Introduction to Logic

Problem 6.1 - Relational Syntax

Say whether each of the following expressions is a syntactically legal sentence of Relational Logic. Assume that *red*, *green*, *jim*, and *molly* are object constants. Assume that *color* is a binary function constant, and assume that *parent* is a binary relation constant.

a. $parent(red, green)$	legal ▼	✓
b. $\neg color(jim, green)$	illegal ▼	✓
c. $color(blue, molly)$	illegal ▼	✓
d. $parent(molly, molly)$	legal ▼	✓
e. $parent(molly, z)$	legal ▼	✓
f. $\exists x.parent(molly, x)$	legal ▼	✓
g. $\exists y.parent(molly, jim)$	legal ▼	✓
h. $\forall z.(z(jim, molly) \Rightarrow z(molly, jim))$	illegal ▼	✓

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Problem 6.2 - Counting

Consider a language with n object constants, no function constants, and a single binary relation constant.

a. How many ground terms are there in this language?

✓ ◉ n ○ n^2 ○ 2^n ○ 2^{n^2} ○ 2^{2^n}

b. How many ground atomic sentences are there in this language?

✓ ○ n ◉ n^2 ○ 2^n ○ 2^{n^2} ○ 2^{2^n}

c. How many distinct truth assignments are possible for this language?

✓ ○ n ○ n^2 ○ 2^n ◉ 2^{n^2} ○ 2^{2^n}

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Problem 6.3 - Relational Evaluation

Consider a language object constants a and b, no function constants, and relation constants p and q where p has arity 1 and q has arity 2. The following is a truth assignment for this language.

$$p(a)^i = 1$$
$$p(b)^i = 0$$
$$q(a,a)^i = 0$$
$$q(a,b)^i = 1$$
$$q(b,a)^i = 1$$
$$q(b,b)^i = 0$$

Say whether each of the following sentences is true or false under with this truth assignment.

a. $\forall x.(p(x) \Rightarrow q(x,x))$ false ▼ ✓

b. $\forall x.\exists y.q(x,y)$ true ▼ ✓

c. $\exists y.\forall x.q(x,y)$ false ▼ ✓

d. $\forall x.(p(x) \Rightarrow \exists y.q(x,y))$ true ▼ ✓

e. $\forall x.p(x) \Rightarrow \exists y.q(y,y)$ true ▼ ✓

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Problem 6.4 - Relational Evaluation

This exercise concerns the interpersonal relations of a small sorority. There are just four members - Abby, Bess, Cody, and Dana; and there is just one type of binary relationship - likes. The following table below shows who likes whom. A check in a box of the table indicates that the girl named at the beginning of the row likes the girl named at the head of the column; the absence of a check means that she does not.

	Abby	**Bess**	**Cody**	**Dana**
Abby		✓		✓
Bess	✓		✓	
Cody		✓		✓
Dana	✓		✓	

The following sentences are constraints that characterize the possibilities. Your job here is to select a truth value for each constraint indicating whether that constraint is satisfied by the table shown above.

a. $likes(dana,cody)$ true ✓

b. $\neg likes(abby,dana)$ false ✓

c. $likes(bess,cody) \vee likes(bess,dana)$ true ✓

d. $\forall y.(likes(bess,y) \Rightarrow likes(abby,y))$ false ✓

e. $\forall y.(likes(y,cody) \Rightarrow likes(cody,y))$ true ✓

f. $\forall x.\neg likes(x,x)$ true ✓

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Problem 6.5 - Relational Satisfaction

Consider a version of the Blocks World with just three blocks - *a*, *b*, and *c*. The *on* relation is axiomatized below.

$\neg on(a,a)$	$on(a,b)$	$\neg on(a,c)$
$\neg on(b,a)$	$\neg on(b,b)$	$on(b,c)$
$\neg on(c,a)$	$\neg on(c,b)$	$\neg on(c,c)$

Let's suppose that the *above* relation is defined as follows. This is *almost* the same as in the notes except that we have replaced an occurrence of *on* with *above* and we have dropped the axiom $\forall x.\neg above(x,x)$.

$$\forall x.\forall z.(above(x,z) \Leftrightarrow on(x,z) \vee \exists y.(above(x,y) \wedge above(y,z)))$$

A sentence φ is consistent with a set Δ of sentences if and only if there is a truth assignment that satisfies all of the sentences in $\Delta \cup \{\varphi\}$. Say whether each of the following sentences is consistent with the sentences about *on* and *above* shown above. Be careful. It's tricky.

a. $above(a,c)$ Consistent ✓

b. $above(a,a)$ Consistent ✓

c. $above(c,a)$ Consistent ✓

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Clausal Form

Consider a language with two object constants a and b and one function constant f. For each of the following sentences in this language, say which of the alternatives shown is its clausal form.

1. $\exists y.\forall x.p(x,y)$ ✓
   - ○ $\{p(a,y)\}$
   - ○ $\{p(c,y)\}$
   - ○ $\{p(x,b)\}$
   - ● $\{p(x,c)\}$
   - ○ $\{p(x,f(x))\}$
   - ○ $\{p(x,g(x))\}$

2. $\forall x.\exists y.p(x,y)$ ✓
   - ○ $\{p(a,y)\}$
   - ○ $\{p(c,y)\}$
   - ○ $\{p(x,b)\}$
   - ○ $\{p(x,c)\}$
   - ○ $\{p(x,f(x))\}$
   - ● $\{p(x,g(x))\}$

3. $\exists x.\exists y.(p(x,y) \wedge q(x,y))$ ✓
   - ○ $\{p(a,b), q(a,b)\}$
   - ○ $\{p(a,b)\}$ and $\{q(a,b)\}$
   - ○ $\{p(c,d), q(c,d)\}$
   - ● $\{p(c,d)\}$ and $\{q(c,d)\}$
   - ○ $\{p(x,y), q(x,y)\}$
   - ○ $\{p(x,y)\}$ and $\{q(x,y)\}$

4. $\forall x.\forall y.(p(x,y) \Rightarrow q(x))$ ✓
   - ○ $\{\neg p(a,b), q(a)\}$
   - ○ $\{\neg p(c,d), q(c)\}$
   - ○ $\{\neg p(x,b), q(x)\}$
   - ○ $\{\neg p(x,d), q(x)\}$
   - ● $\{\neg p(x,y), q(x)\}$
   - ○ $\{\neg p(x,g(x)), q(x)\}$

5. $\forall x.(\exists y.p(x,y) \Rightarrow q(x))$ ✓
   - ○ $\{\neg p(a,b), q(a)\}$
   - ○ $\{\neg p(c,d), q(c)\}$
   - ○ $\{\neg p(x,b), q(x)\}$
   - ○ $\{\neg p(x,d), q(x)\}$
   - ● $\{\neg p(x,y), q(x)\}$
   - ○ $\{\neg p(x,g(x)), q(x)\}$