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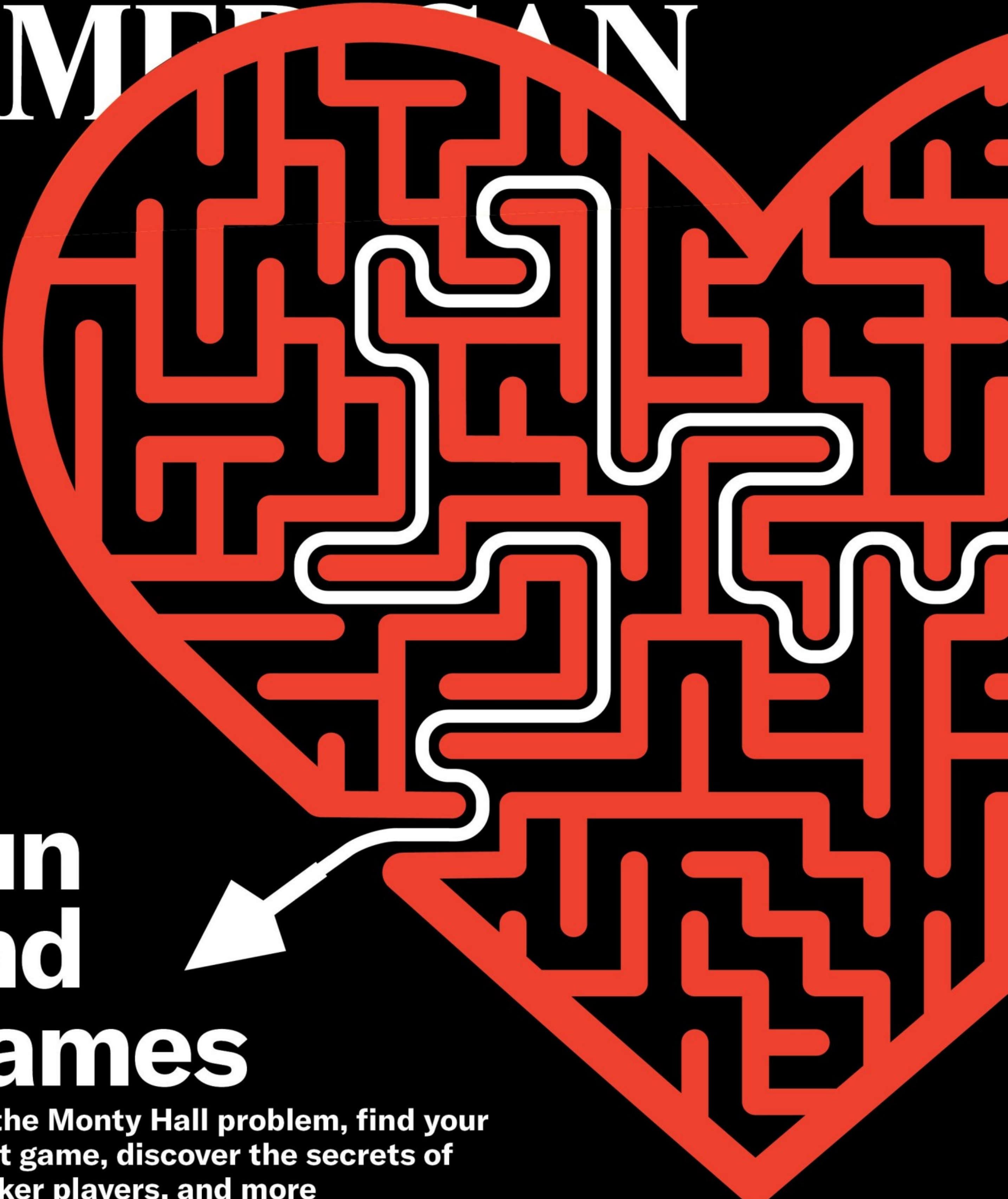

Fun and Games

Solve the Monty Hall problem, find your perfect game, discover the secrets of pro poker players, and more

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Features

- ## What Do Baseball Pitchers Shohei Ohtani, Gerrit Cole and Shane Bieber Have in Common?
  Baseball pitchers are throwing faster than ever—and needing Tommy John surgery
- ## 10 Ancient Games That Are Still Fun to Play
  Find new ways to fill the long summer days with these quick primers on 10 ancient games
- ## How Games Run Everything from Online Dating to Social Media to Stock Markets
  Games have captivated people since ancient times. Now they run our lives
- ## Science Crossword: Pseudoscience
  Your favorite word game with a twist of science
- ## The Nash Equilibrium Is the Optimal Poker Strategy. Expert Players Don’t Always Use It
  Poker players can now employ AI to find the optimal playing strategy, but they often don’t use it. Here’s why
- ## LARPing Made Me a Stronger Person. It Can Do the Same for You, Too
  Thousands of people around the world do live-action role-playing to learn and change themselves. I am one of them
- ## Why Almost Everyone Gets the Monty Hall Probability Puzzle Wrong
  How to finally wrap your mind around the uniquely counterintuitive Monty Hall dilemma
- ## What Game Should I Play? Take This Interactive Quiz to Find Out
  From board games to solo puzzles to fantastical escapes to tricks for running faster, find a game for every mood

How Baseball Got Faster but Riskier

Baseball pitchers are throwing faster than ever—and needing Tommy John surgery

By Abe Streep

Paul Skenes, number 30, of the Pittsburgh Pirates pitches against the Detroit Tigers during game two of a doubleheader at Comerica Park on May 29, 2024, in Detroit, Mich.
Rick Osentoski/Getty Images

In May Paul Skenes, a then 21-year-old pitcher, debuted on a Major League mound, 10 months after the Pittsburgh Pirates selected him as the first overall pick in the professional baseball draft. Before he threw a pitch in the big leagues, Skenes had been anointed as the sport's next great hope on account of his electric fastball; he is one of a rising generation of pitchers who routinely throws above 100 miles per hour, a mark that was seen as an upper physiological limit as recently as a decade ago.

During his first two games, Skenes threw 29 pitches above 100 mph and exhibited the kind of screen-ready cockiness that recalled famed player Roger Clemens. When a reporter asked Skenes how he'd deal with hitters adjusting to his repertoire, he chuckled, "Go ahead and adjust. Good luck.” One writer called him “the perfect baseball prospect through every possible lens.” Rob Friedman, a popular social media personality and analyst who goes by the moniker Pitching Ninja, called him “the filthiest pitcher on freaking Earth.”

But simmering beneath the rhapsodic excitement was a fear that others voiced: How long would it be before Skenes, like so many recent flamethrowers, injured his pitching arm?

Baseball has never been faster. The games fly by in two hours, and more pitchers are throwing harder than ever before. When Aroldis Chapman entered the league in 2010, he was seen as an anomaly for throwing above 100 mph. (He still holds the official record for the fastest pitch ever thrown, at 105.8 mph.) Since Chapman debuted, the average fastball speed has jumped by two miles per hour, and the number of pitches reaching the 100 mile per hour threshold has quadrupled. This growing emphasis on speed appears to be coming at a cost.

In recent years some of the game’s best and most exciting pitchers—including Spencer Strider, Gerrit Cole and Shohei Ohtani, the undisputed face of the game—have suffered injuries to their elbow. “Training has increased, and techniques have gotten better, so as a result, you have a lot more people throwing the ball harder," says Koco Eaton, team surgeon for the Tampa Bay Rays. "There's more stress to the system and more injuries."

How did this happen? From 2012 to 2022, not counting 2020, Major League Baseball (MLB) suffered a 14-percent decline in game attendance. At the same time, the era of optimization was rising, and teams were using data analytics and technology such as video analysis to try to gain any advantage they could. "Analytics are very disruptive to any industry," says Brian Bannister, current director of pitching for the Chicago White Sox, who was at the forefront of the trend while working with the Boston Red Sox and San Francisco Giants. "They distort the game and reprioritize things."

In this case, analytics reprioritized velocity as well as what pitchers refer to as "stuff": the mix of curving, sliding and sweeping pitches, most of them pivoting off a fastball, that can fool the best hitters on Earth. Companies such as Driveline and Tread Athletics—the latter co-founded by Ben Brewster, a former Minor League pitcher and author of *Building the 95 MPH Body*—began training both amateur and professional pitchers with the aim of getting the most out of each arm. At Tread's 33,000-square-foot facility in North Carolina, every pitch is captured on screens and analyzed, so that coaches can refine players' grip and figure out which pitches work best together.

In advance of the 2023 season, MLB instituted new rules to make games more viewer-friendly, with a pitch clock to keep games under three hours and larger bases to incentivize stealing. Viewership shot up, but so, too, it seemed, did injuries to the sport's hardest throwers.

Last summer Ohtani, now star of the Los Angeles Dodgers, was found to have a tear in the ulnar collateral ligament (UCL) of his pitching arm, ending his season. (It was the second time he's suffered the injury.) Shortly afterward, Gerrit Cole, last year's American League Cy Young Award winner and one of the game's hardest throwers, went down with inflammation in his elbow. Then the Cleveland Guardians announced that Shane Bieber, the team's ace, would undergo UCL reconstruction, commonly known as Tommy John surgery. Around the same time, Spencer Strider, ace of the 2023 National League East champion Atlanta Braves, tore his UCL.*

It is accepted that the UCL can withstand 30 pounds of force; throwing a Major League fastball creates twice that much. The human body compensates for that pressure with the complex of muscles and tendons around the ligament. (Pitchers can now emerge from surgery with their UCL sheathed inside a device known as an internal brace—a suture system designed to protect the ligament.)

MLB has disputed that the pitch clock increases injuries. But there is concern among baseball's top minds that the financial pressures to maximize velocity on every pitch, combined with the shortened recovery time between pitches, poses new risks. “What we've done, at all costs, is train pitchers and alter the usage of pitchers and encourage pitchers to go out there [and] throw as hard as possible, as often as possible,” says Bannister, who now counts himself as a skeptic of the leaguewide move toward absolute optimization.

Baseball is still called a national pastime, but the application of the moniker seems to be rooted in nostalgia. The heart of the game has long been in its quiet moments—the thick air of expectation that hovers before a fly ball drops into an outfielder's glove while a runner prepares to try to tag up or the way a pitcher looks at a hitter when the tying run is on third.

These days, though, it's more about fast pitching, long home runs and stolen bases. Bannister says that, along with the potential risk of injury, he’s concerned about a spiritual malaise. “We’ve now reached a saturation point where we’re vulnerable to squeezing the beauty out of the game, the storylines out of the game.”

Not everyone agrees. “There’s still plenty of art to the game,” says Tread’s Brewster. “It’s not purelyalgorithmic.” Friedman, the Pitching Ninja, says there’s never been a better time to be a pitcher. Eaton, the Rays’ surgeon, agrees. “You’re going to have opportunity coming your way,” he says. “Guys around you are going to be on injured reserve, and you’re going to have a chance show your stuff pretty soon.” Eaton envisions a future in which pitchers take the mound in wearable devices that might be able to predict when an elbow is at risk of blowing out. But we’re not there yet. And for the moment, he allows, it’s also a great time to be a surgeon.

**Editor’s Note (8/23/24): This sentence was edited after posting to correct the description of the Atlanta Braves’ championship in 2023.*

Abe Streep is a journalist and author of *Brothers on Three: A True Story of Family, Resistance, and Hope on a Reservation in Montana* (Celadon, 2021). He lives in New Mexico.

These 10 Ancient Games Are Still Fun to Play

Find new ways to fill the long summer days with these quick primers on 10 ancient games

By Stephanie Pappas

Game box for playing Senet and Twenty Squares, circa 1635–1458 B.C.E.
Penta Springs Limited/Alamy Stock Photo

Board games might involve a race to a finish line, as in Sorry! or that ubiquitous first board game for kids, Candyland. Or they might entail a strategic battle for dominance, as in chess or checkers. Some have simple rules (think Clue), while others have a headache-inducing learning curve (such as Feudeum, in which players control a set of medieval characters who must survive in a complex economy).

Whether you like to while away long hours with Monopoly or a quick Snakes and Ladders is more your speed, you're taking part in a tradition that stretches back millennia. Here are 10 of the oldest board games and a quick primer on how to play each one. With these games, you may find new (ancient) ways to fill the long summer days.

[*Play science-inspired games, puzzles and quizzes in our new Games section*]

On supporting science journalism

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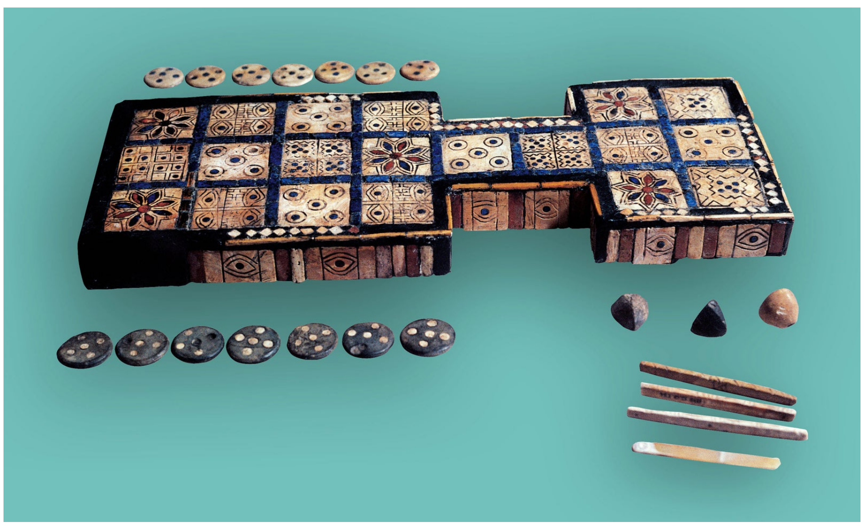


An inlaid game board and playing pieces discovered in a tomb at the southern Iraqi site of Ur. They date back to the second millennium B.C.E.
Zev Radovan/Alamy Stock Photo

1. The Royal Game of Ur

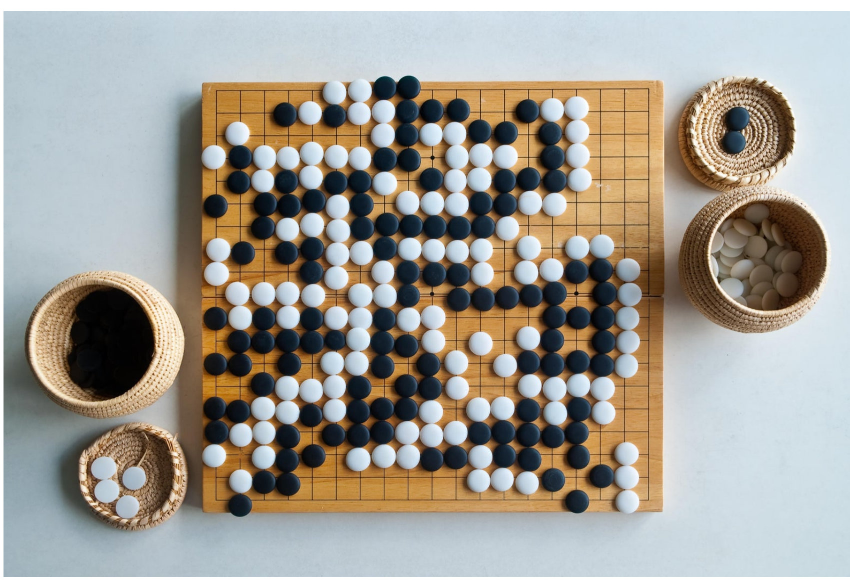
First played: As early as 2600 B.C.E.

Who played it: Ancient Mesopotamians. The game also spread around Central Asia, from Iran to India.

The backstory: No one knows what the people who invented the Royal Game of Ur called it. Modern archaeologists named this game after the site in southern Iraq where the boards and pieces were found in the 1920s. The game was played until at least around 177 B.C.E., but it may have persisted into the 20th century among the Jewish community of Kochi, India, says Walter Crist, an archaeologist and lecturer at Leiden University in the Netherlands, who studies ancient games. Some snippets of the rules are known from ancient texts that describe the game's distinctive I-shaped board but don't say the name of the game, which, Crist notes, "would have been nice."

The rules: Each of the two players has five or seven buttonlike pieces. They throw dice to enter these pieces onto the game board, where they try to beat their opponent in moving all their own pieces to a square at the end of the board. Some squares are marked with symbols denoting them as safe, while others are "combat" squares, where players can capture their opponent's pieces and send them back to the starting square.

Can I play it today? You can get your own version of the Royal Game of Ur for less than $20 online.



An example of a Go board at the end of a game between two people.
Saran Poroong/Alamy Stock Photo

2. Go

First played: Around 500 B.C.E.

Who played it: The ancient Chinese, who also spread the game throughout Asia.

The backstory: Myth and legend hold that a Chinese emperor named Yao taught his son Dan Zhu the game of Go as early as 2100 B.C.E., while other tales trace the game to the mythical Yellow Emperor Huangdi. The oldest archaeological evidence of the game dates to the last few centuries B.C.E., however, Crist says. "It's basically the oldest of the traditional board games that are really still popular today," he adds.

The rules: Go is played on a 19-by-19 board of squares. Each of the two players gets a supply of stones (181 black pieces or 180 white ones) and must place them on the board in turn. The goal is to surround the other player's stones to win more of the board's area. The player with the most area wins.

Can I play today? Oh, you can play all right. Go is a massively popular strategy game with associations dedicated to its play around the world. Even computers are in on it: in March 2016 artificial intelligence AlphaGo beat champion Go player Lee Sedol in a highly publicized match.

One of at least four Senet boards that were found buried with King Tut.
Smith Archive/Alamy Stock Photo

3. Senet

First played: Around 3000 B.C.E.

Who played it: Ancient Egyptians.

