Decomposition Proofs

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Information

Below steps through the situation where either price a or b does not exist. This is not detailed in the paper, but with some input from the author (Michael Webster) and some algebra the equation can be simplified in both cases. Below shows how they are simplified, and in turn is how I wrote the code.

Three important identities

The exponential of a sum is the product of the exponential of the summands.

$$a^{\sum_t^t x} = \prod_n^t a^x$$

Negative exponential

$$a^{-x} = 1/a^x$$

This

$$\frac{1}{\prod_i (x_i)} = \prod_i (\frac{1}{x_i})$$

Where all $w_i(t, a) = 0$ and therefore $(p_i^a)^{w_i(\bullet, a)}$ is equal to 1

$$= \frac{(p_i^b)^{w_i(\bullet,b)}}{(p_i^a)^{w_i(\bullet,a)}} \cdot \left[\prod_t (p_i^t)^{\frac{(w_i(t,a) - w_i(t,b))}{T+1}} \right]$$

$$= \frac{(p_i^b)^{w_i(\bullet,b)}}{1} \cdot \left[\prod_t (p_i^t)^{\frac{0 - w_i(t,b))}{T+1}} \right]$$

$$= \prod_t (p_i^b)^{\frac{w_i(t,b)}{T+1}} \cdot \left[\prod_t (\frac{1}{p_i^t})^{\frac{w_i(t,b)}{T+1}} \right]$$

$$= \prod_t (\frac{p_i^b}{p_i^t})^{\frac{w_i(t,b)}{T+1}}$$

Where all $w_i(t, b) = 0$ and therefore $(p_i^b)^{w_i(\bullet, b)}$ is equal to 1

$$= \frac{1}{(p_i^a)^{w_i(\bullet,a)}} \cdot \left[\prod_t (p_i^t)^{\frac{(w_i(t,a)-0)}{T+1}} \right]$$

$$= \frac{1}{\prod_t (p_i^a)^{\frac{w_i(t,a)}{T+1}}} \cdot \left[\prod_t (p_i^t)^{\frac{w_i(t,a)}{T+1}} \right]$$

$$= \prod_t \frac{(p_i^t)^{\frac{w_i(t,a)}{T+1}}}{(p_i^a)^{\frac{w_i(t,a)}{T+1}}}$$

$$= \prod_t (\frac{p_i^t}{p_i^a})^{\frac{w_i(t,a)}{T+1}}$$