M19CSE483 Mid-sem exam

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TOTAL POINTS

25.5 / 30

QUESTION 1

1 Q1 - Warm up 4 / 5

- ✓ + **1 pts** **Camera calibration**
- ✓ + **1 pts** **Triangulation**
- ✓ + **1 pts** **Stereo rectification**
- ✓ + **1 pts** **PnP**
- \+ **1 pts** Bundle adjustment
- \+ **0 pts** Wrong

QUESTION 2

Q2 - Pose transformations 5 pts

2.1 Q2.1 2 / 2

- \+ **0 pts** Incorrect
- ✓ + **1 pts** **Rotation matrix**
- ✓ + **1 pts** **Translation vector**
- \- **0.5 pts** Proper explanation not given for R
- \- **0.5 pts** Proper explanation not given for translation vector

2.2 Q2.2.a 1 / 1

- \+ **0 pts** Incorrect
- ✓ + **1 pts** **Correct**
- \- **0.5 pts** No explanation
- \+ **0.5 pts** correct explanation

2.3 Q2.2.b 1 / 1

- \+ **0 pts** Incorrect
- ✓ + **1 pts** **Correct**
- \- **0.5 pts** Incorrect order of rotations

2.4 Q2.2.c 1 / 1

- \+ **0 pts** Incorrect
- ✓ + **0.5 pts** **Rotation matrix**
- ✓ + **0.5 pts** **Translation vector**

QUESTION 3

Q3 5 pts

3.1 Q3.1 - Single-view geometry 2 / 2

- \+ **0 pts** Incorrect
- ✓ + **1 pts** **Camera center**
- ✓ + **1 pts** **Rotation matrix**

3.2 Q3.2 - Reconstruction ambiguities 1 / 3

- \+ **0 pts** Incorrect
- ✓ + **0.5 pts** **Requirements for (a)**
- \+ **0.5 pts** Requirements for (b)
- ✓ + **0.5 pts** **Requirements for (c)**
- \+ **0.5 pts** Justification for (a)
- \+ **0.5 pts** Justification for (b)
- \+ **0.5 pts** Justification for (c)

QUESTION 4

4 Q4 - Special Essential matrix 5 / 5

- \+ **1 pts** Equation
- \+ **1 pts** Partial Image Points
- \+ **1 pts** Partial Proof
- \+ **2 pts** Complete Image Points
- \+ **2 pts** Complete proof
- \+ **0 pts** Not correct / Unattempted
- \+ **0.5 pts** Additional Info
- ✓ + **5 pts** **Correct Derivation**

QUESTION 5

Q5 - Homography from pure rotations 5 pts

5.1 Q5.a 2.5 / 2.5

- ✓ + **2.5 pts** **Complete**
- \+ **1 pts** Right approach
- \+ **0 pts** Incorrect / Not attempted

5.2 Q5.b 2.5 / 2.5

- ✓ + **2.5 pts** **Complete**
- + **1 pts** Right approach
- + **0 pts** Incorrect / Not attempted

QUESTION 6

Q6 - Dense Visual Odometry 5 pts

6.1 Q6.a 3 / 3

- ✓ + **1.5 pts** **Found 3D point in C1 frame**
- ✓ + **1.5 pts** **Re-project onto I2**
- + **0 pts** Incorrect / Not attempted

Last line doesn't make sense though

6.2 Q6.b 0.5 / 2

- + **1 pts** Type
- + **1 pts** How to solve
- ✓ + **0.5 pts** **Only name of solver is mentioned**
- + **0 pts** Incorrect / Not attempted

CSE483-Mobile Robotics

Mid-semester exam

Monsoon 2019

September 21st

Maximum points: 30 | Duration: 90 minutes

Instructions

- This is an **open-book** exam. You are allowed to use any paper notes or textbooks that you have brought with you.
- Laptops, tablets, or smartphones are NOT allowed. You also cannot collaborate with other students.
- Your answers must be concise and to-the-point. Verbosity will NOT fetch you additional marks.
- Sufficient space has been provided for each question. Using additional sheets are discouraged, if you need them you're probably doing something wrong.
- You do NOT get credit for replicating whatever is present in the textbook or your notes. Please do not fill your answer scripts with excerpts from such sources.
- Use the last page for rough work or for any of your answers, if necessary.
- State your assumptions clearly if there is any ambiguity with the question(s).

