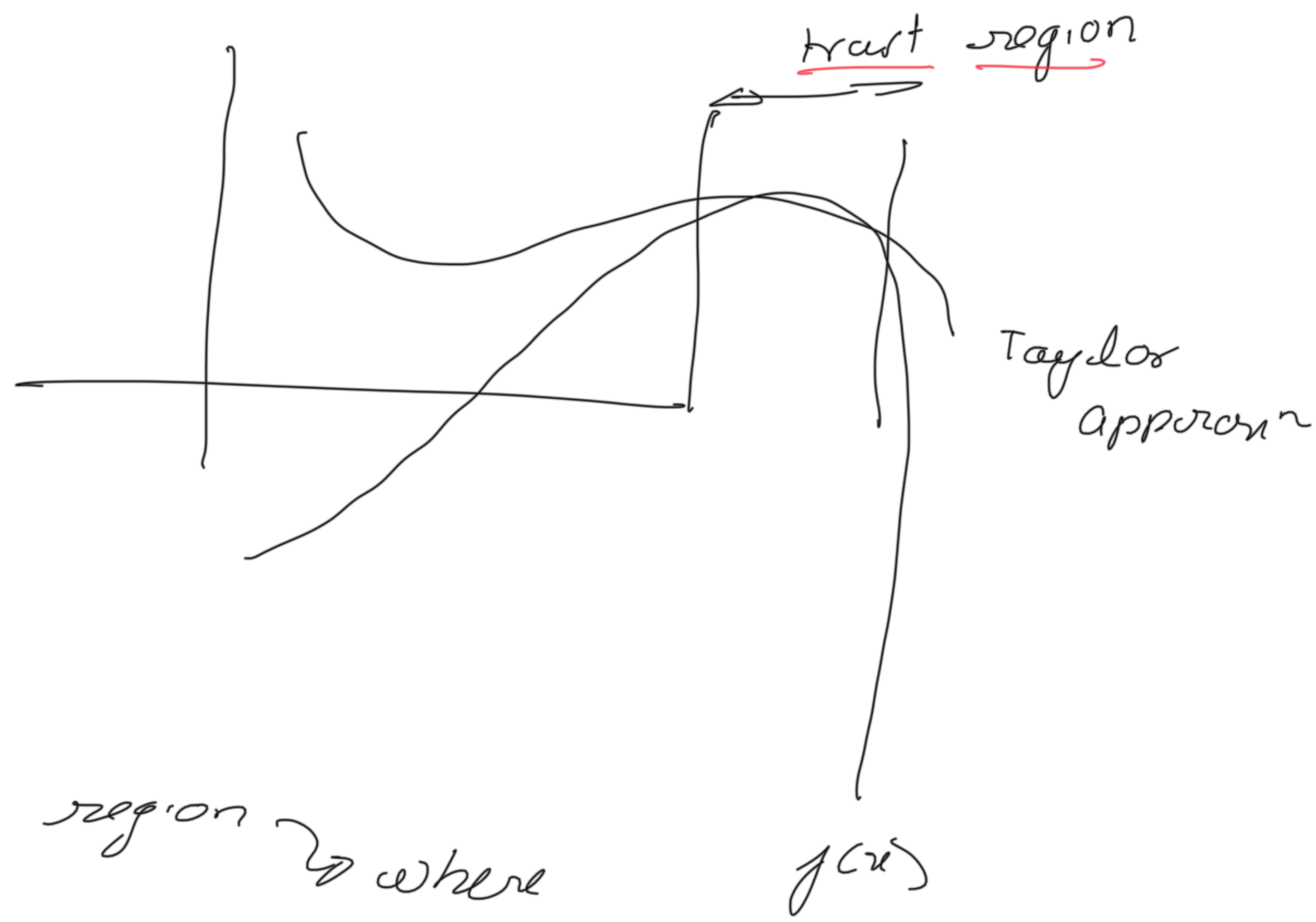
03 – Least Squares (continued)

→ LM (cont.)



trust region → where gradient is in correct direcn & close to correct direcn.

We use trust region indicator (ρ)

$$\boxed{\rho = \frac{f_R(x+\Delta x) - f_R(x)}{J^T \Delta x}}$$

$\dfrac{\text{real descent}}{\text{approxn descent}}$

f_R → real funcn

f_A → approxn.

$$f_A(x+\Delta x) = f_R(x) - J^T \Delta x$$

$$f_A - f_R = -J^T \Delta x$$

$\rho < 1 \rightarrow$ gradient is diverging (reduce descent)

$\rho = 1 \rightarrow$ stay same

$\rho > 1 \rightarrow$ ...

Now, we use a constrained optimization problem

$$\boxed{\|D\,\Delta x\|^2 \le \mu}$$

$\rightarrow D$ is some matrix, usually I

$\rightarrow$ used to enforce trust region.

How?

$\rightarrow$ use Lagrangian

$$\boxed{L(\Delta x, \lambda) = \frac{1}{2}\|f(x) + J^T \Delta x\|_2^2 + \frac{\lambda}{2}\left(\|D\,\Delta x\|^2 - \mu\right)}$$

$\rightarrow$ minimizing Lagrangian.

If we minimize L by diff wrt Δx:

$$\boxed{(J J^T + \lambda D^T D)\,\Delta x = -J f(x)}$$

when $D = I \rightarrow$ L.M.

solves problem of singularity

($J J^T + \lambda D^T D$ is invertible)

We don't optimize λ ($\because$ very expensive)