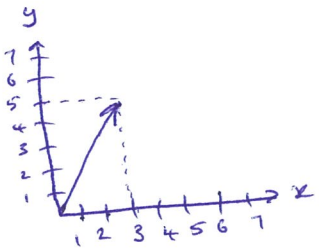
2/12/14

Vectors

A vector $\underline{v}$ is a collection of coordinates, and, in 2 dimensions is written like this:

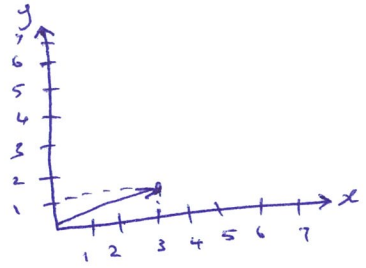
$$\underline{v} = \begin{bmatrix} 3 \\ 5 \end{bmatrix} \text{ or just } v = (3, 5)$$

In cartesian coordinates, v looks like this:



$$\begin{bmatrix} 3 \\ 5 \end{bmatrix}$$ — 3: x coordinate; 5: y coordinate

for $u = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$,



Vectors can be added, subtracted and multiplied by scalar values like 0.3, $\frac{1}{2}$, 1.5 etc.

Adding vectors

For adding and subtracting vectors, everything is done on all coordinates.

Adding vectors continued

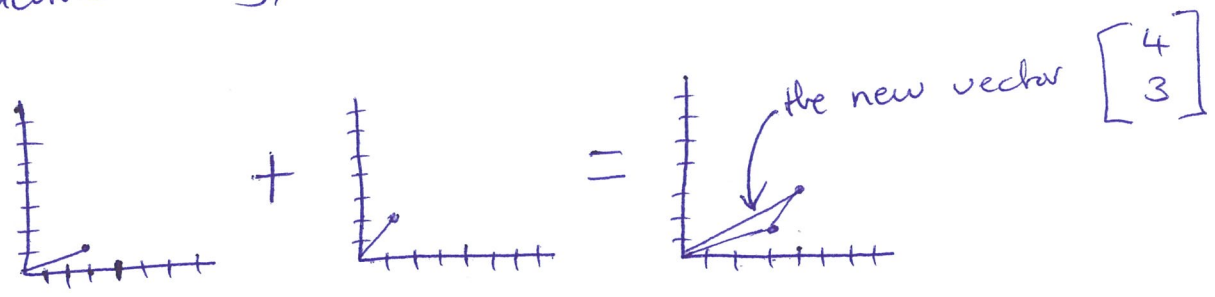
If we have

$$a = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Adding them together looks like this:

$$a + b = \begin{bmatrix} 3 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3+1 \\ 1+2 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

Geometrically, it looks like this:



Multiplying Vectors

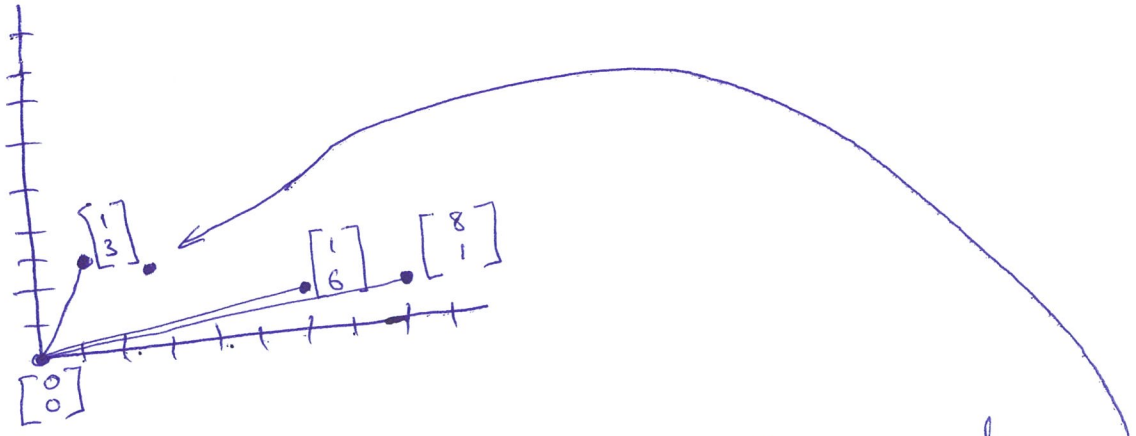
Multiplying vectors together is complicated, unless you just need to multiply by a scalar like 0.5, $\frac{1}{2}$ or 5.9 or 10. This is all we need.

$$u = \begin{bmatrix} 6 \\ 4 \end{bmatrix} \quad \text{so} \quad \frac{1}{2}u = \frac{u}{2} = u \times 0.5 = \begin{bmatrix} 6/2 \\ 4/2 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$1.5u = \begin{bmatrix} 6 \times 1.5 \\ 4 \times 1.5 \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \end{bmatrix}$$

One thing we will do often is average lots of vectors.

If you have $a = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ $b = \begin{bmatrix} 1 \\ 6 \end{bmatrix}$ $c = \begin{bmatrix} 8 \\ 1 \end{bmatrix}$ $d = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, what is the average vector and what does it look like?



Same as averaging a set of numbers.

$$(a + b + c + d) / 4 =$$

$$\left(\begin{bmatrix} 1 \\ 3 \end{bmatrix} + \begin{bmatrix} 1 \\ 6 \end{bmatrix} + \begin{bmatrix} 8 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right) \times \frac{1}{4}$$

$$= \begin{bmatrix} 1 + 1 + 8 + 0 \\ 3 + 6 + 1 + 0 \end{bmatrix} \times \frac{1}{4} = \begin{bmatrix} 10 \\ 10 \end{bmatrix} \times \frac{1}{4} = \begin{bmatrix} 10/4 \\ 10/4 \end{bmatrix} = \begin{bmatrix} 2.5 \\ 2.5 \end{bmatrix}$$