4]

$$\min_{x \in \mathbb{R}^n} \frac{1}{2} x^T Q x + c^T x \quad \text{s.d.} \quad \begin{array}{l} Ax - a \leq 0 \\ Bx - b = 0 \end{array}$$

$$L(x, \lambda, \mu) = \frac{1}{2} x^T Q x + c^T x + \lambda^T (Ax - a) + \mu^T (Bx - b)$$

$$g(\lambda, \mu) = \inf_{x \in \mathbb{R}^n} L(x, \lambda, \mu) \quad \rightsquigarrow \quad \max_{\lambda, \mu} g(\lambda, \mu) \quad \text{s.d. } \lambda \geq 0$$

$$\nabla_x L(x^*, \lambda, \mu) = 0 \quad \Longrightarrow \quad x^* = -Q^{-1}(c + A^T \lambda + B^T \mu)$$

$$\hookrightarrow g(\lambda, \mu) = \frac{1}{2} \begin{pmatrix} \lambda \\ \mu \end{pmatrix}^T P \begin{pmatrix} \lambda \\ \mu \end{pmatrix} + d^T \begin{pmatrix} \lambda \\ \mu \end{pmatrix} - \frac{1}{2} c^T Q^{-1} c$$

$$P = -\begin{pmatrix} AQ^{-1}A^T & AQ^{-1}B^T \\ BQ^{-1}A^T & BQ^{-1}B^T \end{pmatrix}, \quad d = \begin{pmatrix} AQ^{-1}c + a \\ BQ^{-1}c + b \end{pmatrix}$$

2)

$$A = \left(k(x_i, x_j)\right)_{1 \le i,j \le m} \in \mathbb{R}^{m \times m}$$

$$G = \left((\psi_i, \psi_j)_{\mathcal{H}} \right)_{1 \le i,j \le m}, \quad \mathbb{1}_m = \frac{1}{m} \begin{pmatrix} 1 & - & 1 \\ | & \ddots & | \\ 1 & - & 1 \end{pmatrix} \in \mathbb{R}^{m \times m}$$

$$(\psi_i, \psi_j)_{\mathcal{H}} = \left(\Phi(x_i) - \overline{\Phi}, \Phi(x_j) - \overline{\Phi} \right)_{\mathcal{H}}$$

$$= \underbrace{\left(\Phi(x_i), \Phi(x_j) \right)_{\mathcal{H}}}_{= k(x_i, x_j)} - \frac{1}{m} \sum_{s=1}^{m} (\Phi(x_i), \Phi(x_s))_{\mathcal{H}}$$

$$- \frac{1}{m} \sum_{s=1}^{m} (\Phi(x_j), \Phi(x_s))_{\mathcal{H}} + \frac{1}{m^2} \sum_{s,t=1}^{m} (\Phi(x_s), \Phi(x_t))_{\mathcal{H}}$$

$$G = A - \mathbb{1}_m A - A \mathbb{1}_m + \mathbb{1}_m A \mathbb{1}_m$$

· $\lambda \neq 0$ EW of C with EV φ s.t. $\|\varphi\|_{\mathcal{H}} = 1$

$$v_i = \frac{1}{\sqrt{\lambda}} \left(\varphi, \psi_i \right)_{\mathcal{H}} \qquad i = 1, \dots, m$$

$$G v = \lambda v \;, \quad \|v\|_2 = 1$$

$$\|v\|_2 = \sqrt{v^T v} = \sqrt{\sum_{i=1}^{m} \left(\frac{1}{\sqrt{\lambda}} (\varphi, \psi_i)_{\mathcal{H}} \right)^2}$$

$$= \sqrt{\frac{1}{\lambda} \Big(\underbrace{\sum_{i=1}^{m} (\psi_i, \varphi)_{\mathcal{H}} \, \psi_i}_{C\varphi = \lambda \varphi} , \varphi \Big)_{\mathcal{H}}} = \|\psi\|_{\mathcal{H}} = 1$$

- $\lambda \neq 0$ EW of G with EV v s.t. $\|v\|_2 = 1$

$$\varphi = \frac{1}{\sqrt{\lambda}} \sum_{i=1}^{m} v_i \cdot \psi_i$$

$$\hookrightarrow \quad C\varphi = \lambda \varphi \,, \quad \|\varphi\|_{\mathcal{H}} = 1$$

$$\hookrightarrow \quad \beta_j = \begin{pmatrix} (u_1, \psi_j)_{\mathcal{H}} \\ \vdots \\ (u_p, \psi_j)_{\mathcal{H}} \end{pmatrix}$$

$$(u_\ell, \psi_j)_{\mathcal{H}} = \frac{1}{\sqrt{\lambda_\ell}} \sum_{i=1}^{m} v_{\ell_i} \overbrace{(\psi_i, \psi_j)_{\mathcal{H}}}^{= G_{i,j}}$$

3)

$$\rho(z) = \|\underline{\Psi}(z) - P_{\underline{\Psi}}(x)\|_{\mathcal{H}}^2$$

$$= \underbrace{(\underline{\Psi}(z), \underline{\Psi}(z))_{\mathcal{H}}}_{= k(z,z) = 1} - 2(\underline{\Psi}(z), P_{\underline{\Psi}}(x))_{\mathcal{H}} + C$$

$$\Leftrightarrow \max_z \underbrace{(\underline{\Psi}(z), P_{\underline{\Psi}}(x))_H}_{= \sum_{j=1}^m \gamma_j \underbrace{k(z, x_j)}_{= \exp(-c\|z - x_j\|^2)}} =: \tilde{\rho}(z)$$

$\hookrightarrow \quad \nabla_z \rho^2(z) \overset{!}{=} 0$

$$= -2c \sum_{j=1}^{m} \gamma_j \exp(-c\|z-x_j\|^2)(z-x_j)$$

$$\Rightarrow \quad z = \frac{\sum_{j=1}^{m} r_j \exp(-c\|z-x_j\|^2)\, x_j}{\sum_{j=1}^{m} r_j \exp(-c\|z-x_j\|^2)}$$

1] $\Phi(x) = K(\cdot, x) \in \mathbb{R}^{\mathcal{X}}$

$$S = \left\{ \sum_{i=1}^{n} c_i K(\cdot, x_j) \;\middle|\; n \in \mathbb{N},\ c_j \in \mathbb{R},\ x_j \in \mathcal{X} \right\}$$

$$(f, g)_S = \sum_{i=1}^{n_f} \sum_{j=1}^{n_g} \alpha_i \beta_j K(x_i, y_j)$$

$$f = \sum_{i=1}^{n_f} \alpha_i K(\cdot, x_i)$$

$$\mathcal{H} = \overline{S}$$

$$g = \sum_{j=1}^{n_g} \beta_j K(\cdot, y_j)$$