The backstory: Senet was popular in Egypt up through the Roman period. Everyone played, from guards in temples who scratched game boards into the ground to kings who were interred with Senet boards in their tombs, Crist says.

“Pharaohs, commoners—everybody knew the game,” he says.

Ancient Egyptian texts sometimes refer to the game as a metaphor for moving through the afterlife, Crist says, so it may have sometimes had religious connotations. This religious meaning tied the game very closely to Egyptian culture, says Barbara Carè, an archaeologist and senior lecturer at the University of Fribourg in Switzerland. Such deep cultural meaning may be why some games gain popularity locally but don’t spread too far abroad, she says.

“You can move the game somewhere else, but it doesn’t have the same meaning,” Carè adds.

The rules: Each of two players moved at least 10 pawns around a 30-square board and may have determined the number of spaces to move by throwing sets of four two-sided sticks. Fragments of texts and tomb paintings suggest players could block or pass each other, Crist says, but the details are unknown.

Can I play today? You can try. Timothy Kendall, author of *Passing through the Netherworld: The Meaning and Play of Senet, an Ancient Egyptian Funerary Game* (Kirk Game Company, 1978), attempted to reconstruct Senet’s gameplay. Historian R. C. Bell also came up with a version. Would ancient Egyptians recognize these rules? Who knows.

A reconstruction of the ancient Greek board game pente grammai.
WHPicks/Getty Images

4. Pente Grammai

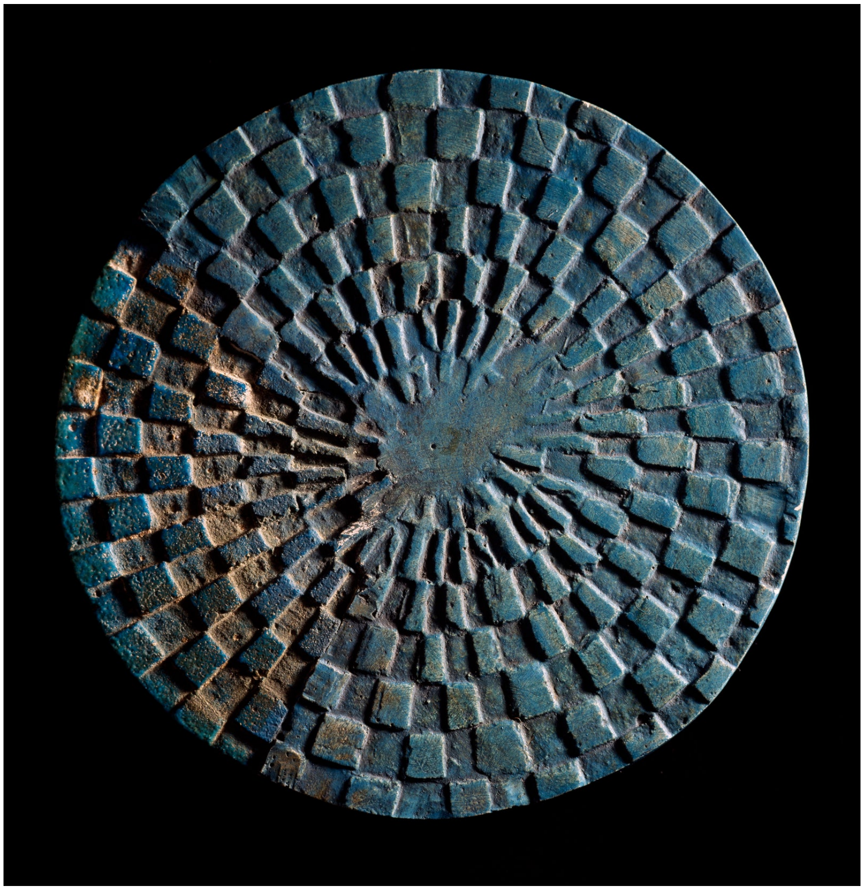
First played: Earlier than 600 B.C.E.

Who played it: Ancient Greeks.

The backstory: Pente grammai, or “five lines,” is an ancient Greek board game whose original name and rules have been lost. The game is known from poems, texts and ancient boards that consist of five parallel lines, each with a small dot or divot at each end. The game was associated with men, strategy and heroics, Carè says. It's most famously depicted in a painting on a vase, reproduced many times in Greece around 500 B.C.E., showing the heroes Ajax and Achilles playing a tense round. Women weren't depicted playing board games in ancient Greece, Carè notes, because of the association of games with male pursuits. In ancient Rome, however, the connotations were different. Women are often depicted playing board games alongside men in ancient Roman paintings, Carè says. "This is again symbolic," she says, "but it's a matter of seduction" rather than military strategy.

The rules: The details are sketchy, but two players competed, each possibly holding five pieces. The middle of the five lines was known as the sacred line, and the goal was to land one's pieces on that line. The players threw dice to determine their move.

Can I play today? There are various reconstructed rules of the game, but how close they are to the ancient Greek version is hard to determine.



A Mehen board discovered in the tomb of the pharaoh Seth-Peribsen at Abydos in Egypt. The spiral board looks like an abstract coiled snake.
DeAgostini/Getty Images

5. Mehen

First played: Before 3000 B.C.E.

Who played it: Ancient Egyptians for about 1,000 years.

The backstory: Mehen is one of the oldest games we still know the name for, Crist says. The board was a spiral, akin to an abstract rendering of a snake. And different examples of that board had different numbers of spaces. The game could be played by up to six players. That made it unique among ancient board games, which tended to be two-player affairs, Crist says.

The rules: Each player had six marblelike pieces and two lion-shaped pieces. Based on modern games from Saharan Africa, the goal may have been to get pieces to the center of the board and back, with the lion pieces perhaps acting to prevent one's opponent's pieces from returning safely. "Whether or not this is the way the ancient game was played is not something we can really say," Crist says, because there are 4,000 years between when ancient Egyptians quit playing the game and when modern people started.

Can I play today? Likely the closest you can get is to play Hyena Chase, or the Hyena Game, which is also known as Li'b el-Merafib in Sudan.

A game board of Hounds and Jackals shows the sticks with jackal or dog heads that players would have moved.
Cultural Archive/Alamy Stock Photo

6. Hounds and Jackals

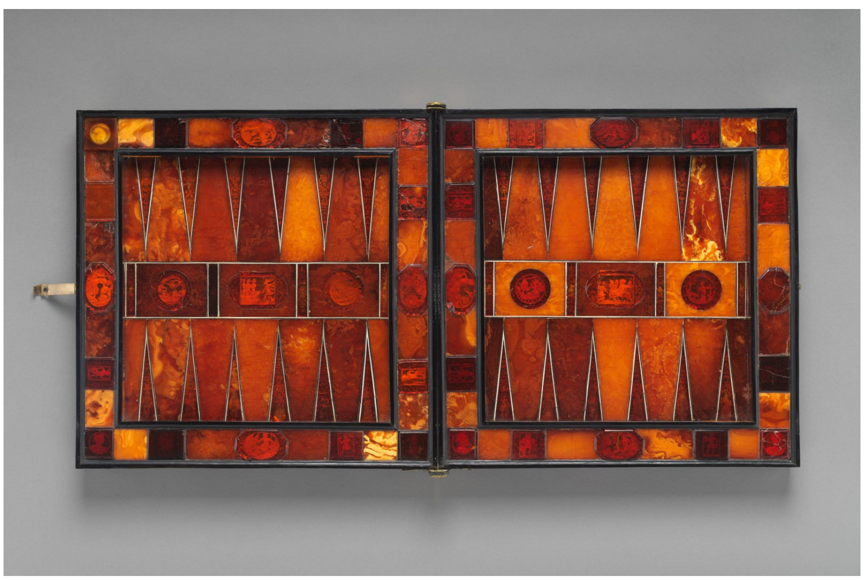
First played: Around 2000 B.C.E.

Who played it: People all over western Asia and Egypt.

The backstory: This game pops up across western Asia and Egypt around the same time, Crist says, so it’s unclear who invented it. Egyptologist Howard Carter gave this game its modern name; no one knows what ancient people called it. There were different variations of the game board across time and space, with a wider variety of gameplay in western Asia than in Egypt, Crist says.

The rules: Players moved sticks with either jackal or dog heads through two tracks made up of 29 holes each. No dice have been found, Crist says, but they must have existed. Most likely, the goal was to get one’s pieces to a hole at the end of the game board that was often either specially marked or larger than the other holes.

Can I play today? Game boards with modern-day reconstructions of rules are available, but no surviving text describes how the game was originally played.



A game board that could be used for playing backgammon, chess, a French variation of backgammon called trictrac and a dice-throwing game called goose.
Gift of Gustavus A. Pfeiffer, 1948/Metropolitan Museum of Art

7. Backgammon

First played: Great question!

Who played it: Persians, Ancient Romans, or maybe other Central Asians.

The backstory: Dating the origins of games is often a tricky business, Carè says. Graffiti game boards are often more recent than the buildings or pavements they're etched into. In ancient Greece and Rome, for example, many buildings that were located in solemn religious regions in the classical period (between the eighth century B.C.E. and the fifth century C.E.) were, in later centuries, decorated with gaming boards etched into the facades. This fact is interesting as far as elucidating how people used a space, Carè says, but can make it hard to determine the true age of a game board.

Backgammon is a good example of a game with a clouded origin. Some argue that it traces back to a Roman game called *ludus duodecim scriptorum,* or "game of 12 lines," Crist says. Others say it was invented in Persia, where it was first known as nard. Archaeologists have found game board graffiti of backgammon from the early eighth century C.E. in what is now Iran, Crist says.

The game likely spread to Europe when Crusaders encountered it during the 10th and 11th centuries.

The rules: Players each get 15 checkers and a pair of dice. The goal is to move all checkers around and off the board along a horseshoe-shaped path.

Can I play today? Backgammon is a standard in game cabinets around the world. A version called tavli is the national game in Greece and Cyprus.


Illustration of a patolli game being played.
The History Collection/Alamy Stock Photo

8. Patolli

First played: As early as 200 B.C.E.

Who played it: Ancient Maya, Ancient Aztecs and other Mesoamerican groups.

The backstory: Patolli, or patole, was played across Central America and southern North America up into what is today Oklahoma, Crist says. Much of what is known of the rules come from the records of Spanish conquistadors who encountered it during their besiegement of the Aztec Empire in the 1500s.

The rules: Players moved six pieces across an X-shaped board and used a handful of marked dried beans as dice. When a player moved all six pieces around the board before their opponent, they won the round. Patolli was a gambling game, so winners got to take wagered items such as blankets or food from the loser of each round.

Can I play today? Tūhura Otago Museum in New Zealand has a printable game board and reconstructed rules.

A replica of the historic Viking board game hnefatafl.
mrpluck/Getty Images

9. Hnefatafl

First played: Popular during the Viking Age in Europe, after C.E. 600, but likely derived from a Roman game called *ludus latrunculorum* with roots in the first century B.C.E.

Who played it: Famously, the Vikings. But versions of hnefatafl, or tafl, games were found all over northern Europe. The Sámi people of northern Scandinavia played a version called tablut as late as 1732.

The backstory: Romans brought early versions of tafl to the "barbarians" along the frontier of the empire, and the games spread and were adapted with different rules over time. Game pieces could be quite elaborate: In 2019 a dig led by the community archaeology firm DigVentures and Durham University in England turned up a candylike game piece at an early medieval monastery on the isle of Lindisfarne. Made of blue glass and decorated with white swirls and dollops, the piece dated to before C.E. 793, when the Vikings raided Lindisfarne, says DigVentures archaeologist Maiya Pina-Dacier.

“We know the Vikings played another branch of the game,” Pina-Dacier says. Had the Vikings not arrived at Lindisfarne with the intention of razing the place, she says, they might have all played together.

The rules: One player took the role of the king and his army, starting with their pieces at the center of the board, with the exact starting positions differing depending on the version of the game. A second player acted as a surrounding army. The player with the king attempted to get him to the side or corner of the board, while the other player attempted to block the king’s escape.

Can I play today? You sure can. When Pina-Dacier and her colleagues realized they’d found a rare king piece from this ancient game, they immediately ran out to buy and try modern sets, she says. “It’s a really good, fun game, and I would recommend people try playing it,” she says.

A painted and inlaid chess board found in India and dating to the late 17th century. Ashtapada was a precursor to chess.

Heritage Art/Heritage Images via Getty Images

10. Ashtapada

First played: Strictly speaking, ashtapada can be used to describe an eight-by-eight board as well as a game played on such a board. The first reference to the game comes from the Buddhist games list, a sixth- or fifth-century B.C.E. list of games that the Buddha purportedly would not play.

Who played it: Not the Buddha. Other ancient Indians seemed to enjoy it, though.

The backstory: Ashtapada is notable because it provides the substrate for chess; early sources describe chess as being played on an ashtapada board, Crist says. One game played on the ashtapada board involved moving pieces in a spiral toward the center, though other games may have used this board, too. Chess showed up around C.E. 600 and supplanted whatever came before as the most wildly popular use of the ashtapada board.

The rules: Players moved an even number of pieces around the board from outside to inside, probably using dice to count their moves. The details of the gameplay have been lost, however.

Can I play today? Plenty of board game enthusiasts speculate about the rules online and make up their own theoretical paths for gameplay. But if you like games with agreed-upon rules, you’re better off joining the seventh century and learning chess.

Stephanie Pappas is a freelance science journalist based in Denver, Colo.

How Game Designers Secretly Run Your Life

Games have captivated people since ancient times. Now they run our lives

By Kelly Clancy


Boris Zhitkov/Getty Images

In an ancient Hindu hymn, a game of dice was compared to an addictive drug. Three thousand years later, when the world's first casino emerged in Renaissance Venice, it prompted a near collapse of the ruling class because they bankrupted themselves gambling.

We can learn a lot about ourselves as a species by studying what fascinates us. People have been captivated by games for millennia, and today the gaming market is about as big as all other forms of entertainment combined. Play is often dismissed as trivial, but it is evolutionarily ancient: most mammals and some birds, reptiles, fish and even insects play. It's been one of the more challenging behaviors for neuroscientists to study because it is so difficult to suppress. If you surgically remove a rat's entire rindlike cortex—the structure believed to be responsible for higher intelligence—the animal will still play.

Why is this behavior so deeply ingrained? The brain builds models of the world to anticipate events, but games are all about uncertainty. Unpredictable events are particularly fascinating because their unpredictability means something is missing in the brain's world model. Uncertainty is like a signpost indicating that there's more to learn. Children stop playing tic-tac-toe once they realize that, played correctly, it always ends in a draw. Through play, we learn how to handle the unknown.

Many animals play to explore the physics of their environment, and social animals also play to understand one another. By isolating young rats from their peers, researchers have gotten a sense of how play contributes to brain development. As adults, play-deprived rats tend to be more aggressive and less attuned to their peers' social cues. Play is how we learn to get along with others and discover one another's boundaries. For instance, while playing, kittens learn to retract their claws and to bite with restraint. How can I grab my friend's attention without hurting them? How much force should I use when I carry my future offspring by their nape?

Long before neuroscientists studied play's role in social learning, Plato argued that games were vital because they taught children to follow rules. In the future, as citizens, they'd respect laws. Medieval aristocrats were required to learn chess because it was thought to act like a mirror for one’s character: they’d learn about themselves. Philosophers similarly celebrated the ancient game of Go for honing its players’ insight. Competitive games are ultimately about cooperation. They train us to obey rules and achieve arbitrary goals in a fair and socially sanctioned way. Games are a kind of domestication.

Given play’s role as a powerful socialization tool, it’s perhaps no surprise that games have also served as a medium for moral lessons. An Indian saint reportedly invented Snakes and Ladders to demonstrate how karma works. The ancient board game Senet taught Egyptian players how to navigate the afterlife. The Mansion of Happiness—precursor to today’s Game of Life—taught Victorian children how virtues and vices could buffet their life trajectory. Games force us to think about other people, to consider what they want and how they’ll try to get it. This isn’t the same thing as empathy, but it’s groundwork for it.

Games also revealed profound truths about reality—or so it seemed. As casinos became popular in Renaissance Europe, gamblers who sought an edge studied how dice worked. Their efforts to mathematize gambling led to the birth of probability theory, one of empiricism’s most astonishing early successes. Chance—then thought to reflect the whim of God—could, in fact, be studied and systematized. It operated according to laws. That the very unpredictability of die throws could be formalized was revolutionary.

As a language suited to expressing what one doesn’t know, probability theory helped fuel the scientific revolution. Yet the randomness of dice is an orderly kind of randomness, much less messy than that of real events. Though this may seem like an abstract problem, it’s thought to be partly responsible for the replication crises in science because researchers can mistakenly use statistics that are better suited to game pieces to characterize their experimental results.

Today the business world uses probability theory as its lingua franca, and economic ventures are reframed as bets. The implications of this ramify through today’s stock markets and their myriad financial derivatives. The connection goes even deeper than that. Because games are so good at shaping our behaviors, they’ve been adopted in the design of many of our modern social and economic systems. Now game design dictates what ads we’re served as we scroll through our feeds, how we’re paired on dating apps and how we’re matched with jobs.

Game designer Reiner Knizia argues that the scoring system is an essential aspect in creating a game. It drives how the game is played. By leveraging rewards, designers can control players’ behaviors to make games work as intended. In Monopoly, for instance, a player must act like a cutthroat capitalist to win, regardless of their personal convictions. Games designed by corporations permeate our lives, even as the rules of those games remain mostly hidden to us. That’s why it’s crucial that we understand how these games influence us so that we can sieve out our actual values from those of the designers. Otherwise, the games are playing us.

Kelly Clancy is a biophysicist and neuroscientist with a Ph.D. from the University of California, Berkeley. Her first book is *Playing with Reality: How Games Have Shaped Our World* (Riverhead Books, 2024). Her writing has appeared in magazines such as *Harper’s Magazine, Wired* and the *New Yorker*.

