

Demystifying ICP-SLAM:

NLS formulation: $\sum_{i=1}^n [y_i - f(x_i, \beta)]^2$

ICP as NLS:

$$\sum_{i=1}^m \sum_{j=1}^n \|x_{ij} - \hat{x}_{ij}\|^2 = \sum_{i=1}^m \sum_{j=1}^n \|x_{ij} - \hat{T}_{i0} x_{j0}\|_2^2$$

NLS solution

$$\delta \beta = \underbrace{[J^T J]^{-1} J^T}_{\text{residual}} [y - f(x, \beta)].$$

$(3 \times 1) \quad (3 \times m) \times (m \times 3) \quad (3 \times m) \quad (m \times 1).$

ICP Solution:

$$\delta \xi x = [J^T J]^{-1} J^T [x_{ij} - \hat{R}_{i0} x_{i0} + \hat{t}_{i0}]$$

$(6m+3n, 1) \quad \left[\begin{matrix} (6m+3n, 3mn) \\ \times (3mn, 6m+3n) \end{matrix} \right]^{-1} (6m+3n, 3mn) \times (3mn, 1).$

→ \mathcal{J} is split into Localization and Mapping Jacobian parts.

→ Has terms of the form $J_{11L}, J_{11M}, \dots, J_{mnL}, J_{mnM}$.

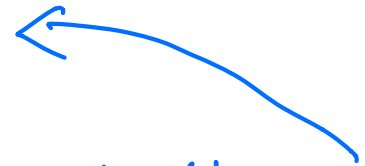
→ J_{11L} is the Jacobian obtained by taking the derivative with respect to T_{10} of the term $[X_{11} - \hat{T}_{10} X_{01}]$ (actually wrt ξ_{10}).

→ J_{11M} is the Jacobian obtained by taking derivative wrt X_{01} of the term $X_{11} - \hat{T}_{10} X_{01}$

→ $J_{12L} \dots$ derivative wrt T_{10} of $X_{12} - \hat{T}_{10} X_{02}$ (actually wrt ξ_{10})

→ $J_{12M} \dots$ wrt X_2 of above term

→ $J_{mnL} \dots$ derivative wrt T_{m0} of (ξ_{m0}) .
 $X_{mn} - \hat{T}_{m0} X_{on}$



→ $J_{mnM} \dots$ wrt X_n of the above term

→ ξ_{10} is the local tangent vector of T_{10} .
obtained from Log Map.

NLS update:

$\beta(q+1) = \beta(q) - \delta\beta$ update from
 q to $q+1$ instances during Gauss Newton's
iterative solution

ICP SLAM Update:

We consider for illustration the
update of \hat{T}_{10} and \vec{X}_{0j} as

the localization and mapping updates are very different for one is the regular Euclidean update and the other is over a manifold.

$X_{oj}(q+1) = X_{oj}(q) - \delta X_{oj}$ is the update of point X_{oj}

Localization Update:

$$\begin{array}{c} T_{io}(q+1) \\ (3 \times 4) \end{array} = \begin{array}{c} T_{io}(q) \\ (3 \times 4) \end{array} \cdot \text{Exp}(\delta \xi_{io}).$$

$\longrightarrow (L1)$

$$\xi_{io}(q+1) = \text{Log}(T_{io}(q+1)) \longrightarrow (L2)$$

$$\text{Exp}(\delta \mathbf{E}_{io}) = \begin{bmatrix} R(\delta \omega_{io}) & R(\delta \omega_{io}) \delta \mathbf{u}_{io} \\ 0 & 1 \end{bmatrix} \quad \longrightarrow (23)$$

$$\begin{aligned} R(\delta \omega_{io}) &= \exp([\delta \omega_{io}]_{\times}) \\ &= \mathbf{I}_{3 \times 3} + \frac{\sin |\delta \omega_{io}|}{|\delta \omega_{io}|} [\delta \omega_{io}]_{\times} \\ &\quad + \frac{1 - \cos |\delta \omega_{io}|}{|\delta \omega_{io}|^2} [\delta \omega_{io}]_{\times}^2 \end{aligned} \quad \longrightarrow (24)$$

where $\delta \omega_{io} = \begin{bmatrix} \delta \omega_{io1} \\ \delta \omega_{io2} \\ \delta \omega_{io3} \end{bmatrix}$ is the

update to the axis angle

and $|\delta \omega_{io}|$ is the magnitude of

$\delta \omega_{io}$ is θ or the rotation

about $\delta \omega_{io}$

(11) is the Exponential map that maps a vector in the local tangent space of T , denoted by ξ back to T .

(12) is the Logarithm Map that maps the transform matrix T to the local tangent vector ξ ,

$$\xi = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \vdots \\ \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}$$

$$\omega = [\omega_1 \ \omega_2 \ \omega_3]^T = \ln(R).$$

$$\nu = [\nu_1 \ \nu_2 \ \nu_3]^T = R^{-1}t. \quad \hookrightarrow (15)$$

$$\hookrightarrow (16)$$

For T_{io} then

$$\omega_{io} = [\omega_{io1} \ \omega_{io2} \ \omega_{io3}]^T = \ln(R_{io}).$$

$$v_{io} = [v_{io1} \ v_{io2} \ v_{io3}]^T = R_{io}^{-1} t_{io}$$

NOTE:

Jacobian is evaluated in the next iteration w.r.t $\xi_{io}(q+1)$,

$$x_{oj}(q+1) \quad i=1 \rightarrow m, \quad j=1 \rightarrow n.$$

→ No Need to Remember all These

→ Solvers like Ceres, G2O, ATSAM do these for you.

→ But you need to appreciate the cost function, the Jacobian structure and the general

notion of why we resort
to Manifold Optimization