2021 European Control Conference (ECC)
June 29 - July 2, 2021. Rotterdam, Netherlands

Online Data-driven Stabilization of Switched Linear Systems

Monica Rotulo, Claudio De Persis, Pietro Tesi

***Abstract*— We consider the stabilization problem of a discrete-time system that switches among a finite set of unknown linear subsystems under unknown switching signal. To this end, we propose a method that uses data to directly design a control mechanism without any explicit identification step. Our approach is online, meaning that the data are collected over time while the system is evolving in closed-loop, and are directly used to iteratively update the controller. A major benefit of the proposed online implementation is therefore the ability of the controller to automatically adjust to changes in the operating mode of the system. We show that the proposed control mechanism guarantees exponential stability of the closed-loop switched system under sufficiently slow switching. The effectiveness of the approach is illustrated via a numerical example.**

I. INTRODUCTION

Switched linear systems consist of a finite number of subsystems described by linear dynamics, together with a switching signal that coordinates the switching between these subsystems. In practical applications, such systems are widely used in many field as power systems, automotive industry, aircraft and traffic control [1].

In the literature, switched systems have been extensively studied during the past decade, and stability issues have always been a major focus in the control community. In fact, the stability of the switched systems depends not only on the dynamics of each subsystem but also on the properties of the switching signals. Among the large variety of problems, one can study the existence of a switching signal that ensures stability of the switched system. Alternatively, one can assume that the switching signal is not known a-priori and look for stability results under arbitrary switching sequences [2]. In this regard, [3] shows that a common Lyapunov function for all subsystems guarantees stability under an arbitrary switching signal. On the other hand, certain classes of switched systems may be stable when restrictions on the switching signals are imposed. For example, by imposing a bound on the time interval between two successive switchings [4], [5]. For switched systems under restricted switching, the multiple Lyapunov-like function approach has proven to be more efficient in demonstrating stability of the system [6]. The reader is referred to the survey paper [7] and the references cited therein for further discussion.

This project is conducted at the Centre for Data Science and Systems Complexity at the University of Groningen and is sponsored with a Marie Skłodowska-Curie COFUND grant, no. 754315.

M. Rotulo, C. De Persis, are with ENTEG, University of Groningen, 9747 AG Groningen, The Netherlands. P. Tesi is with DINFO, University of Florence, 50139 Florence, Italy. Email: {m.rotulo, c.de.persis}@rug.nl, pietro.tesi@unifi.it

Most of the existing work relies on the knowledge of a model of the switched system, which usually involves some identification steps [8]. However, identifying a switched linear system from a collection of input-output data is often computationally demanding. The main challenge is that the data are available only as a mixture of observations generated by a finite set of different interacting linear subsystems so that one does not know a-priori which subsystem has generated which data. In fact, this identification step is known to be NP-hard [9], [10].

More recently, data-driven control methods seek to avoid the identification step by synthesizing a controller directly from experimental data. Various efforts have been made in this direction, and we refer the interested reader to [11] for a survey on data-driven contributions. In the context of switched systems, [12] presents an extension of the virtual-reference feedback tuning method [13]. However, the proposed work relies on a reference model and cannot formally guarantee closed-loop stability.

For discrete-time linear systems, the extensive work of [14] revisits a result by Willems and coauthors [15]. Essentially, [15] stipulates that all possible trajectories of a linear time-invariant system can be obtained from any given single trajectory whose input component is persistently exciting. Among many, this result has been applied in various control problems, including robust state feedback control [16], data-enabled predictive control [17], set-invariance control [18], nonlinear control [19], [20] and time-delay systems [21]. To the best of our knowledge, the only work currently available in the context of switched linear systems is [22]. In particular, stabilization under arbitrary switching is possible at the expense of assuming the existence of a common polyhedral Lyapunov function for the systems as well as having access to the switching signal.