Science Crossword: Pseudoscience

By Aimee Lucido

Play science-inspired games, puzzles and quizzes in our new Games section

Aimee Lucido writes crosswords and trivia puzzles that are published everywhere from the *New Yorker* to the *New York Times* to independent publications such as AVCX. She is also author of the middle-grade novels *Emmy in the Key of Code* and *Recipe for Disaster,* as well as the brand-new picture book *Pasta Pasta Lotsa Pasta*. Lucido lives with her husband, daughter and dog in New York.

Professional Poker Players Know the Optimal Strategy but Don’t Always Use It

Poker players can now employ AI to find the optimal playing strategy, but they often don’t use it. Here’s why

By Jack Murtagh

Andrii Sedykh/Getty Images

“All in.” Your opponent slides a stack of chips across the high-stakes poker table. You glance back at your cards, a pair of sixes. The game is Texas Hold’em. Only two of you remain, and no community (face-up) cards have been dealt yet. Things rarely get simpler than this in poker, and you have a binary decision to make: call (match your opponent’s bet) or fold (give up). To a professional player, though, every detail demands consideration. What was the betting pattern before the all-in push? Who acted first? How many chips does each player have, and how many are in the pot? When will the blinds, or forced bets, increase? And of course, how likely are sixes to win the hand? You’ve studied poker strategy, memorized probability tables and run the numbers in your head. It all points to folding as the objectively best decision. But you’ve noticed over a prolonged tournament that your opponent has a tendency to overbet with mediocre hands. Do you stick with your training and fold or adjust your strategy on the fly to exploit the weakness you’ve observed?

This question of whether to use what’s known as “game theory optimal versus exploitative play” captures a central conversation in high-level poker. Its mathematical underpinnings trace back 80 years, but rapid advances in AI have brought mid-20th-century math to the forefront of modern gaming. New tools teach poker players optimal strategy for the game, so why would they ever decline to use it?

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Optimal Play?

Objectively optimal play might seem counterintuitive in a game based on randomly dealt cards and messy human psychology. Take bluffing—when a player pretends they hold an unbeatable hand to scare an opponent into folding. Lying about one’s cards feels inherently psychological and resists a rigorous mold of objectivity. But we should never underestimate mathematicians’ knack for turning human behavior into tidy equations.

In fact, the foundational 1944 book on mathematical game theory by mathematician John von Neumann and economist Oskar Morgenstern, *Theory of Games and Economic Behavior*, highlighted poker as a central example. The authors analyzed a simplified variant that distilled the game down to its most fundamental dynamics: two players would each receive a number between 0 and 1, with higher numbers representing stronger hands, and then engage in a single round of betting. Von Neumann and Morgenstern proved not only that an optimal strategy exists but also that bluffing is an essential part of that strategy.

Of course, Texas Hold'em packs a great deal more complexity than this toy example. Who's to say that an optimal strategy even exists in full-fledged multiplayer poker? The late mathematician John Nash, that's who. In the 1950s Nash, who went on to win a Nobel prize in economics in 1994 and was later depicted in the 2001 biopic *A Beautiful Mind*, propelled the then nascent field of game theory. His most famous discovery, now called a Nash equilibrium, occurs when no player of a game would benefit by deviating from their chosen strategy (assuming others don't deviate from theirs).

Game theorists consider this condition optimal because if you and I play a game where we each begin with any old strategy, and then I adapt mine to take advantage of what I see you doing, and then you counter-adapt to my change, and so on, we will eventually reach a steady state in which neither of us can keep improving. With a Nash equilibrium, players can even announce their strategies in advance, and still everybody's best course will be to stick with the equilibrium. In a one-page paper in 1950, John Nash proved that *every* finite competitive game—from mahjong to Magic: The Gathering—has at least one Nash equilibrium.

Despite its name, game theory applies to a broad spectrum of topics beyond traditional games, including economic systems, nuclear deterrence and evolutionary biology. To researchers in this field, games refer to any interactions among rational decision-makers whose actions and payoffs can be rigorously defined and analyzed. So Nash's theorem has wide-reaching implications. In poker, it justifies the search for optimal strategies in a game once thought to rely on gut instinct and reading tells.

An AI Poker Revolution

Just because we know that Texas Hold'em has a Nash equilibrium, that doesn't mean we know what it looks like. As games ratchet up in complexity, their optimal strategies tend to become harder to figure out. Anyone could learn how to play perfect tic-tac-toe in one sitting by memorizing a few move sequences. For a more elaborate game such as checkers, which always ends in a draw with perfect play, humans could never memorize enough variations to implement the optimal strategy. Scientists have created unbeatable algorithms that play optimally, however, because computers can store massive databases of positions and extensively search the game tree in a way that humans cannot. Meanwhile chess computers have dominated the best human players since around 1997 (when world champion Garry Kasparov lost a historic match to IBM's Deep Blue), yet chess computers still don't exhibit optimal play—the next generation of chess engines will crush today's.

Unlike chess, poker involves *imperfect information*. Players know their own cards but not their competitors', which makes the game more daunting to model computationally. This explains why the algorithmic revolution in poker didn't come until the recent AI boom. In 2015 computer scientists announced an algorithm that displayed essentially perfect play for a restricted version of the game with only two players and constrained bet sizes. Only four years later, we got the first superhuman AI for multiplayer Texas Hold'em. A flurry of commercially available software tools called "solvers" followed, and in the span of a few years every rounder (person who plays poker for a living) with a few hundred dollars to spare had a card shark at their fingertips who could tell them how to play in almost every situation.

“The game went from being this fuzzy art to a hard science,” says Liv Boeree, a former professional poker player. To stay ahead in today’s environment, advanced players study the game by using computer programs such as PioSOLVER, which approximates optimal strategies. For simple and common situations, pros will memorize the machine’s recommendations, whereas they glean more high-level lessons from its behavior in rare and more complicated situations. For any elite poker player, studying with these solvers is essential. “If you want to play high stakes against the best, absolutely ... you’d get eaten alive [if you didn’t use solvers],” says Boeree, a World Series of Poker champion. “There were some players who just rejected the entire notion, and they didn’t work with solvers..., and for the most part, they got left behind.”

AI has both confirmed some common wisdom about Texas Hold’em strategy and overturned some maxims that players had gotten wrong. For example, computers find success in “donk betting”—initiating the first bet on a round of betting after merely calling another player’s bet on the previous round—despite the folk belief that donk betting is an amateur move. AIs also play a wider variety of hands in situations where expert humans tend to fold. Like chess engines, multiplayer poker solvers don’t literally play optimally, but they dominate humans thoroughly enough that we have a lot to learn from them.

How to Win

In defining the Nash equilibrium, I smuggled in a critical detail: equilibrium occurs when no player would benefit by deviating from their chosen strategy (assuming others don’t deviate from theirs).

When other players *do* deviate despite this, however, it's often wise to deviate in response.

Take rock-paper-scissors as an illustrative example. What is its Nash equilibrium? Think for a moment: What strategy from both players would leave no incentive to deviate? Answer: players should toss rock, paper and scissors perfectly at random; each has a one third chance of appearing, regardless of all previous rounds. You can announce this strategy to your opponent in advance, and they will be helpless to take advantage of your candor.

If you and your opponent both play this equilibrium strategy, you can expect to win half of the decisive rounds (ignoring ties). Now suppose your opponent deviates. In the extreme case, imagine they always play paper. If you stick with the equilibrium strategy, then still you'll win half of the decisive rounds because you play the winning scissors and the losing rock with equal frequency. But you can instead exploit your opponent's deviation by always playing scissors and cutting their paper on every round. Less dramatic deviations still give you opportunities to exploit. For example, empirical research on rock-paper-scissors shows that when people win one round, they are slightly more likely to repeat the throw that they just won with. Knowing this can give you an edge. If you just lost to rock, for example, then play paper next because your opponent is likely to throw rock again. The Nash equilibrium is the only strategy that is not susceptible to exploitation.

The same dynamics play out in poker at a much more complicated scale. As players learn more optimal techniques from their AI collaborators, they also learn how to sniff out when their opponents fall short of optimal play and how best to punish them.

You might think there's a catch here. If your opponent deviates, isn't the optimal decision to exploit them ruthlessly rather than to blindly stick to the Nash equilibrium and leave potential money on the table? If you discover that an opponent deviates from the Nash equilibrium in predictable ways, then deviating yourself to exploit their weakness may net you more money. As soon as you exploit them, however, *you're* now veering from the equilibrium and opening *yourself* up to exploitation. If your opponent always throws paper and you start only throwing scissors, eventually they'll catch on and start rocking your scissors.

As former poker pro Igor Kurganov puts it, "any time you pick up on a mistake by your opponent, you improve your model of how they think about the game, adjust how you play against them to account for that mistake and, by that, become exploitable yourself."

Most players agree that to stay competitive at the top levels of poker they must use a blend of game theory's optimal and exploitative play. Optimal is more defensive, whereas exploitative is more offensive. Some teachers recommend that you should begin a tournament by emulating optimal play—and only after you've had time to observe your opponent's weaknesses should you sprinkle in your exploits. The flexibility to switch between strategies separates the fish from the sharks. "This whole process works better the more certain you are that you're smarter than [your opponent] about the game," Kurganov says, adding that "you do less exploitative adjustments when you feel like they're as good or better than you."

For some, the emergence of superhuman poker engines has sapped the game of its intrigue, while others contend that computers have added a new layer to the game. Boeree, who retired from pro poker in 2019 and now works as a science communicator, philanthropist and podcast host, falls more into the former camp. "It felt like it took a little bit of the magic out of the game, like, 'Oh, okay, the mystery has been solved,'" she says. But Boeree acknowledges that the new age of poker has no shortage of enthusiasts. "Since COVID it's been booming," she adds. "The World Series of Poker got more players than ever before last year. Records are getting smashed. So clearly it has not killed the game." Instead we might say that the changing landscape of poker is still finding its equilibrium.

Jack Murtagh is a freelance math writer and puzzle creator. He writes a column on mathematical curiosities for *Scientific American* and creates daily puzzles for the Morning Brew newsletter. He holds a Ph.D. in theoretical computer science from Harvard University. Follow Jack on X @JackPMurtagh

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| Section menu | Main menu |

How Role-Playing a Sorceress Released My Inner Badass

Thousands of people around the world do live-action role-playing to learn and change themselves. I am one of them

By Ericka Skirpan

Ericka Skirpan dressed as Halvdana Naglisdottir, a shieldmaiden and cursed warrior, during the Dammerung LARP game in 2022.
Miles Lizak

The point of no return was when I started the fire in the university parking lot.

I ripped a pile of reports out of a confused bureaucrat's hands and ran as fast as I could. Once I was outside, I grabbed a lighter out of my pocket and ran it along the parchments' edges, making sure flames caught on every inch of the documents. There was angry shouting behind me, the confused school administrator and several professors now spilling down the front stairs to see the mess, but it was too late. Parchment burns fast, it turns out.

Was this a scene out of my undergraduate studies? No—it was the climax of a live-action role-playing (LARP) game I did in Belgium called Myrddin Emrys College. Though it was “just a game,” it felt every bit as real as my regular life. I really did grab parchments from someone’s unsuspecting hands and decide to burn them in front of many shocked participants. It was one of the most rebellious acts of my life, in-game or out. My incendiary adventure is just one example of how LARPing can unlock facets of our personality we never knew existed and how our characters can make strides we never thought we’d take in life. As we act out the moments of a LARP in our own body, sometimes we surprise ourselves with proof of how strong we can be through the characters we play.

Skirpan in character as Arminda McClinton, a young wife and pleasure robot in a 2024 LARP inspired by the *Westworld* television series.
Kai Simon Fredriksen

LARPing can take lots of forms—it's essentially any storytelling interaction where the participants act out the roles of their characters (role-playing) but are also the only audience present to witness the story the whole group creates together. There are as many kinds of LARP as there are genres of fiction, but put a group of people into a room, give them characters who are not themselves and a setting to play within, as well as some kind of goal, and you've got a LARP. If you've ever played one of those How to Host a Murder dinner party games that come in a box, then you've played one without ever knowing it. In her book *Leaving Mundania: Inside the Transformative World of Live Action Role-Playing Games*, journalist Lizzie Stark describes how "one or more directors ... organize everyone, select the form of the performance, and decide whether the setting resembles, for example, *Lord of the Rings*, *Hamlet*, or *Buffy the Vampire Slayer*.... The outcome of every LARP remains in question, as the characters improvise all their lines.... Essentially, LARP is make-believe on steroids for adults." The first LARPs may have been inspired by the rise in popularity of Dungeons and Dragons in the 1970s. One such LARP was the first large Dagorhir Battle Game, which started near Washington, D.C., in 1977. Participants donned costumes, made fake weapons and took to the field to play out massive mock combats. Shortly after, in England, the Treasure Trap game took place in Peckforton Castle and seems to have developed independently. In the 1990s LARPing started to catch on in the Czech Republic and in some Scandinavian countries, where participants quickly developed their own scenes and styles. More recently, out of the pandemic, a new kind of LARP has grown in China: Jubensha. It's brought tens of thousands of people into LARPing and is one of the most popular growing hobbies in the country. Today there are likely almost half a million live-action role-players across the world.

So why LARP? Aside from offering pure fun and the ability to live out fantasies that would be impossible in reality, LARP is a powerful tool to teach us things about ourselves that would be difficult to learn in everyday life. Think of it as a pressure cooker for your skills, personality and reactions to intense situations. It's rare for most people to be shoved into crisis situations on a regular basis; few experience a field of combat, an escape from a hostile kingdom or the threat of a destroyed academic career regularly. When you are a LARPer, however, you routinely face situations such as these that test your reactions. There's a reason why role-play is often used in training scenarios, psychological treatment and education—it works. I've seen people who want to work on their public speaking create characters who are charismatic leaders just so they can practice talking in front of audiences or people who have issues confronting others play antagonistic characters simply to get better at standing up for themselves. And beyond expanding your personality, the fact that you physically have to perform the skills your character possesses means that players will often learn new talents so their characters can use them during a story. I have personally learned rudimentary knitting, country dancing, foam crafting, sketching and sewing just because I wanted those skills for LARP.

Role-playing games can have a powerful effect on mental health. "I am struck by the similarities between LARP and psychodrama, a type of group therapy that focuses on the exploration of emotion, relationships and the human psyche through improvised dramatic performances," says Nishanthi J. Anthonipillai, a clinical psychology graduate student at the City College of New York, who has studied LARPing. "The benefits of role-playing games such as LARP have been documented in research: studies suggest that those who engage in role-playing games exhibit greater motivation and self-efficacy [and] creativity, as well as empathy. Role-playing also promotes increased social connections and a sense of belonging, known protective factors reducing the likelihood of severe mental health disorders."

Sometimes LARP is a chance to experience something new, but other times it's a way to tackle situations a person may struggle with in their normal life. "I think LARP can serve as a paradoxical intervention: when issues within the game narrative are similar to those the player is working on in their own life, they can lean into their symptoms and issues while playing and find a different perspective," Anthonipillai says. "Doing so in a semi-structured narrative, within a genuinely supportive community, with built-in safety protocols—both physical and psychological—while playing a character may allow some distance for a person to process their emotions in a less confrontational manner than direct therapy."

Beyond personal development, LARP has made me a more empathetic person. It's given me a chance to walk in someone else's shoes, whether it's a character living in a different economic situation, branching into a new stage of life, struggling with addiction or learning about a new religion. When LARP is done respectfully, it can open players' eyes to so many different, difficult facets of the world.

And I have watched LARP help more than one person realize a truth within themselves. Many transgender people found LARP to be their first opportunity to safely explore their real gender. It was a chance to try on a body and gender expression that felt truer for them without making an immediate, life-changing decision. Others have used LARP to try gender fluidity and different sexualities to see if they felt more comfortable before embracing their identities publicly. LARP gives you a sandbox for experimentation where co-players are supportive, respectful and willing to celebrate whatever story you want to tell.

Several years ago I played a prefectlike character at the New World Magischola, a U.S.-based LARP with a setting akin to a magical

British boarding school. I was Quinevere Radcliff-Forsythe, scion of a powerful family. I knew we had several young LARPers at the school, and they all looked up to me, as the senior student in charge of their house, to be not just a mentor but also a guide through this new world. My character spent her time organizing her chaotic house, getting young students ready for classes and protectively stepping up to fight when outside threats arrived at the school. Quinevere, or Quin, embodied many aspects of my own organizing, caring personality but with far more self-confidence and none of my fears of authority figures. She often spoke forcefully with professors and never backed down when she had justified worries about her students. In short, Quin was a badass. After playing her that summer, I came back to a work meeting where we had gotten into the weeds—debating small details and losing sight of the big picture we'd all sat down to discuss. Normally, I'd have kept to my corner, silently taken notes and let everyone else debate—I didn't like confrontations, much less with senior members of my team. But after having Quin in my head for days, I took a breath, adopted her courage and dared to speak. I made a proposal, laying out the details and the justification in an organized way. Everyone looked stunned. Their meek administrator had just solved half an hour of debating in five minutes. I started to realize I could use my voice to change things for the better if I just believed in it when I spoke up.

Skirpan plays Dame Bathshira Themis, Cavalier Primary of the Emperor's Fleet, in a 2024 LARP set in the universe of the *Locked Tomb* book series.
M.G. Norris

The summer after that, I traveled to Belgium for the Myrrdin Emrys College LARP. I decided to challenge myself to play a character so loud and confident that she could not be ignored: Simone. "High-femme, angry, punk" might be the best way to describe her, although she'd hate anyone putting labels on her. Simone was angry at the world, with every right to be. In the narrative of the game, she stood up in front of all her professors and got herself kicked out of school because she couldn't stand by while students were abused there. She spoke up in a way I'd never had the courage to. And on her way out, she ripped some school evaluation paperwork out of a bureaucrat's hands and set it on fire in the parking lot. Her actions inspired students to stand up against the cruelty at the school. One of the last scenes I remember is the entire school standing in that parking lot, telling the teachers they wouldn't come back inside unless things changed. Simone was so proud.

