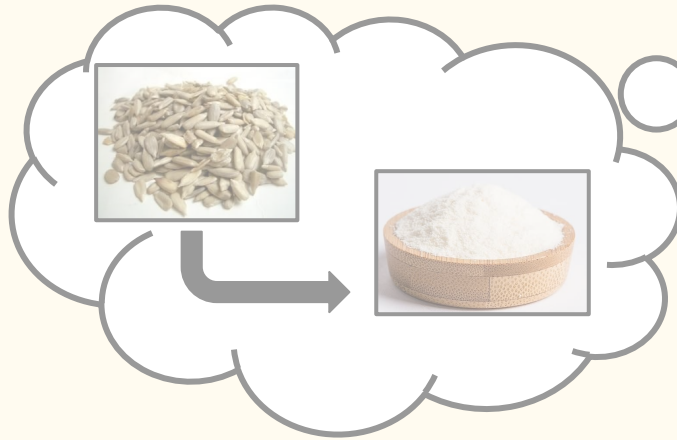


Parte 4

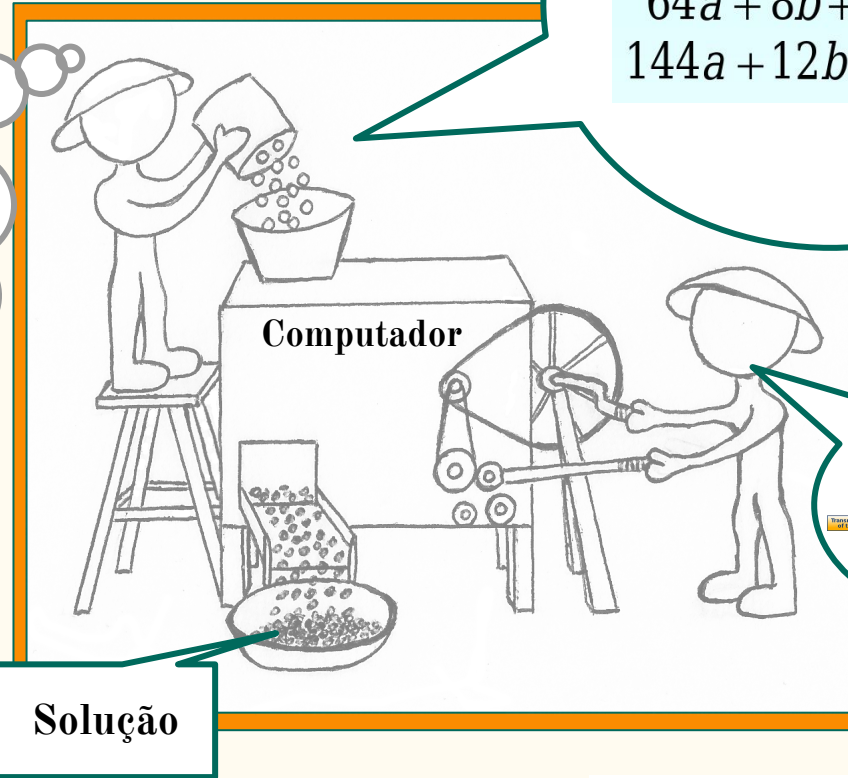
Resolução de Sistemas Lineares - 2

CI1164 - Introdução à Computação Científica
Profs. Armando Delgado e Guilherme Derenievicz
Departamento de Informática - UFPR

Visão geral da disciplina:



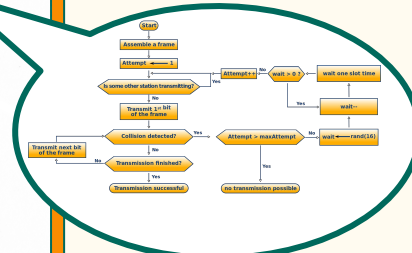
Descrição do Problema



Solução

$$\begin{aligned}25a + 5b + c &= 106 \\64a + 8b + c &= 177 \\144a + 12b + c &= 600\end{aligned}$$

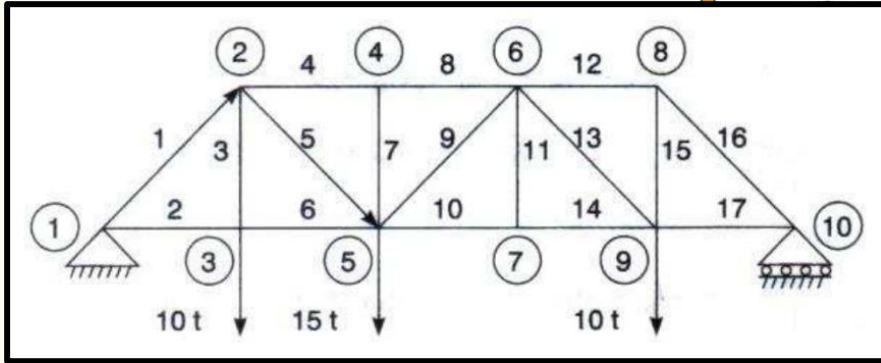
Sistemas Lineares



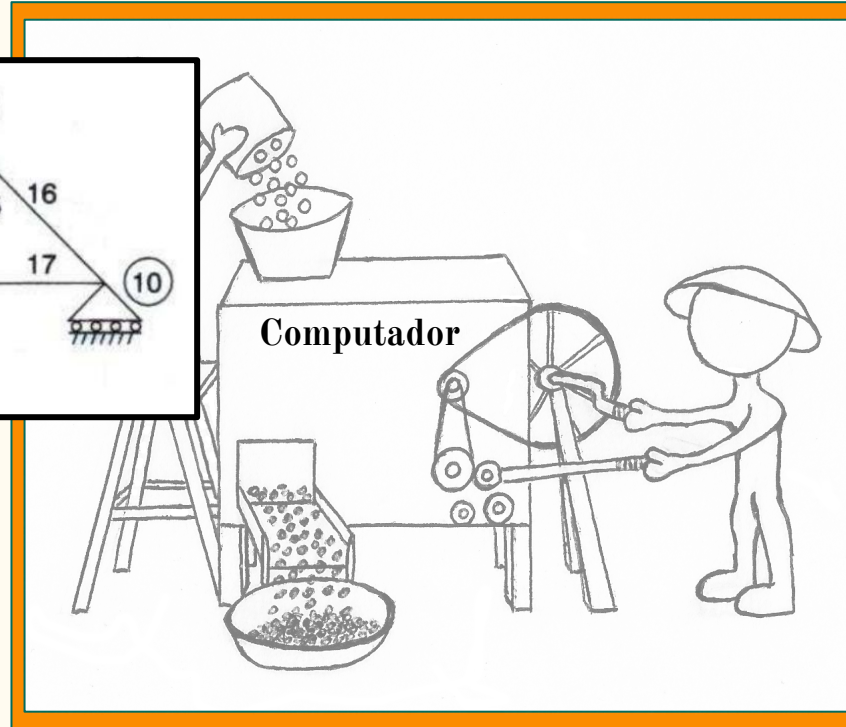
Método Numérico

Resolução de Sistemas Lineares

Fonte: Cálculo Numérico (Ruggiero & Lopes)

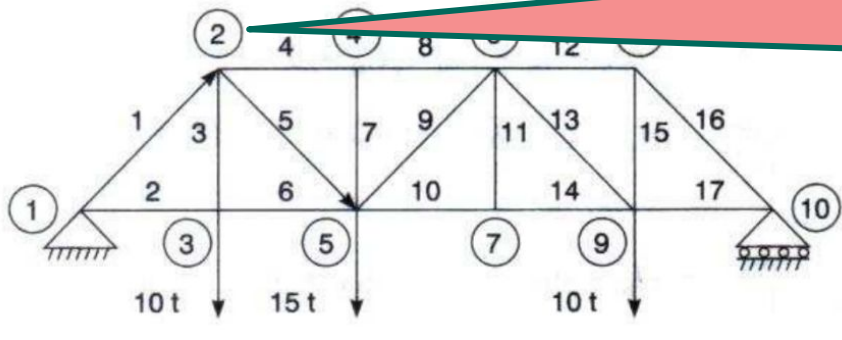


Descrição do Problema



Resolução de Sistemas Lineares

Fonte: Cálculo Numérico (Ruggiero & Lopes)



Descrição do Problema

$$\begin{cases} -\alpha f_1 + f_4 + \alpha f_5 = 0 \\ -\alpha f_1 - f_3 - \alpha f_5 = 0 \\ -f_2 + f_6 = 0 \\ f_3 - 10 = 0 \\ -f_4 + f_8 = 0 \\ -f_7 = 0 \\ -\alpha f_5 - f_6 + \alpha f_9 + f_{10} = 0 \\ \alpha f_5 + f_7 + \alpha f_9 - 15 = 0 \\ -f_8 - \alpha f_9 + f_{12} + \alpha f_{13} = 0 \\ -\alpha f_9 - f_{11} - \alpha f_{13} = 0 \\ -f_{10} + f_{14} = 0 \\ f_{11} = 0 \\ -f_{12} + \alpha f_{16} = 0 \\ -f_{15} - \alpha f_{16} = 0 \\ -\alpha f_{13} - f_{14} + f_{17} = 0 \\ \alpha f_{13} + f_{15} - f_{10} = 0 \\ \alpha f_{16} - f_{17} = 0 \end{cases}$$

$$\alpha f_5 = 0$$

$$\alpha f_5 = 0$$

Resolução de Sistemas Lineares

$$A = \begin{bmatrix} -\alpha & 0 & 0 & 1 & \alpha & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\alpha & 0 & -1 & 0 & -\alpha & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\alpha & -1 & 0 & 0 & \alpha & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha & 0 & 1 & 0 & \alpha & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -\alpha & 0 & 0 & 1 & \alpha & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\alpha & 0 & -1 & 0 & -\alpha & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & \alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -\alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\alpha & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\alpha & -1 \end{bmatrix}$$

Resolução de Sistemas Lineares

$$A = \begin{bmatrix} -\alpha & 0 & 0 & 1 & \alpha & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\alpha & 0 & -1 & 0 & -\alpha & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\alpha & -1 & 0 & 0 & \alpha & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha & 0 & 1 & 0 & \alpha & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -\alpha & 0 & 0 & 1 & \alpha & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\alpha & 0 & -1 & 0 & -\alpha & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & \alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -\alpha & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\alpha & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\alpha & -1 & 0 \end{bmatrix}$$

Matriz Esparsa

Resolução de Sistemas Lineares

$$A = \begin{bmatrix} -\alpha & 0 & 0 & 1 & \alpha & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\alpha & 0 & -1 & 0 & -\alpha & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\alpha & -1 & 0 & 0 & \alpha & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha & 0 & 1 & 0 & \alpha & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -\alpha & 0 & 0 & 1 & \alpha & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\alpha & 0 & -1 & 0 & -\alpha & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & \alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -\alpha & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\alpha & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\alpha & -1 & 0 & 0 \end{bmatrix}$$

Matriz Esparsa

Matrizes k-Diagonais

Matriz de Banda

Matriz Esparsa

Matrizes k-Diagonais

$$A = \begin{pmatrix} -\alpha & 0 & 0 & 1 & \alpha & 0 & 0 & 0 & 0 & 0 \\ -\alpha & 0 & -1 & 0 & -\alpha & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\alpha & -1 & 0 & \alpha & 1 & 0 \\ 0 & 0 & 0 & 0 & \alpha & 0 & 1 & 0 & \alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & -\alpha & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\alpha & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\alpha & -1 \end{pmatrix}$$

Matriz Esparsa

Matriz de Banda

Matriz de Banda

Matriz Esparsa

$$\mathbf{i} = \mathbf{j}$$

Matrizes k-Diagonais

$i = j + 4$ →

Matriz de Banda

Matriz Esparsa

$i = j$

$$A = \begin{bmatrix} -\alpha & 0 & 0 & 1 & \alpha & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\alpha & 0 & -1 & 0 & -\alpha & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\alpha & -1 & 0 & 0 & \alpha & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha & 0 & 1 & 0 & \alpha & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -\alpha & 0 & 0 & 1 & \alpha & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\alpha & 0 & -1 & 0 & -\alpha & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & \alpha & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -\alpha & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\alpha & -1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\alpha & -1 & 0 & 0 & 0 \end{bmatrix}$$

Matrizes k-Diagonais

Matriz de Banda

$i = j + 4$ →

$i = j + 7$ →

Matriz Esparsa

$i = j$

$A =$

$-\alpha$	0	0	1	α	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$-\alpha$	0	-1	0	$-\alpha$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	-1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	-1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	$-\alpha$	-1	0	0	α	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	α	0	1	0	α	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	-1	$-\alpha$	0	0	1	α	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	$-\alpha$	0	-1	0	$-\alpha$	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	-1	0	0	0	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	α	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	$-\alpha$	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	$-\alpha$	-1	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	α	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$-\alpha$	-1	0	0	0

Matrizes k-Diagonais

Matriz de Banda

$i = j + 4$ →

$i = j + 7$ →

**Se $i > j + 3$
Então $A[i][j] = 0$**

Matriz Esparsa

$i = j$

$A =$

$-\alpha$	0	0	1	α	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$-\alpha$	0	-1	0	$-\alpha$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	-1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	-1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	$-\alpha$	-1	0	0	α	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	α	0	1	0	α	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	-1	$-\alpha$	0	0	1	α	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	$-\alpha$	0	-1	0	$-\alpha$	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	-1	0	0	0	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	α	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	-1	$-\alpha$	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	$-\alpha$	-1	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0	0	0	α	0	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	$-\alpha$	-1	0	0	0	0	0

Diagram illustrating the banded structure of a sparse matrix A .

The matrix A is shown with its non-zero elements highlighted. The matrix is banded, with non-zero elements concentrated along the main diagonal and a few off-diagonal elements.

Key observations from the diagram:

- The matrix is sparse, with many zero elements.
- The non-zero elements are concentrated along the main diagonal and a few off-diagonal elements.
- The matrix is banded, with non-zero elements concentrated within a certain distance from the main diagonal.
- The diagram shows the relationship between the indices i and j and the value of $A[i][j]$.
- Condition: $\text{Se } i < j - 4 \text{ Entao } A[i][j] = 0$
- Labels: **Matriz Esparsa** (Sparse Matrix), **Matriz de Banda** (Banded Matrix).

