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An Exchanged 3-Ary n-Cube Interconnection Network for Parallel Computation

Yali Lv*, Cheng-Kuan Lin†,§ and Guijuan Wang‡

*School of Information Technology
Henan University of Chinese Medicine
Zhengzhou, China, 450046

†College of Mathematics and Computer Science
Fuzhou University, Fuzhou, China, 350108

‡School of Computer Science and Technology
Qilu University of Technology, Jiyuan, China, 250353
§cklin@fzu.edu.cn

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The interconnetion network plays an important role in a parallel system. To avoid the edge number of the interconnect network scaling rapidly with the increase of dimension and achieve a good balance of hardware costs and properties, this paper presents a new interconnection network called exchanged 3-ary n-cube ($E3C$). Compared with the 3-ary n-cube structures, $E3C$ shows better performance in terms of many metrics such as small degree and fewer links. In this paper, we first introduce the structure of $E3C$ and present some properties of $E3C$; then, we propose a routing algorithm and obtain the diameter of $E3C$. Finally, we analyze the diagnosis of $E3C$ and give the diagnosibility under PMC model and MM* model.

Keywords: Interconnection network; exchanged 3-ary n-cube; routing; diagnosibility.

1. Introduction

The past decade has witnessed significant progress in parallel computing architectures for the development of large-scale parallel systems. It is well known that the interconnection network is very important in large-scale parallel systems, because its design impacts directly the performance and cost-effectiveness of the system [5, 29]. Topology structure is usually viewed as an important design issue for interconnection networks. In current literatures, many topologies were proposed and studied,

§Corresponding author.

especially the hypercube structure and its varieties [8, 11, 18, 33], such as k-ary n-cube, folded hypercube, crossed cube, and so on. The k-ary n-cube structure [4, 10], especially the 3-ary n-cube structure [15, 26, 35], has received much attention due to its many attractive properties [13, 24, 25], such as its ability to reduce message latency and ease of implementation. A number of parallel and distributed systems have been built with a k-ary n-cube forming the underlying topology, such as the J-machine [30], the Cray T3D [19], the Cray T3E [2] and the IBM super computer BlueGene/L [1]. The underlying topology of the IBM super computer BlueGene/L is the 3-ary n-cube structure.

Among all the features of a network topology, the number of edges and the diameter are two of the most important factors. More edges and less diameter ensure efficient and rapid message transmission. Unluckily, more edges will obviously incur hardware costs, but fewer edges result in longer diameter and higher time complexity of interprocessor communication. Meanwhile, a topology structure is also bad if it scales too rapidly as the dimension increases, because it is difficult to complete the expansion of the system. The challenge is to avoid the edge number of the interconnect network scaling rapidly with the increase of dimension and achieve a good balance of hardware costs and properties. Loh *et al.* constructed an exchanged hypercube by removing some edges from the hypercube [23]. Loh *et al.* also studied the spanning-tree embedding of the exchanged hypercube. In [21], Li *et al.* proposed the exchanged crossed cube and gave the optimal routing and broadcasting algorithms for the new network topology. Qi *et al.* presented the exchanged folded hypercube-based topology structure and gave a routing algorithm and a load-balancing algorithm for this new network topology [32]. Considering the wide application of the 3-ary n-cube network, we plan to change its structure to improve its performance. The exchanged hypercube was arrived by removing two types of edges from different parts of the hypercube. In order to maintain some good properties, we divide the 3-ary n-cube into three parts and remove different types of edges from different parts. Thus we propose a new interconnection topology structure, named exchanged 3-ary n-cube ($E3C$).

$E3C$ has fewer edges than the Q_n^3 with the same dimension. However, it maintains several desirable properties of the Q_n^3. Meanwhile, we investigate that the network diameter of $E3C$ is only 2 more than the diameter of Q_n^3. We also design an efficient routing algorithm for $E3C$. Finally, we analyze the diagnosibility of $E3C$ under PMC model and MM* model. Table 1 presents a straightforward comparison between Q_n^3 and $E3C$ in terms of the following properties: the total number of vertices, the total number of edges, vertex degree, diameter, diagnosibility and connectivity. These properties have significant impact on the performance of a parallel system. Detailed description of these parameters is in the following sections.

The rest of the paper is organized as follows: Section 2 presents the construction of $E3C$. Section 3 discusses its various topological properties and compares the $E3C$ with the Q_n^3. In Sec. 4, an efficient routing algorithm is proposed and the

Table 1. Comparison of Networks.

