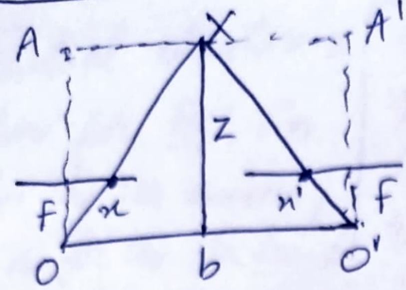
Stereo Rectification:



Position of X wrt $O = X$

Position of X wrt $O' = X - b$

Similarity in OAX,

$\frac{|x|}{f} = \frac{|X|}{Z}$ → length of side

Similarity in $O'A'X$,

$\frac{|x'|}{f} = \frac{|X-b|}{Z}$

⊛ They wrote $b-X$ in slides to have +ve side length

⊛ So, we still technically have

$\frac{x}{f} = \frac{X}{Z}, \quad \frac{x'}{f} = \frac{X-b}{Z}$

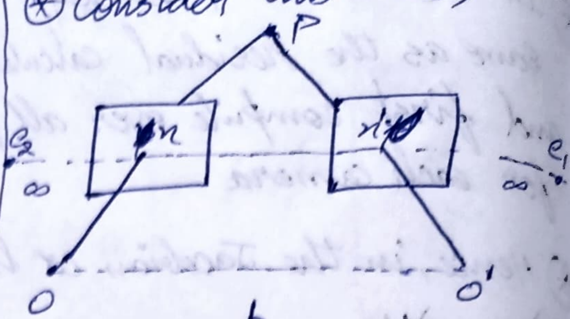
Hence, formula for disparity makes sense

$$d = x - x' = \frac{fX}{Z} - \frac{f(X-b)}{Z} = \frac{bf}{Z}$$

⊛ Motivation for making epipolar lines horizontal:

- Correspondences have to be searched along horizontal axis only because corresponding y would be same
- Triangulation would become just the above discussed d formula, which helps us find Z directly for corresponding pixel locations x & x'

⊛ Consider this case,



t = Distance = T

when the epipolar lines are horizontal, we have for the relative transform params R and t b/w O and O';

$R = I, \quad t = (T, 0, 0)$

$E = [t_x] \cdot R$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -T \\ 0 & T & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -T \\ 0 & T & 0 \end{bmatrix}$$

Note: $[t_x] \equiv S_t$ from Stachniss notation

where $S_t = \begin{bmatrix} 0 & -t_3 & t_2 \\ t_3 & 0 & -t_1 \\ -t_2 & t_1 & 0 \end{bmatrix}$

where $t = [t_1 \; t_2 \; t_3]^T$

⊛ Now, consider the simple case where $K' = K = I$

We will have $x^T E x' = 0$ ← Img coords

$$[u \; v \; 1] \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -T \\ 0 & T & 0 \end{bmatrix} \begin{bmatrix} u' \\ v' \\ 1 \end{bmatrix} = 0$$

$$[u \; v \; 1] [0 \;\; -T \;\; Tv'] = 0$$

$\Rightarrow Tv = Tv' \Rightarrow \boxed{v = v'}$

⇒ y-coordinate in pixel location is same

Will hold for other K, K' as well

* Hence, we want both the image planes and thus, the camera orientations to not have any relative rotation and be aligned with the baseline vector → Imp.

* Consider the setup in slide 35/50

* We get E from which we get R as $E = S_B R$ and we can get S_B and R

* Align both planes to have no relative rotation

* Now, find rotation matrix R_{rect} such that the epipoles e and e' lie on the horizontal axis / epipolar axis / ∥ to the baseline vector ∥ to epipolar axis

* R_{rect} will be same for both (obvious reasons, both have same relative orientation) and then scale them so that these epipoles are at infinity.

* The above happens when these fall on the epipolar axis or along baseline vector. This takes depth from 1 to 0 (same as centers, and so they become pts. at ∞)

* Coming to the formulation for the rotation matrix, we have

$$R_{rect} = \begin{bmatrix} r_1^T \\ r_2^T \\ r_3^T \end{bmatrix}$$

Takes from the orientation with with some angle with horizontal axis but no relative orientation, to horizontal axis aligned planes and epipoles at infinity

* ~~Consider $r_1 = e_1$~~

* We again have $e^T E = 0$ and $Ee'^T = 0$. Get e from svd of E

* We also have translation T from E

* Now consider $r_1 = e = \frac{T}{\|T\|}$, epipole coincides with translation vector, i.e., baseline vector direction is same

* Consider the optical axis direction from O to image plane of O. We have optical axis direction vector as (0, 0, 1). So, we get another rotation vector r_2 as $r_2 = [0\ 0\ 1] \times [T_x\ T_y\ 1]$ [Cross product of optical axis and r_1/T]

$$r_2 = [-T_y\ T_x\ 0] \xrightarrow[\|r_2\| = 1]{\text{Ensure}} r_2 = \frac{1}{\sqrt{T_x^2 + T_y^2}} [-T_y\ T_x\ 0]$$

$$r_3 = r_1 \times r_2$$

* $$R_{rect} e_1 = \begin{bmatrix} r_1^T e_1 \\ r_2^T e_1 \\ r_3^T e_1 \end{bmatrix} = \begin{bmatrix} r_1^T r_1 \\ r_2^T r_1 \\ r_3^T r_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$ Pt. on X-axis at ∞

* Also see the 13 pg pdf I uploaded