Se	$i < j - 4$
Então	$A[i][j] = 0$

```
Se      i > j + 3
Então   A[i][j] = 0
```

Matriz Esparsa

$$\mathbf{i} = \mathbf{j}$$

Matrizes k-Diagonais

Matriz de Banda 8

```
Se      i < j - 4
Então  A[i][j] = 0
```

Se $i > j + 3$
Então $A[i][j] = 0$

Matriz Esparsa

$$\mathbf{i} = \mathbf{j}$$

Matrizes k-Diagonais

Matriz de Banda $p+q+1$

Se $i < j - p$
Então $A[i][j] = 0$

Se $i > j + q$
Então $A[i][j] = 0$

Matriz Esparsa

$i = j + 4$

$i = j + 7$

$i = j$

$A =$

$-\alpha$	0	0	1	α	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$-\alpha$	0	-1	0	$-\alpha$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	-1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	-1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	$-\alpha$	-1	0	0	α	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	α	0	1	0	α	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	-1	$-\alpha$	0	0	1	α	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	$-\alpha$	0	-1	0	$-\alpha$	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	-1	0	0	0	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	α	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	-1	$-\alpha$	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	$-\alpha$	-1	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0	α	0	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	$-\alpha$	-1	0	0	0	0	0	0

Matrizes k-Diagonais

Matriz de Banda $p+q+1$

Se $i < j - p$
Então $A[i][j] = 0$

Se $i > j + q$
Então $A[i][j] = 0$

Matriz Esparsa

$i = j + 4$

$i = j + 7$

$i = j$

$A =$

$-\alpha$	0	0	1	α	0	0	0	0	0	0	0	0	0	0
$-\alpha$	0	-1	0	$-\alpha$	0	0	0	0	0	0	0	0	0	0
0	-1	0	0	0	1	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	-1	0	0	0	1	0	0	0	0	0	0	0
0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0
0	0	0	0	$-\alpha$	-1	0	0	α	1	0	0	0	0	0
0	0	0	0	α	0	1	0	α	0	0	0	0	0	0
0	0	0	0	0	0	-1	$-\alpha$	0	0	1	α	0	0	0
0	0	0	0	0	0	0	-1	0	$-\alpha$	0	0	0	0	0
0	0	0	0	0	0	0	-1	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	-1	0	0	0	α	0	0
0	0	0	0	0	0	0	0	0	0	-1	$-\alpha$	0	0	0
0	0	0	0	0	0	0	0	0	$-\alpha$	-1	0	0	1	0
0	0	0	0	0	0	0	0	α	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	$-\alpha$	-1	0	0

Matrizes Tridiagonais ($p = q = 1$)

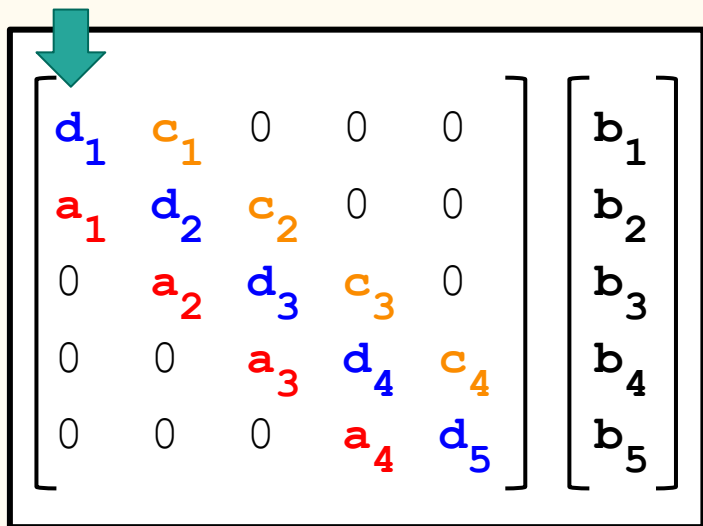
$$\begin{bmatrix} d_1 & c_1 & 0 & 0 & 0 \\ a_1 & d_2 & c_2 & 0 & 0 \\ 0 & a_2 & d_3 & c_3 & 0 \\ 0 & 0 & a_3 & d_4 & c_4 \\ 0 & 0 & 0 & a_4 & d_5 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \end{bmatrix}$$


Matrizes Tridiagonais ($p = q = 1$)

$$\begin{bmatrix} d_1 & c_1 & 0 & 0 & 0 \\ a_1 & d_2 & c_2 & 0 & 0 \\ 0 & a_2 & d_3 & c_3 & 0 \\ 0 & 0 & a_3 & d_4 & c_4 \\ 0 & 0 & 0 & a_4 & d_5 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \end{bmatrix}$$

```
/* Seja um S.L. de ordem 'n'
*/
void eliminacaoGauss( double **A, double *b, u
    /* para cada linha a partir da primeira */
    for (int i=0; i < n; ++i) {
        for(int k=i+1; k < n; ++k) {
            double m = A[k][i] / A[i][i];
            A[k][i] = 0.0;
            for(int j=i+1; j < n; ++j)
                A[k][j] -= A[i][j] * m;
            b[k] -= b[i] * m;
        }
    }
}
```

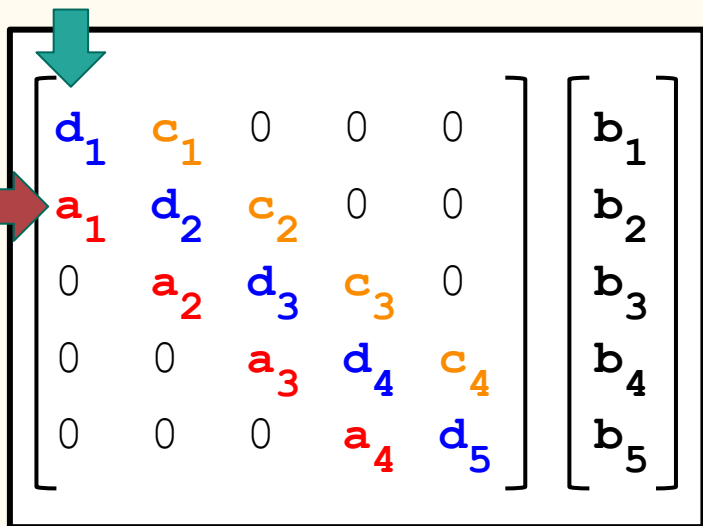
Matrizes Tridiagonais ($p = q = 1$)


$$\begin{bmatrix} d_1 & c_1 & 0 & 0 & 0 \\ a_1 & d_2 & c_2 & 0 & 0 \\ 0 & a_2 & d_3 & c_3 & 0 \\ 0 & 0 & a_3 & d_4 & c_4 \\ 0 & 0 & 0 & a_4 & d_5 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \end{bmatrix}$$



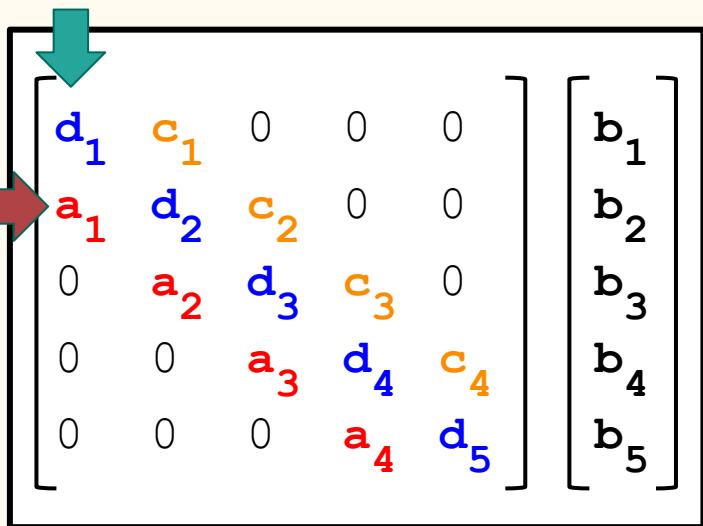
```
/* Seja um S.L. de ordem 'n'
*/
void eliminacaoGauss( double **A, double *b, u
    /* para cada linha a partir da primeira */
    for (int i=0; i < n; ++i) {
        for(int k=i+1; k < n; ++k) {
            double m = A[k][i] / A[i][i];
            A[k][i] = 0.0;
            for(int j=i+1; j < n; ++j)
                A[k][j] -= A[i][j] * m;
            b[k] -= b[i] * m;
        }
    }
}
```

Matrizes Tridiagonais ($p = q = 1$)


$$\begin{bmatrix} d_1 & c_1 & 0 & 0 & 0 \\ a_1 & d_2 & c_2 & 0 & 0 \\ 0 & a_2 & d_3 & c_3 & 0 \\ 0 & 0 & a_3 & d_4 & c_4 \\ 0 & 0 & 0 & a_4 & d_5 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \end{bmatrix}$$

```
/* Seja um S.L. de ordem 'n'
*/
void eliminacaoGauss( double **A, double *b, u
    /* para cada linha a partir da primeira */
    for (int i=0; i < n; ++i) {
        for(int k=i+1; k < n; ++k) {
            double m = A[k][i] / A[i][i];
            A[k][i] = 0.0;
            for(int j=i+1; j < n; ++j)
                A[k][j] -= A[i][j] * m;
            b[k] -= b[i] * m;
        }
    }
}
```


Matrizes Tridiagonais ($p = q = 1$)


$$\begin{bmatrix} d_1 & c_1 & 0 & 0 & 0 \\ a_1 & d_2 & c_2 & 0 & 0 \\ 0 & a_2 & d_3 & c_3 & 0 \\ 0 & 0 & a_3 & d_4 & c_4 \\ 0 & 0 & 0 & a_4 & d_5 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \end{bmatrix}$$

```
/* Seja um S.L. de ordem 'n'
*/
void eliminacaoGauss( double **A, double *b, u
    /* para cada linha a partir da primeira */
    for (int i=0; i < n; ++i) {
        for(int k=i+1; k < n; ++k) { k = i+1
            double m = A[k][i] / A[i][i];
            A[k][i] = 0.0;
            for(int j=i+1; j < n; ++j)
                A[k][j] -= A[i][j] * m;
            b[k] -= b[i] * m;
        }
    }
}
```

Matrizes Tridiagonais ($p = q = 1$)

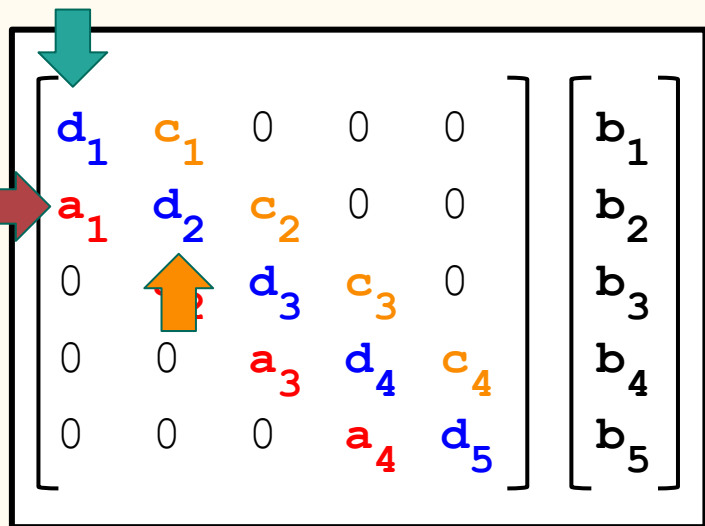



Diagram illustrating a 5x5 tridiagonal matrix system $Ax = b$. The matrix A is shown with elements $d_1, c_1, 0, 0, 0$ in the first row, $a_1, d_2, c_2, 0, 0$ in the second row, $0, d_3, c_3, 0, 0$ in the third row, $0, 0, a_3, d_4, c_4$ in the fourth row, and $0, 0, 0, a_4, d_5$ in the fifth row. The vector b is shown on the right with elements b_1, b_2, b_3, b_4, b_5 . Arrows indicate the structure: a green arrow points to the first row, a red arrow points to the first column, and an orange arrow points to the third row.

```
/* Seja um S.L. de ordem 'n'
*/
void eliminacaoGauss( double **A, double *b, u
    /* para cada linha a partir da primeira */
    for (int i=0; i < n; ++i) {
        for(int k=i+1; k < n; ++k) { k = i+1
            double m = A[k][i] / A[i][i];
            A[k][i] = 0.0;
             for(int j=i+1; j < n; ++j)
                A[k][j] -= A[i][j] * m;
            b[k] -= b[i] * m;
        }
    }
}
```

Matrizes Tridiagonais ($p = q = 1$)

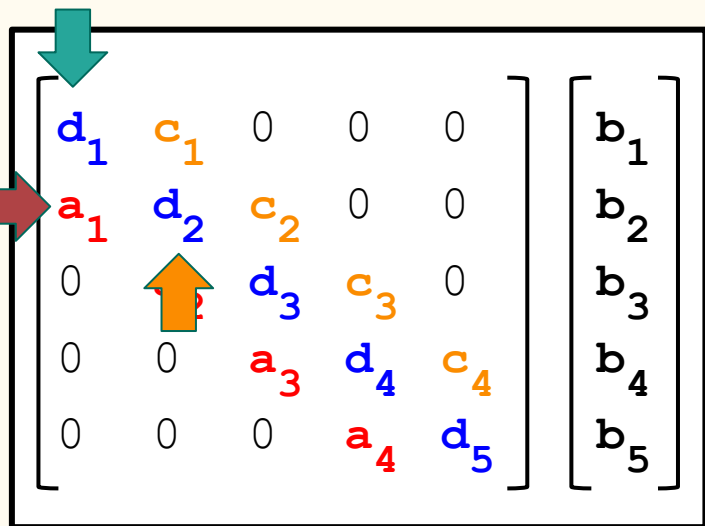
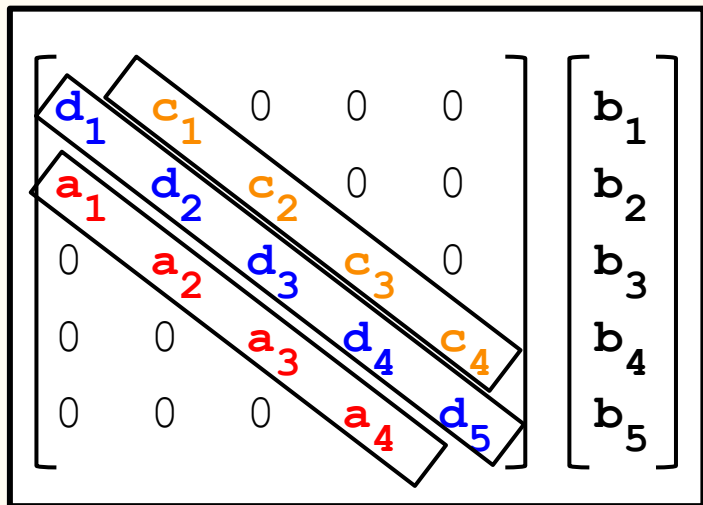


Diagram illustrating a 5x5 tridiagonal matrix system $Ax = b$. The matrix A is shown with elements d_i on the main diagonal, c_i on the super-diagonal, and a_i on the sub-diagonal. The right-hand side vector b is shown. Arrows indicate the structure: a green arrow points to the main diagonal, a red arrow points to the sub-diagonal, and an orange arrow points to the super-diagonal.