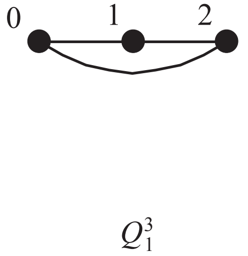
Networks	Q_n^3	$E3C(r,s,t)$ $(r+s+t+1=n)$
Vertices	3^n	3^n
Edges	$n3^n$	$(n+2)3^{n-1}$
Diameter	n	$r+s+t+3=n+2$
Degree	$2n$	$2r+2$, $2s+2$ or $2t+2$
Diagnosibility	$2n$	$\min\{2r+2,\ 2s+2,\ 2t+2\}$
Connectivity	$2n$	$\min\{2r+2,\ 2s+2,\ 2t+2\}$

diameter of $E3C$ is obtained. In Sec. 5, we analyze the diagnosis of $E3C$ and give the diagnosibility under PMC model and MM* model. The paper is then concluded followed by the acknowledgement and references.

2. The Exchanged 3-Ary n-Cube

In this paper, we follow [17] for the graph-theoretical terminology and notation not defined here. Given a graph G, we denote the *vertex set* and the *edge set* as $V(G)$ and $E(G)$, respectively. A *path* P, denoted by $\langle u_1, u_2, \ldots, u_n\rangle$, is a sequence of adjacent vertices where all the vertices are distinct except possibly $u_1 = u_n$. We set $P \oplus v = \langle u_1, u_2, \ldots, u_n, v\rangle$, while v is an adjacent vertex of u_n. For $0 \le i \le j \le k-1$, we use $[i,j]$ to denote a set of integers: $[i,j] = \{l | i \le l \le j\}$. Before constructing an $E3C$, we give the definition of Q_n^3.

For $n \ge 1$, the 3-*ary* n-*cube* Q_n^3 has 3^n vertices, each of which has the form $x = x_{n-1}x_{n-2}\ldots x_0$ where $x_i \in \{0,1,2\}$ for $0 \le i \le n-1$. Two vertices $x = x_{n-1}x_{n-2}\ldots x_0$ and $y = y_{n-1}y_{n-2}\ldots y_0$ in Q_n^3 are adjacent if and only if there exists an integer i such that (1) either $y_i = (x_i + 1) \mod 3$ or $y_i = (x_i - 1) \mod 3$, and (2) $x_j = y_j$ for each $j \ne i$. Note that each vertex has degree $2n$. Note that Q_1^3 is isomorphic to a cycle of length 3. The Q_1^3 and Q_2^3 are illustrated as in Fig. 1.



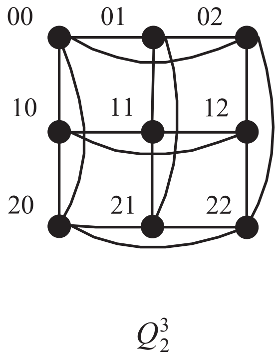


Fig. 1. The Q_1^3 and Q_2^3.

We set $h(a,b) = \begin{cases} 0 \text{ if } a = b \\ 1 \text{ if } a \neq b \end{cases}$. The procedure of constructing an $E3C$ is presented as follow.

The exchanged 3-ary n-cube is defined as an undirected graph $E3C(r,s,t) = (V,E)$ where $r \geq 1, s \geq 1, t \geq 1$ and $r+s+t+1 = n$. V is the set of vertices, $V = \{a_{r-1}a_{r-2}\cdots a_0 b_{s-1}b_{s-2}\cdots b_0 c_{t-1}c_{t-2}\cdots c_0 d | a_i, b_j, c_k, d \in \{0,1,2\}, 0 \leq i \leq r-1, 0 \leq j \leq s-1, 0 \leq k \leq t-1\}$, and E is the set of edges, $E = \{(x,y)|(x,y) \in V \times V\}$, consisting of four types of edges (i.e., E_0, E_1, E_2 and E_3), which are described as follows:

