Transformations between ECEF and ENU coordinates

The relation between the local East, North, Up (ENU) coordinates and the (x, y, z) Earth Centred Earth Fixed (ECEF) coordinates is illustrated in the next figure:

Fundamentals	
Title	Transformations between ECEF and ENU coordinates
Author(s)	J. Sanz Subirana, J.M. Juan Zornoza and M. Hernández-Pajares, Technical University of Catalonia, Spain.
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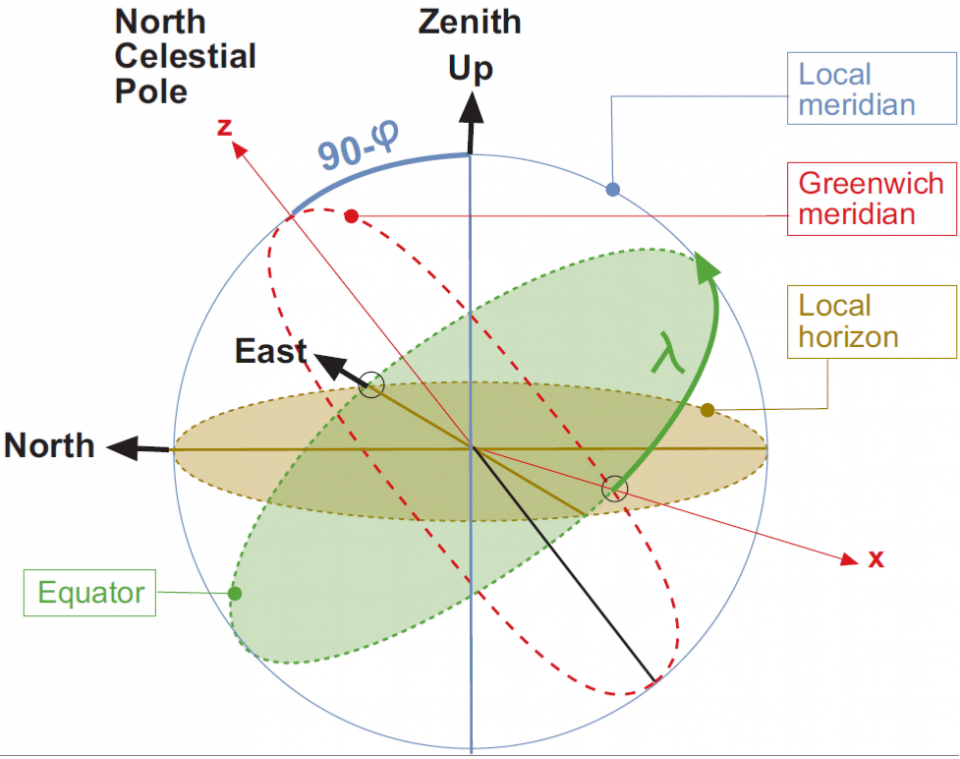

Figure 1:: Transformations between ENU and ECEF coordinates.

From the figure 1 it follows that the ENU coordinates can be transformed to the (x, y, z) ECEF by two rotations, where φ and λ are, respectively, the latitude and longitude from the ellipsoid:

1. A clockwise rotation over east-axis by an angle $90 - \varphi$ to align the up-axis with the z-axis. That is $\mathbf{R}_1[-(\pi/2 - \varphi)]$.

2. A clockwise rotation over the z-axis by an angle $90+\lambda$ to align the east-axis with the x-axis. That is $\mathbf{R}_3[-(\pi/2+\lambda)]$.

That is:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \mathbf{R}_3[-(\pi/2+\lambda)]\,\mathbf{R}_1[-(\pi/2-\varphi)]\begin{bmatrix} E \\ N \\ U \end{bmatrix} \qquad (1)$$

where, according to the expressions (2) (see Transformation between Terrestrial Frames)

$$\mathbf{R}_1[\theta] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} \; ; \; \mathbf{R}_2[\theta] = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$\mathbf{R}_3[\theta] = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (2)$$

yields:

$$\mathbf{R}_3[-(\pi/2+\lambda)]\,\mathbf{R}_1[-(\pi/2-\varphi)] = \begin{pmatrix} -\sin\lambda & -\cos\lambda\sin\varphi & \cos\lambda\cos\varphi \\ \cos\lambda & -\sin\lambda\sin\varphi & \sin\lambda\cos\varphi \\ 0 & \cos\varphi & \sin\varphi \end{pmatrix} \qquad (3)$$

The unit vectors in local East, North and Up directions as expressed in ECEF cartesian coordinates are given by the columns of matrix (3). That is:

$$\begin{aligned} \hat{\mathbf{e}} &= (-\sin\lambda\,,\ \cos\lambda\,,\ 0) \\ \hat{\mathbf{n}} &= (-\cos\lambda\sin\varphi\,,\ -\sin\lambda\sin\varphi\,,\ \cos\varphi) \\ \hat{\mathbf{u}} &= (\cos\lambda\cos\varphi\,,\ \sin\lambda\ \cos\varphi\,,\ \sin\varphi) \qquad (4) \end{aligned}$$

Note: If (λ,φ) are ellipsoidal coordinates, thence, the vector $\hat{\mathbf{u}}$ is orthogonal to the tangent plane to the ellipsoid, which is defined by $(\hat{\mathbf{e}},\hat{\mathbf{n}})$. If (λ,φ) are taken as the spherical longitude and latitude, thence, the vector $\hat{\mathbf{u}}$ is in the radial direction and $(\hat{\mathbf{e}},\hat{\mathbf{n}})$ defines the tangent plane to the sphere.