$$\begin{bmatrix} d_1 & c_1 & 0 & 0 & 0 \\ a_1 & d_2 & c_2 & 0 & 0 \\ 0 & a_2 & d_3 & c_3 & 0 \\ 0 & 0 & a_3 & d_4 & c_4 \\ 0 & 0 & 0 & a_4 & d_5 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \end{bmatrix}$$

```
/* Seja um S.L. de ordem 'n'
*/
void eliminacaoGauss( double **A, double *b, u
/* para cada linha a partir da primeira */
for (int i=0; i < n; ++i) {
    for(int k=i+1; k < n; ++k) { k = i+1
        double m = A[k][i] / A[i][i];
        A[k][i] = 0.0;
        for(int j=i+1; j < n; ++j) j = i+1
            A[k][j] -= A[i][j] * m;
            b[k] -= b[i] * m;
        }
    }
}
```

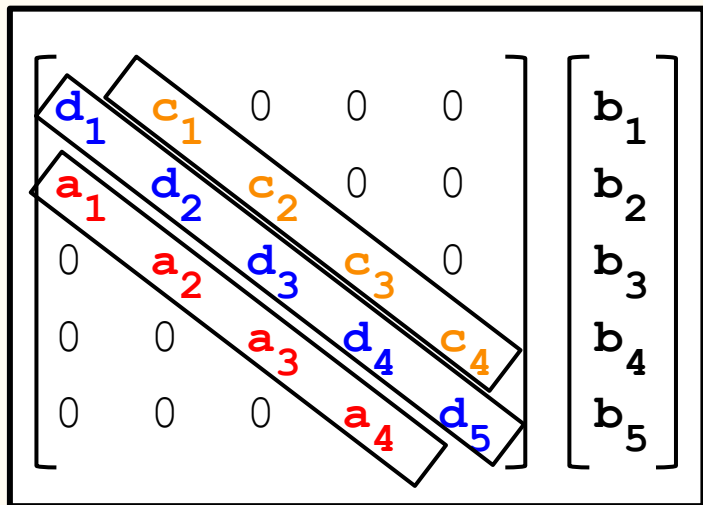
Matrizes Tridiagonais ($p = q = 1$)



```
double b[5];  
double d[5];  
double a[4];  
double c[4];
```

```
/* Seja um S.L. de ordem 'n' */  
void eliminacaoGauss( double **A, double *b, u  
    /* para cada linha a partir da primeira */  
    for (int i=0; i < n; ++i) {  
        for(int k=i+1; k < n; ++k) { k = i+1  
            double m = A[k][i] / A[i][i];  
            A[k][i] = 0.0;  
            for(int j=i+1; j < n; ++j) j = i+1  
                A[k][j] -= A[i][j] * m;  
            b[k] -= b[i] * m;  
        }  
    }  
}
```

Matrizes Tridiagonais ($p = q = 1$)

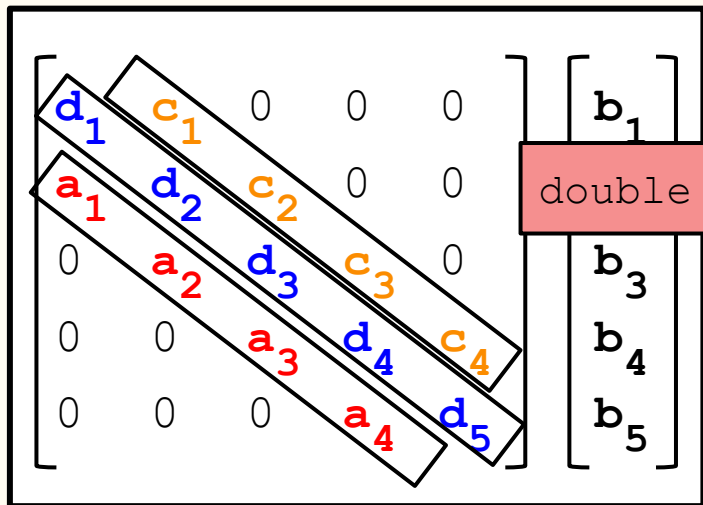


```
double b[5];  
double d[5];  
double a[4];  
double c[4];
```

```
double *d, double *a, double *c
```

```
/* Seja um S.L. de ordem n */  
void eliminacaoGauss( double **A, double *b, unsigned int n )  
{  
    /* para cada linha a partir da primeira */  
    for (int i=0; i < n; ++i) {  
        for(int k=i+1; k < n; ++k) {  
            double m = A[k][i] / A[i][i];  
            A[k][i] = 0.0;  
            for(int j=i+1; j < n; ++j) {  
                A[k][j] -= A[i][j] * m;  
            b[k] -= b[i] * m;  
        }  
    }  
}
```

Matrizes Tridiagonais ($p = q = 1$)



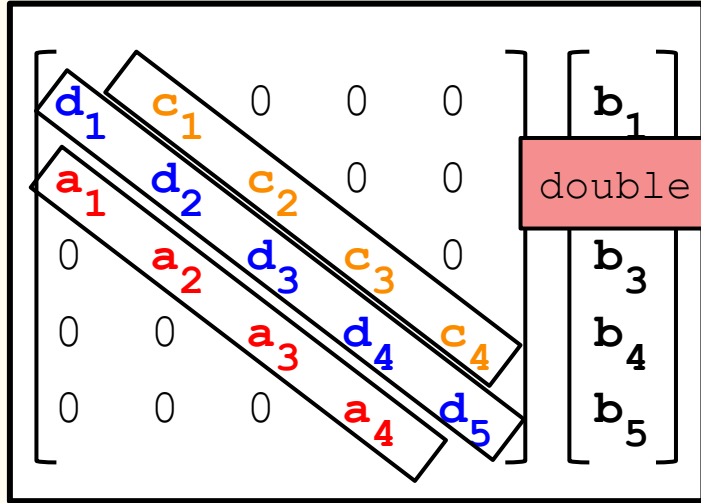
```
double b[5];
double d[5];
double a[4];
double c[4];
```

```
double *d, double *a, double *c
```

```
double m = a[i] / d[i];
```

```
void eliminaGauss( double **A, double *b, u
/* para cada linha a partir da primeira */
for (int i=0; i < n; ++i) {
for(int k=i+1; k < n; ++k) {
    double m = A[k][i] / A[i][i];
    A[k][i] = 0.0;
for(int j=i+1; j < n; ++j)
    A[k][j] -= A[i][j] * m;
    b[k] -= b[i] * m;
}
}
```


Matrizes Tridiagonais ($p = q = 1$)



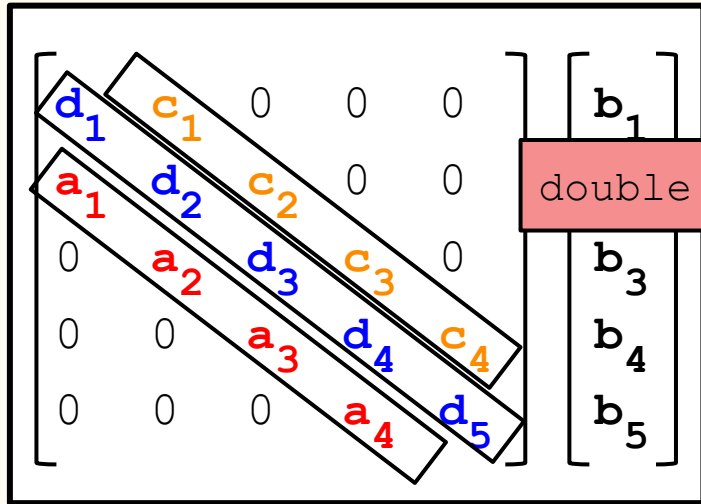
```
double b[5];  
double d[5];  
double a[4];  
double c[4];
```

```
double *d, double *a, double *c
```

```
double m = a[i] / d[i];
```

```
void elimGauss( double **A, double *b, u  
/* para cada linha a primeira */  
for (int i=0; i < n; ++i) {  
    a[i] = 0.0;  
    for(int k=i+1; k < n; ++k) {  
        double m = A[k][i] / A[i][i];  
        A[k][i] = 0.0;  
        for(int j=i+1; j < n; ++j)  
            A[k][j] -= A[i][j] * m;  
        b[k] -= b[i] * m;  
    }  
}
```

Matrizes Tridiagonais ($p = q = 1$)



```
double b[5];
double d[5];
double a[4];
double c[4];
```

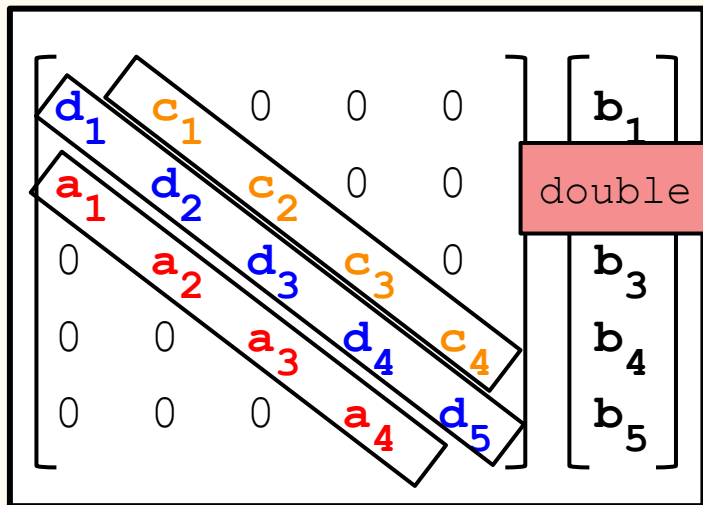
```
double *d, double *a, double *c
```

```
double m = a[i] / d[i];
```

```
void elimGauss( double **A, double *b, u
/* para calcular a primeira */
for (int i=0; i < n; ++i) {
    a[i] = 0.0;
    for(int k=i+1; k < n; ++k) {
        double m = A[k][i] / A[i][i];
        A[k][i] = 0.0;
        for(int j=i+1; j < n; ++j)
            A[k][j] -= A[i][j] * m;
        b[k] -= b[i] * m;
    }
}
```

```
d[i+1] -= c[i]*m;
```

Matrizes Tridiagonais ($p = q = 1$)



```
double b[5];
double d[5];
double a[4];
double c[4];
```

```
double *d, double *a, double *c
```

```
double m = a[i] / d[i];
```

```
void elimGauss( double **A, double *b, u
/* para cada linha a primeira */
for (int i=0; i < n; ++i) {
    a[i] = 0.0;
    for(int k=i+1; k < n; ++k) {
        double m = A[k][i] / A[i][i];
        A[k][i] = 0.0;
        for(int j=i+1; j < n; ++j)
            A[k][j] -= A[i][j] * m;
        b[k] -= b[i] * m;
    }
    b[i+1] -= b[i]*m;
    d[i+1] -= c[i]*m;
}
```

Fatoração LU

Problema: calcular a inversa de uma matriz.

$$\begin{array}{c} \mathbf{A} \\ \left[\begin{array}{cccc} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{array} \right] \end{array} \begin{array}{c} \mathbf{A}^{-1} \\ \left[\begin{array}{cccc} x_{11} & x_{12} & x_{13} & x_{14} \\ x_{21} & x_{22} & x_{23} & x_{24} \\ x_{31} & x_{32} & x_{33} & x_{34} \\ x_{41} & x_{42} & x_{43} & x_{44} \end{array} \right] \end{array} = \begin{array}{c} \mathbf{I} \\ \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right] \end{array}$$

Fatoração LU

$$a_{11}x_{11} + a_{12}x_{21} + a_{13}x_{31} + a_{14}x_{41} = 1$$

Problema: calcular a inversa de uma matriz.

$$\begin{matrix} & \mathbf{A} & & \mathbf{A}^{-1} & & \mathbf{I} \end{matrix}$$
$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} x_{11} & x_{12} & x_{13} & x_{14} \\ x_{21} & x_{22} & x_{23} & x_{24} \\ x_{31} & x_{32} & x_{33} & x_{34} \\ x_{41} & x_{42} & x_{43} & x_{44} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Fatoração LU

Problema: calcular a inversa de uma matriz.

$$a_{11}x_{11} + a_{12}x_{21} + a_{13}x_{31} + a_{14}x_{41} = 1$$

$$a_{21}x_{11} + a_{22}x_{21} + a_{23}x_{31} + a_{24}x_{41} = 0$$

$$\begin{matrix} & \mathbf{A} & & \mathbf{A}^{-1} & & \mathbf{I} \end{matrix}$$
$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} x_{11} & x_{12} & x_{13} & x_{14} \\ x_{21} & x_{22} & x_{23} & x_{24} \\ x_{31} & x_{32} & x_{33} & x_{34} \\ x_{41} & x_{42} & x_{43} & x_{44} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Fatoração LU

Problema: calcular a inversa de uma matriz.

$$a_{11}x_{11} + a_{12}x_{21} + a_{13}x_{31} + a_{14}x_{41} = 1$$

$$a_{21}x_{11} + a_{22}x_{21} + a_{23}x_{31} + a_{24}x_{41} = 0$$

$$a_{31}x_{11} + a_{32}x_{21} + a_{33}x_{31} + a_{34}x_{41} = 0$$

$$\begin{matrix} & A & & A^{-1} & & I \end{matrix}$$
$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} x_{11} & x_{12} & x_{13} & x_{14} \\ x_{21} & x_{22} & x_{23} & x_{24} \\ x_{31} & x_{32} & x_{33} & x_{34} \\ x_{41} & x_{42} & x_{43} & x_{44} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Fatoração LU

Problema: calcular a inversa de uma matriz.

$$\begin{cases} a_{11}x_{11} + a_{12}x_{21} + a_{13}x_{31} + a_{14}x_{41} = 1 \\ a_{21}x_{11} + a_{22}x_{21} + a_{23}x_{31} + a_{24}x_{41} = 0 \\ a_{31}x_{11} + a_{32}x_{21} + a_{33}x_{31} + a_{34}x_{41} = 0 \\ a_{41}x_{11} + a_{42}x_{21} + a_{43}x_{31} + a_{44}x_{41} = 0 \end{cases}$$

$$\begin{matrix} & \mathbf{A} & & \mathbf{A}^{-1} & & \mathbf{I} \end{matrix}$$
$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} x_{11} & x_{12} & x_{13} & x_{14} \\ x_{21} & x_{22} & x_{23} & x_{24} \\ x_{31} & x_{32} & x_{33} & x_{34} \\ x_{41} & x_{42} & x_{43} & x_{44} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Fatoração LU

Problema: calcular a inversa de uma matriz.

