CSE/ECE 483 Mobile Robotics - End-Semester Examination - Monsoon 2017

Place: IIIT Hyderabad **Max. Duration**: 180 minutes
Date: 18th November 2017 (Sat) **Max. Marks**: 45

General Instructions

1. The nature of the examination is *open-book*. You are free to use any reference material such as notes, reference books, research papers, blog articles,, etc. that you have brought to the exam hall. You may NOT, however, collaborate with other students.
2. Unless otherwise specified, your answers must be concise and to-the-point. Verbosity will NOT fetch you additional marks.
3. You do NOT usually get credit for replicating content from the textbook or lecture slides. So, please do NOT fill your answer scripts with excerpts from such sources.

Questions

1. **[Localization with Landmarks with Bounded Uncertainty]** Assume a robot R moving in a map infested with N landmarks. Each such landmark, has a bounded uncertainty, uncorrelated with robot's own uncertainty. The robot at time t, makes measurements on each landmark. Write down the measurement equation, the measurement Jacobian H and the innovation covariance S with Q being the measurement covariance. [5 marks]
2. **[Motion Model Uncertainty]** Consider a differential drive robot whose center is assumed to be at the mean of the distribution at $\mu_t = (\mu_{xt}, \mu_{yt}, 0)$ at time t with initial covariance Σ_t. [2 + 2 + 4 = 8 marks]
    a. Write or expand the Σ_t matrix in terms of its elements.
    b. If the robot's center moves with a velocity $< v, \omega >$ with a radius of curvature R, derive $\mu_{t+1} = f(\mu_t, u_{t+1})$, where $u_t = < v, \omega >$
    c. If the control noise Σ_{ut} is $\Sigma_{ut} = [\sigma_{v^2}, 0; 0, \sigma_{\omega^2}]$, write down the expression for state covariance Σ_{t+1} clearly deriving expressions for all the Jacobian matrices involved.
3. **[Multi-view Geometry]** For each of the following statements, give very brief answers (maximum of 2-3 sentences). [1 + 2 + 2 + 2 + 3 = 10 marks]
    a. If the fundamental matrix between images I1 and I2 is F, what is the fundamental matrix between images I2 and I1? Why are they different?
    b. If the camera undergoes a pure rotational motion, what will the fundamental matrix be?
    c. In order to perform triangulation, you need M views (i.e., M images) of N points. What are the minimum values of M and N here (assuming no degeneracies), given the rotation and translation between the camera coordinates of all the M views?

d. In order to perform resection, you need M views and N points. What are the minimum possible values of M and N here, assuming you already have a 3D reconstruction of the world relative to the first camera's coordinate frame?
e. List out 3 differences between camera calibration using Direct Linear Transform (DLT) vs camera calibration using planes (Zhang's method, i.e., the checkerboard method).

4. **[Nonlinear Least Squares]** For each statement, provide a justification as to why the statement is true. (Note: All the below statements ARE true). We'll use the shorthand GN for Gauss-Newton method and LM for Levenberg-Marquardt method. [1 + 1 + 1 + 1 + 1 = 5 marks]
   a. Gauss-Newton (GN) method is usually orders of magnitude faster than Levenberg-Marquardt (LM).
   b. As GN iterations progress, the value of the cost function, i.e., the total error, need not decrease.
   c. In LM iterations, the error is guaranteed not to increase.
   d. In certain conditions, LM behaves close to a GN routine.
   e. In certain conditions, LM behaves close to a Gradient Descent routine.
5. **[Bundle Adjustment]** With regards to our discussion of the Bundle Adjustment (BA) technique in class, answer the following questions. [1 + 1 + 1 + 4 + 3 = 10 marks]
   a. What are the inputs to the BA algorithm, and what does it optimize for?
   b. Write down the BA cost function for three 3D points X_1, X_2, X_3 and four cameras with projection matrices P_1, P_2, P_3, P_4. Assume that all points are visible in all cameras.
   c. What is the size of the Jacobian matrix and the number of residual terms?
   d. Show the structure of the Jacobian matrix and mention what each populated term in the Jacobian matrix indicates (over here, a coarse, block structure of the Jacobian suffices. You needn't expand all the terms to their fullest).
   e. Expand the Jacobian terms in the first two rows of the matrix.
6. **[Bernstein basis for Trajectory planning]**