(1) $E_0 = \{(x,y)|H_1^{r+s+t}(x,y) = 0 \text{ and } x[0] \neq y[0]\}$,
(2) $E_1 = \{(x,y)|H_{t+1}^{r+s+t}(x,y) = 0, H_1^t(x,y) = 1 \text{ and } x[0] = y[0] = 0\}$,
(3) $E_2 = \{(x,y)|H_{s+t+1}^{r+s+t}(x,y) = 0, H_{t+1}^{s+t}(x,y) = 1, H_1^t(x,y) = 0 \text{ and } x[0] = y[0] = 1\}$, and
(4) $E_3 = \{(x,y)|H_{s+t+1}^{r+s+t}(x,y) = 1, H_1^{s+t}(x,y) = 0 \text{ and } x[0] = y[0] = 2\}$,

where $H(x,y) = \sum_{i=0}^{r+s+t} h(x[i], y[i])$ denotes the *Hamming* distance between vertices x and y, and $H_p^q(x,y) = \sum_{i=p}^{q} h(x[i], y[i])$. This means that an $E3C(r,s,t)$ has four sets of edges: The first set links vertex pairs that exhibit unity Hamming distance in the first bit of their addresses, the second set links vertex pairs that exhibit unity Hamming distance in the first t bits of their addresses, the third set links vertex pairs that exhibit unity Hamming distance in the middle s bits of their addresses, and the fourth set links vertex pairs that exhibit unity Hamming distance in the last r bits of their addresses. For a vertex $x = x_{r+s+t}x_{r+s+t-1}\cdots x_{i+1}x_i x_{i-1}\cdots x_0$, we set $x^{i,j} = x_{r+s+t}x_{r+s+t-1}\cdots x_{i+1} j x_{i-1}\cdots x_0$. For $i \in \{1,2,3\}$, we set $E3C_i(r,s,t)$ being a subgraph of $E3C(r,s,t)$ induced by edges in E_i.

Figure 2 shows an $E3C(1,1,1)$.

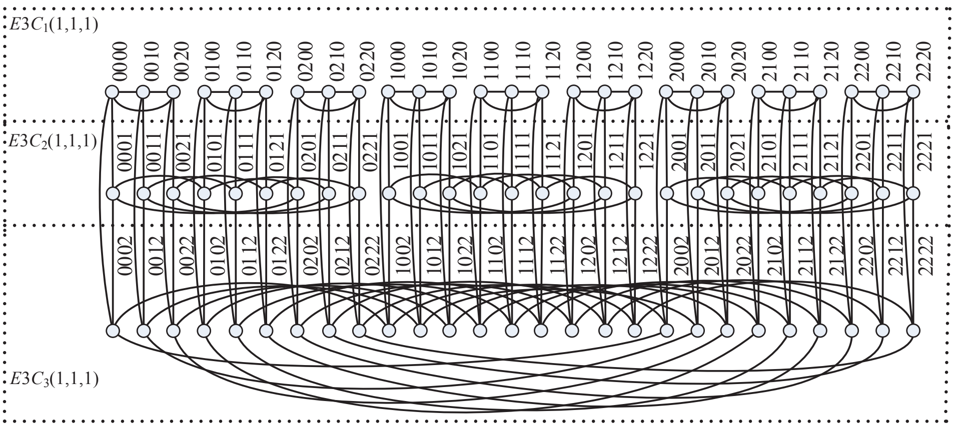


Fig. 2. An $E3C(1,1,1)$.

3. Topological Properties of *E3C*

Now we use the following propositions to illustrate the important topological properties of the $E3C(r,s,t)$ and then compare its performance with the 3-ary n-cube.

3.1. *Number of vertices and edges*

The number of vertices in an $E3C(r,s,t)$ is given by the following proposition:

Proposition 1. *The total number of vertices in* $E3C(r,s,t)$ *is* 3^n, *where* $n = r+s+t+1$.

Proposition 2. *The total number of edges in* $E3C(r,s,t)$ *is* $(r+s+t+3)\times 3^{r+s+t} = (n+2)3^{n-1}$, *which is about one third of that in* Q_n^3.

Proof. The edge set of $E3C(r,s,t)$ is composed of four types, i.e., E_0, E_1, E_2, E_3. The definition of $E3C$ reveals that the number of edges in E_0 is $3^{r+s+t+1}$; the number of edges in E_1 is $t\times 3^{r+s+t}$; the number of edges in E_2 is $s\times 3^{r+s+t}$; the number of edges in E_3 is $r\times 3^{r+s+t}$. The total number of edges in Q_n^3 is $n3^n$, and the total number of edges in $E3C(r,s,t)$ is $(n+2)3^{n-1}$, $r+s+t+1=n$. Hence, we have $\frac{(n+2)3^{n-1}}{n3^n} = \frac{n+2}{3n}$, which approaches $1/3$ as $n\to\infty$. □

3.2. *Isomorphism*

By definition, two graphs G and H are isomorphic if H can be obtained from G by relabeling the vertices [37]. Now we discuss the isomorphism of $E3C$.