$$\begin{cases} a_{11}x_{11} + a_{12}x_{21} + a_{13}x_{31} + a_{14}x_{41} = 1 \\ a_{21}x_{11} + a_{22}x_{21} + a_{23}x_{31} + a_{24}x_{41} = 0 \\ a_{31}x_{11} + a_{32}x_{21} + a_{33}x_{31} + a_{34}x_{41} = 0 \\ a_{41}x_{11} + a_{42}x_{21} + a_{43}x_{31} + a_{44}x_{41} = 0 \end{cases}$$

$$\begin{matrix} & \mathbf{A} & & \mathbf{A}^{-1} & & \mathbf{I} \end{matrix}$$
$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} x_{11} & x_{12} & x_{13} & x_{14} \\ x_{21} & x_{22} & x_{23} & x_{24} \\ x_{31} & x_{32} & x_{33} & x_{34} \\ x_{41} & x_{42} & x_{43} & x_{44} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{A}\mathbf{x}_1 = \mathbf{i}_1$$

Fatoração LU

Problema: calcular a inversa de uma matriz.

$$\begin{cases} a_{11}x_{11} + a_{12}x_{21} + a_{13}x_{31} + a_{14}x_{41} = 1 \\ a_{21}x_{11} + a_{22}x_{21} + a_{23}x_{31} + a_{24}x_{41} = 0 \\ a_{31}x_{11} + a_{32}x_{21} + a_{33}x_{31} + a_{34}x_{41} = 0 \\ a_{41}x_{11} + a_{42}x_{21} + a_{43}x_{31} + a_{44}x_{41} = 0 \end{cases}$$

$$\begin{matrix} & \mathbf{A} & & \mathbf{A}^{-1} & & \mathbf{I} & & \end{matrix}$$
$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} x_{11} & x_{12} & x_{13} & x_{14} \\ x_{21} & x_{22} & x_{23} & x_{24} \\ x_{31} & x_{32} & x_{33} & x_{34} \\ x_{41} & x_{42} & x_{43} & x_{44} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$Ax_1 = i_1$

$Ax_2 = i_2$

$Ax_3 = i_3$

$Ax_4 = i_4$

Fatoração LU

Refinamento

Algoritmo:

Basta aplicar em \mathbf{r} todas as transformações que foram aplicadas em \mathbf{b}

- 1 - Obter solução inicial $\mathbf{x}^{(0)}$ resolvendo $\mathbf{Ax} = \mathbf{b}$ e inicializar $\mathbf{i} = 0$;
- 2 - Calcular o resíduo $\mathbf{r} = \mathbf{b} - \mathbf{Ax}^{(i)}$ e testar critério de parada (a);
- 3 - Obter \mathbf{w} resolvendo $\mathbf{Aw} = \mathbf{r}$;
- 4 - Obter nova solução $\mathbf{x}^{(i+1)} = \mathbf{x}^{(i)} + \mathbf{w}$ e testar critério de parada (b);
- 5 - Incrementar \mathbf{i} e voltar ao passo 2;

$$\mathbf{Ax}_4 = \mathbf{i}_4$$

Fatoração LU

Decompor \mathbf{A} em duas matrizes triangulares $\mathbf{A} = \mathbf{L}\mathbf{U}$.

Fatoração LU

Decompor \mathbf{A} em duas matrizes triangulares $\mathbf{A} = \mathbf{L}\mathbf{U}$.

$$\mathbf{A}\mathbf{x} = \mathbf{b}$$

Fatoração LU

Decompor \mathbf{A} em duas matrizes triangulares $\mathbf{A} = \mathbf{LU}$.

$$\mathbf{Ax} = \mathbf{b}$$

$$\mathbf{LUx} = \mathbf{b}$$

Fatoração LU

Decompor \mathbf{A} em duas matrizes triangulares $\mathbf{A} = \mathbf{L}\mathbf{U}$.

$$\mathbf{A}\mathbf{x} = \mathbf{b}$$

$$\underbrace{\mathbf{L}\mathbf{U}}_{\mathbf{y}}\mathbf{x} = \mathbf{b}$$

Fatoração LU

Decompor \mathbf{A} em duas matrizes triangulares $\mathbf{A} = \mathbf{LU}$.

$$\mathbf{Ax} = \mathbf{b}$$

$$\underbrace{\mathbf{LU}}_{\mathbf{y}} \mathbf{x} = \mathbf{b} \quad \longrightarrow \quad \mathbf{Ly} = \mathbf{b}$$
$$\mathbf{Ux} = \mathbf{y}$$

Fatoração LU

Decompor \mathbf{A} em duas matrizes triangulares $\mathbf{A} = \mathbf{LU}$.

$$\mathbf{Ax} = \mathbf{b}$$

$$\underbrace{\mathbf{LU}}_{\mathbf{y}} \mathbf{x} = \mathbf{b}$$



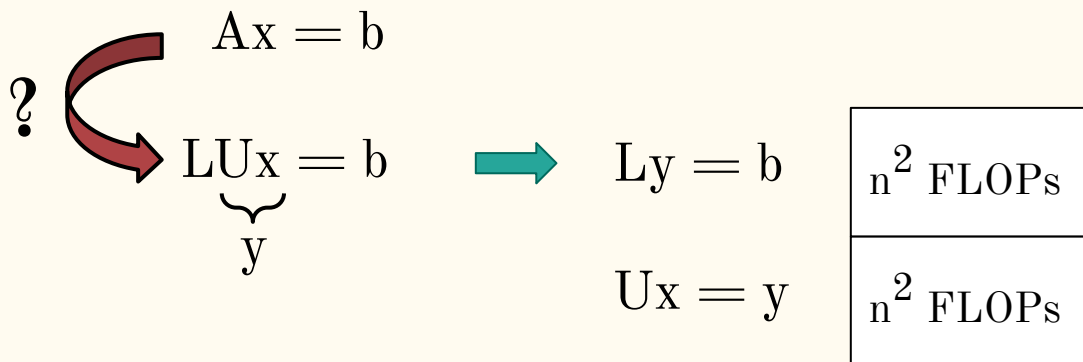
$$\mathbf{Ly} = \mathbf{b}$$

$$\mathbf{Ux} = \mathbf{y}$$

n^2 FLOPs
n^2 FLOPs

Fatoração LU

Decompor \mathbf{A} em duas matrizes triangulares $\mathbf{A} = \mathbf{LU}$.



Fatoração LU

Decompor \mathbf{A} em duas matrizes triangulares $\mathbf{A} = \mathbf{LU}$.


$$\mathbf{A}\mathbf{x}_1 = \mathbf{i}_1$$

$$\mathbf{A}\mathbf{x}_2 = \mathbf{i}_2$$

$$\mathbf{A}\mathbf{x}_3 = \mathbf{i}_3$$

\vdots

$$\mathbf{A}\mathbf{x}_k = \mathbf{i}_k$$

?  $\mathbf{A}\mathbf{x} = \mathbf{b}$

$\underbrace{\mathbf{L}\mathbf{U}}_{\mathbf{y}}\mathbf{x} = \mathbf{b}$



$$\mathbf{L}\mathbf{y} = \mathbf{b}$$

$$\mathbf{U}\mathbf{x} = \mathbf{y}$$

n^2 FLOPs

n^2 FLOPs

$\text{custo}(\text{decomposição}) + 2kn^2 \text{ FLOPs}$

Fatoração LU

Decompor \mathbf{A} em duas matrizes triangulares $\mathbf{A} = \mathbf{L}\mathbf{U}$.

\mathbf{A}

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$$

Fatoração LU

 m_{21} m_{31} m_{41}

Decompor \mathbf{A} em duas matrizes triangulares $\mathbf{A} = \mathbf{LU}$.

 \mathbf{A}

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{matrix} \\ m_{21} \\ m_{31} \\ m_{41} \end{matrix}$$

 \mathbf{A}_1

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ 0 & b_{22} & b_{23} & b_{24} \\ 0 & b_{32} & b_{33} & b_{34} \\ 0 & b_{42} & b_{43} & b_{44} \end{bmatrix}$$

Fatoração LU

Decompor \mathbf{A} em duas matrizes triangulares $\mathbf{A} = \mathbf{LU}$.

$$\begin{array}{cc} m_{21} & \\ m_{31} & m_{32} \\ m_{41} & m_{42} \end{array}$$

$$\begin{array}{c} \mathbf{A}_1 \\ \left[\begin{array}{cccc} a_{11} & a_{12} & a_{13} & a_{14} \\ 0 & b_{22} & b_{23} & b_{24} \\ 0 & b_{32} & b_{33} & b_{34} \\ 0 & b_{42} & b_{43} & b_{44} \end{array} \right] \begin{array}{c} m_{32} \\ m_{42} \end{array} \end{array} \quad \rightarrow \quad \begin{array}{c} \mathbf{A}_2 \\ \left[\begin{array}{cccc} a_{11} & a_{12} & a_{13} & a_{14} \\ 0 & b_{22} & b_{23} & b_{24} \\ 0 & 0 & c_{33} & c_{34} \\ 0 & 0 & c_{43} & c_{44} \end{array} \right] \end{array}$$

Fatoração LU

Decompor \mathbf{A} em duas matrizes triangulares $\mathbf{A} = \mathbf{L}\mathbf{U}$.

$$\begin{array}{ccc} m_{21} & & \\ m_{31} & m_{32} & \\ m_{41} & m_{42} & m_{43} \end{array}$$

$$\mathbf{A}_2 = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ 0 & b_{22} & b_{23} & b_{24} \\ 0 & 0 & c_{33} & c_{34} \\ 0 & 0 & c_{43} & c_{44} \end{bmatrix} \quad m_{43}$$



$$\mathbf{A}_3 = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ 0 & b_{22} & b_{23} & b_{24} \\ 0 & 0 & c_{33} & c_{34} \\ 0 & 0 & 0 & d_{44} \end{bmatrix}$$

Fatoração LU

Decompor \mathbf{A} em duas matrizes triangulares $\mathbf{A} = \mathbf{L}\mathbf{U}$.

$$\begin{array}{ccc} m_{21} & & \\ m_{31} & m_{32} & \\ m_{41} & m_{42} & m_{43} \end{array}$$

$$\begin{array}{c} \mathbf{A}_3 \\ \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ m_{21} & 1 & 0 & 0 \\ m_{31} & m_{32} & 1 & 0 \\ m_{41} & m_{42} & m_{43} & 1 \end{array} \right] \end{array} \begin{array}{c} \mathbf{A} \\ \left[\begin{array}{cccc} a_{11} & a_{12} & a_{13} & a_{14} \\ 0 & b_{22} & b_{23} & b_{24} \\ 0 & 0 & c_{33} & c_{34} \\ 0 & 0 & 0 & d_{44} \end{array} \right] \end{array} = \begin{array}{c} \mathbf{A} \\ \left[\begin{array}{cccc} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{array} \right] \end{array}$$

Fatoração LU

Decompor \mathbf{A} em duas matrizes triangulares $\mathbf{A} = \mathbf{LU}$.

$$\begin{matrix} & & \mathbf{A}_3 & & \mathbf{A} \\ & & & & \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ m_{21} & 1 & 0 & 0 \\ m_{31} & m_{32} & 1 & 0 \\ m_{41} & m_{42} & m_{43} & 1 \end{bmatrix} & \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ 0 & b_{22} & b_{23} & b_{24} \\ 0 & 0 & c_{33} & c_{34} \\ 0 & 0 & 0 & d_{44} \end{bmatrix} & = & \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}
 \end{matrix}$$

$$\begin{aligned}
 m_{21}a_{12} + b_{22} &= (a_{21}/a_{11})a_{12} + (a_{22} - a_{12}m_{21}) \\
 &= (a_{12}a_{21})/a_{11} + a_{22} - (a_{12}a_{21})/a_{11}
 \end{aligned}$$

Fatoração LU

Decompor \mathbf{A} em duas matrizes triangulares $\mathbf{A} = \mathbf{LU}$.

$$\begin{array}{c} \mathbf{L} \\ \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ m_{21} & 1 & 0 & 0 \\ m_{31} & m_{32} & 1 & 0 \\ m_{41} & m_{42} & m_{43} & 1 \end{array} \right] \end{array} \begin{array}{c} \mathbf{U} \\ \left[\begin{array}{cccc} a_{11} & a_{12} & a_{13} & a_{14} \\ 0 & b_{22} & b_{23} & b_{24} \\ 0 & 0 & c_{33} & c_{34} \\ 0 & 0 & 0 & d_{44} \end{array} \right] \end{array} = \begin{array}{c} \mathbf{A} \\ \left[\begin{array}{cccc} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{array} \right] \end{array}$$

Fatoração LU

Exemplo: encontrar a inversa da matriz **A**.

$$\mathbf{A} = \begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix}$$

Fatoração LU

Exemplo: encontrar a inversa da matriz **A**.

$$\begin{array}{c} \mathbf{A} \\ \left[\begin{array}{ccc} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{array} \right] \end{array} \begin{array}{c} \mathbf{X} \\ \left[\begin{array}{ccc} \mathbf{x}_{11} & \mathbf{x}_{12} & \mathbf{x}_{13} \\ \mathbf{x}_{21} & \mathbf{x}_{22} & \mathbf{x}_{23} \\ \mathbf{x}_{31} & \mathbf{x}_{32} & \mathbf{x}_{33} \end{array} \right] \end{array} = \begin{array}{c} \mathbf{B} \\ \left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right] \end{array}$$

Fatoração LU

Exemplo: encontrar a inversa da matriz **A**.

$$\begin{matrix} & \mathbf{A} \\ \left[\begin{array}{ccc} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{array} \right] & \begin{array}{l} m_{21} = 2.56 \\ m_{31} = 5.76 \end{array} \end{matrix}$$

Fatoração LU

Exemplo: encontrar a inversa da matriz **A**.

$$\begin{array}{c} \mathbf{A} \\ \left[\begin{array}{ccc} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{array} \right] \begin{array}{l} m_{21} = 2.56 \\ m_{31} = 5.76 \end{array} \end{array} \rightarrow \left[\begin{array}{ccc} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & -16.8 & -4.76 \end{array} \right]$$

Fatoração LU

Exemplo: encontrar a inversa da matriz **A**.

$$\begin{array}{c} \mathbf{A} \\ \left[\begin{array}{ccc} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{array} \right] \begin{array}{l} m_{21} = 2.56 \\ m_{31} = 5.76 \end{array} \end{array} \rightarrow \begin{array}{c} \left[\begin{array}{ccc} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & -16.8 & -4.76 \end{array} \right] \begin{array}{l} \\ m_{32} = 3.5 \end{array} \end{array}$$

Fatoração LU

Exemplo: encontrar a inversa da matriz **A**.

$$\begin{array}{c} \mathbf{A} \\ \left[\begin{array}{ccc} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{array} \right] \begin{array}{l} m_{21} = 2.56 \\ m_{31} = 5.76 \end{array} \end{array} \rightarrow \begin{array}{c} \left[\begin{array}{ccc} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & -16.8 & -4.76 \end{array} \right] m_{32} = 3.5 \end{array}$$

$$\rightarrow \left[\begin{array}{ccc} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{array} \right]$$

Fatoração LU

Exemplo: encontrar a inversa da matriz **A**.