I played many badasses after her: Audi, a biker gal in a LARP called Dystopia Rising Virginia, who taught me that the best way to leave a bad decision in the dust is on the back of a bike while giving it the middle finger; Bathshira Themis, from a game called The Jaw of Victory, who never backed down from a duel, including one against the best fighter in the universe; Halvdana Naglisdottir from the Dammerung LARP, who stood fighting with her people to the literal end of their world. All of them helped turn me into who I am today.

That person isn't who I was expected to be. I grew up in a place where women were meant to be seen but not heard. As a bigger girl who enjoyed being loudly dramatic from a young age, I was told that I was "too much." I needed to put on a pretty dress, keep my head down and not speak when anyone more important than me was talking. If I performed traditional femininity quietly in a corner, maybe I could be as pretty as the tiny, shy girls who already seemed to be what the world wanted. But I was wrong. Girls like me also deserve to be heard, and every one of our bodies is a beautiful statement of rebellion against a world that wants to keep us contained. It took walking in the shoes of all these badass women to teach me I could be one, too. After all, everything I was doing was still happening in my body—they were still me.

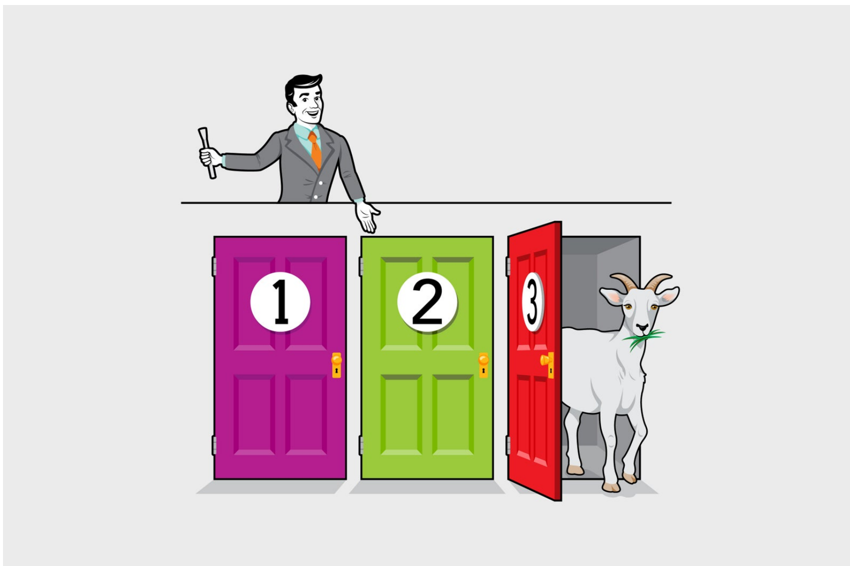
Yes, these were only games. But these LARPs gave me one of the most important lessons of my life, one that I'm still processing years later: sometimes anger inspires change. If people won't listen to you talk, then scream. Taking those feelings back home has taught me to stand up in situations where previously I'd have been silent. Simone made me a badass. And although I might never change the world, she's helped me change parts of mine for the better.

Ericka Skirpan is an experience designer, writer, actor and instructor. She's been in the LARP and interactive theater trenches for more than 20 years. Skirpan has an Honors B.F.A. in theater from the University of Toronto, where she was artistic director for an immersive theater company, In the Moment Productions. She firmly believes in bringing emotionally cathartic transformations to actors and audiences alike through live storytelling and then providing safety mechanisms to ensure a healthy return to reality after such experiences. She writes the blog The Space Between Stories.

See Why Everyone Gets the Monty Hall Puzzle Wrong

How to finally wrap your mind around the uniquely counterintuitive Monty Hall dilemma

By Allison Parshall

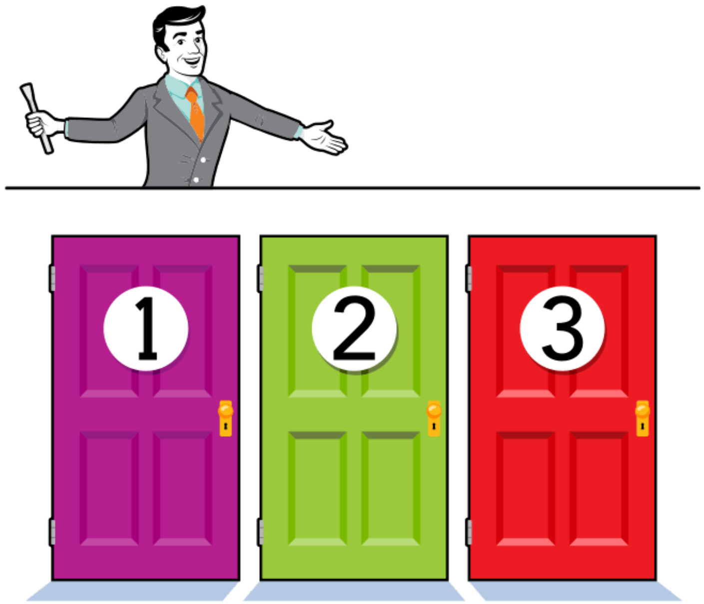


Brown Bird Design

In high school statistics class, my teacher presented a probability problem that haunts me to this day. It was a puzzle inspired by the TV game show *Let's Make a Deal* and named after its longtime host, the late Monty Hall. The setup is simple:

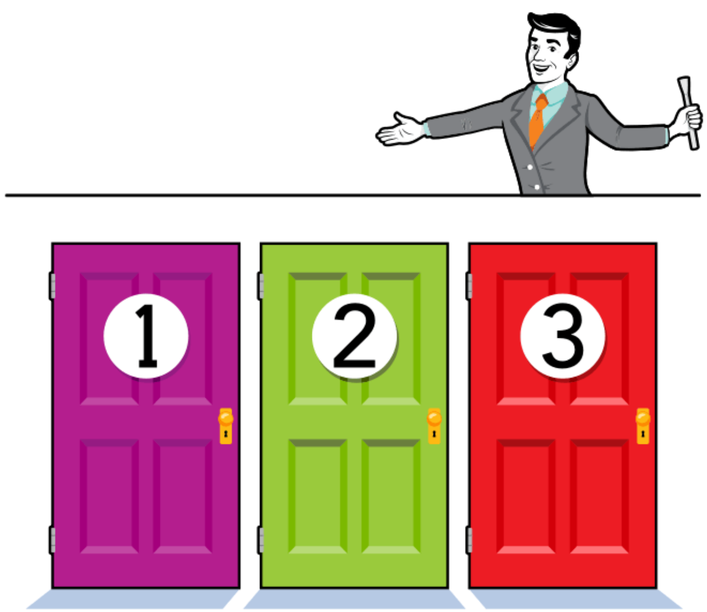
Open in your browser for the full experience.

There are three doors in front of you.

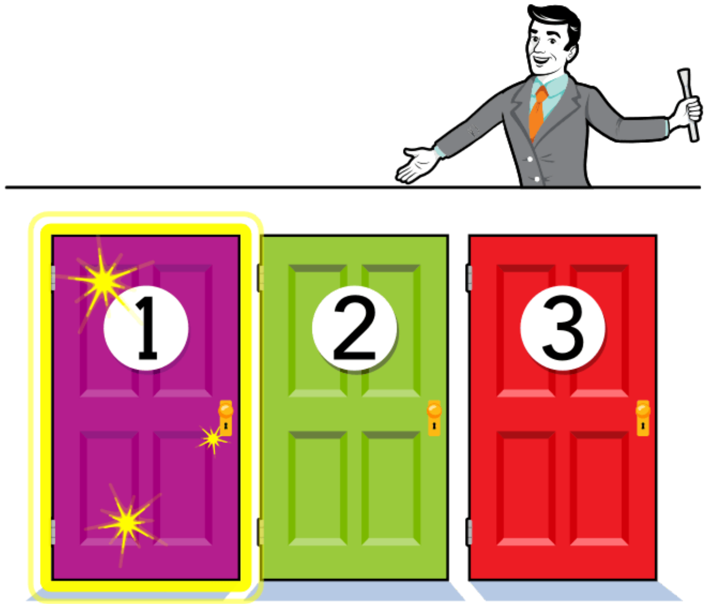


Behind one of the doors is a new car. Behind the other two doors are goats.

The host invites you to pick a door, any door.



Let's say you pick door one.



The host throws open one of the doors you did not select, revealing a goat.

Then the host gives you the option of switching your selection to door two.

Should you stay with door one, or should you switch to door two?

Most people think it doesn't matter whether they stick with their original choice or switch to the other unopened door because the odds are 50–50—that it's nothing more than a coin toss. But you should always switch doors. You win two thirds of the time if you switch and one third of the time if you stay. In other words, switching doors doubles your chance of winning.

This counterintuitive problem, first described in relation to the TV show in 1975, is taught in introductory math and statistics classes across the world. But it was widely popularized in 1990 in *Parade* magazine. After writer Marilyn vos Savant wrote about the puzzle in her Ask Marilyn column, she received an estimated 10,000 furious letters declaring the answer she gave was wrong, including 1,000 or so that were signed by people with a Ph.D. in their title. The whole affair was so spectacular that it made the front page of the *New York Times*.

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How are the odds *not* 50–50? Since that fateful column, mathematicians, psychologists and philosophers have been trying to understand what makes this answer uniquely hard to grasp. They've found that some of the most common cognitive biases may be to blame, along with a core misunderstanding of how probability works. "Pretty much everybody, even people who are well trained in mathematics, takes the wrong intuitive approach to the Monty Hall dilemma," says Walter Herbranson, a comparative psychologist at Whitman College. "And that's [an] indicator that they've got a perfectly normally functioning human brain."

By revisiting the problem again and again over the past decade, I've fallen victim to every one of these biases and errors in reasoning. In the end, understanding why the odds are not 50–50 required a mind-bending shift in perspective.

One of the most straightforward ways to see why you should always switch doors is to draw out the possible outcomes. Say you pick door 1. There are three possible realities: the car is behind door 1, 2 or 3.

If the car is behind door 2, Monty Hall will open door 3 and offer for you to switch to door 2. Switching yields the correct door.

If the car is behind door 3, Monty will open door 2 and offer for you to switch to door 3. Switching yields the correct door.

If the car is indeed behind door 1, Monty will open either door 2 or door 3 and offer for you to switch to the one he didn't open. Switching yields the wrong door.

Put simply, if you selected the correct door initially (which happens $^1/_3$ of the time), you shouldn't switch when you're offered the chance. If you chose the wrong door initially (which happens $^2/_3$ of the time), you should switch.

This basic decision tree proves two things: the probability isn't 50–50, and you're better off if you always switch. Yet psychological research has shown not only that people usually believe the choice is 50–50 but also that they decide to stick with their original door around 85 to 90 percent of the time. What's more, they typically continue to choose to stay put even as evidence surmounts that they shouldn't, says social psychologist John Petrocelli of Wake Forest University. Petrocelli had participants play the Monty Hall game over and over, allowing them to repeatedly observe that the prize is more often behind the "switch" door—but it took many trials for these participants to learn that they should be switching.

He then ran the test with a series of simple coin flips. Imagine you were flipping a coin repeatedly and betting on its outcome each time. How long would it take you to notice that the coin was biased and landed heads-up $^2/_3$ of the time instead of $^1/_2$?

"Most people think they would start to recognize that after about 40 flips. But they don't," Petrocelli says. His research has attributed this lack of learning to what he calls "kick-in-the-pants" thinking, or counterfactual thinking. After we make decisions, we often simulate the outcomes of the choices we didn't make by telling ourselves something like "If only I had chosen differently, I might have won." Such counterfactuals are not all that salient if you stick with your original door and lose, but they're extremely salient when you switch and lose—probably because it feels worse to have had and lost than to never have had at all. Those competing alternatives to reality can cloud your memory, Petrocelli says, and "you're not going to see the pattern that's emerging before you."

Even when they learn how the problem works, many people don't believe they should switch. That's because the explanation above, while elegant and accurate, isn't convincing—and not just for people who encountered it as a statistics student, like me. Even prolific mathematician Paul Erdős was tripped up, according to his childhood friend and fellow mathematician Andrew Vázsonyi. In a 1999 article in the Decision Sciences Institute's publication *Decision Line*, Vázsonyi described Erdős getting flustered when his friend presented him with the problem for the first time, demanding a reason for why he should switch and then responding with "what's the matter with you?" when Vázsonyi did not provide one. "I said that I was sorry, but I didn't have a commonsense explanation," Vázsonyi recounted.

There is one commonsense explanation that statistics professors will often provide if you tell them you're confused about the Monty Hall problem. Imagine there are 100 doors instead of three. Only one has a car behind it, and the other 99 have goats. You select a door, say, number 1, and then Monty walks down the line, flinging open door after door. He skips right over number 72, leaving it closed, before opening the rest. Do you want to stick with number 1 or switch to 72? Here you *really* should switch. Your chance of winning is 99 percent if you do.

"People will usually believe you at that point," says Jason Rosenhouse, a mathematician at James Madison University, who wrote a book on the Monty Hall problem. "But it doesn't quite get at the mathematical issue" at the heart of the problem.

He's right: after hearing the 100 doors explanation, I believed the correct choice was to switch—but only reluctantly. The idea that the chances are 50–50 has a sort of gravity that I felt I couldn't escape. I was constantly getting snagged on one baffling question: Why does my first choice among the three doors seem to “affect” my second choice between switching and sticking? My gut intuition was that the probability should reset between the two choices, with the second having nothing to do with the first.

“That [assumption] is so natural, and that seems so obvious, but it’s actually wrong,” Rosenhouse says. That’s because Monty Hall *knows* what door the car is behind, and he chooses his “tease reveal” door accordingly. If he didn’t do this and instead just opened one of the two unselected doors at random—giving him a genuine chance of accidentally revealing the car—then your “stick or switch” choice would indeed be 50–50. But Monty’s knowledge and constraints change everything. To fully understand why this is true, you need to step into Monty's shoes.

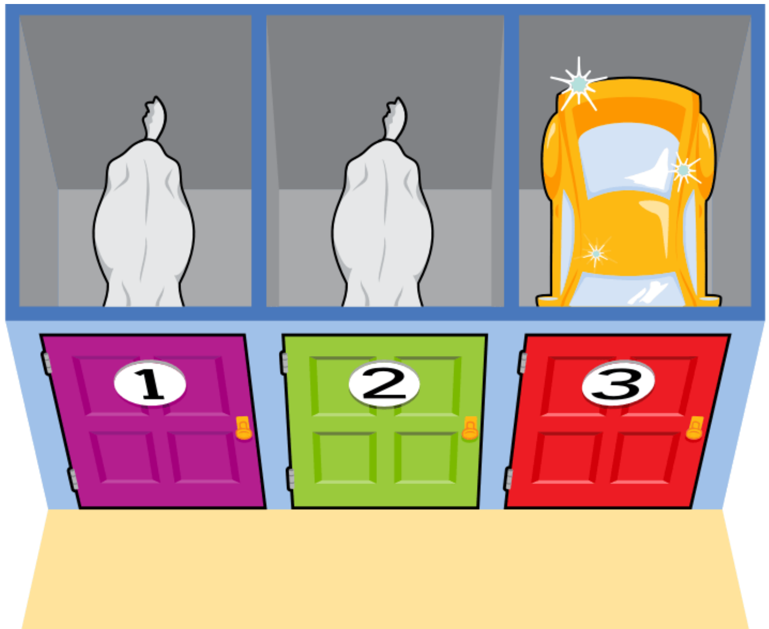
Last year, as I was explaining my love-hate relationship with this problem to my friend and her father—who teaches military strategy and game theory—we decided to set up the game so we could play it in person. I placed a peanut M&M under one of three cups and assumed the role of Monty Hall. After 10 years of intermittent frustration, everything finally clicked.

Try stepping into Monty’s shoes for yourself:

Open in your browser for the full experience.

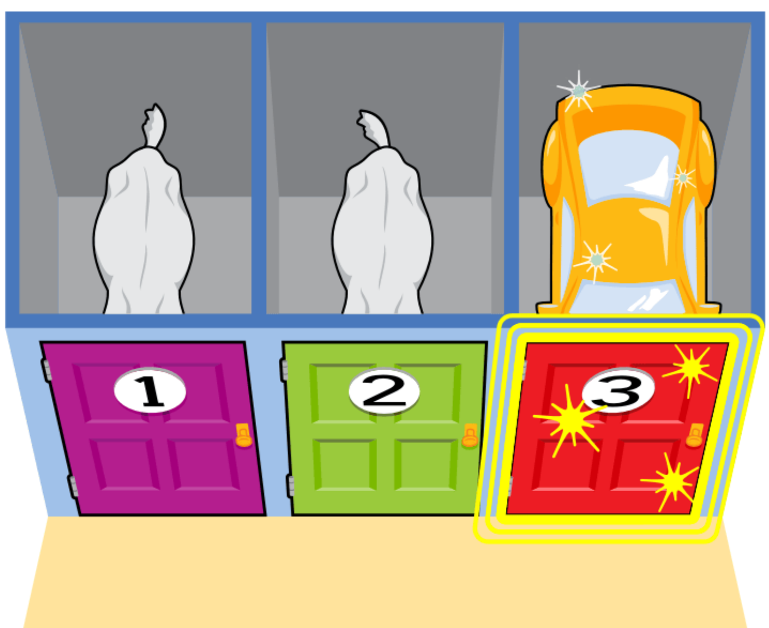
Let’s play again. This time, you are the host.