Roll number: 2019701013

Seat: C61

Invigilator sign: 21|9|19

Q1	Q2	Q3	Q4	Q5	Q6	Total

Q1) *Warm-up:* Fill up the following table by indicating the quantities that are known, to be estimated, or unknown, and the type of measurements that are needed. **(5 points)**

Problem	Structure (Scene geometry)	Motion (Camera parameters)	Measurements
F-matrix estimation	Unknown	Estimate	2D - 2D features
Camera calibration	Known	Estimate	3D - 2D correspondences
Triangulation	Estimate	Known	2D-2D correspondences
Stereo rectification	Unknown	Old known, new ones ~~[illegible]~~ to estimate	R, t_1, K_1, K_2 t_2
PnP	Known	Pose → estimate	3D-2D correspondences
Bundle adjustment	Known	Estimate P	"

Q2) *Transformations:*

(i) Derive the expression for T_W^C if $T_C^W = \begin{bmatrix} R_C^W & P_{CORG}^W \\ 0_{3\times1} & 1 \end{bmatrix}$. **(2 points)**

$$T_W^C = \begin{pmatrix} R_W^C & P_{W\,org}^C \\ 0_{3\times1} & 1 \end{pmatrix}$$

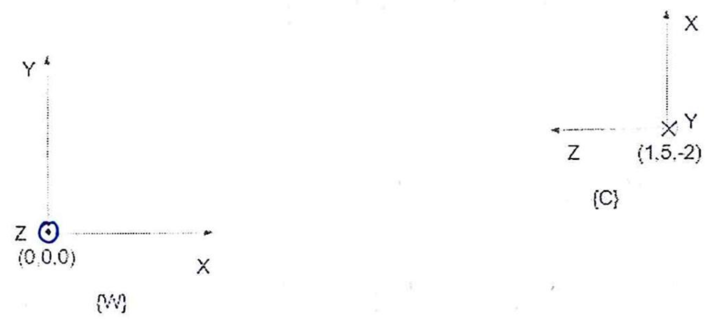
We know that $R_W^{C\,-1} = R_W^{C\,T} = R_C^W$ (or) $\boxed{R_W^C = R_C^{W\,T}}$

Now, $P_{W\,org}^C$ is simply -ve of $P_{C\,org}^W$ after rotating it to camera's frame from world frame.

i.e. $P_W^C = -\left(R_W^C P_{C\,org}^W\right)$

Therefore,
$$\boxed{T_W^C = \begin{bmatrix} R_C^{W\,T} & -R_W^C P_{C\,org}^W \\ 0_{3\times1} & 1 \end{bmatrix}}$$

(ii) Consider the following figure and answer questions (a) to (c).



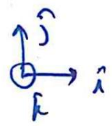


Figure 1: World Frame and Camera Frame

{W} represents the world frame and {C} represents the camera frame. The Z axis of {W} is coming out of the plane. Whereas the Y axis of {C} is going into the plane.

(a) Find R_C^W. **(2 points)**

$$R_C^W = \begin{bmatrix} \hat{x}_c \cdot \hat{x}_w & \hat{y}_c \cdot \hat{x}_w & \hat{z}_c \cdot \hat{x}_w \\ \hat{x}_c \cdot \hat{y}_w & \hat{y}_c \cdot \hat{y}_w & \hat{z}_c \cdot \hat{y}_w \\ \hat{x}_c \cdot \hat{z}_w & \hat{y}_c \cdot \hat{z}_w & \hat{z}_c \cdot \hat{z}_w \end{bmatrix}$$

$$= \begin{bmatrix} \hat{j} \cdot \hat{i} & -\hat{k} \cdot \hat{i} & -\hat{i} \cdot \hat{i} \\ \hat{j} \cdot \hat{j} & -\hat{k} \cdot \hat{j} & -\hat{i} \cdot \hat{j} \\ \hat{j} \cdot \hat{k} & -\hat{k} \cdot \hat{k} & -\hat{i} \cdot \hat{k} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

~~(b)~~ Find the YXZ-Euler angles representation for R_C^W. (2 points)

Rotate {B} first about $\hat{Y}_B$ by β, then $\hat{X}_B$ by γ, finally $\hat{Z}_B$ by α.

$\beta = -90°$, $\gamma = 0$, $\alpha = -90°$

$${}^A_B R = \underline{R_Y(\beta)\, R_X(\gamma)\, R_Z(\alpha)}$$

where $R_Y(\beta) = \begin{bmatrix} c\beta & 0 & s\beta \\ 0 & 1 & 0 \\ -s\beta & 0 & c\beta \end{bmatrix}, \ldots$

~~(c)~~ Find P_{WORG}^C and T_W^C. (1 points)

$$P_C^W = \begin{bmatrix} 1 \\ 5 \\ -2 \end{bmatrix} \to (1)$$

$\cancel{R_{Corg}^W}$ $\boxed{P_{W_{org}}^C = -R_W \, P_C^W}$ from (1) $\to$ (2)

$(R_C^W)^T$ from (a)

So, we found $P_{W\,org}^C$

$$T_W^C = \begin{bmatrix} R_C^{W\,T} & -R_C^W P_C^W \\ 0_{3\times1} & 1 \end{bmatrix}$$

We know all the elements of matrix from (1), (2) & (a).