$$\begin{array}{c} \mathbf{A} \\ \left[\begin{array}{ccc} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{array} \right] \begin{array}{l} m_{21} = 2.56 \\ m_{31} = 5.76 \end{array} \end{array} \quad \longrightarrow \quad \begin{array}{c} \left[\begin{array}{ccc} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & -16.8 & -4.76 \end{array} \right] m_{32} = 3.5 \end{array}$$

$$\longrightarrow \begin{array}{c} \left[\begin{array}{ccc} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{array} \right] \\ \mathbf{U} \end{array}$$

Fatoração LU

Exemplo: encontrar a inversa da matriz **A**.

$$\begin{array}{c} \mathbf{A} \\ \left[\begin{array}{ccc} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{array} \right] \begin{array}{l} m_{21} = 2.56 \\ m_{31} = 5.76 \end{array} \end{array} \longrightarrow \begin{array}{c} \left[\begin{array}{ccc} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & -16.8 & -4.76 \end{array} \right] m_{32} = 3.5 \end{array}$$

$$\longrightarrow \begin{array}{c} \left[\begin{array}{ccc} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{array} \right] \\ \mathbf{U} \end{array} \quad \begin{array}{c} \left[\begin{array}{ccc} 1 & 0 & 0 \\ 2.56 & 1 & 0 \\ 5.76 & 3.5 & 1 \end{array} \right] \\ \mathbf{L} \end{array}$$

Fatoração LU

Exemplo: encontrar a inversa da matriz **A**.

$$Ax = b$$

$$L\underset{y}{U}x = b \quad \longrightarrow \quad Ly = b$$

$$Ux = y$$

$$\begin{matrix} & \mathbf{A} \\ \begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} & \begin{matrix} m_{21} = 2.56 \\ m_{31} = 5.76 \end{matrix} \end{matrix} \quad \longrightarrow \quad \begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & -16.8 & -4.76 \end{bmatrix} \begin{matrix} \\ \\ m_{32} = 3.5 \end{matrix}$$

$$\longrightarrow \begin{matrix} \begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{bmatrix} & \begin{bmatrix} 1 & 0 & 0 \\ 2.56 & 1 & 0 \\ 5.76 & 3.5 & 1 \end{bmatrix} \\ \mathbf{U} & \mathbf{L} \end{matrix}$$

Fatoração LU

Exemplo: encontrar a inversa da matriz **A**.

$$Ax = b$$

$$L\underset{y}{U}x = b \quad \rightarrow \quad Ly = b$$

$$Ux = y$$

L

$$\begin{bmatrix} 1 & 0 & 0 \\ 2.56 & 1 & 0 \\ 5.76 & 3.5 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$



$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -2.56 \\ 3.2 \end{bmatrix}$$

Aplica o mesmo
escalamento
no vetor b

U

$$\begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -2.56 \\ 3.2 \end{bmatrix}$$



$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0.04762 \\ -0.9524 \\ 4.571 \end{bmatrix}$$

Fatoração LU

Exemplo: encontrar a inversa da matriz **A**.

$$Ax = b$$

$$L \underbrace{Ux}_y = b \quad \rightarrow \quad Ly = b$$

$$Ux = y$$

L

$$\begin{bmatrix} 1 & 0 & 0 \\ 2.56 & 1 & 0 \\ 5.76 & 3.5 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$



$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -3.5 \end{bmatrix}$$

Aplica o mesmo
escalonamento
no vetor b

U

$$\begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -3.5 \end{bmatrix}$$



$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -0.08333 \\ 1.417 \\ -5.000 \end{bmatrix}$$

Fatoração LU

Exemplo: encontrar a inversa da matriz **A**.

$$Ax = b$$

$$L \underbrace{Ux}_y = b \quad \rightarrow \quad Ly = b$$

$$Ux = y$$

L

$$\begin{bmatrix} 1 & 0 & 0 \\ 2.56 & 1 & 0 \\ 5.76 & 3.5 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$



$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Aplica o mesmo
escalonamento
no vetor b

U

$$\begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$



$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0.03571 \\ -0.4643 \\ 1.429 \end{bmatrix}$$

Fatoração LU

Exemplo: encontrar a inversa da matriz A .

$$A^{-1}$$

$$\begin{bmatrix} 0.04762 & -0.08333 & 0.03571 \\ -0.9524 & 1.417 & -0.4643 \\ 4.571 & -5.000 & 1.429 \end{bmatrix}$$

$$Ax = b$$

$$L \underbrace{Ux}_y = b \quad \rightarrow \quad Ly = b$$

$$Ux = y$$

Fatoração LU

Exemplo: encontrar a inversa da matriz \mathbf{A} .

Para pivoteamento parcial, efetuar as mesmas trocas de linhas no vetor \mathbf{b} .

$$\mathbf{Ax} = \mathbf{b}$$

$$\mathbf{L}\underbrace{\mathbf{Ux}}_{\mathbf{y}} = \mathbf{b} \quad \longrightarrow \quad \mathbf{Ly} = \mathbf{b}$$

$$\mathbf{Ux} = \mathbf{y}$$

Fatoração LU

Exemplo: encontrar a inversa da matriz \mathbf{A} .

Para pivoteamento parcial, efetuar as mesmas trocas de linhas no vetor \mathbf{b} .

$$\mathbf{A}\mathbf{x} = \mathbf{b}$$

$$\mathbf{L}\underbrace{\mathbf{U}\mathbf{x}}_{\mathbf{y}} = \mathbf{b} \quad \longrightarrow \quad \mathbf{L}\mathbf{y} = \mathbf{b}$$

$$\mathbf{U}\mathbf{x} = \mathbf{y}$$

Exemplo:

$$\begin{array}{c} \mathbf{A} \\ \left[\begin{array}{ccc} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{array} \right] \end{array} \quad \longrightarrow \quad \begin{array}{c} \mathbf{A}' \\ \left[\begin{array}{ccc} 144 & 12 & 1 \\ 64 & 8 & 1 \\ 25 & 5 & 1 \end{array} \right] \end{array} \quad \longrightarrow \quad \mathbf{A}' = \mathbf{L}\mathbf{U}$$

Fatoração LU

$$Ax = b$$

Exemplo: encontrar a inversa da matriz A

Para pivoteamento parcial, efetuar as m de linhas no vetor b .

$$LU \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Exemplo:

$$\begin{matrix} & A & & A' & \\ \begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} & \longrightarrow & \begin{bmatrix} 144 & 12 & 1 \\ 64 & 8 & 1 \\ 25 & 5 & 1 \end{bmatrix} & \longrightarrow & A' = LU \end{matrix}$$

Fatoração LU

Exemplo: encontrar a inversa da matriz A

Para pivoteamento parcial, efetuar as m de linhas no vetor \mathbf{b} .

Exemplo:

$$A = \begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix}$$



$$A' = \begin{bmatrix} 144 & 12 & 1 \\ 64 & 8 & 1 \\ 25 & 5 & 1 \end{bmatrix}$$



$$A' = LU$$

$$Ax = b$$

LU

$$\begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Atenção: além do vetor \mathbf{b} , as mesmas trocas de linhas devem ser feitas a cada iteração na matriz L .

Fatoração LU

Exemplo:

$$\begin{matrix} & \mathbf{A} & & \mathbf{x} & & \mathbf{b} \\ \begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} & \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} & = & \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \end{matrix}$$

Fatoração LU

Exemplo:

$$A = \begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix}$$

L

$$\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

b

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Fatoração LU

Exemplo:

$$A = \begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix}$$

Escolhendo o primeiro pivô:
trocar as linhas 1 e 3.

L

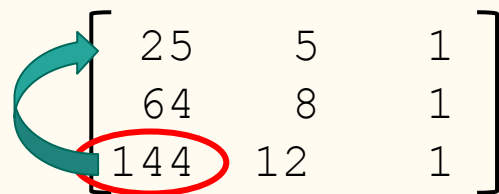
$$\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

b

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Fatoração LU

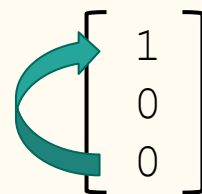
Exemplo:

$$A = \begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix}$$


L

$$\begin{bmatrix} \\ \\ \end{bmatrix}$$

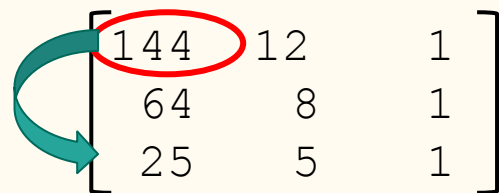
b

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$


A mesma troca deve ser feita no vetor **b**.

Fatoração LU

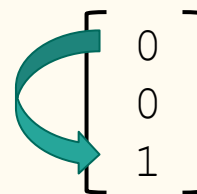
Exemplo:

$$A = \begin{bmatrix} 144 & 12 & 1 \\ 64 & 8 & 1 \\ 25 & 5 & 1 \end{bmatrix}$$


L

$$\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

b

$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$


A mesma troca deve ser feita no vetor **b**.

Fatoração LU

Exemplo:

$$\begin{array}{c} A \\ \left[\begin{array}{cc|c} 144 & 12 & 1 \\ 64 & 8 & 1 \\ 25 & 5 & 1 \end{array} \right] \end{array} \quad \begin{array}{l} m_{21} = 0.4444 \\ m_{31} = 0.1736 \end{array}$$

Definindo multiplicadores
da primeira iteração.

L

$$\left[\begin{array}{c} \\ \\ \\ \end{array} \right]$$

b

$$\left[\begin{array}{c} 0 \\ 0 \\ 1 \end{array} \right]$$

Fatoração LU

Exemplo:

$$\begin{array}{c} A \\ \left[\begin{array}{cc|c} 144 & 12 & 1 \\ 64 & 8 & 1 \\ 25 & 5 & 1 \end{array} \right] \end{array} \quad \begin{array}{l} m_{21} = 0.4444 \\ m_{31} = 0.1736 \end{array}$$

Armazenando na matriz L.

$$\begin{array}{c} L \\ \left[\begin{array}{c} 0.4444 \\ 0.1736 \end{array} \right] \end{array}$$
$$\begin{array}{c} b \\ \left[\begin{array}{c} 0 \\ 0 \\ 1 \end{array} \right] \end{array}$$

Fatoração LU

Exemplo:

$$\begin{array}{ccc} & A & \\ \left[\begin{array}{cc|c} 144 & 12 & 1 \\ 64 & 8 & 1 \\ 25 & 5 & 1 \end{array} \right] & \begin{array}{l} m_{21} = 0.4444 \\ m_{31} = 0.1736 \end{array} & \end{array}$$



Aplicando operações sobre as equações.

$$\left[\begin{array}{cc|c} 144 & 12 & 1 \\ 0 & 2.6672 & 0.5556 \\ 0 & 2.9168 & 0.8264 \end{array} \right]$$

L

$$\left[\begin{array}{c} 0.4444 \\ 0.1736 \end{array} \right]$$

b

$$\left[\begin{array}{c} 0 \\ 0 \\ 1 \end{array} \right]$$

Fatoração LU

Exemplo:

A

$$\begin{bmatrix} 144 & 12 & 1 \\ 0 & 2.6672 & 0.5556 \\ 0 & 2.9168 & 0.8264 \end{bmatrix}$$

Na segunda iteração é necessário trocar as linhas 2 e 3.

L

$$\begin{bmatrix} 0.4444 \\ 0.1736 \end{bmatrix}$$


b

$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$


Fatoração LU

Exemplo:

A



$$\begin{bmatrix} 144 & 12 & 1 \\ 0 & 2.6672 & 0.5556 \\ 0 & 2.9168 & 0.8264 \end{bmatrix}$$

L


$$\begin{bmatrix} 0.4444 \\ 0.1736 \end{bmatrix}$$

A mesma troca deve ser feita no vetor **b** e nos elementos já preenchidos da matriz **L**.


b


$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Fatoração LU


Exemplo:

A



$$\begin{bmatrix} 144 & 12 & 1 \\ 0 & 2.9168 & 0.8264 \\ 0 & 2.6672 & 0.5556 \end{bmatrix}$$

A mesma troca deve ser feita no vetor **b** e nos elementos já preenchidos da matriz **L**.

L


$$\begin{bmatrix} 0.1736 \\ 0.4444 \end{bmatrix}$$

b


$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

Fatoração LU

Exemplo:

A

$$\begin{bmatrix} 144 & 12 & 1 \\ 0 & 2.9168 & 0.8264 \\ 0 & 2.6672 & 0.5556 \end{bmatrix} m_{32} = 0.9144$$

Definindo multiplicadores
da segunda iteração.

L

$$\begin{bmatrix} 0.1736 \\ 0.4444 \end{bmatrix}$$

b

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

Fatoração LU

Exemplo:

A

$$\begin{bmatrix} 144 & 12 & 1 \\ 0 & 2.9168 & 0.8264 \\ 0 & 2.6672 & 0.5556 \end{bmatrix} m_{32} = 0.9144$$

Armazenando na matriz L.

L

$$\begin{bmatrix} 0.1736 & & \\ 0.4444 & 0.9144 & \end{bmatrix}$$

b

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

Fatoração LU

Exemplo:

$$A = \begin{bmatrix} 144 & 12 & 1 \\ 0 & 2.9168 & 0.8264 \\ 0 & 0 & -0.2001 \end{bmatrix}$$



Aplicando operações sobre as equações.

$$\begin{bmatrix} 144 & 12 & 1 \\ 0 & 2.9168 & 0.8264 \\ 0 & 2.6672 & 0.5556 \end{bmatrix} m_{32} = 0.9144$$

L

$$\begin{bmatrix} 0.1736 & & \\ 0.4444 & 0.9144 & \end{bmatrix}$$

b

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

Fatoração LU

Exemplo:

A

$$\begin{bmatrix} 144 & 12 & 1 \\ 0 & 2.9168 & 0.8264 \\ 0 & 0 & -0.2001 \end{bmatrix}$$

U



Completando matriz L.

L

$$\begin{bmatrix} 1 & 0 & 0 \\ 0.1736 & 1 & 0 \\ 0.4444 & 0.9144 & 1 \end{bmatrix}$$

b

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

Na prática: efetuar as trocas no(s) vetor(es) **b** posteriormente. No momento da fatoração, apenas armazenar as trocas em uma estrutura de dados.

Métodos de Resolução de Sistemas Lineares

- **Métodos Exatos ou Diretos:** permitiram a solução exata com um número finito de operações, se não fosse por erros numéricos.
- **Métodos Iterativos:** permitem uma solução aproximada através de um processo infinito convergente.