You know what’s behind each door, but the player doesn’t.



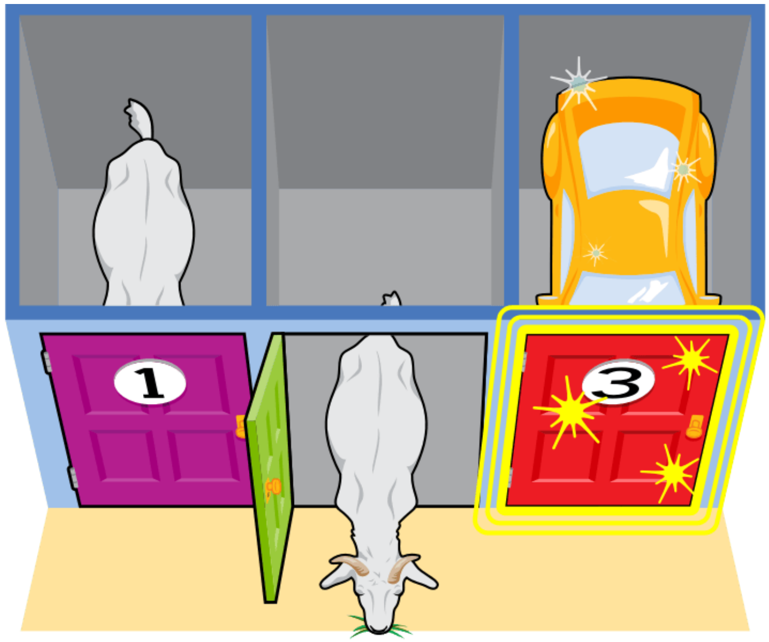
You ask the player to pick a door.

Occasionally—one out of three times, on average—the player will randomly pick a door hiding a car as their first choice. In this round, that's door three.

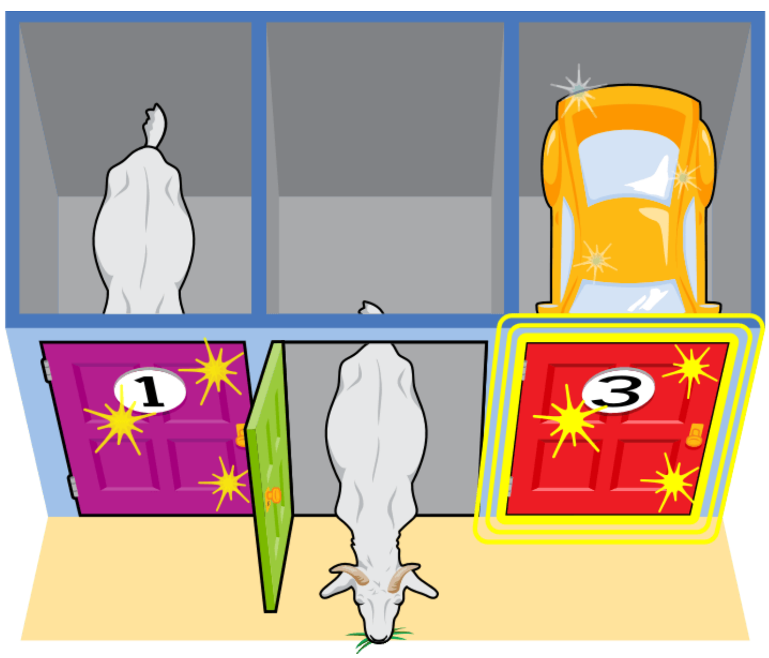


As host, you can then open either of the other two doors because they both hide goats. (The host can only choose to reveal goats.)

You throw open door two, revealing a goat.



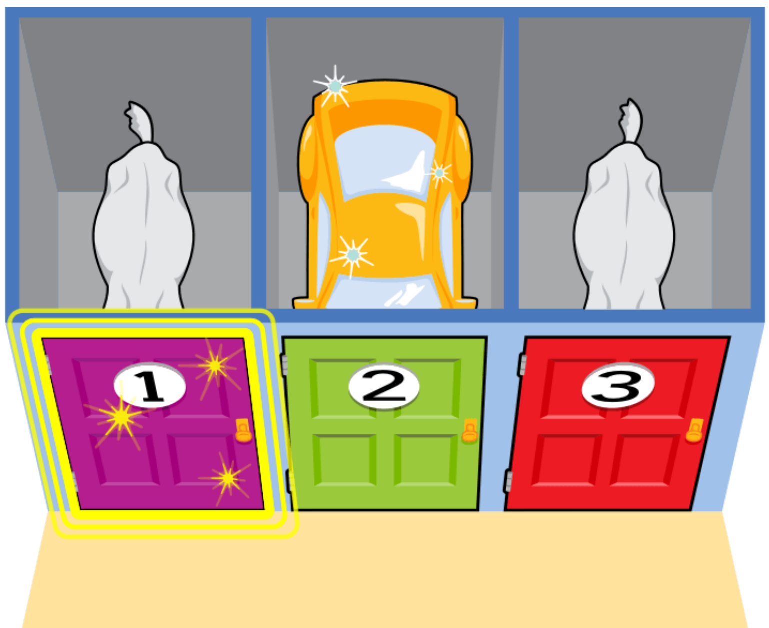
Then you give the player the option of switching their selection to door one.



In this scenario, the player will win if they remain with their original choice, door three.

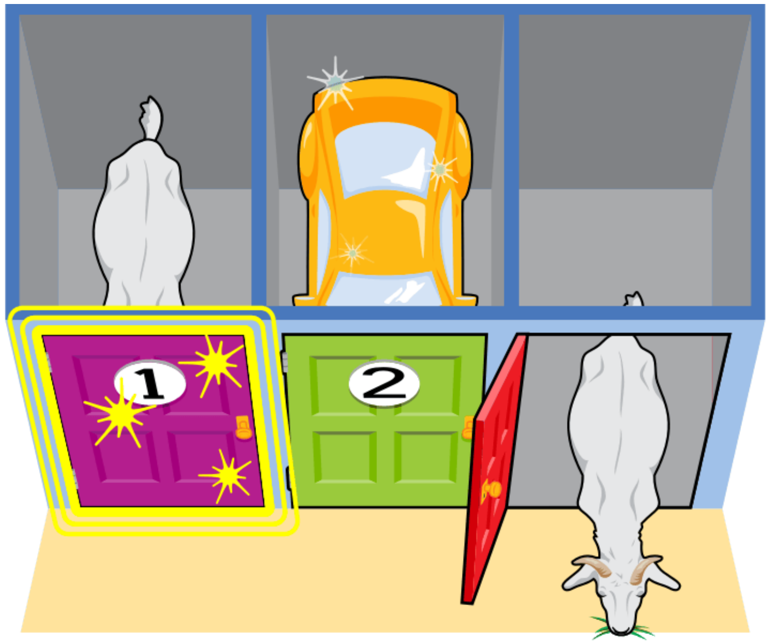
But more often—two out of three times—the player will pick a door hiding a goat as their first choice.

Let’s reset the game. In this round, the player starts by choosing door one.

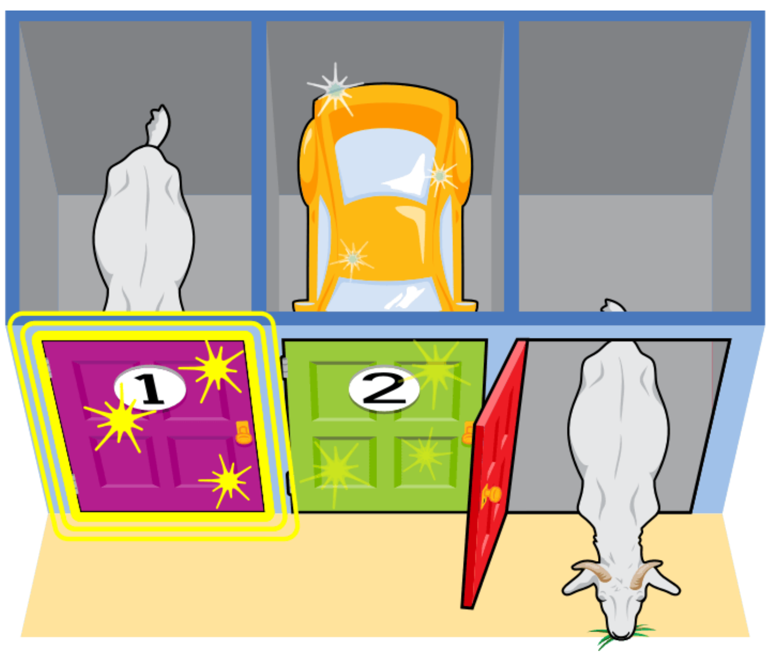


As host, you must now open one of the other two doors. You can’t choose the car.

Thus, your only option as host is door three. You open door three and reveal a goat.

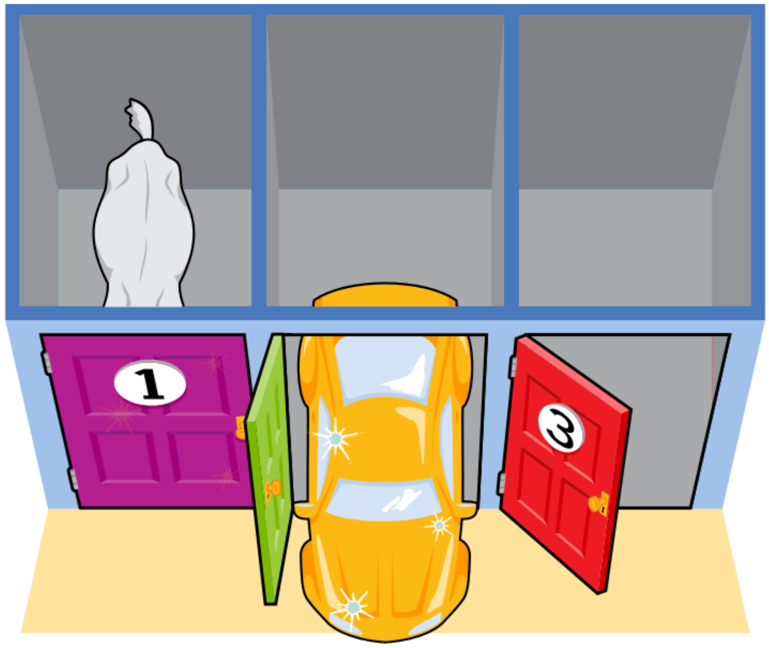


Now you give the player the option to remain with door one or switch to door two.



If they stay, they lose. If they switch, they win.

So, yes, even though one in three times, the player might choose the car first and benefit from not changing their mind ...



...statistically speaking, it’s in the player’s best interest to always switch their door choice.

Shifting from playing the contestant to playing the host helps many people understand why they should switch doors. In a 2003 study researchers found that people who played the game from Monty’s perspective were more likely to say that game show participants should switch doors than those that played as the participant. (They also had to be prompted to think through what would happen if the participant had picked each of the three doors.) Petrocelli and his colleagues had the same experience with their own research. Programming the problem as part of their study made it “crystal clear” why switching has such an advantage, he says.

Here’s what you can see when you play as the host: Monty Hall isn’t some mysterious figure making shadowy decisions to manipulate, deceive or otherwise trick the contestant. In fact, Monty makes practically no meaningful decisions at all—to the point that playing as the host is practically boring. In two out of three games, the participant picks a goat the first time, meaning the host doesn’t have a choice in what door they open to tease them. In one out of three games, the participant picks the car the first time, so the host picks between the two goat doors at random for the tease reveal.

While the tease reveal feels like new and important information to the contestant, Monty sees that it is simply a distraction. The host is telling the contestant something they already know: “Hey, one of these two doors definitely doesn’t have the car.” As Monty, you see that by opening a goat door, you aren’t giving them information—you’re giving them an opportunity.

That opportunity is two doors’ probability for the price of one. In a game like this, you can’t pick two doors. But that is effectively what the contestant is doing when they decide to switch.

Imagine it like this: You’re the contestant, and you pick door 3. Then the host asks if you want to switch to *both* doors 1 and 2. Heck, yeah—of course you’re going to take two doors if that’s an option. Then the host opens door 1 and reveals a goat. Do you still want to stick with both doors 1 and 2 or switch to just door 3? You stick with doors 1 and 2 because nothing has changed. You already knew one of your two doors contained a goat. That’s old news. But you’ve still got a $^2/_3$ chance that your selection contains the winning door. The fact that one of your two doors was opened and all of your hopes now rest in the other is completely irrelevant.

When the tease-reveal door is opened, “people think that everything has changed, but the situation is the same as in the beginning,” says Elisabet Tubau, a psychologist at the University of Barcelona. That “is the illusion in the problem.”

“It’s almost like a magic trick, right?” says Christopher Pynes, a philosopher who studies logic and the philosophy of science at Western Illinois University. You may see two possible doors in front of you, but that’s just some clever sleight of hand. What you’re really betting on is not one door versus the other but the probability that you were right the first time versus the probability that you weren't. You're being given the chance to bet against your original choice, with the probability of two doors collapsed into one thanks to the throwaway tease reveal.

Here the Monty Hall dilemma is preying on something called the equiprobability bias: we assume that all presented outcomes are equally likely. In an exaggerated form, it's like rolling a die and asking someone what the odds are that you rolled a one or "not a one," explains Patrick Onghena, who studies scientific methodology and statistics at the university KU Leuven in Belgium. If you didn't understand how dice worked, you might incorrectly think that the odds are 50–50 because there are two options being presented to you. But once you understand that a die has six sides, you see that these two choices aren't equivalent.

Though I've now achieved Monty Hall enlightenment, thinking about the problem in the "correct" way still feels like bending my mind out of shape. I keep stubbornly returning to that one question: Why *can't* the probabilities just reset once there are only two doors left?

When I ask Pynes about this, his answer reveals a misunderstanding I've had about probability for my whole life. "Imagine you turned on the television [during the middle of this game] and there are only two doors. You don't know anything else, and you pick a door," he says. "That's a true 50–50 choice. You didn't have the option of the third one because you're coming into the situation after."

But unlike you, the game show contestant still has a $^2/_3$ probability of getting it right if they switch. That's because the probabilities we have been talking about this entire time are not tied to the doors and the car and the goats but to the observer. Probability, in other words, is in the eye of the beholder.

This more subjective way of thinking about probability is at the heart of so-called Bayesian reasoning. Reverend Thomas Bayes, a theologian who lived during the 1700s, thought about probability in terms of degrees of certainty based on evidence. His theorem can take prior knowledge of the probability that an event will occur and update it when new information comes to light. This approach to statistics is centered around a person making an estimate—and what that person knows about the likelihoods of different outcomes.

Today Bayes's theorem is used to test vaccines, map the cosmos and train machine-learning algorithms. But it is not the default way that scientists and mathematicians approach questions of probability. That honor goes to something called frequentist reasoning, which treats probability more like a physical property that can be revealed through repeated tests or simulations. *Is this coin biased?* The frequentist flips it many times to find out. *Should I switch doors or stay put?* The frequentist runs many computer simulations and sees that the car is behind the "switch" door $^2/_3$ of the time. Here the probability of these events is treated as a physical thing, a ground truth to be discovered.

Unfortunately, this approach does little to reveal the nuances of the Monty Hall dilemma. A frequentist, looking at the game show stage, might attach probability to the doors and the car and not to the participant doing the guessing. "My hypothesis," Vázsonyi said in *The Man Who Loved Only Numbers*, by Paul Hoffman, "is that Erdős had this idea of probability as being attached to physical things and that's why he couldn't understand why it made sense to switch doors."

When I ask vos Savant, the writer of the 1990 *Parade* column, what she thinks the enduring lesson of the Monty Hall dilemma is, her answer doesn't involve competing theories in statistics. The puzzle, she says, is about humility. "It promotes a dandy demonstration of a human frailty," vos Savant explains, "the disbelief that we could be wrong and the tenacity, sometimes aggrieved, with which we hold our earlier judgments, especially when we feel certain."

Allison Parshall is an associate news editor at *Scientific American* who often covers biology, health, technology and physics. She edits the magazine's Contributors column and has previously edited the Advances section. As a multimedia journalist, Parshall contributes to *Scientific American*'s podcast *Science Quickly*. Her work includes a three-part miniseries on music-making artificial intelligence. Her work has also appeared in *Quanta Magazine* and Inverse. Parshall graduated from New York University's Arthur L. Carter Journalism Institute with a master's degree in science, health and environmental reporting. She has a bachelor's degree in psychology from Georgetown University. Follow Parshall on X (formerly Twitter) @parshallison

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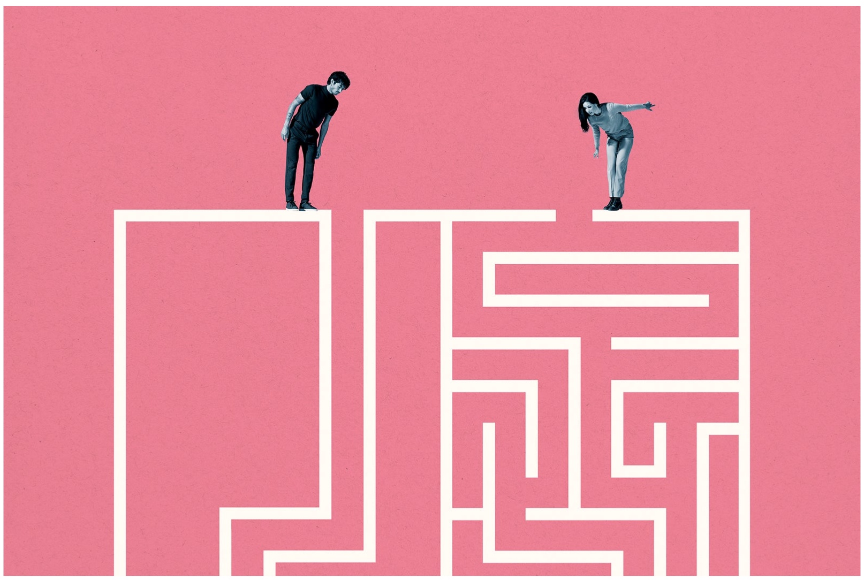
Find the Perfect Game to Play with This Interactive Quiz

From board games to solo puzzles to fantastical escapes to tricks for running faster, find a game for every mood

By Diana Kwon



Klaus Vedfelt/Getty Images

Are you a seasoned video gamer or a board-game geek looking to expand your repertoire? Or are you more of a game-hesitant newbie overwhelmed by the options (and scarred by a childhood Monopoly marathon)? Perhaps you just want to try a fun, new activity for an upcoming party? Whatever the conditions, mood and experience level, there’s a game to suit your needs.