Q3.1) *Single-view geometry:* Given a camera matrix P, detail how you can obtain the camera (i) center and the rotation matrix R (ii) without knowing the intrinsic parameter matrix K. **(2 points)**.

$$\lambda \vec{x} = P \vec{X}$$

$$x = \underbrace{KR\,[I_3 \mid -X_0]}_{P}\, X$$

We need to decompose P into $\overset{\to (i)}{X_0}$ & $\overset{(ii)}{R}$ without knowing K.

$$\hat{P} = \hat{K}\hat{R}\,(I_3 \mid -\hat{X}_0) = [\underset{3\times3}{\hat{H}_\infty} \mid \underset{3\times1}{\hat{h}}]$$

$$\hat{H}_\infty = \hat{K}\hat{R}, \quad \hat{h} = -\hat{K}\hat{R}\hat{X}_0$$

Now, X_0 is simple : $\boxed{\hat{X}_0 = -\hat{H}_\infty^{-1}\,\hat{h}}$ → found (i)

Consider QR decomposition of $\hat{H}_\infty^{-1} = (\hat{K}\hat{R})^{-1} = \hat{R}^{-1}\hat{K}^{-1} = \hat{R}^T\hat{K}^{-1}$

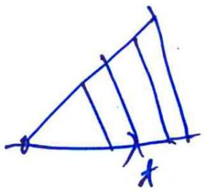
($\because$ QR decomposition : $A = \underset{\text{Rotation/orthogonal}}{M} D$, D^u matrix)

Therefore, $\boxed{\text{we found } \hat{R} \text{ as well (ii)}}$

Q3.2) *Reconstruction:* State and justify the cases when the 3D reconstruction obtained from two views is (a) Unambiguous (b) Up to an unknown scaling factor (c) Up to an unknown projective transformation. **(3 points)**

(a) When K, R, t all are known, it is unambiguous

(b) When only direction of t is known, it is upto a scaling factor.



$\lambda x = (R\ t)\ \vec{X}$

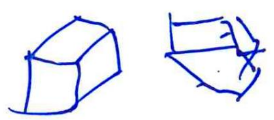
if $t = k$, $\lambda \vec{X} = m$

$t = k/2$, $\lambda \vec{X} = m/2$

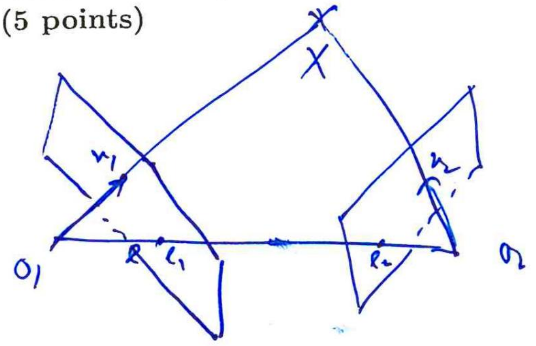
$t = 2k$, $\lambda = 2m$

the idea is that depth λ changes upto a scaling factor as 't''s direction is only known

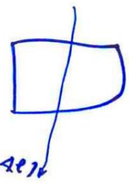
(c) If K is known but R, t are unknown, it is upto an unknown projective transformation.

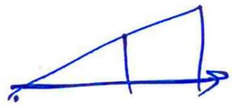


Q4) *Essential matrix:* Two cameras fixate on a point P in 3D space such that their *optical axes* intersect at this point. Show that the E_{33} element of their associated Essential matrix E is zero. **(5 points)**



Here, O_1X & O_2X are optical axes





As $O_1 x_1$ & $O_2 x_2$ are optical axes,

normalized coordinates ray would be $\vec{x_1} = \begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix}$ & $\vec{x_2} = \begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix}$

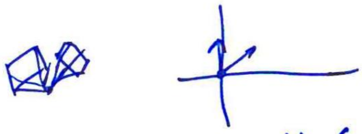
$$\begin{pmatrix} 0 & 0 & f \end{pmatrix} \begin{bmatrix} E_{11} & E_{12} & E_{13} \\ E_{21} & E_{22} & E_{23} \\ E_{31} & E_{32} & E_{33} \end{bmatrix} \begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix} = 0$$

$$\begin{pmatrix} 0 & 0 & f \end{pmatrix} \begin{pmatrix} f E_{13} \\ f E_{23} \\ f E_{33} \end{pmatrix} = 0$$

$$f^2 E_{33} = 0$$

$$f^2 \neq 0 \Rightarrow \boxed{E_{33} = 0}$$

Q5) *Homography*: Suppose a camera, with intrinsic matrix K, rotates about its optical centre by a rotation matrix R. (a) Show that its two views are related by a homography H such that $x_2 = Hx_1$. **(2.5 points)** (b) Also show that if θ is the rotation between the two views then the angle 2θ corresponds to the homography H^2. **(2.5 points)**



Let origin coincide with ~~first~~ camera at initial position.