Contribution. In this paper, we consider the data-driven control problem of switched linear systems with unknown subsystems dynamics and unknown switching signals. In particular, we propose a data-driven control mechanism where the controller is itself parametrized through data. The data are collected over time while the system is evolving in closed-loop, and are directly used for updating the controller. This way, we are able to capture any changes in the dynamics of the system and adjust the controller accordingly to stabilize the overall closed-loop system. The main features of the proposed online implementation are twofold. First, we guarantee that the data generated online are *persistently exciting*. Second, we analytically prove that the proposed control mechanism *exponentially stabilizes* the closed-loop switched system when the switching is slow enough. This result shows the potential of the data-driven paradigm in solving problems that could not be solved using conventional control schemes. The proofs are omitted for space reasons and will appear somewhere else.

The paper is organized as follows. Section II provides preliminaries on the data-driven framework and introduces the problem of interest. In Section III, the proposed online data-based control mechanism is presented and the stability analysis of the closed-loop system is provided. A practical numerical example is discussed in Section IV. The paper ends with some concluding remarks in Section V.

II. Preliminaries and Problem Statement

A. Notations

We denote the set of real numbers, integers, and natural numbers including 0 by $\mathbb{R}$, $\mathbb{Z}$, and $\mathbb{N}$, respectively. Throughout the paper, for simplicity of the notation, we write $[i, j]$ to denote the discrete interval $[i, j] \cap \mathbb{Z}$. The standard Euclidean norm is denoted by $\|\cdot\|$. Given a matrix A, the notations $A \succ 0$ and $A \succeq 0$ respectively denote that $A = A^\top \in \mathbb{R}^{n\times n}$ is positive definite and semi-definite. For $A \succ 0$, $\lambda_{min}(A)$ and $\lambda_{max}(A)$ stand for the minimal and maximum eigenvalue of A, respectively. Given a sequence $z(i), z(i+1), \ldots \in \mathbb{R}^\sigma$, we denote its Hankel matrix of depth ℓ as

$$Z_{i,\ell,j} := \begin{bmatrix} z(i) & \cdots & z(i+j-1) \\ \vdots & \ddots & \vdots \\ z(i+\ell-1) & \cdots & z(i+\ell+j-2) \end{bmatrix},$$

where $i \in \mathbb{Z}$, and $\ell, j \in \mathbb{N}$. For $\ell = 1$, we can simply write

$$Z_{i,j} := \begin{bmatrix} z(i) & z(i+1) & \cdots & z(i+j-1) \end{bmatrix}.$$

Definition 1. *The signal $z(i), \ldots, z(i+T-1) \in \mathbb{R}^\sigma$ is persistently exciting of order L if the Hankel matrix $Z_{i,L,T-L+1}$ has full row rank σL.*

For a signal to be persistently exciting of order L, it must be sufficiently long in the sense that $T \geq (\sigma+1)L - 1$.

B. Preliminaries on data-driven framework

Consider the linear time-invariant system

$$x(k+1) = Ax(k) + Bu(k), \quad k \in \mathbb{N} \tag{1}$$

with state $x(k) \in \mathbb{R}^n$ and input $u(k) \in \mathbb{R}^m$. Let the pair (A, B) be controllable. During an experiment of duration $T > 0$, a sequence $u_d(0), \ldots, u_d(T-1)$ of inputs is applied to the system and the corresponding values $x_d(0), \ldots, x_d(T)$ of the state response are measured. The subscript d emphasizes that these are *offline data*. These data are organized in Hankel matrices as

$$\begin{aligned} U_{0,T} &:= \begin{bmatrix} u_d(0) & u_d(1) & \ldots & u_d(T-1) \end{bmatrix}, \\ X_{0,T} &:= \begin{bmatrix} x_d(0) & x_d(1) & \ldots & x_d(T-1) \end{bmatrix}, \\ X_{1,T} &:= \begin{bmatrix} x_d(1) & x_d(2) & \ldots & x_d(T) \end{bmatrix}. \end{aligned}$$

A main observation that emerges from [14] is that controllers can be directly parametrized in terms of data provided the following condition is satisfied:

$$\operatorname{rank} \begin{bmatrix} U_{0,T} \\ X_{0,T} \end{bmatrix} = m + n. \tag{2}$$

Condition (2) guarantees that any T-long input-state trajectory of the system can be expressed as a linear combination of the collected input-state data. It is possible to guarantee (2) when persistently exciting inputs are injected to the system.