From ECEF to ENU coordinates

Taking into account the properties of the rotation matrices $\mathbf{R}_i(\alpha)$,i.e., $\mathbf{R}_i^{-1}(\alpha) = \mathbf{R}_i(-\alpha) = \mathbf{R}_i^T(\alpha)$, thence, the inverse transformation of (1) is given by:

$$\begin{bmatrix} E \\ N \\ U \end{bmatrix} = \mathbf{R}_1[\pi/2 - \varphi]\,\mathbf{R}_3[\pi/2 + \lambda]\begin{bmatrix} x \\ y \\ z \end{bmatrix} \qquad (5)$$

where the transformation matrix of (5) is the transpose of matrix (3):

$$\mathbf{R}_1[\pi/2 - \varphi]\,\mathbf{R}_3[\pi/2 + \lambda] = \begin{pmatrix} -\sin\lambda & \cos\lambda & 0 \\ -\cos\lambda\sin\varphi & -\sin\lambda\sin\varphi & \cos\varphi \\ \cos\lambda\cos\varphi & \sin\lambda\cos\varphi & \sin\varphi \end{pmatrix} \qquad (6)$$

The unit vectors in the ECEF $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$ and $\hat{\mathbf{z}}$ directions, as expressed in ENU coordinates, are given by the columns of matrix (6). That is:

$$\begin{array}{l} \hat{\mathbf{x}} = (-\sin\lambda\,,\,-\cos\lambda\sin\varphi\,,\,\cos\lambda\cos\varphi) \\ \hat{\mathbf{y}} = (\cos\lambda\,,\,-\sin\lambda\sin\varphi\,,\,\sin\lambda\cos\varphi) \\ \hat{\mathbf{z}} = (0\,,\,\cos\varphi\,,\,\sin\varphi) \end{array} \qquad (7)$$

Elevation and azimuth computation

Given the line of sight unit vector

$$\hat{\boldsymbol{\rho}} = \frac{\mathbf{r}^{sat} - \mathbf{r}_{rcv}}{\|\mathbf{r}^{sat} - \mathbf{r}_{rcv}\|} \qquad (8)$$

where $\mathbf{r}^{sat}$ and $\mathbf{r}_{rcv}$ are the geocentric position of the satellite and receiver, respectively, the elevation and azimuth in the local system coordinates (ENU), defined by the unit vectors $\hat{\mathbf{e}}$, $\hat{\mathbf{n}}$ and $\hat{\mathbf{u}}$ can be computed from (see figure 2):

$$\begin{array}{l} \hat{\boldsymbol{\rho}}\cdot\hat{\mathbf{e}} = \cos E\sin A \\ \hat{\boldsymbol{\rho}}\cdot\hat{\mathbf{n}} = \cos E\cos A \\ \hat{\boldsymbol{\rho}}\cdot\hat{\mathbf{u}} = \sin E \end{array} \qquad (9)$$

Thence the elevation and azimuth of satellite in the local coordinates system are given by:

$$E = \arcsin(\hat{\boldsymbol{\rho}}\cdot\hat{\mathbf{u}}) \qquad (10)$$

$$A = \arctan\left(\frac{\hat{\boldsymbol{\rho}}\cdot\hat{\mathbf{e}}}{\hat{\boldsymbol{\rho}}\cdot\hat{\mathbf{n}}}\right) \qquad (11)$$

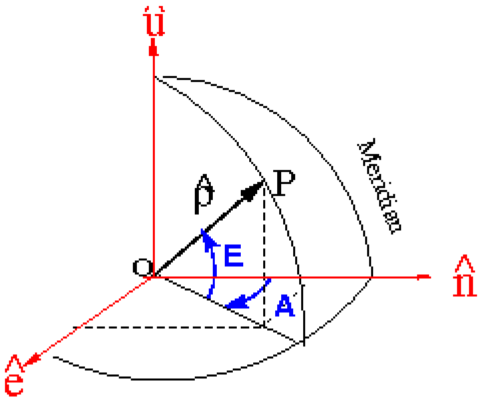

Figure 2:: Local coordinate frame showing the elevation (E) and azimuth (A).

Note: If (λ, φ) are ellipsoidal coordinates, thence, the vector $\hat{\mathbf{u}}$ is orthogonal to the tangent plane to the ellipsoid, which is defined by $(\hat{\mathbf{e}}, \hat{\mathbf{n}})$. If (λ, φ) are taken as the spherical longitude and latitude, thence, the vector $\hat{\mathbf{u}}$ is in the radial direction and $(\hat{\mathbf{e}}, \hat{\mathbf{n}})$ defines the tangent plane to the sphere.