(a) $\lambda_1 x_1 = K \vec{X}_{4\times1}$ $\quad$ $\lambda_2 x_2 = \underset{3\times3}{K}\underset{3\times4}{(R_{21}\ t_{21})} \cdot \vec{X}_{4\times1}$ $\quad$ $t_{21} = \begin{pmatrix}0\\0\\0\\0\end{pmatrix}$

$\lambda_1 x_1 = K \vec{X}_{3\times1}$ $\quad$ $\lambda_2 x_2 = K_{3\times3}(R_{21})_{3\times1}\vec{X}_{3\times1}$

$\lambda_1 K^{-1} x_1 = \vec{X}_{3\times1}$ substitute

$$\lambda_2 x_2 = \left[K_{3\times3}(R_{21})_{3\times1}(\lambda_1)K^{-1}\right] x_1$$

$$\boxed{x_2 = H x_1}$$

(b) $H^2 = K_{3\times3} R_{21} \cancel{K^{-1}} \cancel{K} R_{21} K^{-1}$

$$H^2 = K_{3\times3}\left((R_{21})^2\right)K^{-1}$$

Rotating by θ once, again by $\theta \Rightarrow \theta+\theta = 2\theta$.

(or)

$$\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix} = \begin{pmatrix}\cos^2\theta-\sin^2\theta & -2\cos\theta\sin\theta\\ 2\cos\theta\sin\theta & \cos^2\theta-\sin^2\theta\end{pmatrix} = \begin{pmatrix}\cos 2\theta & -\sin 2\theta\\ \sin 2\theta & \cos 2\theta\end{pmatrix}$$

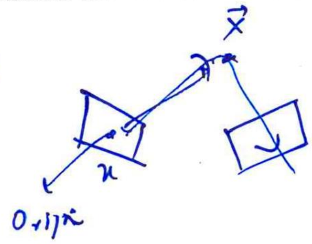
(other elements remain ~~same~~ same)

not showing other elements as it is obvious ~~that~~,

$$\begin{bmatrix}- & - & 0\\ - & - & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}- & - & 0\\ - & - & 0\\ - & - & 1\end{bmatrix}$$

Q6) *Dense-VO*: Dense-VO is one other type of visual odometry where the camera motion is estimated by aligning consecutive image frames and then finding the transformation that best minimizes the *photometric error* between them. Suppose there is a camera C with known intrinsics K, and it captures two images I_1, I_2 from two views separated by a rotation R and translation t. The photometric error between these two views is given as $\sum_{x \in I_1} \|I_1(x) - I_2(w(x, (R|t))\|^2$ where $w(x, (R|t))$ is a function that maps a point x in the first image to a point in the second image given the camera motion R, t. (a) Assuming d is the depth of the point x in I_1, describe the steps involved to map this point to the second image, and hence provide a mathematical expression for $w(x, (R|t))$. **(3 points)** (b) What is the nature of this photometric error? Very briefly in words mention how it can be solved for to find the best camrera motion. **(2 points)**

(a)



$$\lambda \vec{P}_1 = K\vec{X}$$

$$\lambda = d$$

$$d\vec{P}_1 = K\vec{X}$$

$$\boxed{\vec{X} = d K^{-1} \vec{P}_1} \text{ or } \boxed{\vec{X} = d K^{-1} \hat{x}} \quad (1)$$

$$\lambda_2 \vec{P}_2 = K[R_{21}, t_{21}]\vec{X}$$

$$\lambda_2 \vec{x}_2 = K[R_{21}, t_{21}]\vec{X} \quad (2)$$

(1) in (2) $\Rightarrow \lambda_2 \vec{x}_2 = K[R_{21}, t_{21}]\, d K^{-1} \vec{x}$

$$\therefore \boxed{w(x, (R|t)) = K[R_{21}, t_{21}]\, d K^{-1} \vec{x}}$$

If in first coordinate frame, it will be $R_{12} K[R_{21} t_{21}] d K^{-1} \vec{x}$.

(b) Least squares minimization, can be solved using Gauss-Newton.

Extra space

Extra space