[*Play science-inspired games, puzzles and quizzes in our new Games section*]

Scientific American created a quiz to help guide you toward the experience you’re seeking, from quiet, meditative puzzles and fantastical journeys to novel fitness challenges and bonding experiences with friends. Some of these games will take you on adventures in the great outdoors; others will offer you an escape from reality without leaving the comfort of your sofa. There are games you can play alone and games you can play with others; games that take a matter of minutes and games that require hours—or weeks—of commitment. Choose a game now, and come back for another round when you’re seeking something new.

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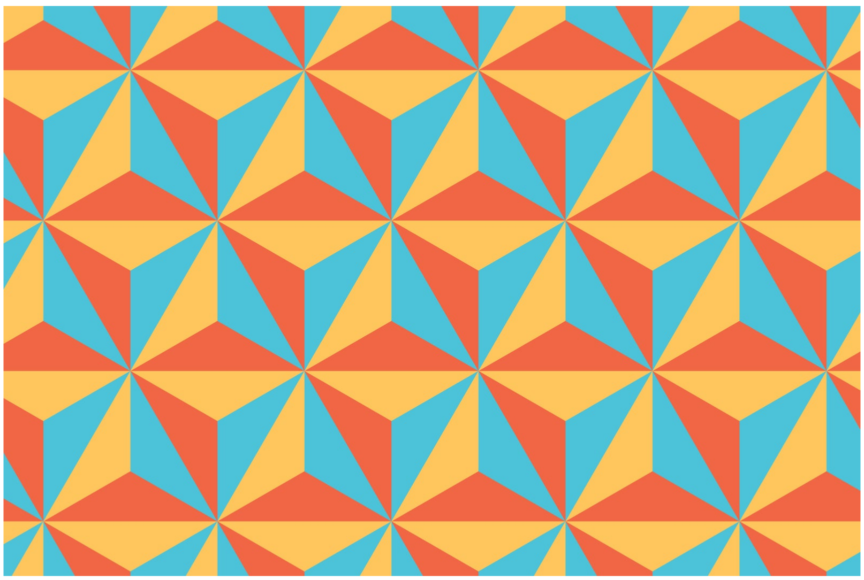
Diana Kwon is a freelance journalist who covers health and the life sciences. She is based in Berlin.

Mathematics

Here’s How to Make Flexagons—Folded Strips of Paper with Strange and Surprising Properties

In which strips of paper are used to make hexagonal figures with unusual properties

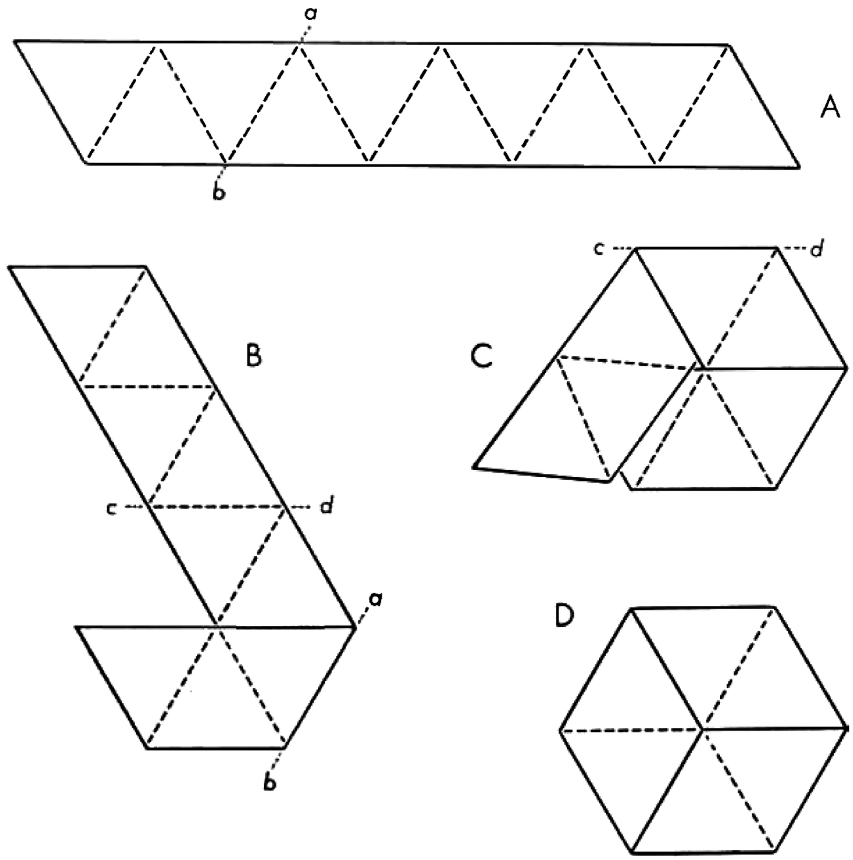
By Martin Gardner



PytyCzech/Getty Images

Editor’s Note: Published in 1956, this article proved so popular that it served as the inspiration for Martin Gardner’s legendary Scientific American *column Mathematical Games. Read more in our special digital issue, Fun and Games.*

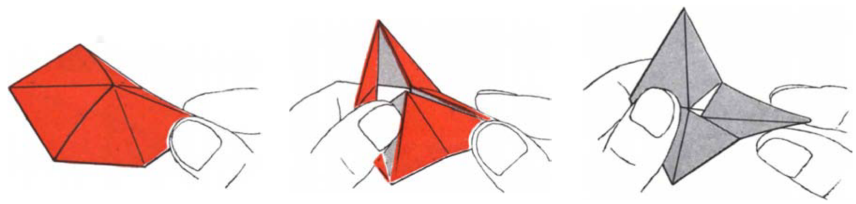
Mathematics owes a lot to games, and vice versa. There is an engaging little exercise with strips of paper which has fascinated some first-class brains in recent years. It was discovered in an idle moment by a British mathematics student at Princeton University. The whole thing grew out of the trivial circumstance that British and American notebook paper are not the same size. Arthur H. Stone, a 23-year-old English graduate student who came to Princeton on a fellowship in the fall of 1939, found that he had to trim an inch off American notebook sheets to fit them into his English binder. For amusement he began to fold the trimmed-off strips of paper in various ways, and one of the figures he made turned out to be particularly intriguing. He had folded the strip diagonally at three places and joined the ends so that it made a hexagon [*see illustration below*]. When he pinched two adjacent triangles together and pushed the opposite vertex of the hexagon toward the center, the hexagon would open out again, like a budding flower, and show a completely new face. If, for instance, the top and bottom faces of the original hexagon were painted different colors, the new face would come up blank and one of the colored faces would disappear!



Trihexaflexagon is constructed by cutting a strip of paper so that it may be marked off in 10 equilateral triangles (A). The strip is folded backward along the line ab and turned over (B). It is then folded backward again along the line cd and the next to the last triangle placed on top of the first (C). The last triangle is now folded backward and glued to the other side of the first (D). The figure may be flexed as shown below. It is not meant to be cut out. Fairly stiff paper at least an inch and a half wide is recommended.
Bunji Tagawa

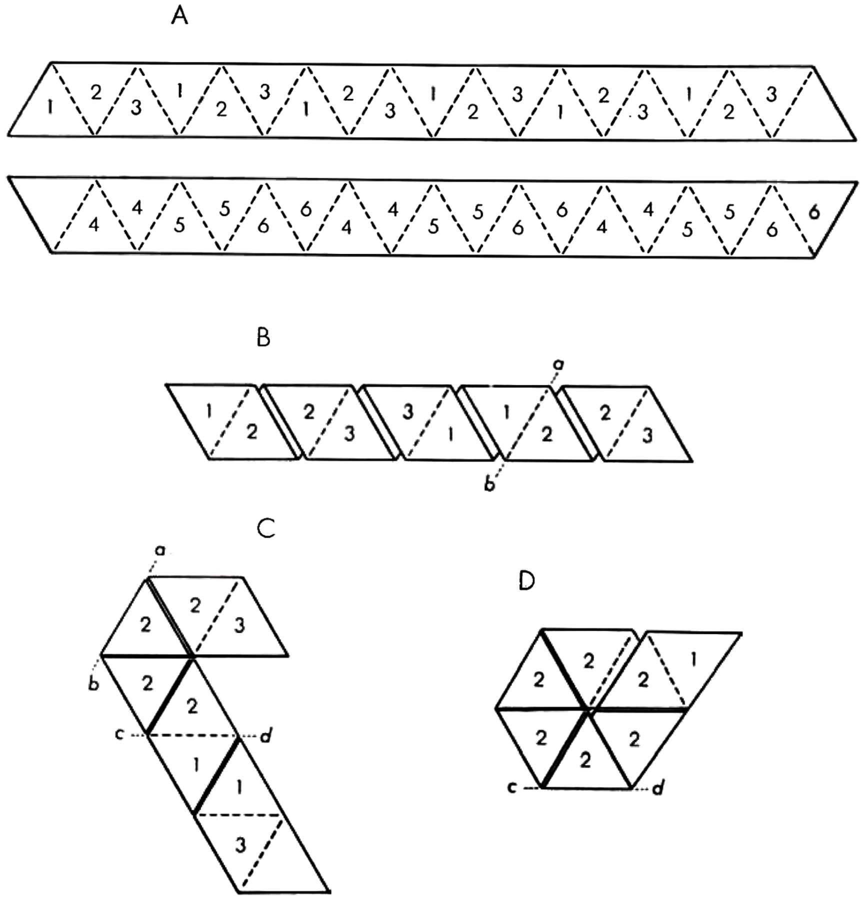
This figure had three faces—say, red, green and blank. Stone experimented further with longer strips, and, with considerable patience and creative insight, was able to construct a model which had the same hexagonal shape but could be opened to six different faces instead of only three. At this point Stone found the game so interesting that he showed his paper models to friends in the graduate school. Soon “flexagons” were appearing in profusion at the lunch and dinner tables. A “Flexagon Committee” was organized to probe further into the mysteries of flexigation. The other members besides Stone were Bryant Tuckerman, a graduate student of mathematics; Richard P. Feynman, a graduate student in physics; and John W. Tukey, a young mathematics instructor.

The models were named hexaflexagons—“hexa” for the six triangles that form the hexagonal face, and “flexagon” for the structure’s ability to flex. Stone’s first model is a trihexaflexagon (“tri” for the three different faces that can be brought into view); his elegant second structure is a hexahexaflexagon (for its six faces).



Trihexaflexagon is flexed by pinching together two of its triangles. The inner edge of the two opposite triangles may be opened with the other hand (*middle*). If the figure cannot be opened, the adjacent pair of triangles is pinched. If the figure opens, it can be turned inside out, revealing a side that was not visible before.
Bunji Tagawa

To make a hexahexaflexagon you start with a strip of paper (the tape used in adding machines serves admirably) which is divided into 19 triangles [*see illustration below*]. You number the triangles on one side of the strip 1, 2 and 3, leaving the 19th triangle blank, as shown in the drawing. On the opposite side of the triangles are numbered 4, 5 and 6, according to the scheme shown. Now you fold the strip so that the same underside numbers face each other—4 on 4, 5 on 5, 6 on 6 and so on. The resulting folded strip, illustrated by the second drawing below, is then folded back on the lines *ab* and *cd* [*third drawing in illustration*], forming the hexagon [*fourth drawing in illustration*]; finally the blank triangle is turned under and pasted to the corresponding blank triangle on the other side of the strip. All this is easier to carry out with a marked strip of paper than it is to describe.

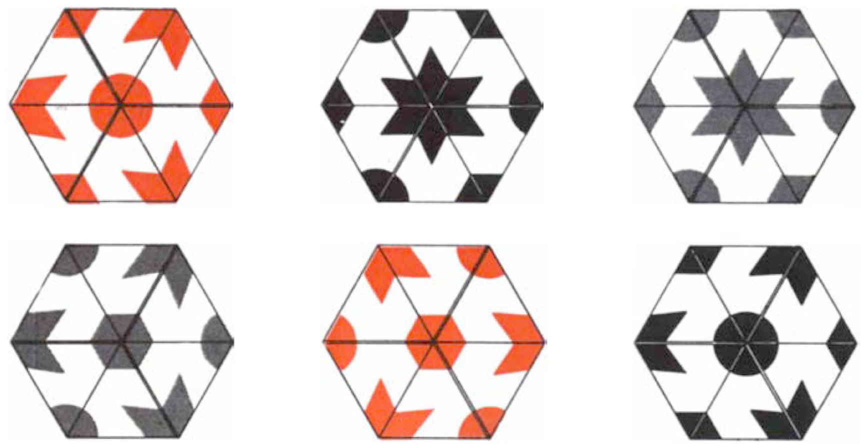


Hexahexaflexagon is constructed by cutting a strip of paper so that it may be marked off in 19 triangles (A). The triangles on one side are numbered 1, 2 and 3; the triangles on the other, 4, 5 and 6. A similar pattern of colors or geometrical figures may also be used. The hexagon is then folded as shown. The figure can be flexed to show six different faces.
Bunji Tagawa

If you have made the folds properly, the triangles on one visible face of the hexagon are all numbered 1, and on the other face all are numbered 2. Your hexahexaflexagon is now ready for flexing. You pinch two adjacent triangles together, bending the paper along the line between them, and push in the opposite vertex; the figure may then open up to face 3 or 5. By random flexing you should be able to find the other faces without much difficulty. Faces 4, 5 and 6 are a bit harder to uncover than 1, 2 and 3. At times you may find yourself trapped in an annoying cycle that keeps returning the same three faces over and over again.

Tuckerman quickly discovered that the simplest way to bring out all the faces of any flexagon was to keep flexing it at the same vertex until it refused to open, then to shift to an adjacent vertex.

This procedure, known as the “Tuckerman traverse,” will bring up the six faces of a hexahexa in a cycle of 12 flexes, but 1, 2 and 3 turn up three times as often as 4, 5 and 6.



“Tuckerman’s Traverse” exposes all six faces of a hexahexaflexagon in 12 flexes. Here the numbers of the flexagon in the preceding image have been replaced by geometrical figures in the same pattern. Faces 1, 2 and 3 turn up three times as often as faces 4, 5 and 6.
Bunji Tagawa

By lengthening the chain of triangles, the Committee discovered, one can make flexagons with 9, 12, 15 or more faces: Tuckerman managed to make a workable model with 48! He also found that with a strip of paper cut in a zigzag pattern (i.e., a strip with sawtooth rather than straight edges) it was possible to produce a tetrahexaflexagon (four faces) or a pentahexaflexagon. There are three different hexahexaflexagons—one folded from a straight strip, one from a chain bent into a hexagon and one from a form that somewhat resembles a three-leaf clover. The decahexaflexagon (10 faces) has 82 different variations, all folded from weirdly bent strips. Flexagons can be formed with any desired number of faces, but beyond 10 the number of different species for each increases at an alarming rate. All even-numbered flexagons, by the way, are made of strips with two distinct sides, but those with an odd number of faces have only a single side, like a Moebius surface.

A complete mathematical theory of flexigation was worked out in 1940 by Tukey and Feynman. It shows, among other things, exactly how to construct a flexagon of any desired size or species. The theory has never been published, though portions of it have since been rediscovered by other mathematicians. Among the flexigators is Tuckerman's father, the distinguished physicist Louis B. Tuckerman, who was formerly with the National Bureau of Standards. Tuckerman senior devised a simple but efficient diagrammatic notation for the theory.

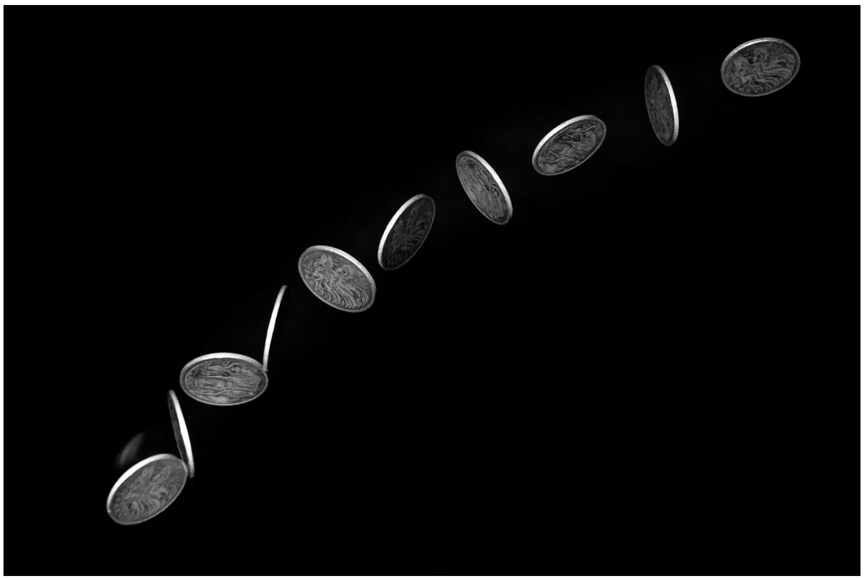
Pearl Harbor called a halt to the Committee's flexigation program, and war work soon scattered the four charter members to the winds. Stone is now a lecturer in mathematics at the University of Manchester in England. Feynman, now at the California Institute of Technology, is a famous theoretical physicist. Tukey, a professor of mathematics at Princeton, has made brilliant contributions to topology and to statistical theory which have brought him worldwide recognition. Tuckerman is a well-known mathematician at the Institute for Advanced Study in Princeton, where he works on the Institute's electronic computer project.

