

# CSE483-Mobile Robotics

Mid-semester exam

Monsoon 2019

September 21<sup>st</sup>

Maximum points: 30

Duration: 90 minutes

## Instructions

- This is an **open-book** exam. You are allowed to use any paper notes or textbooks that you have brought with you.
- Laptops, tablets, or smartphones are NOT allowed. You also cannot collaborate with other students.
- Your answers must be concise and to-the-point. Verbosity will NOT fetch you additional marks.
- Sufficient space has been provided for each question. Using additional sheets are discouraged, if you need them you're probably doing something wrong.
- You do NOT get credit for replicating whatever is present in the textbook or your notes. Please do not fill your answer scripts with excerpts from such sources.
- Use the last page for rough work or for any of your answers, if necessary.
- State your assumptions clearly if there is any ambiguity with the question(s).

**Roll number:**

**Seat:**

**Invigilator sign:**

Q1	Q2	Q3	Q4	Q5	Q6	Total



**Q1)** *Warm-up:* Fill up the following table by indicating the quantities that are known, to be estimated, or unknown, and the type of measurements that are needed. **(5 points)**

Problem	Structure (Scene geometry)	Motion (Camera parameters)	Measurements
F-matrix estimation	Unknown	Estimate	2D - 2D features
Camera calibration			
Triangulation			
Stereo rectification			
PnP			
Bundle adjustment			

**Q2)** *Transformations:*

- (i) Derive the expression for  $T_W^C$  if  $T_C^W = \begin{bmatrix} R_C^W & P_{CORG}^W \\ \mathbf{0}_{3 \times 1} & 1 \end{bmatrix}$ . **(2 points)**

(ii) Consider the following figure and answer questions (a) to (c).

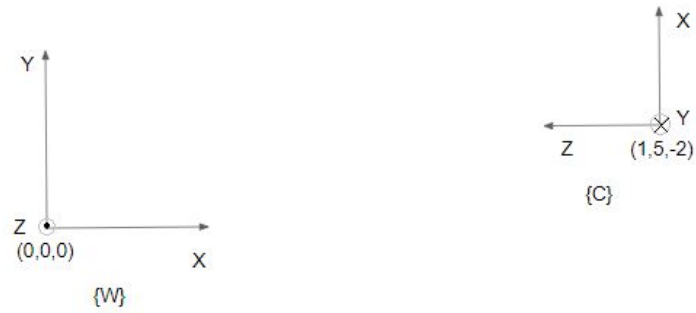


Figure 1: World Frame and Camera Frame

$\{W\}$  represents the world frame and  $\{C\}$  represents the camera frame. The Z axis of  $\{W\}$  is coming out of the plane. Whereas the Y axis of  $\{C\}$  is going into the plane.

(a) Find  $R_C^W$ . (1 points)

(b) Find the YXZ-Euler angles representation for  $R_C^W$ . **(1 points)**

(c) Find  $P_{WORG}^C$  and  $T_W^C$ . **(1 points)**

**Q3.1)** *Single-view geometry:* Given a camera matrix  $P$ , detail how you can obtain the camera center and the rotation matrix  $R$  without knowing the intrinsic parameter matrix  $K$ . **(2 points)**.

**Q3.2) *Reconstruction:*** State and justify the cases when the 3D reconstruction obtained from two views is (a) Unambiguous (b) Up to an unknown scaling factor (c) Up to an unknown projective transformation. **(3 points)**





**Q4)** *Essential matrix:* Two cameras fixate on a point  $P$  in 3D space such that their *optical axes* intersect at this point. Show that the  $E_{33}$  element of their associated Essential matrix  $E$  is zero.  
**(5 points)**



**Q5) Homography:** Suppose a camera, with intrinsic matrix  $K$ , rotates about its optical centre by a rotation matrix  $R$ . (a) Show that its two views are related by a homography  $H$  such that  $x_2 = Hx_1$  where  $x_1$  is a point in the first image and  $x_2$  is its corresponding point in the second image. **(2.5 points)** (b) Also show that if  $\theta$  is the rotation between the two views, then the angle  $2\theta$  corresponds to the homography  $H^2$ . **(2.5 points)**



**Q6) Dense-VO:** Dense-VO is one other type of visual odometry where the camera motion is estimated by aligning consecutive image frames and then finding the transformation that best minimizes the *photometric error* between them. Suppose there is a camera C with known intrinsics K, and it captures two images  $I_1, I_2$  from two views separated by a rotation R and translation t. The photometric error between these two views is given as  $\sum_{x \in I_1} \|I_1(x) - I_2(w(x, (R|t)))\|^2$  where  $w(x, (R|t))$  is a function that maps a point x in the first image to a point in the second image given the camera motion R, t. (a) Assuming  $d$  is the depth of the point x in  $I_1$  relative to the first view, describe the steps involved to map this point to the second image, and hence provide a mathematical expression for  $w(x, (R|t))$ . **(3 points)** (b) What is the nature of this photometric error? Very briefly in words mention how it can be solved for to find the best camera motion. **(2 points)**



Extra space



Extra space

