L8 07 Feb 2022

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Rules of Inference :-

By an argument, we mean a sequence of statements that end with a conclusion. By valid, we mean that the conclusion, or final statement of the argument, must follow from the truth of the preceding statements, or premises, of the argument. That is, an argument is valid if and only if it is impossible for all the premises to be true and the conclusion to be false. To deduce new statements from statements we already have, we use rules of inference which are templates for constructing valid arguments. Rules of inference are our basic tools for establishing the truth of statements.

An *argument* in propositional logic is a sequence of propositions. All but the final proposition in the argument are called *premises* and the final proposition is called the *conclusion*. An argument is *valid* if the truth of all its premises implies that the conclusion is true.

An *argument form* in propositional logic is a sequence of compound propositions involving propositional variables. An argument form is *valid* no matter which particular propositions are substituted for the propositional variables in its premises, the conclusion is true if the premises are all true.

$p_1 \quad p_2 \quad p_3 \; --- \; p_4$

q

"If you have a current password, then you can log onto the network."
p $\qquad$ q

"You have a current password."
p

Therefore,

"You can log onto the network."

$$p \to q$$
$$p$$
$$\therefore q$$

valid

$((p \to q) \wedge p) \to p$ is a tautology

Argument form with premises $p_1, p_2, p_3, --- p_n$ and conclusion q is valid when

$(p_1 \wedge p_2 \wedge p_3 \wedge p_4 - - \wedge p_n) \rightarrow q$ is tautology

TABLE 1 Rules of Inference.

Rule of Inference	*Tautology*	*Name*
p $p \rightarrow q$ $\therefore q$	$(p \wedge (p \rightarrow q)) \rightarrow q$	Modus ponens or Law of Detachment
$\neg q$ $p \rightarrow q$ $\therefore \neg p$	$(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$	Modus tollens
$p \rightarrow q$ $q \rightarrow r$ $\therefore p \rightarrow r$	$((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$	Hypothetical syllogism
$p \vee q$ $\neg p$ $\therefore q$	$((p \vee q) \wedge \neg p) \rightarrow q$	Disjunctive syllogism
p $\therefore p \vee q$	$p \rightarrow (p \vee q)$	Addition
$p \wedge q$ $\therefore p$	$(p \wedge q) \rightarrow p$	Simplification
p q $\therefore p \wedge q$	$((p) \wedge (q)) \rightarrow (p \wedge q)$	Conjunction
$p \vee q$ $\neg p \vee r$ $\therefore q \vee r$	$((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$	Resolution

p
$p \rightarrow q$
$\therefore q$

$(p \wedge (p \rightarrow q)) \rightarrow q$ is tautology

p	q	$p \rightarrow q$	$p \wedge (p \rightarrow q)$	$(p \wedge (p \rightarrow q)) \rightarrow q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

$\rightarrow$ Tautology

State which rule of inference is the basis of the following argument: "It is below freezing now. Therefore, it is either below freezing or raining now."

p:. It is below freezing now

q: It is below raining now.

$$\frac{p}{\therefore\ p \vee q}$$

. Find the argument form for the following argument and determine whether it is valid. Can we conclude that the conclusion is true if the premises are true?

If Socrates is human, then Socrates is mortal.
Socrates is human.

∴ Socrates is mortal.

$$\frac{\begin{array}{c} p \to q \\ p \end{array}}{\therefore\ q}$$

$((p \to q) \wedge p) \to q$ is tautology

Find the argument form for the following argument and determine whether it is valid. Can we conclude that the conclusion is true if the premises are true?

If George does not have eight legs, then he is not a spider.
George is a spider.

∴ George has eight legs.

$$\frac{\begin{array}{c} p \to q \\ \neg q \end{array}}{\therefore\ \neg p}$$

$$\frac{\begin{array}{c} \neg q \to \neg p \\ p \end{array}}{\therefore\ q}$$

p:- George is a spider

q:- George has eight legs

$$\boxed{((\neg q \to \neg p) \wedge p) \to q}$$

p	q	$\neg p$	$\neg q$	$\neg q \to \neg p$	$(\neg q \to \neg p) \wedge p$	$((\neg q \to \neg p) \wedge p) \to q$
T	T	F	F	T	T	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	T	T	F	T