|\hat{K^{-1}e_2}\|^2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Then the image of the point $[1\ 0\ 0]^T$

$$= K[I\ \ 0]\begin{bmatrix}1\\0\\0\\1\end{bmatrix} = K\begin{bmatrix}1\\0\\0\end{bmatrix} = \begin{bmatrix}f_x\\0\\0\end{bmatrix}$$

$$= f_x\begin{bmatrix}1\\0\\0\end{bmatrix}$$ = point at x_∞ in the image.

→ This is precisely what we want, where the image of the 2nd camera or the epipole is at infinity

So what did we do?

We found the homography $\boxed{K R_{rect} K^{-1} e_2}$ that takes the epipole $e_2 = [e_{2x}\ \ e_{2y}\ \ 1]^T$ to $[1\ \ 0\ \ 0]^T$ that lie at infinity