Lemma 1. [15, Corollary 2] *Let system* (1) *be controllable. If the input sequence $u_d(0), \ldots, u_d(T-1)$ is persistently exciting of order $n+1$, then condition* (2) *holds.*

In the context of stabilization, condition (2) enables a data-based parametrization of all stabilizing state feedback controllers in the form $u = Kx$. In particular, [23] formulate the Linear Quadratic Regulator (LQR) problem as an $\mathcal{H}_2$ problem and derive a data-based solution based on convex programming. Specifically, consider the problem of designing a state feedback controller K that renders $A+BK$ Hurwitz and minimizes

$$\operatorname{trace}(P) + \operatorname{trace}(KPK^\top), \tag{3}$$

where P is the unique solution to

$$(A+BK)P(A+BK)^\top - P + I = 0. \tag{4}$$

It is known [24, Sec. 6.4] that the state feedback controller minimizing the $\mathcal{H}_2$-norm of (3), here denoted by K_{opt}, is unique. The work in [23] establishes that K_{opt} can be parametrized directly in terms of data. Specifically, we formulate the following semidefinite program (SDP)[1]:

$$\begin{aligned} &\min_{(\gamma,Q,P,L)} \ \gamma \\ &\text{subject to} \\ &\begin{cases} X_{1,T}\, QP^{-1}Q^\top X_{1,T}^\top - P + I \preceq 0 \\ P \succeq I \\ X_{0,T} Q = P \\ L - U_{0,T}\, QP^{-1}Q^\top U_{0,T}^\top \succeq 0 \\ \operatorname{trace}(P) + \operatorname{trace}(L) \leq \gamma \end{cases} \end{aligned} \tag{5}$$

which is only based on data.

Lemma 2. [23, Thm. 1] *Let condition* (2) *holds. Then problem* (5) *is feasible. Also, any optimal solution $(\gamma_o, Q_o, P_o, L_o)$ satisfies $K_{opt} = U_{0,T} Q_o P_o^{-1}$, where K_{opt} is the unique state feedback controller that minimizes* (3).

Lemma 2 establishes that problem (5) is an equivalent data-based formulation of the classic LQR problem, where by "equivalent" we mean both problems yield the same solution. For a discussion on the properties related to this formulation the interested reader is referred to [23].

[1] With some abuse of terminology we refer to (5) and subsequent derivations as an SDP, with the understanding that by using standard manipulations they can be written as SDP.

C. Problem formulation

We consider the discrete-time switched linear system

$$x(k+1) = A_{\sigma(k)}x(k) + B_{\sigma(k)}u(k) \tag{6}$$

where $x(k) \in \mathbb{R}^n$ and $u(k) \in \mathbb{R}^m$. The switching signal $\sigma : \mathbb{N} \to \mathcal{I}$ is a piecewise constant function of time that selects its values in the finite set $\mathcal{I} := \{1, 2, \ldots, M\}$, with $M > 1$ being the number of modes. Here, $(A_{\sigma(k)}, B_{\sigma(k)})$ are constant matrices of appropriate dimensions which are allowed to take values, at an arbitrary discrete time, in the finite set $\{(A_i, B_i) : i \in \mathcal{I}\}$.

Throughout this note, the following assumption holds.

Assumption 1. *The pairs (A_i, B_i) for $i \in \mathcal{I}$ are controllable.*

We denote by k_s the time instants of the s-th switching, i.e. $k_0 = 0$ and $k_{s+1} := \min\{k > k_s : \sigma(k) \neq \sigma(k_s)\}$ for all $s \in \mathbb{N}$. We formulate the following problem.

Problem 1. *Consider the switched system* (6)*. The pairs (A_i, B_i) for all $i \in \mathcal{I}$, the switching signal $\sigma(\cdot)$ and the switching instants k_s are assumed to be unknown. Design a data-driven feedback control law to ensure exponential stability of the origin of the closed-loop switched system.*

III. Main Results

In this section, we address Problem 1 by applying the data-driven framework in an *online* setting. By "online" we refer to the operation of collecting new data and accordingly modifying the control law while the system is evolving.