Proposition 3. *Let* r, s, t *be any three positive integers and let* $\{r',s',t'\}=\{r,s,t\}$. *Then* $E3C(r,s,t)$ *is isomorphic to* $E3C(r',s',t')$.

Proof. According to the definition of $E3C$, we have $E3C(r,s,t)=(V_1,E_1)$, where $V_1=\{a_{r-1}a_{r-2}\cdots a_0b_{s-1}b_{s-2}\cdots b_0c_{t-1}c_{t-2}\cdots c_0d \mid a_i,b_j,c_k,d\in\{0,1,2\}, i\in[0,r-1], j\in[0,s-1], k\in[0,t-1]\}$. We consider the following six cases.

Case 1: $r=r'$, $s=s'$ and $t=t'$. It is clear that $E3C(r,s,t)=E3C(r',s',t')$.

Case 2: $r=r'$, $s=t'$ and $t=s'$. $E3C(r',s',t')=E3C(r,t,s)$. Note that $E3C(r,t,s)=(V_2,E_2)$, where $V_2=\{a_{r-1}a_{r-2}\cdots a_0b_{t-1}b_{t-2}\cdots b_0c_{s-1}c_{s-2}\cdots c_0d \mid a_i,b_j,c_k,d\in\{0,1,2\}, i\in[0,r-1], j\in[0,t-1], k\in[0,s-1]\}$. Considering an arbitrary vertex $u=a_{r-1}a_{r-2}\cdots a_0b_{s-1}b_{s-2}\cdots b_0c_{t-1}c_{t-2}\cdots c_0d$ in $E3C(r,s,t)$, we can find a vertex $v=a'_{r-1}a'_{r-2}\cdots a'_0b'_{t-1}b'_{t-2}\cdots b'_0c'_{s-1}c'_{s-2}\cdots c'_0d'$ in $E3C(r,t,s)$, obtained by the following bijection f: $a_{r-1}=a'_{r-1}$, $a_{r-2}=a'_{r-2},\ldots,a_0=a'_0$, $b_{s-1}=c'_{s-1}$, $b_{s-2}=c'_{s-2},\ldots,b_0=c'_0$, $c_{t-1}=b'_{t-1}$, $c_{t-2}=b'_{t-2},\ldots,c_0=b'_0$, if $d=0$, then $d'=1$, if $d=1$, then $d'=0$, if $d=2$, then $d'=2$.

Similarly, we can find that every vertex v' in $E3C(r,t,s)$ has a corresponding vertex u' in $E3C(r,s,t)$. An $E3C(r,s,t)$ has the same number of vertices as an $E3C(r,t,s)$. Hence, f is a bijection from V_1 to V_2.

Based on the definition of $E3C$, if (u, v) is an arbitrary edge in $E3C(r, s, t)$, then $(f(u), f(v))$ is an edge in $E3C(r, t, s)$. Similarly, if (u', v') is an arbitrary edge in $E3C(r, t, s)$, then $(f(u'), f(v'))$ is also an edge in $E3C(r, s, t)$. Hence, there is a bijection g from E_1 to E_2 that maps each edge (u, v) to $(f(u), f(v))$. Based on the above analysis and the definition of isomorphic graphs, we can conclude that $E3C(r, s, t)$ and $E3C(r, t, s)$ are isomorphic.

Case 3: $r = s'$, $s = r'$ and $t = t'$. Similar to Case 2, we can get the conclusion.

Case 4: $r = s'$, $s = t'$ and $t = r'$. $E3C(r', s', t') = E3C(t, r, s)$. Note that $E3C(t, r, s) = (V_2, E_2)$, where $V_2 = \{a_{t-1}a_{t-2} \cdots a_0 b_{r-1} b_{r-2} \cdots b_0 c_{s-1} c_{s-2} \cdots c_0 d | a_i, b_j, c_k, d \in \{0, 1, 2\}, i \in [0, t-1], j \in [0, r-1], k \in [0, s-1]\}$. Considering an arbitrary vertex $u = a_{r-1}a_{r-2} \cdots a_0 b_{s-1} b_{s-2} \cdots b_0 c_{t-1} c_{t-2} \cdots c_0 d$ in $E3C(r, s, t)$, we can find a vertex $v = a'_{t-1}a'_{t-2} \cdots a'_0 b'_{r-1} b'_{r-2} \cdots b'_0 c'_{s-1} c'_{s-2} \cdots c'_0 d'$ in $E3C(t, r, s)$, obtained by the following bijection f: $a_{r-1} = b'_{r-1}$, $a_{r-2} = b'_{r-2}$, ..., $a_0 = b'_0$, $b_{s-1} = c'_{s-1}$, $b_{s-2} = c'_{s-2}$, ..., $b_0 = c'_0$, $c_{t-1} = a'_{t-1}$, $c_{t-2} = a'_{t-2}$, ..., $c_0 = a'_0$, $d = (d' + 1) \mod 3$.

