01 — Spatial Descriptions

→ Intro

* $$^A R_B = \begin{bmatrix} ^A\hat{X}_B & ^A\hat{Y}_B & ^A\hat{Z}_B \end{bmatrix}$$

$$= \begin{bmatrix} X_B \cdot X_A & Y_B \cdot X_A & \cdots \\ X_B \cdot Y_A & \ddots & \\ X_B \cdot Z_A & & \ddots \end{bmatrix}$$

→ rot^n matrix of {B} relative to {A}

Also called $^A_B R$

* $$\{B\} = \{ ^A_B R, \; \underbrace{^A P_{BORG}}_{\text{origin of } \{B\} \text{ in } \{A\}} \}$$

* $$^A P = {^B P} + {^A P_{BORG}}$$

when $^A_B R = I$

Now for when $^A P_{BORG} = 0$:

$$\left\{ \begin{aligned} ^A P_x &= {^B\hat{X}_A} \cdot {^B P} \\ ^A P_y &= {^B\hat{Y}_A} \cdot {^B P} \\ ^A P_z &= {^B\hat{Z}_A} \cdot {^B P} \end{aligned} \right\}$$

→ Why?

x component of $^A P$ is just projection of $^B P$ onto x axis of frame {A} in terms of {B}'s coordinate

$$\Rightarrow {}^{A}P = \begin{bmatrix} {}^{B}x_A \\ {}^{B}y_A \\ {}^{B}z_A \end{bmatrix} {}^{B}P = \begin{bmatrix} {}^{A}x_B & {}^{A}y_B & {}^{A}z_B \end{bmatrix} {}^{B}P$$

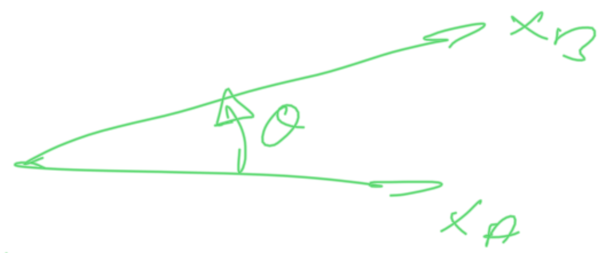
$$= {}^{A}_{B}R \; {}^{B}P$$

* Together,

$$\left\{ \begin{aligned} {}^{A}P &= {}^{A}_{B}R \; {}^{B}P + {}^{A}P_{BORG} \\ &= {}^{A}_{B}T \; {}^{B}P \end{aligned} \right.$$

$\Rightarrow$ If we calc θ for this

* the $\theta \rightarrow$ angle rotated anti clockwise to get {B} by rotating {A}



why? transforming something from {B}'s coordinate to {A}'s coordinates is the same as rotating the pt's coords by θ

$\rightarrow$ Operators:

- $\hookrightarrow$ translation
- $\hookrightarrow$ rotation

→ Compound transformation

$$ {}^{A}_{C}T = {}^{A}_{B}T \; {}^{B}_{C}T $$

$$ {}^{A}_{C}T = \left[\begin{array}{ccc|c} \multicolumn{3}{c|}{{}^{A}_{B}R \; {}^{B}_{C}R} & {}^{A}P_{BORG} + {}^{A}_{B}R \; {}^{B}P_{CORG} \\ \hline 0 & 0 & 0 & 1 \end{array}\right]_{4\times 4} $$

→ Inverting

$$ {}^{A}_{B}T^{-1} = {}^{B}_{A}T = \left[\begin{array}{ccc|c} \multicolumn{3}{c|}{{}^{A}_{B}R^{T}} & ({}^{A}_{B}R^{T})(-{}^{A}P_{BORG}) \\ \hline 0 & 0 & 0 & 1 \end{array}\right] $$

→ $\underbrace{X}_{\text{roll}}-\underbrace{Y}_{\text{pitch}}-\underbrace{Z}_{\text{yaw}}$ fixed angles

* describe orientation of frame {B} as:

start with frame coincident with {A}

rotate {B} about $\hat{X}_A$ by γ

then about $\hat{Y}_A$ by β

then about $\hat{Z}_A$ by α

* ${}^{A}_{B}R_{XYZ}(\gamma, \beta, \alpha)$

$= R_z(\alpha)\, R_y(\beta)\, \underline{R_x(\gamma)}$

Apply X rot^n first

→ ZYX Euler Angles

* Start with frame coincident with $\{B\}$

rotate about $\hat{Z}_B$ by α

then about $\hat{Y}_B$ by β

then about $\hat{X}_B$ by γ

* $${}^A_B R_{Z'Y'X'} = \underbrace{R_z(\alpha)\, R_y(\beta)\, R_x(\gamma)}_{\text{same formulation}}$$

as fixed angle.

why?

If about Z_B given B'

$Y_B \longrightarrow B''$

$X_B \longrightarrow B$

$${}^A_B R = {}^A_{B'}R \; {}^{B'}_{B''}R \; {}^{B''}_{B}R$$

Together w/ $R_x(\alpha)$.

non intuitive but makes sense mathematically