# Triangulation & PnP (3D-2D)

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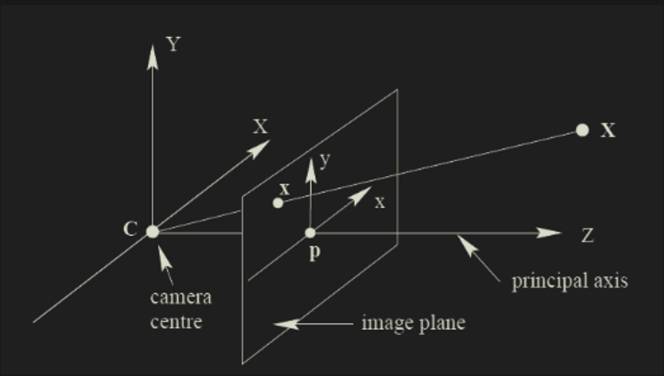
## Oct 23rd agenda

1. Triangulation
2. PnP

Next class: revisiting epipolar geometry and then, computation of F.

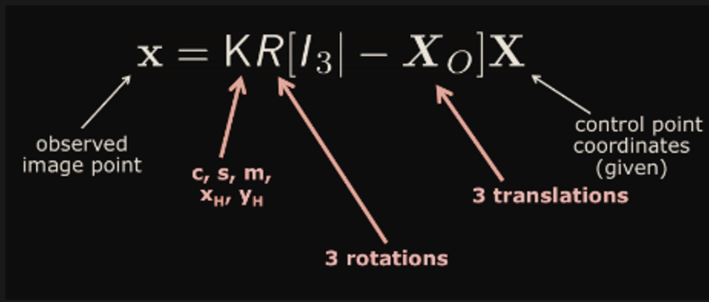
## 0. Revisiting single view geometry

### 0.1 Difference between the ray and the image coordinates




Pin hole camera

Few simplications:




$$\begin{bmatrix} c & 0 & 0 \\ 0 & c & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Coordinates of the image point in the camera coordinate system? Let's call it **normalized image coordinates**.

  Ans: $[x, y, f]^T$

- What is the relation between **normalized image coordinates** and **3D object point** $\mathbf{X}$?

  Related by a scalar

  $$\lambda \mathbf{x} = \mathbf{K}\mathbf{X}$$

- How to arrive at the above vector from homogenous **image coordinates** $[x, y, 1]$? Forget about the scaling factor. Clue ⇒

  $$\text{Ans: } \mathbf{K}^{-1}\text{. Consider } \mathbf{K}^{-1} \times [x, y, 1]^T = ?$$

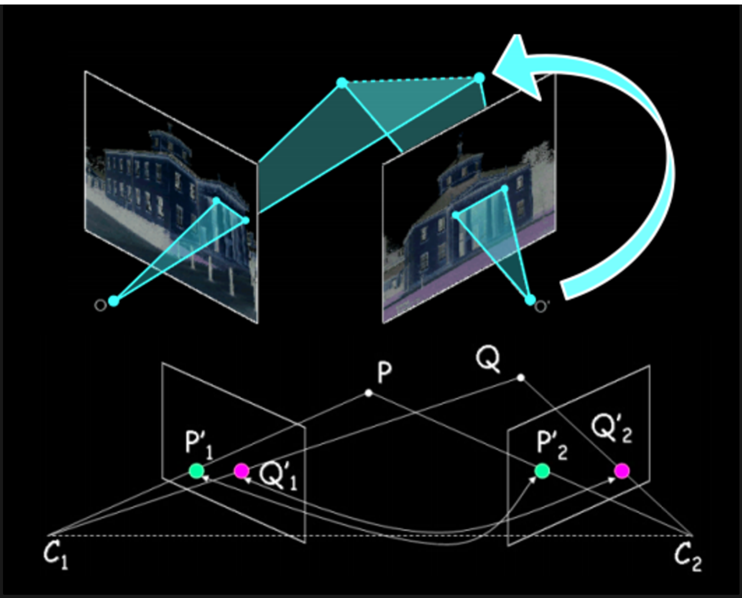
  So $\beta\, \mathbf{K}^{-1}\mathbf{x}$ is a ray connecting camera center and 3D point.