Similarly, we can find that every vertex $v' = a_{t-1}a_{t-2} \cdots a_0 b_{r-1} b_{r-2} \cdots b_0 c_{s-1} c_{s-2} \cdots c_0 d$ in $E3C(t, r, s)$ has a corresponding vertex $u' = a'_{r-1}a'_{r-2} \cdots a'_0 b'_{s-1} b'_{s-2} \cdots b'_0 c'_{t-1} c'_{t-2} \cdots c'_0 d'$ in $E3C(r, s, t)$, obtained by the following bijection f': $a_{t-1} = c'_{t-1}$, $a_{t-2} = c'_{t-2}$, ..., $a_0 = c'_0$, $b_{r-1} = a'_{r-1}$, $b_{r-2} = a'_{r-2}$, ..., $b_0 = a'_0$, $c_{s-1} = b'_{s-1}$, $c_{s-2} = b'_{s-2}$, ..., $c_0 = b'_0$, $d = (d' + 2) \mod 3$.

An $E3C(r, s, t)$ has the same number of vertices as an $E3C(t, r, s)$. Hence, f is a bijection from V_1 to V_2, f' is a bijection from V_2 to V_1.

Based on the definition of $E3C$, if (u, v) is an arbitrary edge in $E3C(r, s, t)$, then $(f(u), f(v))$ is an edge in $E3C(t, r, s)$. Similarly, if (u', v') is an arbitrary edge in $E3C(t, r, s)$, then $(f'(u'), f'(v'))$ is also an edge in $E3C(r, s, t)$. Hence, there is a bijection g from E_1 to E_2 that maps each edge (u, v) to $(f(u), f(v))$, and a bijection g' from E_2 to E_1 that maps each edge (u', v') to $(f'(u'), f'(v'))$ Based on the above analysis and the definition of isomorphic graphs, we can conclude that $E3C(r, s, t)$ and $E3C(t, r, s)$ are isomorphic.

Case 5: $r = t'$, $s = s'$ and $t = r'$. Similar to Case 2, the conclusion can be arrived.

Case 6: $r = t'$, $s = r'$ and $t = s'$. Similar to Case 4, we can get the conclusion.

Based on the above analysis, we have $E3C(r, s, t)$ is isomorphic to $E3C(r', s', t')$ when $\{r', s', t'\} = \{r, s, t\}$. □

3.3. *Decomposition*

Proposition 4. *Let r, s, t be any three positive integers with $r \geq 2$. $E3C(r, s, t)$ can be decomposed into three disjoint subgraphs such that each of them is isomorphic to $E3C(r-1, s, t)$.*

Proof. For $i \in \{0, 1, 2\}$, we set $G^i_{r,s,t}$ being a subgraph of $E3C(r, s, t)$ induced by $\{x = x_{r+s+t}x_{r+s+t-1} \cdots x_0 \in V(E3C(r, s, t)) | x_{r+s+t} = i\}$. Then, a vertex u in each

$G^i_{r,s,t}$ can be expressed as $u = \{a_{r-2}a_{r-3}\cdots\ a_0b_{s-1}b_{s-2}\cdots\ b_0c_{t-1}c_{t-2}\cdots\ c_0d|a_i,$ b_j, $c_k, d \in \{0,1,2\}$, $i \in [0, r-2]$, $j \in [0, s-1]$, $k \in [0, t-1]\}$. Then, vertices x, y in each $G^i_{r,s,t}$ for $i \in \{0,1,2\}$ are connected by the following four types of edges:

(1) $E_0 = \{(x, y)|H_1^{r+s+t-1}(x, y) = 0$ and $x[0] = y[0]\}$.
(2) $E_1 = \{(x, y)|H_{t+1}^{r+s+t-1}(x, y) = 0, H_1^t(x, y) = 1$ and $x[0] = y[0] = 0\}$.
(3) $E_2 = \{(x, y)|H_{s+t+1}^{r+s+t-1}(x, y) = 0, H_{t+1}^{s+t}(x, y) = 1, H_1^t(x, y) = 0$ and $x[0] = y[0] = 1\}$.
(4) $E_3 = \{(x, y)|H_{s+t+1}^{r+s+t-1}(x, y) = 1, H_1^{s+t}(x, y) = 0$ and $x[0] = y[0] = 2\}$.