A. Online data-driven control

We propose the following feedback control law:

$$u(k) = K(k)x(k) + \varepsilon(k)\|x(k)\|, \tag{7}$$

where $K(k) \in \mathbb{R}^{m\times n}$ is the state feedback gain and $\varepsilon(k) \in \mathbb{R}^m$ is an auxiliary input signal that belongs to the ball

$$B_\delta := \{\varepsilon \in \mathbb{R}^m : \|\varepsilon\| \leq \delta\}$$

for every k and some $\delta > 0$. At each time $k \geq 0$, the following matrices of data are available:

$$\begin{aligned} U_{k-1} &:= U_{k-T,T} \\ &= \begin{bmatrix} u(k-T) & u(k-T+1) & \ldots & u(k-1) \end{bmatrix}, \\ X_{k-1} &:= X_{k-T,T} \\ &= \begin{bmatrix} x(k-T) & x(k-T+1) & \ldots & x(k-1) \end{bmatrix}, \\ X_k &:= X_{k-T+1,T} \\ &= \begin{bmatrix} x(k-T+1) & x(k-T+2) & \ldots & x(k) \end{bmatrix}. \end{aligned}$$

In the above definitions, we shift the window of the dataset one-step ahead, where an old data sample is discarded each time a new one is added. Note that the state response is generated according to (6) interconnected with (7). If the index of the sample is negative, it refers to data obtained from some offline *open-loop* experiments, that is without having (7) in the loop. In particular, we apply to system (6) an initial input sequence $u(-T), \ldots, u(-1)$ and collect the corresponding state sequence $x(-T), \ldots, x(0)$. Hence, at time $k = 0$ we construct the initial matrices of data X_{-1}, U_{-1}, X_0.

Throughout the paper, the following condition plays an important role:

$$\operatorname{rank} \begin{bmatrix} U_{k-1} \\ X_{k-1} \end{bmatrix} = m + n, \tag{8}$$

for all $k \geq 0$. Condition (8) guarantees that as long as the T-long data matrices U_{k-1}, X_{k-1} are generated by a single controllable subsystem, they encode all the information regarding the dynamics of that subsystem. On the path of guaranteeing this rank condition, inspired by Lemma 1, we require the sequence $u(k-T), \ldots, u(k-1)$ to be persistently exciting of order $n+1$ for any k. Without the auxiliary input ε in the structure of (7), this persistence of excitation condition would not necessarily hold. The reason is that the input signal at each time k would be merely restricted to $u(k) = K(k)x(k)$. Note that this relation can result in loosing the persistence of excitation condition since the role of $K(k)$ is solely to stabilize the closed-loop system. Therefore, the auxiliary input ε is added to overcome the possible lack of excitation caused by the feedback. This is stated in the following lemma.

Lemma 3. *For any $k \geq 0$ let the input sequence $u(k-T), \ldots, u(k-1)$ be persistently exciting of order $n+1$ and $\|x(k)\| \neq 0$. Then, there exists some $\varepsilon(k) \in B_\delta$ such that the sequence $u(k-T+1), \ldots, u(k)$ with $u(k) = K(k)x(k) + \varepsilon(k)\|x(k)\|$ is persistently exciting of order $n+1$.*

Proof. See the Appendix. □

Lemma 3 shows that for any $k \geq 1$ there exists some $\varepsilon(k-1) \in B_\delta$ such that the input sequence $u(k-T), \ldots, u(k-1)$ is persistently exciting of order $n+1$. To satisfy the requirement of the Lemma for $k = 0$, we choose the initial input sequence $u(-T), \ldots, u(-1)$ persistently exciting of order $n+1$. Note that the condition $\|x(k)\| \neq 0$ is not restrictive since the origin is the equilibrium of the closed-loop system.