O Método de Gauss-Jacobi

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \dots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \dots + a_{2n}x_n = b_2 \\ \vdots \\ a_{n1}x_1 + a_{n2}x_2 + a_{n3}x_3 + \dots + a_{nn}x_n = b_n \end{cases}$$

O Método de Gauss-Jacobi

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \dots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \dots + a_{2n}x_n = b_2 \\ \vdots \\ a_{n1}x_1 + a_{n2}x_2 + a_{n3}x_3 + \dots + a_{nn}x_n = b_n \end{cases}$$

O Método de Gauss-Jacobi

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \dots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \dots + a_{2n}x_n = b_2 \\ \vdots \\ a_{n1}x_1 + a_{n2}x_2 + a_{n3}x_3 + \dots + a_{nn}x_n = b_n \end{cases}$$

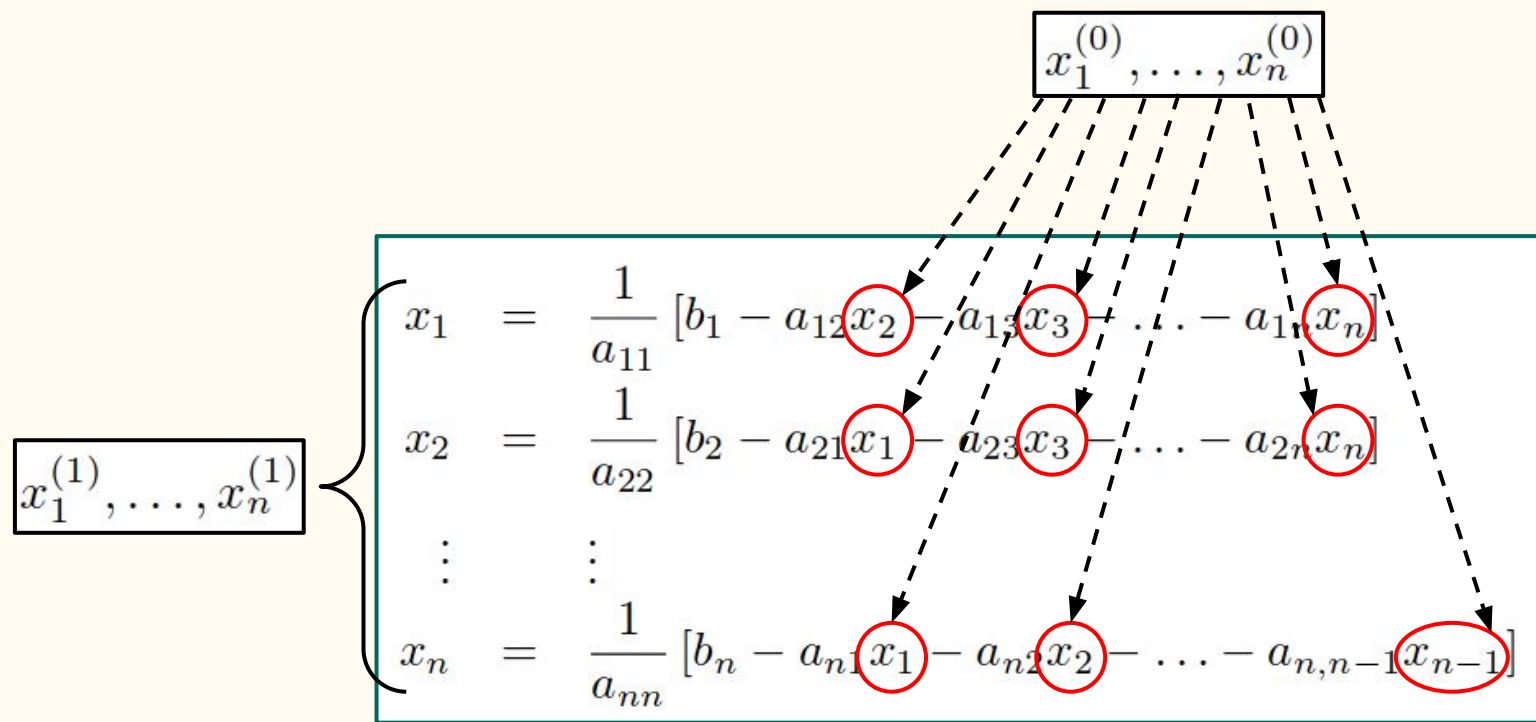
$$\begin{aligned} x_1 &= \frac{1}{a_{11}} [b_1 - a_{12}x_2 - a_{13}x_3 - \dots - a_{1n}x_n] \\ x_2 &= \frac{1}{a_{22}} [b_2 - a_{21}x_1 - a_{23}x_3 - \dots - a_{2n}x_n] \\ &\vdots \\ x_n &= \frac{1}{a_{nn}} [b_n - a_{n1}x_1 - a_{n2}x_2 - \dots - a_{n,n-1}x_{n-1}] \end{aligned}$$

O Método de Gauss-Jacobi

Diagram illustrating the Gauss-Jacobi method. The initial guess vector $x^{(0)} = [x_1^{(0)}, \dots, x_n^{(0)}]$ is used to compute the next iteration's values x_1, x_2, \dots, x_n using the following equations:

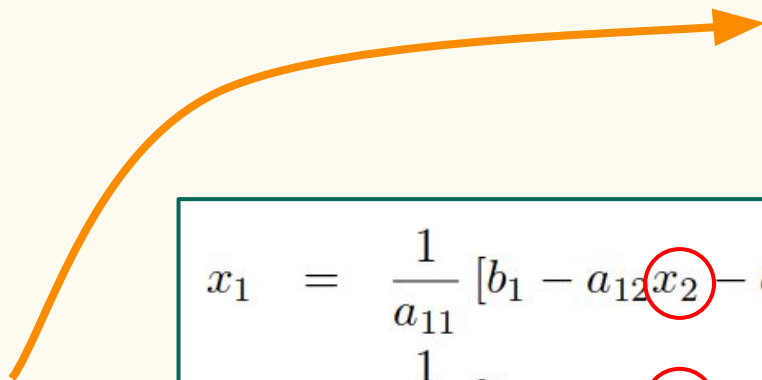
$$\begin{aligned} x_1 &= \frac{1}{a_{11}} [b_1 - a_{12}x_2 - a_{13}x_3 - \dots - a_{1n}x_n] \\ x_2 &= \frac{1}{a_{22}} [b_2 - a_{21}x_1 - a_{23}x_3 - \dots - a_{2n}x_n] \\ &\vdots \\ x_n &= \frac{1}{a_{nn}} [b_n - a_{n1}x_1 - a_{n2}x_2 - \dots - a_{n,n-1}x_{n-1}] \end{aligned}$$

O Método de Gauss-Jacobi



O Método de Gauss-Jacobi

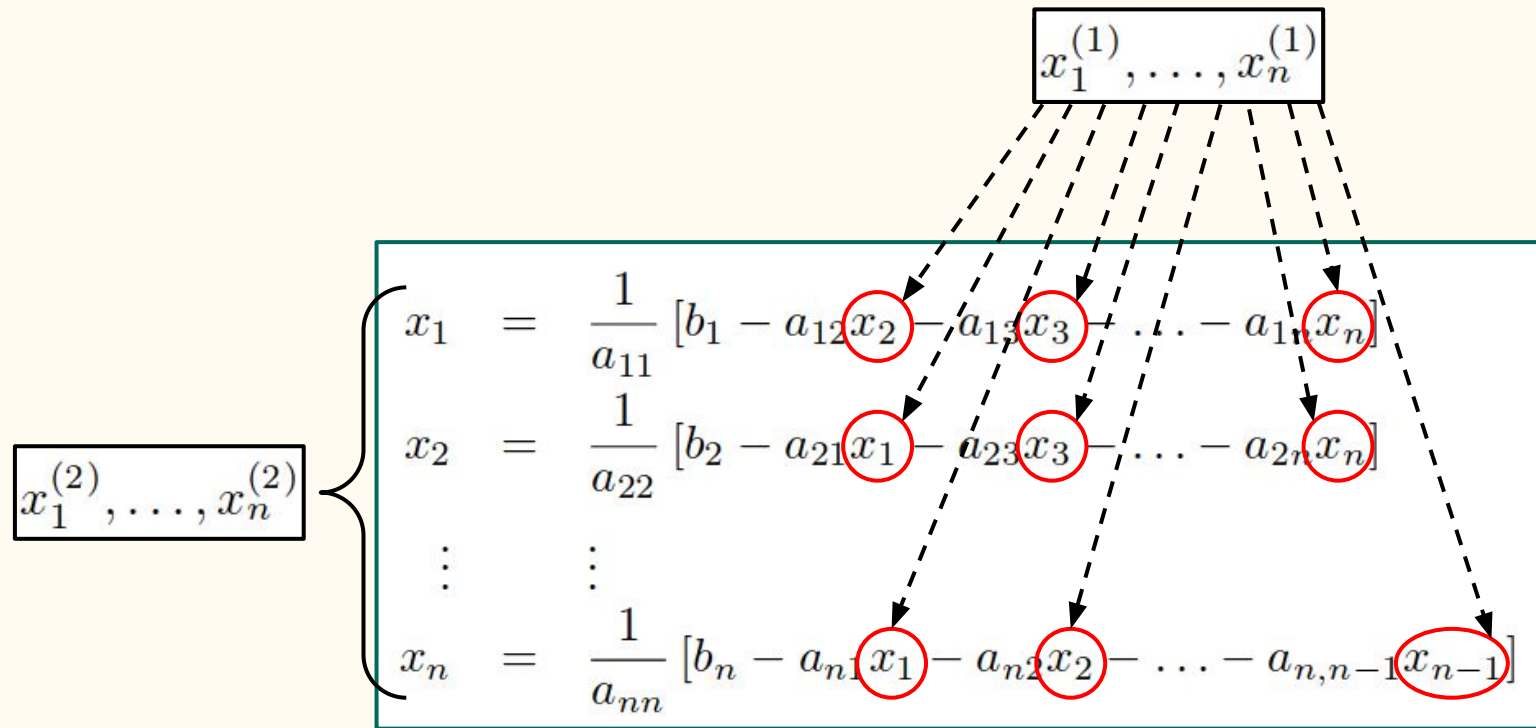
$$x_1^{(1)}, \dots, x_n^{(1)}$$



$$x_1^{(1)}, \dots, x_n^{(1)}$$

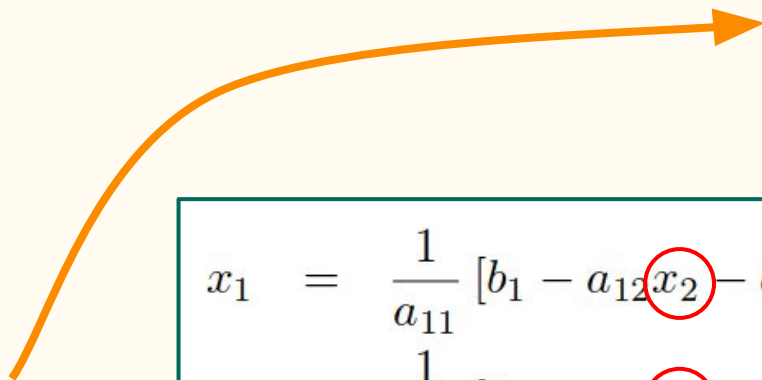
$$\begin{aligned} x_1 &= \frac{1}{a_{11}} [b_1 - a_{12}x_2 - a_{13}x_3 - \dots - a_{1n}x_n] \\ x_2 &= \frac{1}{a_{22}} [b_2 - a_{21}x_1 - a_{23}x_3 - \dots - a_{2n}x_n] \\ &\vdots \\ x_n &= \frac{1}{a_{nn}} [b_n - a_{n1}x_1 - a_{n2}x_2 - \dots - a_{n,n-1}x_{n-1}] \end{aligned}$$