By the definition of $E3C$, $G^i_{r,s,t}$ is isomorphic to $E3C(r-1, s, t)$. □

Proposition 5. *An $E3C(r, s, t)$ can be decomposed into 3^{r+s} topological networks of Q^3_t, 3^{r+t} topological networks of Q^3_s, and 3^{s+t} topological networks of Q^3_r.*

Proof. According to the definition of $E3C$, an $E3C(r, s, t)$ has one set of vertices $V = \{a_{r-1}a_{r-2}\cdots\ a_0b_{s-1}b_{s-2}\ \cdots\ b_0c_{t-1}c_{t-2}\cdots\ c_0d|a_i,\ b_j,\ c_k, d \in \{0,1,2\}$, $i \in [0, r-1]$, $j \in [0, s-1]$, $k \in [0, t-1]\}$ and four disjoint sets of edges E_0, E_1, E_2 and E_3. Furthermore, based on the definition of Q^3_n, E_1, E_2 and E_3 can compose the edge set of a Q^3_t, a Q^3_s, a Q^3_r, respectively. $V_1 = \{c_{t-1}c_{t-2}\cdots c_0|c_k \in \{0,1,2\}, k \in [0, t-1]\}$ is the vertex set of Q^3_t, $V_2 = \{b_{s-1}c_{s-2}\cdots b_0|b_j \in \{0,1,2\}, j \in [0, s-1]\}$ is the vertex set of Q^3_s, and $V_3 = \{a_{r-1}a_{r-2}\cdots a_0|a_i \in \{0,1,2\}, i \in [0, r-1]\}$ is the vertex set of Q^3_r.

There are 3^{r+s} vertex sets V_1, 3^{r+t} vertex sets V_2 and 3^{s+t} vertex sets V_3 in V. The set of V_1 and E_1 can compose a Q^3_t, the set of V_2 and E_2 can compose a Q^3_s while the set of V_3 and E_3 can compose a Q^3_r.

Thus, the subgraph $E3C_1(r, s, t)$ contains 3^{r+s} disjoint copies of Q^3_t, the subgraph $E3C_2(r, s, t)$ contains 3^{r+t} disjoint copies of Q^3_s, the subgraph $E3C_3(r, s, t)$ contains 3^{s+t} disjoint copies of Q^3_r. So an $E3C(r, s, t)$ can be decomposed into 3^{r+s} topological networks of Q^3_t, 3^{r+t} topological networks of Q^3_s and 3^{s+t} topological networks of Q^3_r. □

3.4. *Degree*

Proposition 6. *The degree of a vertex x in $E3C(r, s, t)$ is*

$$deg(x) = \begin{cases} 2t+2, & \text{if } x \in E3C_1(r,s,t) \\ 2s+2, & \text{if } x \in E3C_2(r,s,t) \\ 2r+2, & \text{if } x \in E3C_3(r,s,t) \end{cases}$$

Moreover, the minimum degree of $E3C(r, s, t)$ is $min\{2r+2, 2s+2, 2t+2\}$.

Proof. From the definition of $E3C$, the edge set E of $E3C(r, s, t)$ is composed of four types, i.e., E_0, E_1, E_2, E_3. By the definition of each type, a vertex $x \in E3C_1(r, s, t)$ has $2t$ neighboring vertices connected by edges of type E_1, and 2 neighboring vertices connected by edges of type E_0, and the vertex x has no neighbor vertex connected by edges of type E_2 or E_3. So for a vertex $x \in E3C_1(r,s,t)$, the degree of x is $2t+2$. Similarly, a vertex $y \in E3C_2(r,s,t)$ has $2s$ neighboring vertices connected by edges of type E_2, and 2 neighboring vertices connected by edges of type E_0. The degree of y is $2s+2$. A vertex $z \in E3C_3(r,s,t)$ has $2r$ neighboring vertices connected by edges of type E_3, and 2 neighboring vertices connected by edges of type E_0. The degree of z is $2r+2$. Moreover, the minimum degree of $E3C(r,s,t)$ is $\min\{2r+2, 2s+2, 2t+2\}$. □