We now exploit the rank condition (8) for designing the state feedback gain at the next step. We formulate the following SDP:

$$\begin{aligned} &\min_{(\gamma,Q,P,L)} \ \gamma \\ &\text{subject to} \\ &\begin{cases} X_k QP^{-1}Q^\top X_k^\top - P + I \preceq 0 \\ P \succeq I \\ X_{k-1}Q = P \\ \kappa^2 I - U_{k-1}QP^{-1}Q^\top U_{k-1}^\top \succeq 0 \\ L - U_{k-1}QP^{-1}Q^\top U_{k-1}^\top \succeq 0 \\ \operatorname{trace}(P) + \operatorname{trace}(L) \leq \gamma \end{cases} \end{aligned} \tag{9}$$

where $\kappa > 0$ is a design parameter. Then the corresponding controller is computed as:

$$K(k) = \begin{cases} U_{k-1}Q^*(k)P^*(k)^{-1} & \text{if (9) is feasible,} \\ K(k-1) & \text{otherwise,} \end{cases} \tag{10}$$

where the tuple $(\gamma^*(k), Q^*(k), P^*(k), L^*(k))$ is any optimal solution to (9).

Remark 1 (Implementation of (9))**.** *Problem* (9) *can be written in the equivalent SDP form:*

$$\min_{(\gamma,Q,P,L)} \gamma$$
subject to
$$\begin{cases} \begin{bmatrix} I-P & X_kQ \\ Q^\top X_k^\top & -P \end{bmatrix} \preceq 0 \\ \begin{bmatrix} \kappa^2 I & U_{k-1}Q \\ Q^\top U_{k-1}^\top & P \end{bmatrix} \succeq 0 \\ \begin{bmatrix} L & U_{k-1}Q \\ Q^\top U_{k-1}^\top & P \end{bmatrix} \succeq 0 \\ X_{k-1}Q = P \\ \mathrm{trace}(P) + \mathrm{trace}(L) \le \gamma \end{cases} \tag{11}$$

B. Stability analysis

We now investigate the stability of the switched system (6) under the feedback law (7). In this regard, we will exclude fast switchings. Thus we assume having a minimum interval between any two consecutive switchings, which is known in the literature as dwell time [4]. Formally, we define the dwell time as $\tau := \min_{s\in\mathbb{N}} k_{s+1} - k_s$. To guarantee that during each switching interval we correctly collect T samples of the active subsystem, we will assume that the switching is sufficiently slow such that $\tau > T$.

Consider any switching interval $[k_s, k_{s+1}-1]$ with $s \ge 0$. We refer to $[k_s, k_s+T-1]$ as the *transient* interval. Within the transient interval, recalling the definition of U_{k-1}, X_{k-1}, and X_k, the Hankel matrices contain data generated by both the active subsystem $\sigma(k_s)$ and the subsystem active at the previous switching interval, i.e., subsystem $\sigma(k_{s-1})$. Because of the inconsistent dataset, there is no guarantee that problem (9) is feasible or provides stabilizing controller. By the second and fourth constraints in (9), we enforce $\|K(k)\| \le \kappa$ to guarantee that the system state remains bounded during the transient interval. In particular, the system is evolving as

$$x(k+1) = (A_{\sigma(k)} + B_{\sigma(k)}K(k))x(k) + B_{\sigma(k)}\varepsilon(k)\|x(k)\|,$$

which implies

$$\|x(k+1)\| \le \Big(\|A_{\sigma(k)}+B_{\sigma(k)}K(k)\|+\|B_{\sigma(k)}\varepsilon(k)\|\Big)\|x(k)\|.$$

Let

$$C := \max_{i\in\mathcal{I}} \Big(\|A_i\| + \|B_i\|(\kappa+\delta) \Big). \tag{12}$$

It then follows from $\|K(k)\| \le \kappa$ and $\varepsilon(k) \in B_\delta$ that

$$\|x(k+1)\| \le C\|x(k)\|.$$

Consider now $[k_s+T, k_{s+1}-1]$. In this interval, T samples of the current subsystem are finally collected into the corresponding data matrices. Noting Assumption 1 and Lemma 3, it follows from an analogous argument to Lemma 1, presented in [15], that condition (8) holds. Then, choosing a suitable design parameter κ guarantees that problem (9) is feasible and returns a stabilizing controller for the pair $(A_{\sigma(k_s)}, B_{\sigma(k_s)})$. We formalize this in the following Lemma.