One of these days the Committee hopes to get together on a paper or two which will be the definitive exposition of flexagon theory. Until then the rest of us are free to flex our flexagons and see how much of the theory we can discover for ourselves.

Mind-Bending Probability Paradoxes

Paradoxes dealing with birthdays, playing cards, coins, crows and red-haired typists

By Martin Gardner



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Editor's Note: Published in 1957, this article comes from Martin Gardner's legendary Scientific American *column Mathematical Games. Read more in our special digital issue, Fun and Games.*

A paradox is a truth which cuts so strongly against the grain of common sense that it is hard to believe even when you are confronted with the proof. This quality of incredibility is particularly true of paradoxes in probability—a field of mathematics especially rich in paradoxes.

Consider the paradox of birthdays. What would you estimate to be the probability that, in any group of 24 persons, two or more were born on the same day of the same month? Offhand you would say it will be very low. In fact, the probability is $^{27}/_{50}$, or better than one half! In other words, if you were to bet even money on there being at least one coincidence of birthdays in a random collection of 24 persons, you would have a better than even chance of winning—over the long run.

These odds are so unexpected that you can make an entertaining, as well as profitable, game of the thing at parties or other gatherings of 24 or more people. Let each person write his birthday on a slip of paper. More often than not, at least two of the birthdays will be the same—sometimes to the surprise of the parties concerned, though they may have known each other for years.

You don’t need a party of 24 to play the game: you can merely take 24 names at random out of *Who’s Who* or some other biographical dictionary. I looked up the birthdays of the 33 Presidents of the U.S. and am happy to report that they obeyed the law of averages. Two Presidents had the same birthday: James Polk and Warren Harding were born on November 2.

The calculation of these odds from probability principles is perfectly simple but rather tedious. One method of calculating them is given by George Gamow in his book *One Two Three ... Infinity*. The probability that the birthdays of two persons will not coincide is $^{364}/_{365}$, since there are 364 chances in 365 of their birthdays being different. The probability that a third person will have a birthday different from the first two is $^{363}/_{365}$; for a fourth person the probability of a still different birthday is $^{362}/_{365}$, and so on to $^{342}/_{365}$ for the 24th person. To compute the probability that all 24 persons have different birthdays, you multiply all these probabilities together, and the result is a fraction which reduces to $^{23}/_{50}$. This means that you would win 27 out of every 50 bets on a coincidence of birthdays in groups of 24 persons.

Even more startling is the paradox of the second ace. Suppose that a bridge player were to look at his freshly dealt hand and announce: “I have an ace.” What is the probability that he also has a second ace? This can be calculated precisely, and it proves to be a little under $^{1}/_{3}$. But suppose he announced that he had a particular ace, say the ace of spades, selected by agreement in advance of the deal. The probability that the player holding the ace of spades also had another ace would be 11,686/20,825, or slightly better than $^{1}/_{2}$! Why should naming the ace affect the odds?

To simplify the work of computation, we can illustrate the situation with a more elementary game using only four cards—the ace of spades, the ace of hearts, the deuce of clubs and the jack of diamonds. If two cards are dealt to each of two players, there are only six possible combinations that a player can hold: (1) ace of spades and ace of hearts, (2) ace of spades and jack of diamonds, (3) ace of spades and deuce of clubs, (4) ace of hearts and jack of diamonds, (5) ace of hearts and deuce of clubs, (6) jack of diamonds and deuce of clubs. Now five of these six two-card hands permit the player to say, “I have an ace,” and in one of the five instances he has a second ace. Consequently in this game the probability of the second ace is $^{1}/_{5}$. But observe that if the player is able to declare that he holds the ace of spades, the probability that the second ace is in his hand goes up to $^{1}/_{3}$, because there are only three combinations containing the ace of spades and one includes the second ace.

The most famous of all probability paradoxes is the St. Petersburg paradox, first set forth in a paper by the famous mathematician Daniel Bernoulli before the St. Petersburg Academy. Suppose I toss a penny and agree to pay you a dollar if it falls heads. If it comes tails, I toss again, this time paying you two dollars if the coin is heads. If it is tails again, I toss a third time and pay four dollars if it falls heads. In short, I offer to double the penalty with each toss and I continue until I am obliged to pay off. What should you pay for the privilege of playing this one-sided game with me?

The unbelievable answer is that you could pay me any amount, say a million dollars, for each game and still come out ahead. In any single game there is a probability of $^1/_2$ that you will win a dollar, $^1/_4$ that you will win two dollars, $^1/_8$ that you will win four dollars, and so on. Therefore the total you may expect to win is $(1 \text{ x } ^1/_2) + (2 \text{ x } ^1/_4) + (4 \text{ x } ^1/_8)$.... The sum of this endless series is infinite. As a result, no matter what finite sum you paid me in advance per game, you would win in the end if we played enough games. You would be paid something in every game and you would also have a chance, albeit small, of winning an astronomical sum each time the game was played. This paradox is involved in every “doubling” system of gambling. Its full analysis leads into all sorts of intricate byways.

Carl C. Hempel, a leading figure in the “logical positivist” school and now a professor of philosophy at Princeton University, discovered another astonishing probability paradox. Ever since he first explained it in 1937 in the Swedish periodical *Theoria*, “Hempel’s paradox” has been a subject of much pleasant and learned argument among philosophers of science, for it reaches to the very heart of scientific method.

Let us assume, Hempel began, that a scientist wishes to investigate the hypothesis “All crows are black.” His research consists of examining as many crows as possible. The more black crows he finds, the more probable the hypothesis becomes. Each black crow can therefore be regarded as a "confirming instance" of the hypothesis. Hempel asserted that the existence of a brown stone also is a "confirming instance" of the hypothesis! He proved his paradox with ironclad logic.

The statement "All crows are black" can be transformed to the logically equivalent statement, "All not-black objects are not-crows." The second statement is identical in meaning with the original. Consequently the discovery of any object that "confirms" the second statement must also confirm the first.

Suppose, then, that the scientist, searching about for not-black objects, comes upon a brown stone. This object is a confirming instance of "All not-black objects are not-crows." It therefore must add to the probable truth of the equivalent hypothesis "All crows are black." The same applies to a white elephant, or a red herring, or the scientist's green necktie. As one philosopher recently remarked, on rainy days an ornithologist investigating the black-crow hypothesis could carry on his research without getting his feet wet. He need only explore his room and note instances of not-black objects that are not-crows!

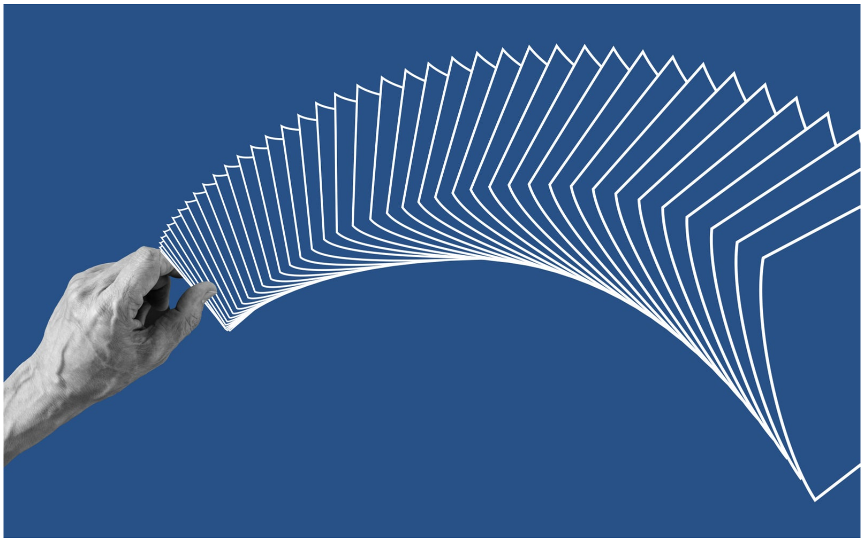
We find it hard to accept the validity of this paradox, says Hempel, because of a "misguided intuition." But it begins to make sense when we consider a simpler problem. Let us say that we wish to test the hypothesis that all red-haired typists working for a certain large company are married. We could investigate this directly by seeking out every red-haired typist and asking her if she has a husband. But there is another test, which might actually be more efficient. We could get a list of all the unmarried typists in the company from the personnel department and then investigate whether any of the girls on this list has red hair. If it turns out that no unmarried typist has red hair, we have completely confirmed our hypothesis that all of the red-headed typists are married. And each not-married typist with not-red hair serves effectively as a confirming instance of the hypothesis. If there are fewer unmarried than married typists, we could save time by this approach.

The only real difference between the problem of the red-headed typists and the one of the black crows is in the relative sizes of the classes. There are so many not-black objects in the world that checking them would be an extremely inefficient method of testing the hypothesis that all crows are black. Nonetheless most logicians agree that Hempel's logic is unassailable. And although we may be tempted to dismiss Hempel's paradox with a smile and a shrug, we must remember that many logical paradoxes which were long regarded as mere mental exercises proved to be highly useful in the development of symbolic logic. Analyses of Hempel's paradox have already provided valuable insights into the obscure nature of inductive logic, the tool by which all scientific knowledge is obtained.

Learn the Math Behind Your Favorite Card Tricks

Concerning various card tricks with a mathematical message

By Martin Gardner



Boris Zhitkov/Getty Images

Editor's Note: Published in 1957, this article comes from Martin Gardner's legendary Scientific American *column Mathematical Games. Read more in our special digital issue, Fun and Games.*

Somerset Maugham's short story "Mr. Know-All" contains the following dialogue:

> "Do you like card tricks?"
>
> "No, I hate card tricks."
>
> "Well, I'll just show you this one."

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After the third trick, the victim finds an excuse to leave the room. His reaction is understandable. Most card magic is a crashing bore, unless it is performed by skillful professionals. There are, however, some “self-working” card tricks that are intensely interesting from a mathematical standpoint.

Consider the following trick. The magician, who is seated at a table directly opposite a spectator, first reverses 20 cards anywhere in the deck. That is, he turns them face up in the pack. The spectator thoroughly shuffles the deck so that these reversed cards are randomly distributed. He then holds the deck underneath the table, where it is out of sight to everyone, and counts off 20 cards from the top. This packet of 20 cards is handed under the table to the magician.

The magician takes the packet but continues to hold it beneath the table so that he cannot see the cards. “Neither you nor I,” he says, “knows how many cards are reversed in this group of 20 which you handed me. However, it is likely that the number of such cards is less than the number of reversed cards among the 32 which you are holding. Without looking at my cards I am going to turn a few more face-down cards face up and attempt to bring the number of reversed cards in my packet to exactly the same number as the number of reversed cards in yours.”

The magician fumbles with his cards for a moment, pretending that he can distinguish the fronts and backs of the cards by feeling them. Then he brings the packet into view and spreads it on the table. The face-up cards are counted. Their number proves to be identical with the number of face-up cards among the 32 held by the spectator!

This remarkable trick can best be explained by reference to one of the oldest mathematical brain-teasers. Imagine that you have before you two beakers, one containing a liter of water; the other, a liter of wine. One cubic centimeter of water is transferred to the beaker of wine and the wine and water mixed thoroughly. Then a cubic centimeter of the mixture is transferred back to the water. Is there now more water in the wine than wine in the water? Or *vice versa?*

The answer is that there is just as much wine in the water as water in the wine. The amusing thing about this problem is the extraordinary amount of irrelevant information involved. It is not necessary to know how much liquid there is in each beaker, how much is transferred, or how many transfers are made. It does not matter whether the mixtures are thoroughly stirred or not. It is not even essential that the two vessels hold equal amounts of liquid at the start! The only significant condition is that at the end each beaker must hold exactly as much liquid as it did at the beginning. When this obtains, then obviously if x amount of wine is missing from the wine beaker, the space previously occupied by this wine must now be filled with x amount of water.

If the reader is troubled by this reasoning, he can quickly clarify it with a deck of cards. Place 26 cards face down on the table to represent wine. Beside them put 26 cards face up to represent water. Now you may transfer cards back and forth in any manner you please from any part of one pile to any part of the other, provided you finish with exactly 26 in each pile. You will then find that the number of face-down cards in either pile will match the number of face-up cards in the other pile.

Now try a similar test beginning with 32 face-down cards and 20 face up. Make as many transfers as you wish, ending with 20 cards in the smaller pile. The number of face-up cards in the large pile will of necessity exactly equal the number of face-down cards among the 20. Now turn over the small pile. This automatically turns its face-down cards face up and its face-up cards face down. The number of face-up cards in both groups will therefore be the same.

The operation of the trick should now be clear. At the beginning the magician reverses exactly 20 cards. Later, when he takes the packet of 20 cards from the spectator, it will contain a number of face-down cards equal to the number of face-up cards remaining in the deck. He then pretends to reverse some additional cards, but actually all he does is turn the packet over. It will then contain the same number of reversed cards as there are reversed cards in the group of 32 held by the spectator. The trick is particularly puzzling to mathematicians, who are apt to think of all sorts of complicated explanations.

Many card effects known in the conjuring trade as “spellers” are based on elementary mathematical principles. Here is one of the best. With your back to the audience, ask someone to take from one to 12 cards from the deck and hide them in his pocket without telling you the number. You then tell him to look at the card at that number from the top of the remainder of the deck and remember it.

Turn around and ask for the name of any individual, living or dead. For example, someone suggests Marilyn Monroe (the name, by the way, must have more than 12 letters). Taking the deck in your hand, you say to the person who pocketed the cards: “I want you to deal the cards one at a time on the table, spelling the name Marilyn Monroe like this.” To demonstrate, deal the cards from the top of the deck to form a face-down pile on the table, taking one card for each letter until you have spelled the name aloud. Pick up the small pile and replace it on the deck.

“Before you do this, however,” you continue, “I want you to add to the top of the deck the cards you have in your pocket.” Emphasize the fact, which is true, that you have no way of knowing how many cards this will be. Yet in spite of this addition of an unknown number of cards, after the spectator has completed spelling Marilyn Monroe, the next card (that is, the card on top of the deck) will invariably turn out to be his chosen card!

The operation of the trick yields easily to analysis. Let x be the number of cards in the spectator’s pocket and also the position of the chosen card from the top of the deck. Let y be the number of letters in the selected name. Your demonstration of how to spell the name automatically reverses the order of y cards, bringing the chosen card to a position from the top that is y minus x. Adding x cards to the deck therefore puts y minus x plus x cards above the selected one. The x’s cancel out, leaving exactly y cards to be spelled before the desired card is reached.

A more subtle compensatory principle is involved in the following effect. A spectator is asked to select any three cards and place them face down on the table without letting the magician see them. The remaining cards are shuffled and handed to the magician.

“I will not alter the position of a single card,” the magician explains. “All I shall do is remove one card which will match in value and color the card you will select in a moment.” He then takes a single card from the pack and places it face down at one side of the table.

The spectator is now asked to take the remaining cards in hand and to turn face up the three cards he previously placed on the table. Let us assume that they are a nine, a queen and an ace. The magician requests that he start dealing cards face down on top of the nine, counting aloud as he does so, beginning the count with 10 and continuing until he reaches 15. In other words, the spectator deals six cards face down on the nine. The same procedure is followed with the other two cards. The queen, which has a value of 12 (jacks are 11, kings 13), will require three cards to bring the count from 12 to 15. The ace (1) will require 14 cards.

The magician now has the spectator total the values of the three original face-up cards, and note the card at that position from the top of the remainder of the deck. In this case the total is 22 (9 plus 12 plus 1), so he looks at the 22nd card. The magician turns over his “prediction card.” The two cards match in value and color!

How is it done? When the magician glances through the deck to find a “prediction card,” he notes the fourth card from the bottom and then removes another card which matches it in value and color. The rest of the trick works automatically. I leave to the reader the easy task of working out an algebraic proof of why the trick cannot fail.

The ease with which cards can be shuffled makes them peculiarly appropriate for demonstrating a variety of probability theorems, many of which are startling enough to be called tricks. For example, let us imagine that two people each hold a shuffled deck of 52 cards. One person counts aloud from 1 to 52; on each count both deal a card face up on the table. What is the probability that at some point during the deal two identical cards will be dealt simultaneously?

Most people would suppose the probability to be low, but actually it is better than $^1/_2$! The probability there will be *no* coincidence is 1 over the transcendental number *e*. (This is not precisely true, but the error is less than 1 over 10 to the 69th power. The reader may consult page 47 in the current edition of W. Rouse Ball’s *Mathematical Recreations and Essays* for a method of arriving at this figure.) Since *e* is 2.718..., the probability of a coincidence is roughly $^{17}/_{27}$ or almost $^2/_3$. If you can find someone who is willing to bet you even odds that no coincidence will occur, you stand a rather good chance to pick up some extra change.