4. Routing of the $E3C(r,s,t)$

In [6], Bose *et al.* gave a routing algorithm to construct the shortest path between any two distinct vertices in Q_n^k. In this section, we develop an optimal one-to-one routing algorithm for $E3C(r,s,t)$. An optimal routing algorithm is to find the shortest path between a source and destination pair, where the source sends a message to the destination. Suppose the source is $x = a_{r-1}a_{r-2}\cdots a_0 b_{s-1}b_{s-2}\cdots b_0 c_{t-1}c_{t-2}\cdots c_0 d$ and the destination is $y = a'_{r-1}a'_{r-2}\cdots a'_0 b'_{s-1}b'_{s-2}\cdots b'_0 c'_{t-1}c'_{t-2}\cdots c'_0 d'$. There are two cases to be considered. For two vertices x and y, let

$$s^i(x,y) = \begin{cases} H_1^t(x,y), & \text{if } i=0 \\ H_{t+1}^{s+t}(x,y), & \text{if } i=1 \\ H_{s+t+1}^{r+s+t}(x,y), & \text{if } i=2 \end{cases}$$

Case 1: $x[0] \neq y[0]$. Let $a = x[0]$, $c = y[0]$, and $b = \{0,1,2\} - \{a,c\}$. We assume that $a = 2$, $b = 1$, and $c = 0$(Otherwise we can do with the similar method), we have the following steps.

Step 1: If $s^a(x,y) > 0$, let $z = a'_{r-1}a'_{r-2}\cdots a'_0 b_{s-1}b_{s-2}\cdots b_0 c_{t-1}c_{t-2}\cdots c_0 d$. Based on Proposition 5, the two vertices x and z are in the same Q_r^3. Routing in the same Q_r^3 can be done by the routing algorithm developed in [6]. If $s^a(x,y) = 0$, let $z = x$.

Step 2: If $s^b(x,y) > 0$, let $z' = a'_{r-1}a'_{r-2}\cdots a'_0 b_{s-1}b_{s-2}\cdots b_0 c_{t-1}c_{t-2}\cdots c_0 b$ and $z^b = a'_{r-1}a'_{r-2}\cdots a'_0 b'_{s-1}b'_{s-2}\cdots b'_0 c_{t-1}c_{t-2}\cdots c_0 b$. It is clear that z' and z are connected by one edge E_0. Based on Proposition 5, the two vertices z' and z^b are in the same Q_s^3. Routing in the same Q_s^3 can be done by the routing algorithm developed in [6]. If $s^b(x,y) = 0$, let $z^b = z$.

Step 3: Let $z'' = a'_{r-1}a'_{r-2}\cdots a'_0 b'_{s-1}b'_{s-2}\cdots b'_0 c_{t-1}c_{t-2}\cdots c_0 c$. So the two vertices z^b and z'' are connected by one edge E_0. If $s^c(x,y) = 0$, the vertex z'' is the same vertex with y. If $s^c(x,y) > 0$, based on Proposition 5, the two vertices z'' and y are in the same Q_t^3. Routing in the same Q_t^3 can be done by the routing algorithm developed in [6]. The path from x to y has been arrived.

In the case the distance between x and y is noted by $dis(x,y)$.

$$dis(x,y)=\begin{cases} H_1^{r+s+t}(x,y)+2, & \text{if } s^b(x,y)>0 \\ H_1^{r+s+t}(x,y)+1, & \text{if } s^b(x,y)=0 \end{cases}$$

Case 2: $x[0]=y[0]$. Let $a=x[0]$. Set c being an element of $\{0,1,2\}-\{a\}$. Then let $b=\{0,1,2\}-\{a,c\}$. We assume that $a=2$, $b=1$, and $c=0$(Otherwise we can do with the similar method), we have the following steps.

Step 1: If $s^a(x,y)>0$, let $z=a'_{r-1}a'_{r-2}\cdots a'_0b_{s-1}b_{s-2}\cdots b_0c_{t-1}c_{t-2}\cdots c_0d$. Based on Proposition 5, the two vertices x and z are in the same Q_r^3. Routing in the same Q_r^3 can be done by the routing algorithm developed in [6]. If $s^a(x,y)=0$, let $z=x$.

Step 2: If $s^b(x,y)>0$, let $z'=a'_{r-1}a'_{r-2}\cdots a'_0b_{s-1}b_{s-2}\cdots b_0c_{t-1}c_{t-2}\cdots c_0b$ and $z^b=a'_{r-1}a'_{r-2}\cdots a'_0b'_{s-1}b'_{s-2}\cdots b'_0c_{t-1}c_{t-2}\cdots c_0b$. It is clear that z' and z are connected by one edge E_0. Based on Proposition 5, the two vertices z' and z^b are in the same Q_s^3. Routing in the same Q_s^3 can be done by the routing algorithm developed in [6]. If $s^b(x,y)=0$, let $z^b=z$.