Lemma 4. *Let $i \in \mathcal{I}$ denote the active subsystem in the time interval $[k_s, k_{s+1}-1]$, i.e. $\sigma(k_s) = i$, and consider $k \in [k_s+T, k_{s+1}-1]$. Then, there exists some $\bar\kappa > 0$ such that for all $\kappa \ge \bar\kappa$ problem* (9) *is feasible and any optimal solution $(\gamma^*(k), Q^*(k), P^*(k), L^*(k))$ satisfies $K^i_{opt} = U_{k-1}Q^*(k)(P^*(k))^{-1}$, where K^i_{opt} is the unique LQR controller of subsystem i.*

Lemma 4 shows that, by choosing κ sufficiently large, in the interval $[k_s+T, k_{s+1}-1]$ the solution of problem (9) returns the unique LQR controller for the active subsystem i. For the rest of the paper, we assume that $\kappa \ge \bar\kappa$. Hence, for $k \in [k_s+T, k_{s+1}-1]$ the controller is

$$u(k) = K^i_{opt}\, x(k) + \varepsilon(k)\|x(k)\|. \tag{13}$$

We can now tackle the stability analysis of the closed-loop system. In particular, the finite set $\{K^i_{opt} : i \in \mathcal{I}\}$ allows us to approach the stability analysis by using multiple Lyapunov functions [6]. The key point of this approach is to construct a set of Lyapunov functions $\{V_i : i \in \mathcal{I}\}$ such that, considering suitable choice of design parameters, the value of V_i decreases on each time interval $[k_s+T, k_{s+1}-1]$ where the i-th subsystem is active. Then, the closed-loop switched system is exponentially stable under sufficiently slow switching. This is discussed in the following Theorem.

Theorem 1. *Consider the switched system* (6) *with unknown (A_i, B_i) for all $i \in \mathcal{I}$, unknown switching law $\sigma(\cdot)$ and unknown switching instants k_s. Also, consider the data-based feedback law* (7) *with the state feedback gain as in* (10)*. Then, there exist some $\bar\delta > 0$ and $\bar\tau > 0$ such that if $\delta \le \bar\delta$ and $\tau > \bar\tau$, the closed-loop system is exponentially stable.*

In the present work, $\bar\delta$ and $\bar\tau$ depend on some knowledge of the norms $\|\mathcal{A}_i\|$ and $\|B_i\|$ for $i \in \mathcal{I}$. We do not need to rely on the knowledge of $\mathcal{A}_i$ and B_i, but one can assume to work under the condition that these quantities belong to a set whose norm bounds are known to the system designer. The relaxation of such knowledge is left for future work.

IV. Illustrative Example

In this section we consider the problem of stabilizing a continuous stirred tank reactor (CSTR) with arbitrary switching between two modes [25]. Using a sampling rate of $h = 0.2s$, we write the CSTR in the form of a discrete-time switched linear system (6). Without causing confusion, we will refer to the time instant k instead of kh. We consider $M = 2$ subsystems with the matrices:

$$A_1 = \begin{bmatrix} 1.1052 & 0.2103 \\ 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1.4918 & 0.2459 \\ 0 & 1 \end{bmatrix},$$
$$B_1 = B_2 = \begin{bmatrix} 0.0207 \\ 0.2000 \end{bmatrix}.$$

Note that all the subsystems are controllable and open-loop unstable. Our purpose is to design a data-based control mechanism structured as (7) to stabilize the closed-loop switched system under unknown switchings.

We generate an initial T-long set of data with random initial condition and by applying an initial T-long sequence of input u in the form of (7) with initial controller gain $K(0) = \begin{bmatrix} 0 & 0 \end{bmatrix}$. In addition, we add to the control input u the term $\varepsilon \|x\|$ with ε as a random variable uniformly distributed on $[-\delta, \delta] = [-0.001, 0.001]$. We organize the samples into appropriate Hankel matrices of length T. We consider $T = 5$ since a necessary condition to have persistently exciting input data as in Definition 1 is $T \geq (m+1)n + m$ (see [14]).

Once the first T samples are collected into the Hankel matrices, we solve (9) by using CVX [26]. In particular, for the case study in hand, we fix the norm bound to $\kappa = 50$. The computed controller is then applied to the system following the structure in (7). At every iteration k, the Hankel matrices are updated by removing the oldest sample each time a new measurement is added. Based on this online stream of data, the data-based convex program (9) is solved online as well.

