8-point Algorithm:

⊛ Normalization needed for stability

⊛ $\boxed{x_n'^T F x_n'' = 0}$

$$\Rightarrow \begin{bmatrix} x_n' & y_n' & 1 \end{bmatrix} \begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix} \begin{bmatrix} x_n'' \\ y_n'' \\ 1 \end{bmatrix} = 0$$

$\Rightarrow$ Can be written as: $\boxed{a_n^T f = 0}$

where $a_n = \begin{bmatrix} x_n'' x_n', x_n'' y_n', x_n'', \\ y_n'' x_n', y_n'' y_n', y_n'', \\ x_n', y_n', 1 \end{bmatrix}_{1\times 12}$

$f = \begin{bmatrix} F_{11}, F_{12}, F_{13}, F_{21}, F_{22}, F_{23}, F_{31}, F_{32}, F_{33} \end{bmatrix}^T_{1\times 12}$

$a_n = (x_n'' \otimes x_n')^T$

$\hookrightarrow$ Kronecker Product

⊛ In general $\boxed{x^T F y = (y \otimes x)^T \cdot vec(F)}$

holds in general

Not only for fundamental matrix, any vec-Mat-vec form.

⊛ $\boxed{\begin{array}{l} a_n^T f = 0, \forall i = 1, \dots, N \\ Af = 0 \Rightarrow SVD \rightarrow f \text{ corr. to min: singular value gives the eig-vector} \end{array}}$

⊛ $\boxed{\begin{array}{l} \dim(f) = 9 \\ \text{Rank}(A) \le 8 \end{array}}$

$\Rightarrow$ 8 corresponding points

⊛ But rank(F) = 2 must be ensured

⊛ Get $UDV^T = svd(\hat{F})$

$D_{11} \ge D_{22} \ge D_{33}$; $\hat{F}$: Previously computed

Now, $F = UD^a V^T$ ~~diag~~

$= U \, diag(D_{11}, D_{22}, 0) V^T$

This 0 ensures rank(F) = 2

⊛ Normalize coordinates to stabilise

New origin at center of mass of all points and scale the coords to $[-1, 1]$

⊛ Call this transformation T.

Then $x'^T F x'' = 0$ and $\boxed{Tx = \hat{x}}$

$\Rightarrow (T^{-1}\hat{x}')^T F (T^{-1}\hat{x}'') = 0$

$\Rightarrow \hat{x}'^T T^{-T} F T^{-1} \hat{x}'' = 0$

$\Rightarrow \hat{x}'^T \hat{F} \hat{x}'' = 0$

$\Rightarrow \boxed{\hat{F} = T^{-T} F T^{-1}}$ and $\boxed{F = T^T \hat{F} T}$

⊛ Avoid coplanar pts. (close to plane also)

⊛ Singularity in projection matrix when translation is absent

8-point algo:

- Build A as before
- $Ae = 0$
- E from SVD
- Ensure rank$(E) = 2$
- Do $UDV^T = \text{svd}(\hat{E})$ — Predicted previously
- $\boxed{E = U\,\text{diag}(1,1,0)\,V^T}$

To model the constraints of essential matrix (reason not given)

- Transformation for stabilisation remains the same.

$$\boxed{\hat{E} = T'^{T} E T^{-1}}$$
$$\boxed{E = T'^{T} \hat{E} T}$$

⊛ Properties of E:

- Homogeneous
- Singular $|E| = 0$
- Two identical non-zero singular values
- $2EE^TE - tr(E^TE)E = 0_{3\times3}$

due to skew-sym. matrix

⊛ We have $E = U\,\text{diag}(1,1,0)\,V^T$

$$E = U\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} V^T$$

Define $Z = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow$ Skew-symmetric matrix

$$W = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

we have $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = ZW$

Now $E = UZWV^T$

$= UZU^TUWV^T$ [$\because U$ is orthogonal]

$\underbrace{\begin{pmatrix}\text{Rot} \\ \text{mat}\end{pmatrix} \times \begin{pmatrix}\text{Skew} \\ \text{symp} \\ \text{mat}\end{pmatrix} \times \begin{pmatrix}\text{Rot} \\ \text{mat} \\ \text{inv.}\end{pmatrix}}_{\text{Skew-sym mat}} \quad \underbrace{\begin{pmatrix}\text{Rot} \\ \text{mat}\end{pmatrix} \times \begin{pmatrix}\text{Rot} \\ \text{mat}\end{pmatrix} \times \begin{pmatrix}\text{Rot} \\ \text{mat}\end{pmatrix}}_{\text{Rot mat}}$

$E = \underbrace{UZU^T}_{S_B}\ \underbrace{UWV^T}_{R^T}$

S_B → From this we can get baseline vector direction (2 Params)

R^T → Relative orientation b/w 2 cams (3 Params)

Total 5 params → 2+3

$\text{diag}(1,1,0) = ZW = -Z^TW = -ZW^T = Z^TW^T$

$Z = -Z^T$ } Total 2 possible vals for Z: Z, Z^T

" " " " " W: W, W^T

4 possible solutions for $(S_B R^T)$ and hence for E

⊛ Solution where all points lie in front of both cams

⊛ This approach coupled with RANSAC used as initial guess for iterative method