T5 G2 03 Feb 2022

03 February 2022 11:03

Negating Quantified Expressions :-

"Every student in your class has taken a course in calculus"

$\forall x\, P(x)$

where $P(x)$ is the statement "x has taken a course in calculus" and the domain consists of all the students in your class.

Negation :- It is not the case that every student in your class has taken a course in calculus.

$\equiv$

Equivalent

✓ There is a student in your class who has not taken a course in calculus

$\exists x \neg P(x)$ ✓

$\neg(p \wedge q) \equiv \neg p \vee \neg q$

$\neg \forall x P(x) \equiv \exists x \neg P(x)$ ✓

$\neg \exists x Q(x) \equiv \forall x \neg Q(x)$

↑ De Morgan's Laws for Quantifiers

$\neg \forall x P(x) \equiv \exists x \neg P(x)$

$\neg \exists x P(x) \equiv \forall x \neg P(x)$

$\neg \forall x P(x) \not\equiv \exists x P(x)$

What is the negation of the statement $\forall x (x^2 > x)$ ✓

$\neg \forall x (x^2 > x) \equiv \exists x \neg (x^2 > x)$

$\equiv \exists x (x^2 \leq x)$ ✓

Negation of the statement $\exists x (x^2 = 2)$

$\neg \exists x (x^2 = 2) \equiv \forall x \neg (x^2 = 2)$

$\equiv \forall x (x^2 \neq 2)$

"No rabbit knows Maths"

$P(x)$:- x knows Maths.

Domain:- Rabbits

$\neg \forall x P(x)$

$\forall x \neg P(x)$

$\equiv \neg \exists x P(x)$

"There is no dog that can talk"

$P(x)$:- x can talk.

Domain:- Dogs

Every dog

$\forall x \neg P(x)$

What is the negation of the statement $\forall x (x^2 > x)$?

(a) $\forall x (x^2 < x)$

(b) $\exists x (x^2 = x)$

(c) $\exists x (x^2 \leq x)$

(d) $\forall x (x^2 \leq x)$

$\neg \forall x (x^2 > x)$

$\equiv \exists x \neg (x^2 > x)$

$\equiv \exists x (x^2 \leq x)$

Nested Quantifiers

Two quantifiers are nested if one is within the scope of the other such as

$\forall x \, \exists y (x+y=0)$

$\forall x \, Q(x)$

Whereee

$Q(x)$ $\quad \exists y (x+y=0)$

$\exists y \, P(x,y)$

$\forall x \, \exists y \underbrace{(x+y=0)}_{P(x,y)}$

$\forall x \, \underbrace{\exists y \, P(x,y)}_{Q(x)}$

$\forall x \, Q(x)$

\# $\forall x \, \forall y \big((x>0) \wedge (y<0) \rightarrow (xy<0) \big)$

(Domain is the set of real nos)