Mind & Brain

- **Mathematical Games: Möbius Bands**
  Curious figures descended from the Möbius band, which has only one side and one edge

| Next section | Main menu | Previous section |

Making Möbius Strips and Other Strange Structures

Curious figures descended from the Möbius band, which has only one side and one edge

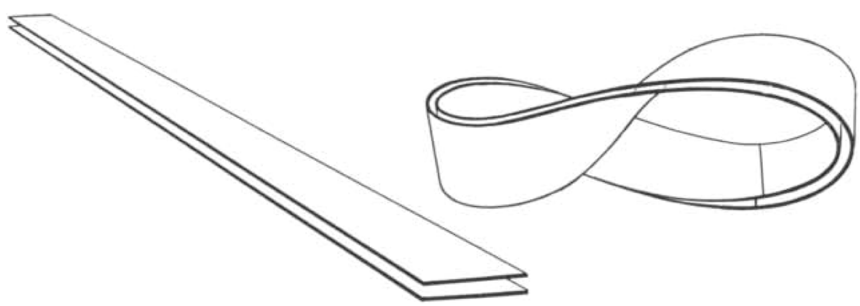
By Martin Gardner

gadost/Getty Images

Editor's Note: Published in 1957, this article comes from Martin Gardner's legendary Scientific American *column Mathematical Games. Read more in our special digital issue, Fun and Games.*

As many readers of this magazine are aware, a Möbius band is a geometrical curiosity which has only one surface and one edge. Such figures are the concern of the branch of mathematics called topology. People who have a casual interest in mathematics may get the idea that a topologist is a mathematical playboy who spends his time making Möbius bands and other diverting topological models. If they were to open any recent textbook of topology, they would be surprised. They would find page after page of symbols, seldom relieved by a picture or diagram. It is true that topology grew out of the consideration of geometrical puzzles, but today it is a jungle of abstract theory. Topologists are suspicious of theorems that must be visualized in order to be understood.

Serious topological studies nonetheless produce a constant flow of weird and amusing models. Consider, for example, the double Möbius band. This is formed by placing two strips of paper together, giving them a single half-twist as if they were one strip, and joining their ends as shown in the illustration below.



Double Möbius band is made by placing two strips of paper together (*left*), giving both of them a half twist and joining them as indicated by the straight lines at right.
James Egleson

We now have what appears to be two nested Möbius bands. Indeed, you can “prove” that there are two separate bands by putting your finger between the bands and running it all the way around them until you come back to the point at which you started. A bug crawling between the bands could circle them indefinitely, always walking on one strip with the other strip sliding along its back. At no point would he find the “floor” meeting the “ceiling.” An intelligent bug would conclude that he was walking between the surfaces of two separate bands.

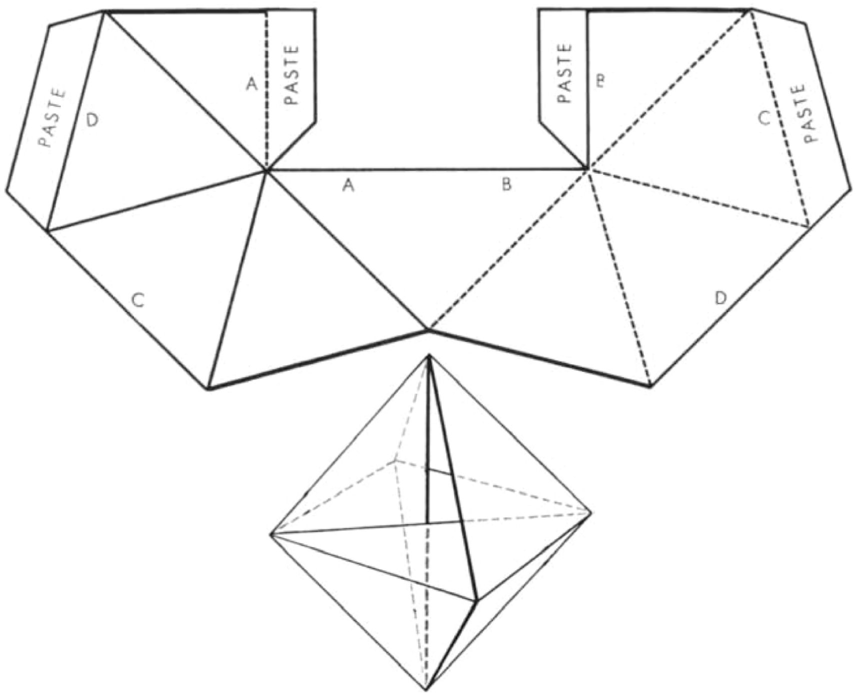
Suppose, however, that the bug made a mark on the floor, and circled the bands until he reached the mark again. It would find the mark not on the floor but on the ceiling, and it would require a second trip around the bands to find it on the floor again! The bug would need considerable imagination to comprehend that both floor and ceiling were one side of a single strip. What appears to be two nested bands is actually one large band! When you open the model into the large band, you will find it difficult to restore it to its original form.

When the band is in its double form, two separate edges of it run parallel to each other; they circle the model twice. Imagine that these edges are joined and that the band is made of thin rubber. You would then have a tube which could be inflated to make a torus (the topologist’s term for the surface of a doughnut). The joined edges would form a closed curve that coiled twice around the torus. This means that a torus can be cut along such a curved line to form the double Möbius band.

The double band is identical, in fact, with a single band that is given four half-twists before its ends are joined. It is possible to cut a torus into a band with any desired even number of half-twists, but impossible to cut it so as to produce bands with an odd number of such twists. This is because the torus is a two-sided surface and only bands with an even number of half-twists are two-sided. Although two-sided surfaces can be made by cutting one-sided ones, the reverse is not possible. If we wish to obtain one-sided bands (bands with an odd number of half-twists) by cutting a surface without edges, we must resort to cutting a Klein bottle. This remarkable one-sided bottle is described in “Topology,” by Albert

W. Tucker and Herbert S. Bailey, Jr. [*Scientific American;* January 1950].

The simple Möbius band is made by giving a strip one half-twist before joining the ends. Can the band somehow be stretched until this edge is a triangle? The answer is yes. The first man to devise such a model was Bryant Tuckerman, one of the four pioneers in the art of folding flexagons [“Mathematical Games”; December 1956]. The illustration below shows how a piece of paper can be cut, folded and pasted to create Tuckerman’s model.



Möbius band with triangular edge was devised by Bryant Tuckerman. If the flat figure is redrawn, preferably on a larger scale, the dimensional polyhedral model may be assembled as follows. First, cut out the figure. Second, fold it “down” along the solid lines. Third, fold it in the opposite direction along the broken lines. Fourth, by applying paste to the four tabs, join edges A and A, B and B, C and C, and D and D. The heavy lines in the finished polyhedron trace the triangular boundary of the Möbius surface.
James Egleson

Surfaces may not only have one or two sides; they may also differ topologically in the number and structure of their edges. Such traits cannot be altered by distorting the surface; hence they are called topological invariants. Let us consider surfaces with no more than two edges, and edges that are either simple closed curves or in the form of an ordinary single knot. If the surface has two edges, they may be independent of each other or linked. Within these limits we can list the following 16 kinds of surfaces (excluding edgeless surfaces such as the sphere, the torus and the Klein bottle):

ONE-SIDED, ONE-EDGED

1. Edge is a simple closed curve.

2. Edge is knotted.

TWO-SIDED, ONE-EDGED

3. Edge is a simple closed curve.

4. Edge is knotted.

ONE-SIDED, TWO-EDGED

5. Both edges are simple closed curves, unlinked.

6. Both edges are simple closed curves, linked.

7. Both edges are knotted, unlinked.

8. Both edges are knotted, linked.

9. One edge is simple; one knotted, unlinked.

10. One edge is simple; one knotted, linked.

TWO-SIDED, TWO-EDGED

11. Both edges are simple closed curves, unlinked.

12. Both edges are simple closed curves, linked.

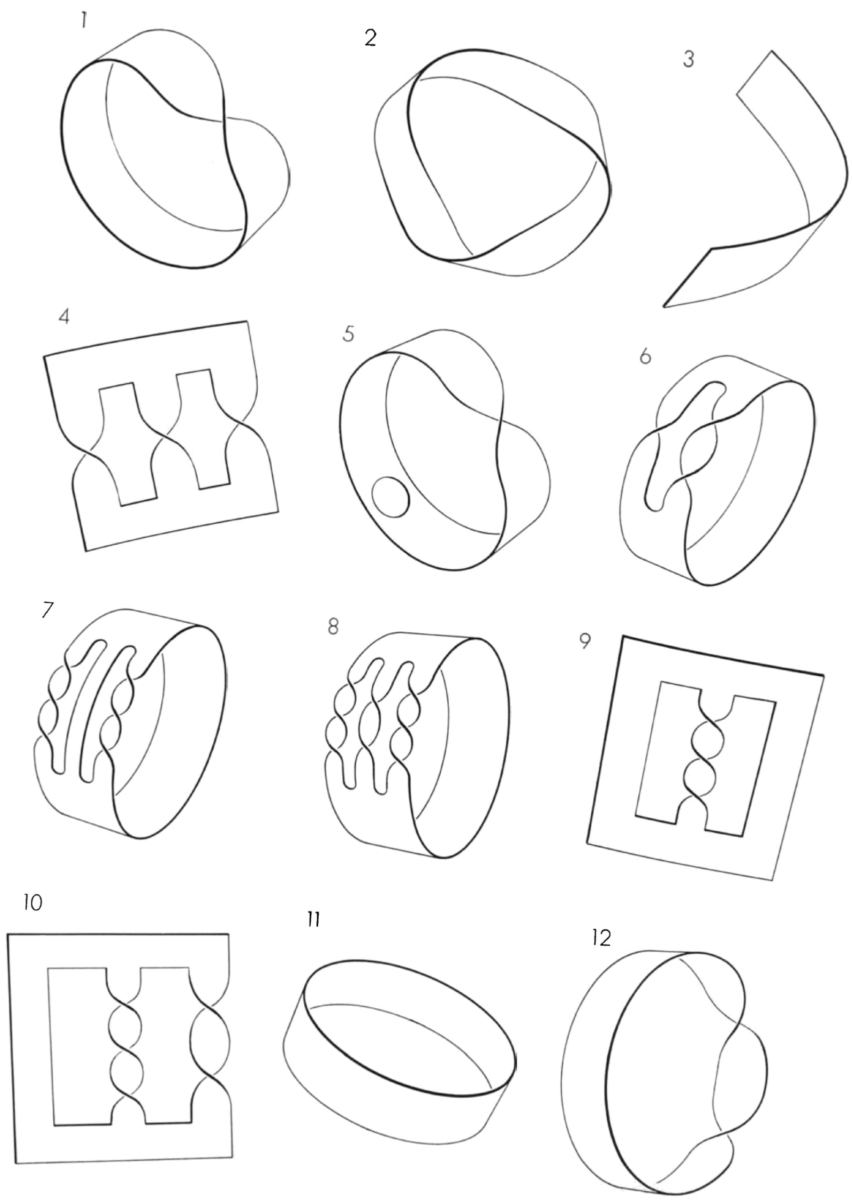
13. Both edges are knotted, unlinked.

14. Both edges are knotted, linked.

15. One edge is simple; one knotted, unlinked.

16. One edge is simple; one knotted, linked.

Paper models are easily constructed to illustrate examples of each of these 16 surfaces. Models for surfaces 1 through 12 are depicted below. The reader may enjoy trying to construct models of the remaining four surfaces, drawings of which will appear in this department next month.

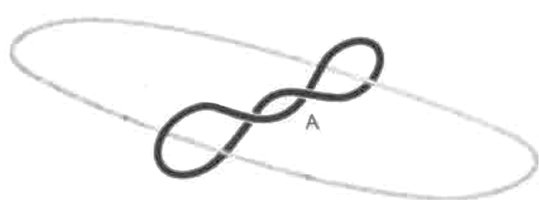


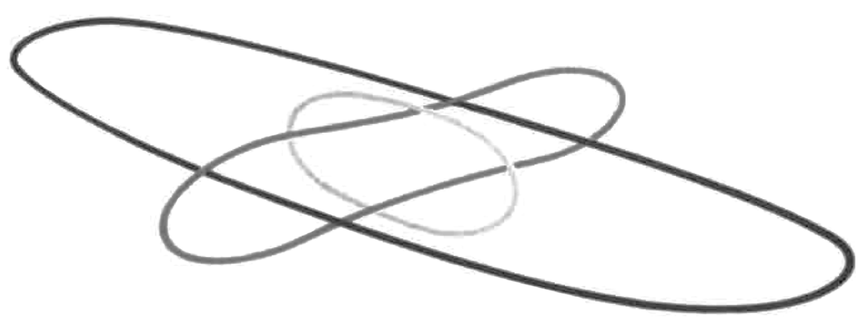
Topological models are 12 of the 16 that can be made from surfaces with no more than two edges, and edges which are a simple closed curve or a single knot. The simple Möbius band is 1.
James Egleson

When some of these models are cut with scissors in certain ways, the results are startling. As almost everyone who has played with a Möbius band knows, cutting the band in half lengthwise does not produce two separate bands, as one might expect, but one large band. (The large band has four half-twists; thus it can be made into the double Möbius band described earlier.) Not so well known is the fact that if you start the cut a third of the way between one edge and the other, and cut until you return to the starting point, the Möbius band opens into a large band linked with a smaller one.

Cutting surface 12 in half yields two interlocked bands of the same size, each exactly like the original one. Cutting surface 2 in half results in a large band that has a knot in it. This latter stunt was the subject of a booklet that enjoyed a wide sale in Vienna in the 1880s. The booklet revealed the secret of forming a knot in a cloth band without resorting to magical trickery.

In saying that two edges are “linked” we mean linked in the manner of two links in a chain. To separate the links it is necessary to open one link and pass the other through the opening. It is possible, however, to interlock two closed curves in such a manner that in order to separate them it is not necessary to pass one through an opening in the other. The simplest way to do this is shown by the upper curves in the illustration below. These curves can be separated by passing one band through *itself* at point A.





Interlocked curves may be separated without passing one through an opening in the other. The curves at the top may be separated by passing black curve through itself at A.
James Egleson

The three closed curves at the bottom of the illustration are also inseparable without being linked. If you remove any one curve, the other two are free; if you link any pair of curves, it frees the third one. This structure, by the way, is topologically identical with the familiar three-ring trademark of a well-known brand of beer. These are called Borromean rings because they formed the coat of arms for the Renaissance Italian family of Borromeo.

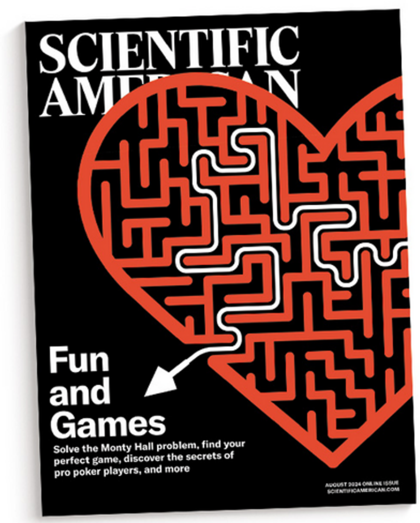
Culture

- **Games Rule the World—And Always Have**
  Why we’re fascinated by solving mind-bending puzzles, pitching faster fastballs, embodying new characters, playing ancient games, and much more

Games Rule the World—And Always Have

Why we're fascinated by solving mind-bending puzzles, pitching faster fastballs, embodying new characters, playing ancient games, and much more

By Jen Schwartz



Scientific American, August 2024

For something that often gets dismissed as trivial, games are a serious part of our lives. Play is evolutionary ancient, and this deeply ingrained behavior helps us understand not just our environment but also the way we relate to one another. Kelly Clancy, a neuroscientist and author, says it goes even deeper than that. "Games are a kind of domestication," she writes in her essay for our special digital issue, Fun and Games, and throughout history, they have been used to imbue moral lessons and teach people to achieve goals. Today game designers are behind so much of our daily lives, creating hidden rules that run our dating apps, social feeds and financial systems.

Beyond a powerful tool for socialization, games help us understand ourselves. In an essay regarding her experiences in live-action role-play (LARP), Ericka Skirpan writes about how embodying different characters revealed surprising facets of herself. Games that are challenging to figure out can get lodged in our mind—and stay stuck there for years. *Scientific American*'s associate news editor Allison Parshall takes us on her journey of how she finally got to the bottom of the infamous Monty Hall problem—a uniquely befuddling puzzle popularized by the game show *Let's Make a Deal*.

[*Play science-inspired games, puzzles and quizzes in our new Games section*]

Virtual reality might have brought us stunning, immersive game experiences, but the world's oldest games are still delightful to play. Writer Stephanie Pappas has collected 10 ancient games—from Go to the Royal Game of Ur—and can teach you how to play them. We also enjoyed digging into the *Scientific American* archive to bring you articles from the late Martin Gardner's legendary column, "Mathematical Games."

Advances in technology don't necessarily make games better, nor do they render computers the undisputed champion over people. Jack Murtagh, *Scientific American*'s math columnist, dives deep into how artificial intelligence has allowed poker players to find the optimal strategy. You'd think everyone would always use it, but the best players still win by staying flexible and exploiting their opponents’ mistakes. In baseball, the push toward optimization and faster, more exciting play has made pitchers throwing at more than 100 miles per hour a common occurrence. Yet as writer Abe Streep explains, pitchers’ elbows are having a hard time keeping up.

As serious as games can be, they are also just *fun*. It can be overwhelming to know what to play when there are so many types to choose from. So we made our own interactive game to help you decide! Writer and game lover Diana Kwon has provided options for every style and mood. (I personally fell in love with the manipulation tactics in Codenames. And Wavelength made my friends and I laugh so hard that we literally cried.) Crossword nerds are in for a treat with a custom, science-themed puzzle. And when you’re done reading and playing your way through this special issue, head over to our brand-new Games page, where you’ll find a word game called Spellements, quirky science news and trivia quizzes, plus jigsaw and math puzzles. Come back daily to play something new.

Jen Schwartz is a senior features editor at *Scientific American*. She produces stories and special projects about how society is adapting—or not—to a rapidly changing world.
