Demystifying ICP-SLAM:

NLS formulation: $\sum_{i=1}^{n} [y_i - f(x_i, \beta)]^2$

ICP as NLS:

$$\sum_{i=1}^{m} \sum_{j=1}^{n} \| X_{ij} - \hat{X}_{ij} \|^2 = \sum_{i=1}^{m} \sum_{j=1}^{n} \| X_{ij} - \hat{T}_{i0} X_{0j} \|_2^2$$

NLS solution

$$\underset{(3\times 1)}{\delta\beta} = \underset{(3\times m)\times(m\times 3)}{[J^T J]^{-1}} \underset{(3\times m)}{J^T} \underset{(m\times 1)}{\overbrace{[y - f(x, \beta)]}^{\text{residual}}}$$

ICP Solution:

$$\underset{(6m+3n,1)}{\delta_{\xi X}} = [J^T J]^{-1} J^T [X_{ij} - \hat{R}_{i0} X_{i0} + \hat{t}_{i0}]$$

$$\left[(6m+3n, 3mn) \times (3mn, 6m+3n) \right]^{-1} (6m+3n, 3mn) \times (3mn, 1).$$

→ J is split into Localization and Mapping Jacobian parts.

→ Has terms of the form $J_{11L}, J_{11M}, \ldots, J_{mnL}, J_{mnM}$.

→ J_{11L} is the Jacobian obtained by taking the derivative with respect to T_{10} of the term $[X_{11} - \hat{T}_{10} X_{01}]$
(actually wrt ξ_{10}).

→ J_{11M} is the Jacobian obtained by taking derivative wrt X_{01} of the term $X_{11} - \hat{T}_{10} X_{01}$

→ J_{12L} ... derivative wrt T_{10} of
$X_{12} - \hat{T}_{10} X_{02}$ (actually wrt ξ_{10})

→ J_{12M} ... wrt X_2 of above term

→ J_{mnL} ... derivative wrt T_{mo} of (ξ_{mo}).

$X_{mn} - \hat{T}_{mo} X_{on}$

→ J_{mnM} ... wrt $\dot{X}_n$ of the above term

→ ξ_{io} is the local tangent vector of T_{io}. obtained from Log Map.

NLS update:

$\beta(q+1) = \beta(q) - \delta\beta$ update from q to $q+1$ instances during Gauss Newton's iterative solution

ICP SLAM Update:

We consider for illustration the update of $\hat{T}_{io}$ and $\vec{X}_{oj}$ as the localization and mapping updates are very different for one is the regular Euclidean update and the other is over a manifold.

$X_{oj}(q+1) = X_{oj}(q) - \delta X_{oj}$ is the update of point X_{oj}

<u>Localization Update:</u>

$$\underset{(3\times4)}{T_{io}}(q+1) = \underset{(3\times4)}{T_{io}}(q) . Exp(\delta \xi_{io}) . \longrightarrow (L1)$$

$$\xi_{io}(q+1) = Log(T_{io}(q+1)) \longrightarrow (L2)$$

$$\operatorname{Exp}(\delta \xi_{io}) = \begin{bmatrix} R(\delta\omega_{io}) & R(\delta\omega_{io})\,\delta u_{io} \\ 0 & 1 \end{bmatrix} \quad (2.3)$$

$$R(\delta\omega_{io}) = \exp([\delta\omega_{io}]_\times)$$

$$= I_{3\times3} + \frac{\sin|\delta\omega_{io}|}{|\delta\omega_{io}|}[\delta\omega_{io}]_\times + \frac{1-\cos|\delta\omega_{io}|}{|\delta\omega_{io}|^2}[\delta\omega_{io}]^2_\times \quad (2.4)$$

where $\delta\omega_{io} = \begin{bmatrix} \delta\omega_{io1} \\ \delta\omega_{io2} \\ \delta\omega_{io3} \end{bmatrix}$ is the update ~~to~~ to the axis angle and $|\delta\omega_{io}|$ is the magnitude of $\delta\omega_{io}$ is θ or the rotation about $\delta\omega_{io}$

(L1) is the Exponential map that maps a vector in the local tangent space of T, denoted by ξ back to T.

(L2) is the Logarithm Map that maps the transform matrix T to the local tangent vector ξ,

$$\xi = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \upsilon_1 \\ \upsilon_2 \\ \upsilon_3 \end{bmatrix}$$

$$\omega = [\omega_1 \;\; \omega_2 \;\; \omega_3]^T = \ln(R).$$

(L5)

$$\upsilon = [\upsilon_1 \;\; \upsilon_2 \;\; \upsilon_3]^T = R^{-1} t.$$

(L6)

For T_{io} then

$$\omega_{io} = [\omega_{io1} \ \omega_{io2} \ \omega_{io3}]^T = \ln(R_{io}).$$

$$v_{io} = [v_{io1} \ v_{io2} \ v_{io3}]^T = R_{io}^{-1} t_{io}$$

NOTE:

Jacobian is evaluated in the next iteration w.r.t $\xi_{io}(q+1)$, $X_{oj}(q+1)$ $i = 1 \rightarrow m$, $j = 1 \rightarrow n$.

→ No Need to Remember all These

→ Solvers like Ceres, G2O, GTSAM do these for you.

→ But you need to appreciate the cost function, the Jacobian structure and the general

notion of why we resort
to manifold Optimization