  $x = KTX$

  $K^{-1}x = T_W^C X_W$ = $X_C$

# 4. Triangulation

> How to compute the position of a point in 3-space given its image in two views and the camera matrices of those views?




We know $C_1, C_2$ and $P_1', P_2'$ and camera matrix $K$. We want to find $P$, the world point.

**Equations of Lines:**

▶ Revisiting what vector and position vectors are.

Here, I have 2 lines and want to find its intersection.

Every line is determined by a point (*position vector*) and a direction (*vector*).

1. Is there a point?
2. Is there a direction?

$$\boldsymbol{f} = \boldsymbol{P} + \lambda \boldsymbol{r}$$
$$\boldsymbol{g} = \boldsymbol{Q} + \mu \boldsymbol{s}$$

▶ **Knowns:**

**Unknown:**

- scalars $\mu$ and $\lambda$. Which will in turn give us the world points.

**Rays from the camera to the 3D point in the world:**

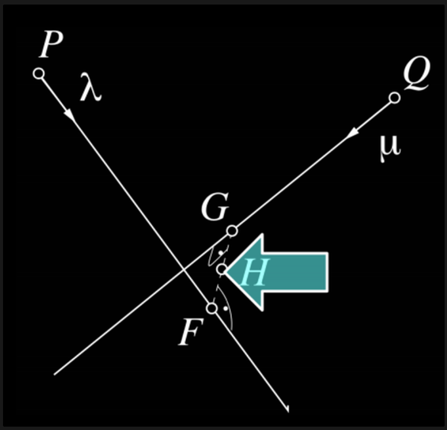
$$\boldsymbol{r} = K_1^{-1}\boldsymbol{x_1}$$
$$\boldsymbol{s} = R_2^1 K_2^{-1}\boldsymbol{x_2}$$

▶ How $s$?

- $\boldsymbol{x_1}, \boldsymbol{x_2}$: image pixel coordinate (homogeneous) in camera 1 & 2.
- $K_1$ and $K_2$ are intrinsic matrices for camera 1 and 2.
- $R_2^1$ is the relative orientation of camera 2 with respect to camera 1.

## If they intersected

$$1. \quad \vec{f} = \vec{g} \iff ||f|| = ||g||$$
$$2. \quad \frac{\vec{f} \cdot \vec{g}}{||f||||g||} = 1$$




▸ **Knowns:**

**Unknown:**

$\mathrm{F}$ and $\mathrm{G}$ are the world points.

- scalars $\mu$ and $\lambda$.

### If they don't intersect

- Ensure distance is minimum.
- Line $(\mathrm{F}-\mathrm{G})$ perpendicular to both lines $\mathrm{r}$ and $\mathrm{s}$.

$$(\boldsymbol{F}-\boldsymbol{G}) \cdot \boldsymbol{r}=0$$
$$(\boldsymbol{F}-\boldsymbol{G}) \cdot \boldsymbol{s}=0$$

$$\boldsymbol{f}=\boldsymbol{P}+\lambda \boldsymbol{r}$$
$$\boldsymbol{g}=\boldsymbol{Q}+\mu \boldsymbol{s}$$

We have two equations and two unknowns $\lambda$ and $\mu$.

