

(a) matrices representing the operator:  $J^2, J_x, J_y$  and  $J_z$  in the  $|3/2, m\rangle$  basis.

$$J^2 = \begin{pmatrix} \frac{15}{4} & 0 & 0 & 0 \\ 0 & \frac{15}{4} & 0 & 0 \\ 0 & 0 & \frac{15}{4} & 0 \\ 0 & 0 & 0 & \frac{15}{4} \end{pmatrix} \quad (1)$$

$$J_x = \begin{pmatrix} 0 & \frac{\sqrt{3}}{2} & 0 & 0 \\ \frac{\sqrt{3}}{2} & 0 & 1 & 0 \\ 0 & 1 & 0 & \frac{\sqrt{3}}{2} \\ 0 & 0 & \frac{\sqrt{3}}{2} & 0 \end{pmatrix} \quad (2)$$

$$J_y = \begin{pmatrix} 0 & -\frac{\sqrt{3}}{2}i & 0 & 0 \\ \frac{\sqrt{3}}{2}i & 0 & -i & 0 \\ 0 & i & 0 & -\frac{\sqrt{3}}{2}i \\ 0 & 0 & \frac{\sqrt{3}}{2}i & 0 \end{pmatrix} \quad (3)$$

$$J_z = \begin{pmatrix} \frac{3}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & -\frac{3}{2} \end{pmatrix} \quad (4)$$

(b) RHS result of the commutator:  $[J_x, J_y] = i * J_z$

$$[J_x, J_y] = \begin{pmatrix} \frac{3}{2}i & 0 & 0 & 0 \\ 0 & \frac{1}{2}i & 0 & 0 \\ 0 & 0 & -\frac{1}{2}i & 0 \\ 0 & 0 & 0 & -\frac{3}{2}i \end{pmatrix} \quad (5)$$

(c)  $\langle J_x \rangle$  and  $\langle (J_x)^2 \rangle$  with respect to the state  $|\psi\rangle = (0, 0, 1, 0)$ :

$$\langle J_x \rangle = 0 \quad (6)$$

$$\langle J_x^2 \rangle = \frac{7}{4} \quad (7)$$

(d)  $\delta J_x$  and  $\delta J_y$  with respect to the state  $|\psi\rangle = (0, 0, 1, 0)$ :

$$\delta J_x = \frac{\sqrt{7}}{2} \quad (8)$$

$$\delta J_y = \frac{\sqrt{7}}{2} \quad (9)$$