We generate the arbitrary switching signal σ by choosing randomly the switching instants between subsystems 1 and 2. In particular, we impose the restriction $\tau > T$. We simulate the switched system under the switching signal σ with $\tau > 3T$. The corresponding state response is shown in Figure 1.

We additionally perform multiple simulations with different arbitrary switching signals. Here we relax the concept of dwell time, allowing the possibility of switching fast occasionally, provided this does not occur too frequently. The concept of average dwell time from [5] serves this purpose. We say that σ has average dwell time τ_{avg} if there exist two positive numbers N_0 and τ_{avg} such that

$$N_\sigma(t_2, t_1) \leq N_0 + \frac{t_2 - t_1}{\tau_{avg}}, \quad \forall\, t_2 \geq t_1 \geq 0,$$

where $N_\sigma(t_2, t_1)$ denotes the number of discontinuities of σ on an interval $[t_1, t_2]$. In our example, we set $\tau_{avg} > 3T$. Figure 2 shows the convergence of the norm of the state. The arbitrary switching signals are constructed such that on average the dwell time between any two consecutive switchings is no smaller than τ_{avg}.

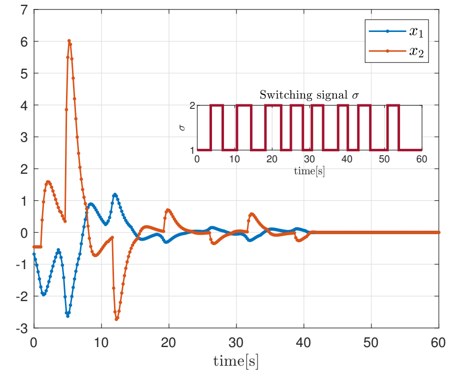

Fig. 1: State response of the closed-loop switched system under arbitrary switching signal σ and dwell time $\tau = 3.2s$.

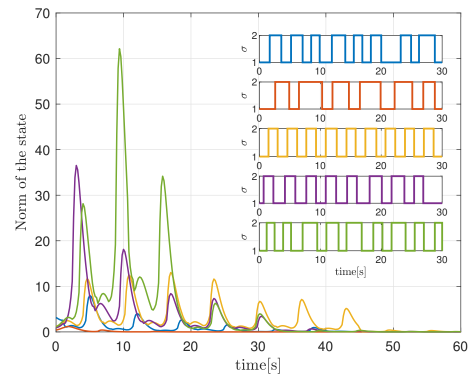

Fig. 2: We perform 5 simulations for arbitrary switched signals σ with average dwell time $\tau_{avg} = 3.2s$. The plot shows the norm of the states.

V. Conclusions

We have considered the design of a data-based feedback controller for switched discrete-time linear systems. The dynamics of each subsystem and the switching signal are assumed to be unknown. We have proposed a framework which requires no intermediate identification steps and provides stability guarantees. The key idea relies on an online scheme where input-state data are collected over time as the system is evolving. The control mechanism is directly parametrized through data and iteratively updated via a computationally tractable data-dependent semidefinite program. The resulting controller is guaranteed to exponentially stabilize the closed-loop system under sufficiently slow switching.

Appendix

Proof of Lemma 3. Without loss of generality, consider $k = 0$. Let $u(-T), \ldots, u(-1)$ be persistently exciting of order $n+1$, meaning that the corresponding Hankel matrix $U_{-T,n+1,T-n}$ has full rank $m(n+1)$. We partition this matrix as follows

$$\begin{aligned} U_{-T,n+1,T-n} &= \left[\begin{array}{c|c} U_{-T,n+1,1} & S \end{array}\right] \\ &= \left[\begin{array}{c} U_{-T,1,T-n} \\ \hline R \end{array}\right] \end{aligned} \tag{14}$$

where

$$\begin{aligned} S &:= \left[\begin{array}{c} U_{1-T,n,T-n-1} \\ \hline U_{1-T+n,1,T-n-1} \end{array}\right] \\ R &:= \left[\begin{array}{c|c} U_{1-T,n,T-n-1} & U_{-n,n,1} \end{array}\right]. \end{aligned}$$