$$(\boldsymbol{P}+\lambda \boldsymbol{r}-(\boldsymbol{Q}+\mu \boldsymbol{s})) \cdot \boldsymbol{r}=0$$
$$(\boldsymbol{P}+\lambda \boldsymbol{r}-(\boldsymbol{Q}+\mu \boldsymbol{s})) \cdot \boldsymbol{s}=0$$

$$\left[\begin{array}{l} r \cdot r-s \cdot r \\ r \cdot s-s \cdot s \end{array}\right]\left[\begin{array}{l} \lambda \\ \mu \end{array}\right]=\left[\begin{array}{l} (\boldsymbol{Q}-\boldsymbol{P}) \cdot r \\ (\boldsymbol{Q}-\boldsymbol{P}) \cdot s \end{array}\right]$$

$$\left[\begin{array}{l} \lambda \\ \mu \end{array}\right]=\left[\begin{array}{l} r \cdot r-s \cdot r \\ r \cdot s-s \cdot s \end{array}\right]^{-1}\left[\begin{array}{l} (\boldsymbol{Q}-\boldsymbol{P}) \cdot r \\ (\boldsymbol{Q}-\boldsymbol{P}) \cdot s \end{array}\right]$$

- $\lambda$ and $\mu$ found.
- Obtain $\boldsymbol{F}$ and $\boldsymbol{G}$ from right equation.
- Mid-point of this line segment $\boldsymbol{H}=\boldsymbol{F}-\boldsymbol{G}$ is the final estimate for the $3\mathrm{D}$ triangulated world point.

$$\boldsymbol{f}=\boldsymbol{P}+\lambda \boldsymbol{r}$$
$$\boldsymbol{g}=\boldsymbol{Q}+\mu \boldsymbol{s}$$

$$\vec{h}=\vec{f}+\frac{\|FG\|}{2} \widehat{fg}$$

# 5. PnP

## 5.0 Introduction

### 5.0.1 What is the Perspective n Points (PnP) problem?

> 💡 **Given:** known 3D landmarks positions in the **world frame** and given their 2D image correspondences in the **camera frame**.

> 💡 **Determine:** 6DOF pose of the camera (or camera motion) in the world frame (including the intrinsic parameters if uncalibrated).

- However, if the 3D position of the feature points is known, then at least 3 point pairs (and at least one additional point verification result) are needed to estimate camera motion. (This is P3P)

▶ The 2D–2D epipolar geometry method

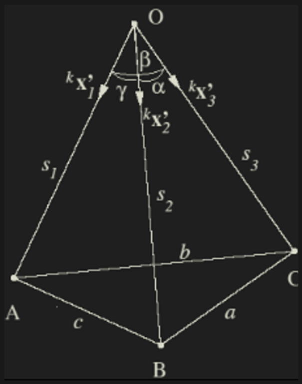
### 5.0.2 The P3P/Spatial Resection Problem

Given:

- $3D$ coordinates of object points $X_i$
- 2D image coordinates $x_i$ of corresponding object points
- $K$ matrix, it is a **calibrated camera.**

Find:

- Extrinsic parameters $R, X_O$ of the **calibrated** camera (unlike DLT)

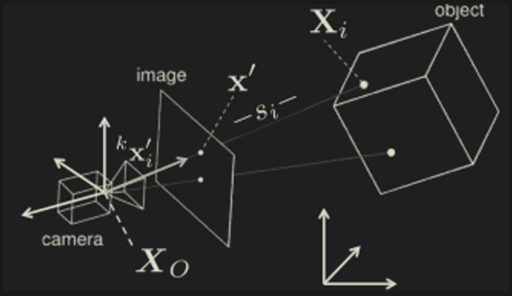



### 5.0.3 Difference between P3P and DLT

- P3P/Spatial Resection for calibrated cameras
  - 6 unknowns, so at least 3 points are needed
- DLT for uncalibrated cameras (seen)
  - 11 unknowns, so at least 6 points are needed