O Método de Gauss-Jacobi



O Método de Gauss-Jacobi

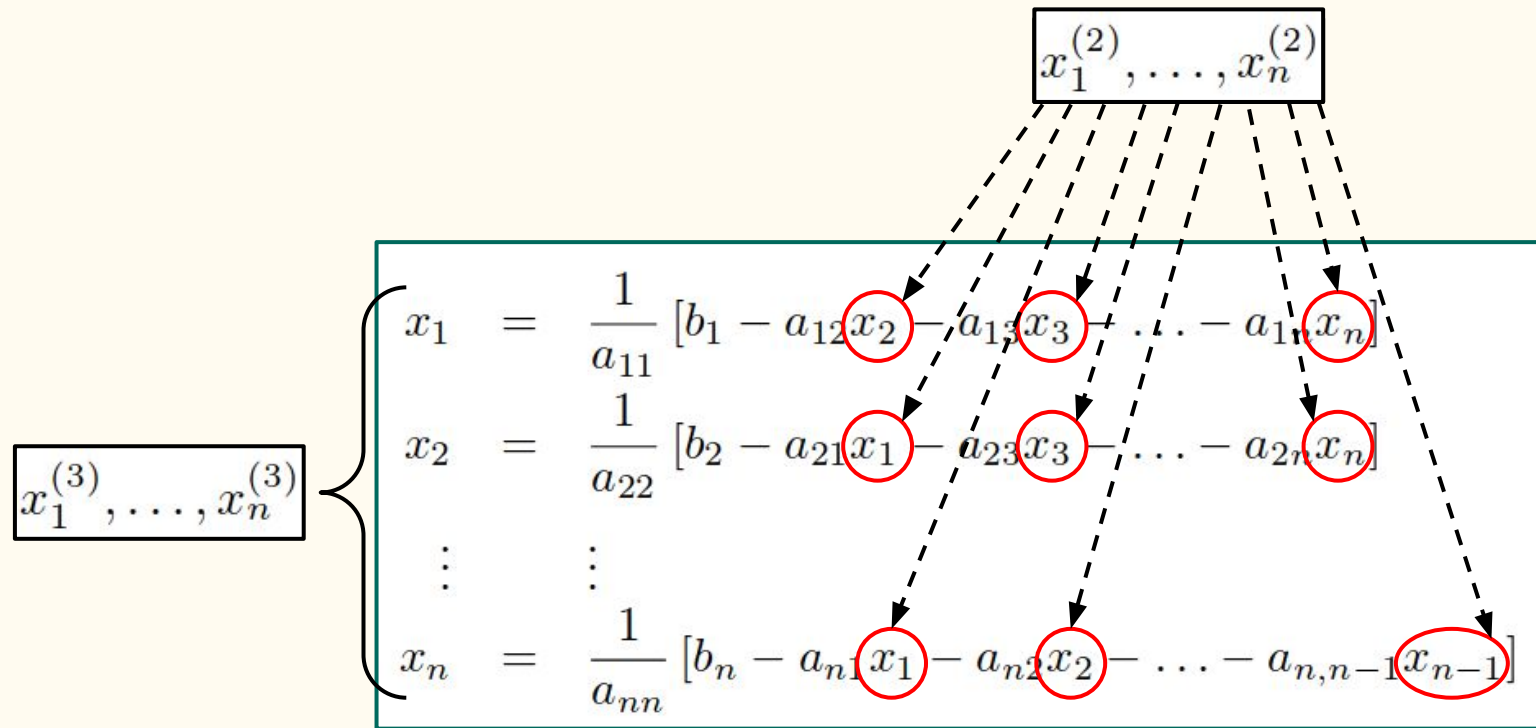
$$x_1^{(2)}, \dots, x_n^{(2)}$$



$$x_1^{(2)}, \dots, x_n^{(2)}$$

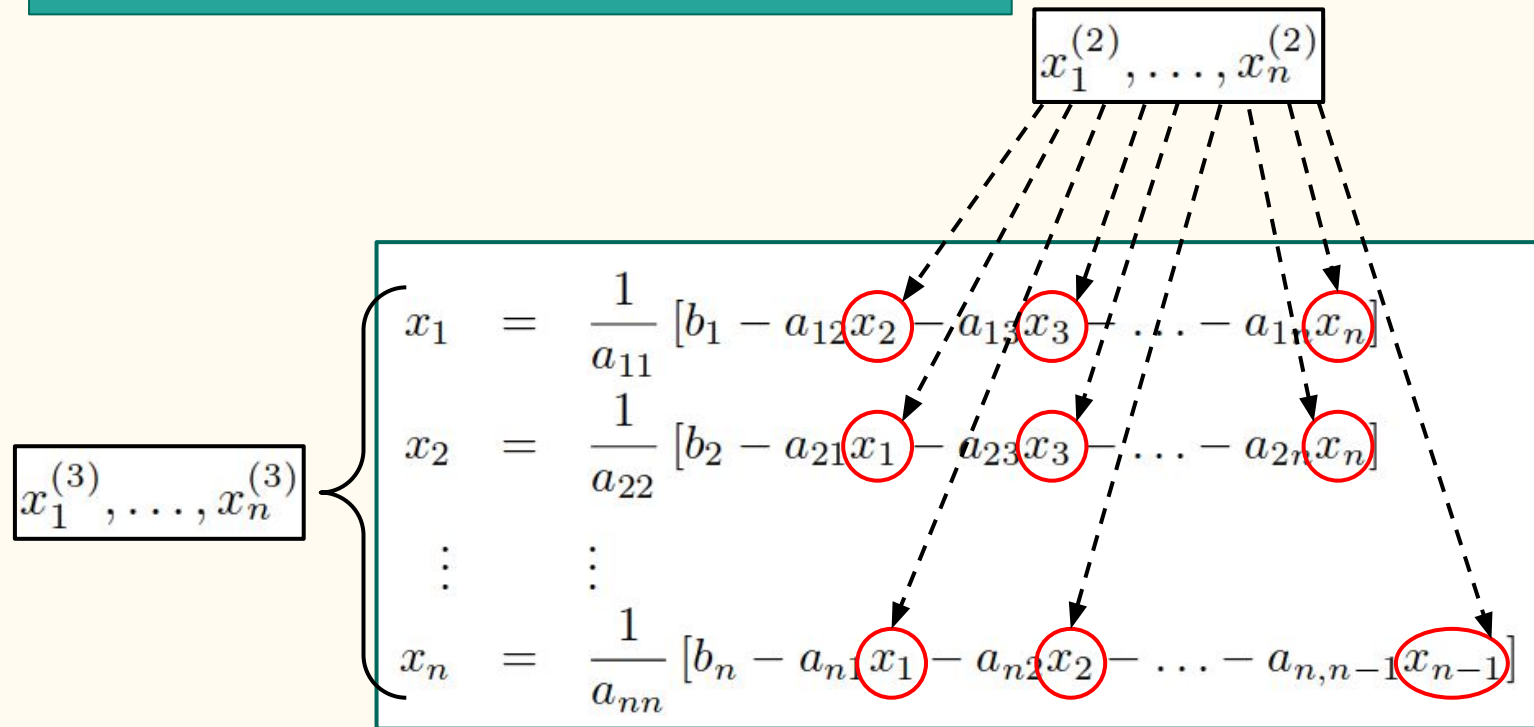
$$\begin{aligned} x_1 &= \frac{1}{a_{11}} [b_1 - a_{12}x_2 - a_{13}x_3 - \dots - a_{1n}x_n] \\ x_2 &= \frac{1}{a_{22}} [b_2 - a_{21}x_1 - a_{23}x_3 - \dots - a_{2n}x_n] \\ &\vdots \\ x_n &= \frac{1}{a_{nn}} [b_n - a_{n1}x_1 - a_{n2}x_2 - \dots - a_{n,n-1}x_{n-1}] \end{aligned}$$

O Método de Gauss-Jacobi



O Método de Gauss-Jacobi

$$\text{Max } \left\{ \left| x_i^{(k+1)} - x_i^{(k)} \right| \right\} \leq \varepsilon \quad i = 1, 2, 3, \dots, n$$



O Método de Gauss-Jacobi

$$\text{Max} \left\{ \left| \frac{x_i^{(k+1)} - x_i^{(k)}}{x_i^{(k+1)}} \right| \right\} \leq \varepsilon \quad i = 1, 2, 3, \dots, n$$

$$x_1^{(2)}, \dots, x_n^{(2)}$$

$$x_1^{(3)}, \dots, x_n^{(3)}$$

$$\begin{cases} x_1 = \frac{1}{a_{11}} [b_1 - a_{12}x_2 - a_{13}x_3 - \dots - a_{1n}x_n] \\ x_2 = \frac{1}{a_{22}} [b_2 - a_{21}x_1 - a_{23}x_3 - \dots - a_{2n}x_n] \\ \vdots \\ x_n = \frac{1}{a_{nn}} [b_n - a_{n1}x_1 - a_{n2}x_2 - \dots - a_{n,n-1}x_{n-1}] \end{cases}$$

O Método de Gauss-Jacobi

$$\text{Max } \left\{ \left| r_i^{(k+1)} \right| \right\} \leq \varepsilon \quad i = 1, 2, 3, \dots, n$$

$$r_i^{(k+1)} = b_i - \sum_{j=1}^n a_{ij} x_j^{(k+1)}$$

$$x_1^{(2)}, \dots, x_n^{(2)}$$

$$x_1^{(3)}, \dots, x_n^{(3)}$$

$$\begin{cases} x_1 = \frac{1}{a_{11}} [b_1 - a_{12}x_2 - a_{13}x_3 - \dots - a_{1n}x_n] \\ x_2 = \frac{1}{a_{22}} [b_2 - a_{21}x_1 - a_{23}x_3 - \dots - a_{2n}x_n] \\ \vdots \\ x_n = \frac{1}{a_{nn}} [b_n - a_{n1}x_1 - a_{n2}x_2 - \dots - a_{n,n-1}x_{n-1}] \end{cases}$$

Exemplo

$$\begin{cases} 3x_1 - x_2 - x_3 = 1 \\ x_1 + 3x_2 + x_3 = 5 \\ x_1 - x_2 + 2x_3 = 2 \end{cases}$$

Exemplo

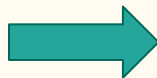
$$\begin{cases} 3x_1 - x_2 - x_3 = 1 \\ x_1 + 3x_2 + x_3 = 5 \\ x_1 - x_2 + 2x_3 = 2 \end{cases}$$



$$\begin{cases} x_1^{(k+1)} = (1 + x_2^{(k)} + x_3^{(k)}) / 3 \\ x_2^{(k+1)} = (5 - x_1^{(k)} - x_3^{(k)}) / 3 \\ x_3^{(k+1)} = (2 - x_1^{(k)} + x_2^{(k)}) / 2 \end{cases}$$

Exemplo

$$\begin{cases} 3x_1 - x_2 - x_3 = 1 \\ x_1 + 3x_2 + x_3 = 5 \\ x_1 - x_2 + 2x_3 = 2 \end{cases}$$



$$\begin{cases} x_1^{(k+1)} = (1 + x_2^{(k)} + x_3^{(k)}) / 3 \\ x_2^{(k+1)} = (5 - x_1^{(k)} - x_3^{(k)}) / 3 \\ x_3^{(k+1)} = (2 - x_1^{(k)} + x_2^{(k)}) / 2 \end{cases}$$

k	$x_1^{(k)}$	$x_2^{(k)}$	$x_3^{(k)}$	$ x_1^{(k+1)} - x_1^{(k)} $	$ x_2^{(k+1)} - x_2^{(k)} $	$ x_3^{(k+1)} - x_3^{(k)} $
0	0	0	0	-	-	-
1	0.333	1.667	1	0.333	1.667	1
2	1.222	1.222	1.667	0.889	0.555	0.667
3	1.296	0.704	1	0.074	0.518	0.667
4	0.901	0.901	0.704	0.395	0.197	0.296
5	0.868	1.132	1	0.033	0.197	0.296
6	1.044	1.044	1.132	0.176	0.088	0.132

Exemplo

$$\begin{cases} 3x_1 - x_2 - x_3 = 1 \\ x_1 + 3x_2 + x_3 = 5 \\ x_1 - x_2 + 2x_3 = 2 \end{cases}$$



$$\begin{cases} x_1^{(k+1)} = (1 + x_2^{(k)} + x_3^{(k)}) / 3 \\ x_2^{(k+1)} = (5 - x_1^{(k)} - x_3^{(k)}) / 3 \\ x_3^{(k+1)} = (2 - x_1^{(k)} + x_2^{(k)}) / 2 \end{cases}$$

k	$x_1^{(k)}$	$x_2^{(k)}$	$x_3^{(k)}$	$ x_1^{(k+1)} - x_1^{(k)} $	$ x_2^{(k+1)} - x_2^{(k)} $	$ x_3^{(k+1)} - x_3^{(k)} $
0	0	0	0	-	-	-
1	0.333	1.667	1	0.333	1.667	1
2	1.222	1.222	1.667	0.889	0.555	0.667
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6	1.044	1.044	1.132	0.176	0.088	0.132

O Método de Gauss-Seidel

Diagram illustrating the Gauss-Seidel method. The initial values $x_1^{(0)}, \dots, x_n^{(0)}$ are used to update the variables x_1, x_2, \dots, x_n iteratively. The equations shown are:

$$\begin{aligned}x_1 &= \frac{1}{a_{11}} [b_1 - a_{12}x_2 - a_{13}x_3 - \dots - a_{1n}x_n] \\x_2 &= \frac{1}{a_{22}} [b_2 - a_{21}x_1 - a_{23}x_3 - \dots - a_{2n}x_n] \\&\vdots \\x_n &= \frac{1}{a_{nn}} [b_n - a_{n1}x_1 - a_{n2}x_2 - \dots - a_{n,n-1}x_{n-1}]\end{aligned}$$

O Método de Gauss-Seidel

Diagram illustrating the Gauss-Seidel method. A box at the top contains the initial guess vector $x_1^{(0)}, \dots, x_n^{(0)}$. Dashed arrows point from this box to the terms x_2 , x_3 , and x_n in the first equation of a system below. A bracket on the left groups the equations, with an arrow pointing to $x_1^{(1)}$ in a box.

$$\begin{aligned} x_1 &= \frac{1}{a_{11}} [b_1 - a_{12}x_2 - a_{13}x_3 - \dots - a_{1n}x_n] \\ x_2 &= \frac{1}{a_{22}} [b_2 - a_{21}x_1 - a_{23}x_3 - \dots - a_{2n}x_n] \\ &\vdots \\ x_n &= \frac{1}{a_{nn}} [b_n - a_{n1}x_1 - a_{n2}x_2 - \dots - a_{n,n-1}x_{n-1}] \end{aligned}$$

O Método de Gauss-Seidel

The diagram illustrates the Gauss-Seidel method iteration. It shows the calculation of $x_1^{(1)}$ from the previous iteration's values $x_1^{(0)}, \dots, x_n^{(0)}$. The equations for x_1 , x_2 , and x_n are shown, with $x_1^{(1)}$ being calculated from the previous iteration's values. The term $a_{21}x_1$ is circled in red, indicating the update of x_1 in the second equation.

$$\begin{aligned} x_1 &= \frac{1}{a_{11}} [b_1 - a_{12}x_2 - a_{13}x_3 - \dots - a_{1n}x_n] \\ x_2 &= \frac{1}{a_{22}} [b_2 - a_{21}x_1 - a_{23}x_3 - \dots - a_{2n}x_n] \\ &\vdots \\ x_n &= \frac{1}{a_{nn}} [b_n - a_{n1}x_1 - a_{n2}x_2 - \dots - a_{n,n-1}x_{n-1}] \end{aligned}$$

O Método de Gauss-Seidel

The diagram illustrates the Gauss-Seidel method for solving a system of linear equations. It shows the iterative update of variables x_1 and x_2 .

Initial values: $x_1^{(0)}, \dots, x_n^{(0)}$

Iterative updates:

$$\begin{aligned} x_1 &= \frac{1}{a_{11}} [b_1 - a_{12}x_2 - a_{13}x_3 - \dots - a_{1n}x_n] \\ x_2 &= \frac{1}{a_{22}} [b_2 - a_{21}x_1 - a_{23}x_3 - \dots - a_{2n}x_n] \\ &\vdots \\ x_n &= \frac{1}{a_{nn}} [b_n - a_{n1}x_1 - a_{n2}x_2 - \dots - a_{n,n-1}x_{n-1}] \end{aligned}$$

The diagram shows the iterative update of x_1 and x_2 . The initial values $x_1^{(0)}, \dots, x_n^{(0)}$ are used to calculate $x_1^{(1)}$. Then, $x_1^{(1)}$ is used to calculate $x_2^{(1)}$. The equations are shown with red circles highlighting the terms being updated.