It follows that $\text{rank}(R) = mn$ and

$$m(n+1) - 1 \leq \text{rank}(S) \leq m(n+1), \tag{15}$$

Consider $u(0) = K(0)x(0) + \varepsilon(0)\|x(0)\|$ with $\varepsilon(0) \in B_\delta$. We aim to show that there exists some $\varepsilon(0) \in B_\delta$ such that $\text{rank}(U_{1-T,n+1,T-n}) = m(n+1)$. We use the definition of S and partition this matrix as

$$U_{1-T,n+1,T-n} = \left[\begin{array}{c|c} S & \begin{array}{c} U_{-n,n,1} \\ \hline u(0) \end{array} \end{array}\right].$$

Noting (15), we consider two cases: (i) $\mathrm{rank}(S) = m(n+1)$ and (ii) $\mathrm{rank}(S) = m(n+1) - 1$. For (i), it follows from the above equation that $U_{1-T,n+1,T-n}$ is full rank for any $\varepsilon(0) \in B_\delta$. For (ii), we have

$$m(n+1) - 1 \leq \mathrm{rank}(U_{1-T,n+1,T-n}) \leq m(n+1).$$

We now proceed by contradiction. Suppose that $U_{1-T,n+1,T-n}$ has rank $m(n+1) - 1$ for all $\varepsilon(0) \in B_\delta$. This means that for all points inside the ball B_δ, the last column of $U_{1-T,n+1,T-n}$ must lie inside the column space of the matrix S, i.e.,

$$\left[\frac{U_{-n,n,1}}{f_0 + \varepsilon_0 \|x(0)\|}\right] \in \mathrm{im}\, S, \quad \forall \varepsilon_0 \in B_\delta, \tag{16}$$

where $\mathrm{im}\, S$ denotes the image of S and $f_0 := K(0)x(0)$, which implies for $\varepsilon_0 = 0$ that

$$\left[\frac{U_{-n,n,1}}{f_0}\right] \in \mathrm{im}\, S.$$

Let some $0 < \rho \leq \delta \|x(0)\|$, then any point $\frac{\rho}{\|x(0)\|} e_i$ with e_i the i-th unit vector of $\mathbb{R}^m$ belongs to the ball B_δ. Therefore, it follows from (16) that

$$\left[\frac{U_{-n,n,1}}{f_0 + \rho e_i}\right] \in \mathrm{im}\, S, \quad \forall i = 1, \ldots, m.$$

We then deduce that the augmented matrix

$$\left[\begin{array}{c|c|ccc} S & \dfrac{U_{-n,n,1}}{f_0} & \dfrac{U_{-n,n,1}}{f_0 + \rho e_1} & \ldots & \dfrac{U_{-n,n,1}}{f_0 + \rho e_m} \end{array}\right]$$

has rank equal to $m(n+1) - 1$. By elementary column operations, the rank of the following matrix

$$M := \left[\begin{array}{c|c|c} S & \dfrac{U_{-n,n,1}}{f_0} & \dfrac{\mathbb{0}}{\rho I_m} \end{array}\right],$$

is equal to $m(n+1) - 1$ as well. We use the definitions of S and R to get

$$M = \left[\begin{array}{c|c} R & \mathbb{0} \\ \hline \left[\begin{array}{c|c} U_{1-T+n,1,T-n-1} & f_0 \end{array}\right] & \rho I_m \end{array}\right].$$

Note that the above matrix is block lower triangular and $\mathrm{rank}(M) = \mathrm{rank}(R) + m = m(n+1)$. Thus we have reached to a contradiction, which means that $U_{1-T,n+1,T-n}$ is full rank for $\varepsilon(0) \in B_\delta$. By similar reasonings, it holds that for any $k > 0$ and any input sequence $u(k-T), \ldots, u(k-1)$ such that $U_{k-T,n+1,T-n}$ has full rank, there exists some $\varepsilon(k) \in B_\delta$ such that the Hankel matrix $U_{k-T+1,n+1,T-n}$ has full row rank, i.e. the input sequence $u(k-T+1), \ldots, u(k)$ with $u(k) = K(k)x(k) + \varepsilon(k)\|x(k)\|$ is persistently exciting of order $n+1$ which concludes the lemma. $\square$

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