## 5.1 Solution to P3P

### 5.1.1 Revisiting normalized coordinates

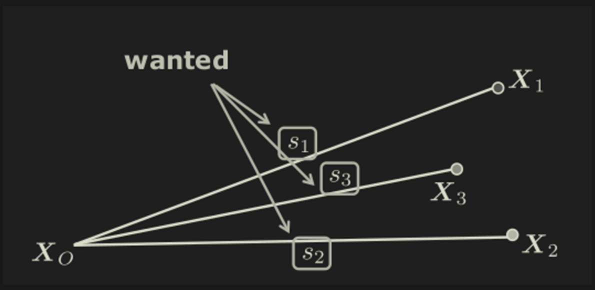


$$\mathrm{x} = \mathrm{K}R\left[I_3 \middle| -X_O\right]\mathrm{X}$$

$${}^{k}\mathbf{x}_i' = \mathrm{K}^{-1}\mathbf{x}_i'$$

### 5.1.2 Two step process

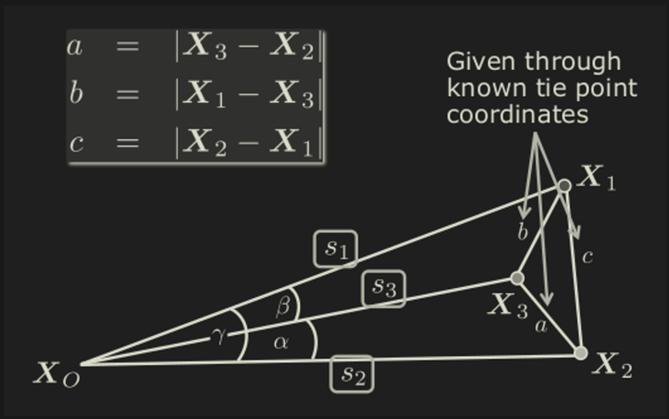
1. Length of projection rays
2. Orientation




💡 Clarity about camera frame and world frame: angles and distances between points

### 5.1.3 Length of projection rays

1. **Do we know a, b, c?**

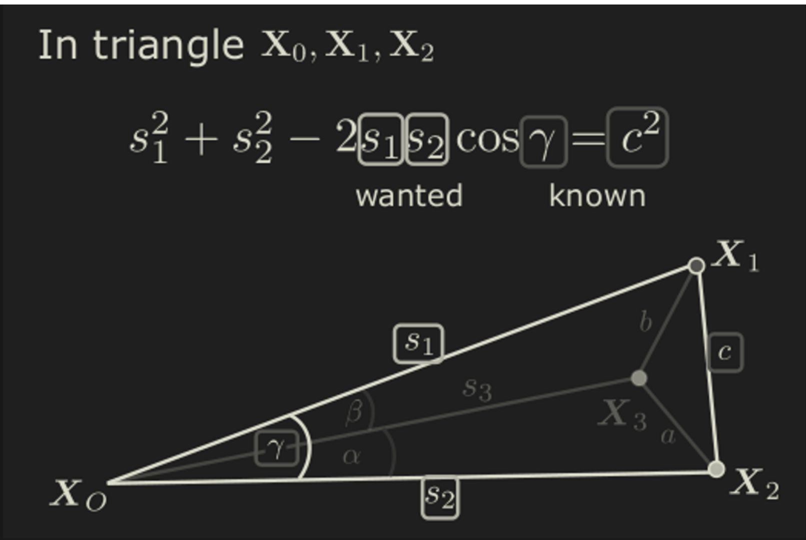



**2. Do we know angles?**

$$\cos\gamma = \frac{(X_1 - X_0)\cdot(X_2 - X_0)}{\|X_1 - X_0\|\,\|X_2 - X_0\|}$$

$$\text{Clue: Normalized Coords}$$

**Cosine rule:**




$$
\begin{aligned}
a^2 &= s_2^2 + s_3^2 - 2s_2s_3\cos\alpha \qquad &-(1)\\
b^2 &= s_1^2 + s_3^2 - 2s_1s_3\cos\beta \qquad &-(2)\\
c^2 &= s_1^2 + s_2^2 - 2s_1s_2\cos\gamma \qquad &-(3)
\end{aligned}
$$