O Método de Gauss-Seidel

$$x_1^{(0)}, \dots, x_n^{(0)}$$

The diagram illustrates the Gauss-Seidel method for solving a system of linear equations. It shows the iterative update of variables x_1, x_2, \dots, x_n from their initial values $x_1^{(0)}, \dots, x_n^{(0)}$ to their first iteration values $x_1^{(1)}, x_2^{(1)}, \dots, x_n^{(1)}$. The equations are presented within a green-bordered box, and the updated values are shown in boxes to the left. Dashed arrows indicate the sequential update process: $x_1^{(1)}$ is used in the equation for x_2 , which is then used in the equation for x_3 , and so on, up to x_n . In the final equation for x_n , the terms $a_{n1}x_1$, $a_{n2}x_2$, and $a_{n,n-1}x_{n-1}$ are circled in red, with arrows pointing from the corresponding $x_1^{(1)}, x_2^{(1)}, \dots, x_{n-1}^{(1)}$ boxes, highlighting that these values are already updated and thus used in their latest iteration.

$$\begin{aligned} x_1^{(1)} &= \frac{1}{a_{11}} [b_1 - a_{12}x_2 - a_{13}x_3 - \dots - a_{1n}x_n] \\ x_2^{(1)} &= \frac{1}{a_{22}} [b_2 - a_{21}x_1 - a_{23}x_3 - \dots - a_{2n}x_n] \\ &\vdots \\ x_n^{(1)} &= \frac{1}{a_{nn}} [b_n - a_{n1}x_1 - a_{n2}x_2 - \dots - a_{n,n-1}x_{n-1}] \end{aligned}$$

O Método de Gauss-Seidel

$x_1^{(1)}$

$x_2^{(1)}$

\vdots

$x_n^{(1)}$

$x_1 = \frac{1}{a_{11}} [b_1 - a_{12}x_2 - a_{13}x_3 - \dots - a_{1n}x_n]$

$x_2 = \frac{1}{a_{22}} [b_2 - a_{21}x_1 - a_{23}x_3 - \dots - a_{2n}x_n]$

\vdots

$x_n = \frac{1}{a_{nn}} [b_n - a_{n1}x_1 - a_{n2}x_2 - \dots - a_{n,n-1}x_{n-1}]$

$x_1^{(1)}, \dots, x_n^{(1)}$

O Método de Gauss-Seidel

$x_1^{(1)}, \dots, x_n^{(1)}$

$$\begin{aligned} x_1 &= \frac{1}{a_{11}} [b_1 - a_{12}x_2 - a_{13}x_3 - \dots - a_{1n}x_n] \\ x_2 &= \frac{1}{a_{22}} [b_2 - a_{21}x_1 - a_{23}x_3 - \dots - a_{2n}x_n] \\ &\vdots \\ x_n &= \frac{1}{a_{nn}} [b_n - a_{n1}x_1 - a_{n2}x_2 - \dots - a_{n,n-1}x_{n-1}] \end{aligned}$$

O Método de Gauss-Seidel

Diagram illustrating the Gauss-Seidel method. A box at the top contains the vector $x_1^{(1)}, \dots, x_n^{(1)}$. Three dashed arrows point from this box to the terms x_2 , x_3 , and x_n in the first equation of a system below. The terms x_2 , x_3 , and x_n are circled in red. A bracket on the left groups the equations, with $x_1^{(2)}$ next to the first equation.

$$\begin{aligned} x_1 &= \frac{1}{a_{11}} [b_1 - a_{12}x_2 - a_{13}x_3 - \dots - a_{1n}x_n] \\ x_2 &= \frac{1}{a_{22}} [b_2 - a_{21}x_1 - a_{23}x_3 - \dots - a_{2n}x_n] \\ &\vdots \\ x_n &= \frac{1}{a_{nn}} [b_n - a_{n1}x_1 - a_{n2}x_2 - \dots - a_{n,n-1}x_{n-1}] \end{aligned}$$

O Método de Gauss-Seidel

Diagram illustrating the Gauss-Seidel method. The initial guess vector $x_1^{(1)}, \dots, x_n^{(1)}$ is used to calculate the next iteration's values. The updated value $x_1^{(2)}$ is calculated using the first equation. This updated value is then used in the second equation to calculate x_2 , which is then used in the third equation to calculate x_3 , and so on, until x_n is calculated. The terms x_1 , x_3 , and x_n in the second equation are circled in red, indicating they are the most recent values available for calculation.

$$\begin{aligned} x_1 &= \frac{1}{a_{11}} [b_1 - a_{12}x_2 - a_{13}x_3 - \dots - a_{1n}x_n] \\ x_2 &= \frac{1}{a_{22}} [b_2 - a_{21}x_1 - a_{23}x_3 - \dots - a_{2n}x_n] \\ &\vdots \\ x_n &= \frac{1}{a_{nn}} [b_n - a_{n1}x_1 - a_{n2}x_2 - \dots - a_{n,n-1}x_{n-1}] \end{aligned}$$

O Método de Gauss-Seidel

Diagram illustrating the Gauss-Seidel method. The system of equations is shown within a box:

$$\begin{aligned}x_1 &= \frac{1}{a_{11}} [b_1 - a_{12}x_2 - a_{13}x_3 - \dots - a_{1n}x_n] \\x_2 &= \frac{1}{a_{22}} [b_2 - a_{21}x_1 - a_{23}x_3 - \dots - a_{2n}x_n] \\&\vdots \\x_n &= \frac{1}{a_{nn}} [b_n - a_{n1}x_1 - a_{n2}x_2 - \dots - a_{n,n-1}x_{n-1}]\end{aligned}

Variables $x_1^{(2)}$ and $x_2^{(2)}$ are shown on the left, with dashed arrows pointing to the corresponding terms in the equations. The variables $x_1^{(1)}, \dots, x_n^{(1)}$ are shown at the top, with dashed arrows pointing to the corresponding terms in the equations. The variables x_1, x_3, x_n in the second equation are circled in red, indicating they are updated using the latest values.$$

O Método de Gauss-Seidel

$$x_1^{(1)}, \dots, x_n^{(1)}$$

The diagram illustrates the Gauss-Seidel iteration for the n -th equation. On the left, a column of variables $x_1^{(2)}$, $x_2^{(2)}$, and $x_n^{(2)}$ is shown, each enclosed in a box. Dashed lines connect these boxes to the corresponding terms in the equation for x_n on the right. Specifically, dashed lines connect $x_1^{(2)}$ to $a_{n1}x_1$, $x_2^{(2)}$ to $a_{n2}x_2$, and $x_n^{(2)}$ to $a_{nn}x_n$. The equation for x_n is
$$x_n = \frac{1}{a_{nn}} [b_n - a_{n1}x_1 - a_{n2}x_2 - \dots - a_{n,n-1}x_{n-1}]$$
 The terms x_1 , x_2 , and x_{n-1} in the brackets are circled in red. Arrows point from the circled x_1 and x_2 to the circled x_{n-1} , indicating that the most recent values are used in the calculation.

$$\begin{aligned} x_1^{(2)} & \quad x_1 = \frac{1}{a_{11}} [b_1 - a_{12}x_2 - a_{13}x_3 - \dots - a_{1n}x_n] \\ x_2^{(2)} & \quad x_2 = \frac{1}{a_{22}} [b_2 - a_{21}x_1 - a_{23}x_3 - \dots - a_{2n}x_n] \\ & \quad \vdots \\ x_n^{(2)} & \quad x_n = \frac{1}{a_{nn}} [b_n - a_{n1}x_1 - a_{n2}x_2 - \dots - a_{n,n-1}x_{n-1}] \end{aligned}$$

O Método de Gauss-Seidel

$x_1^{(2)}$
 $x_2^{(2)}$
 \vdots
 $x_n^{(2)}$

$x_1 = \frac{1}{a_{11}} [b_1 - a_{12}x_2 - a_{13}x_3 - \dots - a_{1n}x_n]$
 $x_2 = \frac{1}{a_{22}} [b_2 - a_{21}x_1 - a_{23}x_3 - \dots - a_{2n}x_n]$
 \vdots
 $x_n = \frac{1}{a_{nn}} [b_n - a_{n1}x_1 - a_{n2}x_2 - \dots - a_{n,n-1}x_{n-1}]$

$x_1^{(2)}, \dots, x_n^{(2)}$

Comparação

Gauss-Jacobi

$$\begin{aligned}x_1^{(1)} &= \frac{1}{a_{11}} [b_1 - a_{12}x_2^{(0)} - a_{13}x_3^{(0)} - \dots - a_{1n}x_n^{(0)}] \\x_2^{(1)} &= \frac{1}{a_{22}} [b_2 - a_{21}x_1^{(0)} - a_{23}x_3^{(0)} - \dots - a_{2n}x_n^{(0)}] \\&\vdots \\x_n^{(1)} &= \frac{1}{a_{nn}} [b_n - a_{n1}x_1^{(0)} - a_{n2}x_2^{(0)} - \dots - a_{n,n-1}x_{n-1}^{(0)}]\end{aligned}$$

Gauss-Seidel

$$\begin{aligned}x_1^{(1)} &= \frac{1}{a_{11}} [b_1 - a_{12}x_2^{(0)} - a_{13}x_3^{(0)} - \dots - a_{1n}x_n^{(0)}] \\x_2^{(1)} &= \frac{1}{a_{22}} [b_2 - a_{21}x_1^{(1)} - a_{23}x_3^{(0)} - \dots - a_{2n}x_n^{(0)}] \\&\vdots \\x_n^{(1)} &= \frac{1}{a_{nn}} [b_n - a_{n1}x_1^{(1)} - a_{n2}x_2^{(1)} - \dots - a_{n,n-1}x_{n-1}^{(1)}]\end{aligned}$$

Exemplo

$$\begin{cases} 3x_1 - x_2 - x_3 = 1 \\ x_1 + 3x_2 + x_3 = 5 \\ x_1 - x_2 + 2x_3 = 2 \end{cases}$$

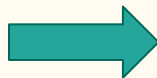


$$\begin{cases} x_1^{(k+1)} = (1 + x_2^{(k)} + x_3^{(k)}) / 3 \\ x_2^{(k+1)} = (5 - x_1^{(k+1)} - x_3^{(k)}) / 3 \\ x_3^{(k+1)} = (2 - x_1^{(k+1)} + x_2^{(k+1)}) / 2 \end{cases}$$

k	$x_1^{(k)}$	$x_2^{(k)}$	$x_3^{(k)}$	$ x_1^{(k+1)} - x_1^{(k)} $	$ x_2^{(k+1)} - x_2^{(k)} $	$ x_3^{(k+1)} - x_3^{(k)} $
0	0	0	0	-	-	-
1	0.333	1.555	1.611	0.333	1.555	1.611
2	1.388	0.666	0.638	1.055	0.888	0.972
3	0.768	1.197	1.214	0.620	0.531	0.575
4	1.137	0.882	0.872	0.368	0.314	0.341
5	0.918	1.069	1.075	0.218	0.186	0.202
6	1.048	0.958	0.955	0.129	0.110	0.120

Exemplo

$$\begin{cases} 3x_1 - x_2 - x_3 = 1 \\ x_1 + 3x_2 + x_3 = 5 \\ x_1 - x_2 + 2x_3 = 2 \end{cases}$$



$$\begin{cases} x_1^{(k+1)} = (1 + x_2^{(k)} + x_3^{(k)}) / 3 \\ x_2^{(k+1)} = (5 - x_1^{(k+1)} - x_3^{(k)}) / 3 \\ x_3^{(k+1)} = (2 - x_1^{(k+1)} + x_2^{(k+1)}) / 2 \end{cases}$$

k	$x_1^{(k)}$	$x_2^{(k)}$	$x_3^{(k)}$	$ x_1^{(k+1)} - x_1^{(k)} $	$ x_2^{(k+1)} - x_2^{(k)} $	$ x_3^{(k+1)} - x_3^{(k)} $
0	0	0	0	-	-	-
1	0.333	1.555	1.611	0.333	1.555	1.611
2	1.388	0.666	0.638	1.055	0.888	0.972
3	0.768	1.197	1.214	0.620	0.531	0.575
4	1.137	0.882	0.872	0.368	0.314	0.341
5	0.918	1.069	1.075	0.218	0.186	0.202
6	1.048	0.958	0.955	0.129	0.110	0.120

Critérios de Convergência

Se o sistema $\mathbf{Ax} = \mathbf{b}$ tiver **diagonal dominante**, os métodos Gauss-Jacobi e Gauss-Seidel convergem para a solução do sistema.

$$\alpha_i = \frac{\sum_{j=1, j \neq i}^n |a_{ij}|}{|a_{ii}|}$$

$$\alpha = \max(\alpha_i) < 1$$

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Exemplo:

$$\begin{cases} 3x_1 - x_2 - x_3 = 1 & |3| \geq |-1| + |-1| & V \\ x_1 + 3x_2 + x_3 = 5 & |3| \geq |1| + |1| & V \\ x_1 - x_2 + 2x_3 = 2 & |2| \geq |1| + |-1| & V \end{cases}$$

Critérios de Convergência

Para o método de Gauss-Seidel, o critério pode ser relaxado (**Critério de Sassenfeld**).

$$\beta_i = \frac{\sum_{j=1}^{i-1} |a_{ij}| \beta_j + \sum_{j=i+1}^n |a_{ij}|}{|a_{ii}|}$$

$$\beta = \max(\beta_i) < 1$$

Critérios de Convergência

Para o método de Gauss-Seidel, o critério pode ser relaxado (**Critério de Sassenfeld**).

Exemplo:

$$\left\{ \begin{array}{ll} 3x_1 - x_2 - x_3 = 1 & |3| \geq |-1| + |-1| \quad V \\ x_1 + 3x_2 + x_3 = 5 & |3| \geq |1| + |1| \quad V \\ x_1 - x_2 + 2x_3 = 2 & |2| \geq |1| + |-1| \quad V \end{array} \right. \Rightarrow 2/3 = 0,6666 = \beta_1$$

Critérios de Convergência

Para o método de Gauss-Seidel, o critério pode ser relaxado (**Critério de Sassenfeld**).

Exemplo:

$$\begin{cases} 3x_1 - x_2 - x_3 = 1 & |3| \geq |-1| + |-1| & V \rightarrow 2/3 = 0,6666 = \beta_1 \\ x_1 + 3x_2 + x_3 = 5 & |3| \geq |1| + |1| & V \rightarrow (0,6666+1) \\ x_1 - x_2 + 2x_3 = 2 & |2| \geq |1| + |-1| & V \end{cases}$$

Critérios de Convergência

Para o método de Gauss-Seidel, o critério pode ser relaxado (**Critério de Sassenfeld**).

Exemplo:

$$\begin{cases} 3x_1 - x_2 - x_3 = 1 & |3| \geq |-1| + |-1| & V \rightarrow 2/3 = 0,6666 = \beta_1 \\ x_1 + 3x_2 + x_3 = 5 & |3| \geq |1| + |1| & V \rightarrow (0,6666 + 1)/3 \\ x_1 - x_2 + 2x_3 = 2 & |2| \geq |1| + |-1| & V \rightarrow = 0,5555 = \beta_2 \end{cases}$$

Critérios de Convergência

Para o método de Gauss-Seidel, o critério pode ser relaxado (**Critério de Sassenfeld**).

Exemplo:

$$\left\{ \begin{array}{ll} 3x_1 - x_2 - x_3 = 1 & |3| \geq |-1| + |-1| \quad V \\ x_1 + 3x_2 + x_3 = 5 & |3| \geq |1| + |1| \quad V \\ x_1 - x_2 + 2x_3 = 2 & |2| \geq |1| + |-1| \quad V \end{array} \right. \begin{array}{l} \rightarrow 2/3 = 0,6666 = \beta_1 \\ \rightarrow (0,6666 + 1)/3 \\ = 0,5555 = \beta_2 \\ \rightarrow (0,6666 + 0,5555)/2 \\ = 0,61105 = \beta_3 \end{array}$$

Critérios de Convergência

Se os critérios de convergência forem satisfeitos, os métodos Gauss-Jacobi e Gauss-Seidel convergem para a solução do sistema para qualquer $\mathbf{X}^{(0)}$.

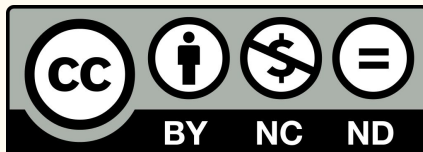
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