# CSE-483, Mobile Robotics - 2020

# Take Home End Semester Examination

**Course Instructor: Madhava Krishna (mkrishna@iiit.ac.in)** **Max Marks: 55**

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## General Instructions

- **You will have 55 hours to complete this examination. The examination starts from 23-11-2020, 5:00 PM IST to 25-11-2020, 11:55 PM IST.**
- **Your answers must be concise and to-the-point. Verbosity will NOT fetch additional marks.**
- **Your responses must be handwritten on clean white A4 sheets. In case an answer is not legible/readable, it shall not be considered for evaluation.**
- **You are supposed to scan and upload your answer script on Moodle and send them via email to the TAs in charge.**
- **You do NOT get credit for replicating whatever is present in the textbook or your notes. Please do not fill your answer scripts with excerpts from such sources.**
- **State your assumptions clearly if there is any ambiguity with the question(s).**
- **Plagiarism is disallowed. Any attempts at mindless plagiarism will be scrutinized and punished.**

1. Fill up the following table by indicating the quantities that are known, to be estimated, or unknown, and the type of measurements (e.g. 2D-2D / 2D-3D correspondences) that are needed. **[5 points]**

| Problem | Structure (Scene geometry) | Motion (Camera parameters) | Measurements |
|---|---|---|---|
| F-matrix estimation | **Unknown** | **Estimate** | **2D-2D Correspondences** |
| Camera calibration | | | |
| Triangulation | | | |
| Stereo rectification | | | |
| PnP | | | |
| Bundle adjustment | | | |

2. **Fundamental Matrix [7 points]**

a. Derive Fe = 0, where e is the epipole of the 1st camera seen in the second image **[2 points]**
b. If the camera undergoes a pure rotational motion, what will the fundamental matrix be? **[2 points]**
c. If the fundamental matrix between images I1 and I2 is F, what is the fundamental matrix between images I2 and I1? Why are they different? **[2 points]**
d. What is the difference between a fundamental matrix and an essential matrix? How are they related? **[1 point]**

**3. Camera Calibration [3 points]**

a. List out 3 differences between camera calibration using Direct Linear Transform (DLT) vs camera calibration using planes (Zhang's method, i.e., the checkerboard method). **[3 points]**

**4. Homography [5 points]**

Suppose a camera, with intrinsic matrix K, rotates about its optical centre by a rotation matrix R.

**a.** Show that its two views are related by a homography H such that x2 = Hx1 where x1 is a point in the first image and x2 is its corresponding point in the second image. **[2.5 points]**
**b.** Also show that if θ is the rotation between the two views, then the angle 2θ corresponds to the homography H2 . **[2.5 points]**

**5. Single View and Multi View Geometry [10 points]**

a. Given a camera matrix P, detail how you can obtain the camera center and the rotation matrix R without knowing the intrinsic parameter matrix K. **[2 points]**
b. State and justify the cases when the 3D reconstruction obtained from two views is (a) Unambiguous (b) Up to an unknown scaling factor (c) Up to an unknown projective transformation. **[3 points]**
c. What does it mean to say that the recovered 3D points from two views is known only up to a scale factor ambiguity? Argue geometrically with figures and with precise reasoning. **[3 points]**
d. Argue the scale factor ambiguity mathematically now. That is, mathematically demonstrate that the recovered 3D points are known only up to a similarity transform **[2 points]**

6. **SLAM [15 points]**

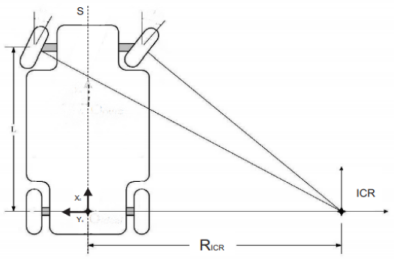
Given 3D points observed in *m* observations and the relative pose between any two observations are known as (the homogenous transform matrix between *i* and *j* observations) write down the optimization formulation for

a. ICP like SLAM clearly mentioning what each term in the cost function stands for **[2 points]**
b. For monocular SLAM (assume you know the camera intrinsics) **[3 points]**
c. The size of the Jacobian for both the formulations **[2 points]**
d. The variables being solved for in both the formulations **[3 points]**
e. The update rule for both the formulations **[5 points]**

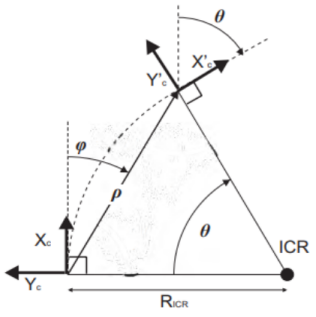
7. **Transformations [5 points]**
   a. Consider a car at a latitude of 30 and longitude of 30, aligned to the north east moves by 500 m. What is its location with respect to the center of the earth. Assume local planarity **[5 points]**

8. **Essential Matrix**

Consider a non-holonomic car that is equipped with a camera. The camera axes are denoted by Xc and Yc, and is placed such that Xc is perpendicular to the rear-wheel axis as show in figure(a). Due to the motion constraints of the car, the camera's motion can be perceived to be a circular motion, as shown in figure(b). X'c and Y'c are the camera axes in motion, whose origin with respect to the first frame, in polar coordinates, is at ρ, ϕ, and is rotated by θ.



(a)



(b)

a. Compute the essential matrix E for this configuration. (3 marks)
b. How many parameters does it have? How many corresponding image points are required for estimating this E? (Hint: It is not 8)(2 marks)