$$\text{We have:} \quad a^2 = s_2^2 + s_3^2 - 2s_2s_3\cos\alpha$$

$$\text{Define:} \quad u = \tfrac{s_2}{s_1} \quad v = \tfrac{s_3}{s_1} \qquad -(4)$$

$$\implies a^2 = s_1^2\left(u^2 + v^2 - 2uv\cos\alpha\right)$$

$$
\begin{aligned}
s_1^2 &= \frac{a^2}{u^2 + v^2 - 2uv\cos\alpha}\\
&= \frac{b^2}{1 + v^2 - 2v\cos\beta}\\
&= \frac{c^2}{1 + u^2 - 2u\cos\gamma}
\end{aligned}
\tag{5}
$$

$$
\begin{aligned}
b^2 &= s_1^2 + s_3^2 - 2s_1s_3\cos\beta\\
c^2 &= s_1^2 + s_2^2 - 2s_1s_2\cos\gamma
\end{aligned}
$$

Substitute ***u*** in other equation — **4th degree polynomial:**

$$A_4v^4 + A_3v^3 + A_2v^2 + A_1v + A_0 = 0$$

$$
\begin{aligned}
A_4 &= \left(\tfrac{a^2-c^2}{b^2}-1\right)^2 - \tfrac{4c^2}{b^2}\cos^2\alpha \\
A_3 &= 4\left[\frac{a^2-c^2}{b^2}\left(1-\frac{a^2-c^2}{b^2}\right)\cos\beta\right. \\
&\quad \left. -\left(1-\frac{a^2+c^2}{b^2}\right)\cos\alpha\cos\gamma + 2\frac{c^2}{b^2}\cos^2\alpha\cos\beta\right]
\end{aligned}
$$

$$
\begin{aligned}
A_2 = 2&\left[\left(\frac{a^2-c^2}{b^2}\right)^2 - 1 + 2\left(\frac{a^2-c^2}{b^2}\right)^2\cos^2\beta\right. \\
&+ 2\left(\frac{b^2-c^2}{b^2}\right)\cos^2\alpha \\
&- 4\left(\frac{a^2+c^2}{b^2}\right)\cos\alpha\cos\beta\cos\gamma \\
&\left. +2\left(\frac{b^2-a^2}{b^2}\right)\cos^2\gamma\right]
\end{aligned}
$$

$$
\begin{aligned}
A_1 = 4&\left[-\left(\frac{a^2-c^2}{b^2}\right)\left(1+\frac{a^2-c^2}{b^2}\right)\cos\beta\right. \\
&+ \frac{2a^2}{b^2}\cos^2\gamma\cos\beta \\
&\left. -\left(1-\left(\frac{a^2+c^2}{b^2}\right)\right)\cos\alpha\cos\gamma\right]
\end{aligned}
$$

$$
A_0 = \left(1+\frac{a^2-c^2}{b^2}\right)^2 - \frac{4a^2}{b^2}\cos^2\gamma
$$

But upto 4 possible solutions possible. So we consider 4th point to confirm the right solution:

So say we know 2D-3D correspondence of $(x, X)$ of 4th point, say $(x_4, X_4)$. Just substitute $X$ of 4th point (we know the K matrix) and the possible solutions of $R, t$ in our camera equation and only one solution will give you the right $(x_4)$.

### 5.1.4 Transformation between camera frame and world frame

md.kalesha@students.i
18, 2020
P3P while calculating the respect to 'v' as (-b+/- sqrt((b^2)-4ab)/2a) we v
1 more comment

Shubodh Sai Nov 19, 202
After finding $s1$ and $s3$ y (1) and (3) equations to f you could multiply (1) wi

KaTeX parse error: Got group of unknown type: 'internal'

$\hat{X}$ are unit (direction) vectors..

$$\lambda \mathbf{x} = \mathbf{K}\mathbf{X}$$

($\mathbf{K^{-1}x_1}/its\ norm$)= √x Invalid equation gives direction in camera's frame. Divide by its norm to get the unit vector.