Step 3: If $s^c(x,y)>0$, let $z''=a'_{r-1}a'_{r-2}\cdots a'_0b'_{s-1}b'_{s-2}\cdots b'_0c_{t-1}c_{t-2}\cdots c_0c$ and $z^c=a'_{r-1}a'_{r-2}\cdots a'_0b'_{s-1}b'_{s-2}\cdots b'_0c'_{t-1}c'_{t-2}\cdots c'_0c$. It is clear that z'' and z^b are connected by one edge E_0. Based on Proposition 5, the two vertices z'' and z^c are in the same Q_t^3. Routing in the same Q_t^3 can be done by the routing algorithm developed in [6]. If $s^c(x,y)=0$, let $z^c=z^b$.

Step 4: If $z^c[0]\neq a$, z^c and y are connected by one edge E_0. If $z^c[0]=a$, then z^c is the same with the vertex y. So the path has been arrived from x to y.

In the case the distance between x and y is:

$$dis(x,y)=\begin{cases} H_1^{r+s+t}(x,y)+3, & \text{if } \min\{s^b(x,y),s^c(x,y)\}>0 \\ H_1^{r+s+t}(x,y)+2, & \text{if } \max\{s^b(x,y),s^c(x,y)\}>0 \text{ and} \\ & \min\{s^b(x,y),s^c(x,y)\}=0 \\ H_1^{r+s+t}(x,y), & \text{if } \max\{s^b(x,y),s^c(x,y)\}=0. \end{cases}$$

Algorithm:
Input: two distinct vertices x and y in $E3C(r,s,t)$.
Output: the shortest path between x and y.
Begin
(1) set $A_1=\{i|x[i]\neq y[i],1\le i\le t\}$, $A_2=\{i|x[i]\neq y[i],t+1\le i\le s+t\}$, $A_3=\{i|x[i]\neq y[i],s+t+1\le i\le r+s+t\}$, and a path $P=<x>$;
(2) set $a=x[0]$;
(3) If $x[0]\neq y[0]$ then {set $c=y[0]$; }
(4) Else {set $c\in\{0,1,2\}-\{a\}$; }
(5) set $b=\{0,1,2\}-\{a,c\}$;
(6) If $s^a(x,y)>0$ then {
(7) While($A_a\neq\phi$) {
(8) select an element $i\in A_a$;

```
            set z be the latest vertex of P;
              P = P ⊕ z^{i,y[i]};
              A_a = A_a − {i};
        }endWhile
    }endIf
    If s^b(x, y) > 0 then {
        set z be the latest vertex of P;
        P = P ⊕ z^{0,b};
        While(A_b ≠ φ) {
            select an element i ∈ A_b;
            set w be the latest vertex of P;
            P = P ⊕ w^{i,y[i]};
            A_b = A_b − {i};
        }endWhile
    }endIf
    If s^c(x, y) > 0 then {
        set z be the latest vertex of P;
        P = P ⊕ z^{0,c};
        While(A_c ≠ φ) {
            select an element i ∈ A_c;
            set w be the latest vertex of P;
            P = P ⊕ w^{i,y[i]};
            A_c = A_c − {i};
        }endWhile
    }endIf
    set z be the latest vertex of P;
    If z[0] ≠ y[0] then {
        P = P ⊕ z^{0,y[0]}
    }EndIf
    Return P;
End
```

Based on the routing and this algorithm, we can arrive the following proposition.

Proposition 7. *The distance of two distinct vertices x and y in $E3C(r, s, t)$ is*

$$dis(x,y) = \begin{cases} H_1^{r+s+t}(x,y)+3, & \text{if } x[0]=y[0] \text{ and } \min\{s^b(x,y), s^c(x,y)\} > 0 \\ H_1^{r+s+t}(x,y)+2, & \text{if } x[0]\neq y[0] \text{ and } s^b(x,y) > 0 \text{ or } x[0]=y[0], \\ & \max\{s^b(x,y), s^c(x,y)\} > 0 \text{ and} \\ & \min\{s^b(x,y), s^c(x,y)\} = 0 \\ H_1^{r+s+t}(x,y)+1, & \text{if } x[0]\neq y[0] \text{ and } s^b(x,y) = 0 \\ H_1^{r+s+t}(x,y), & \text{if } x[0]=y[0] \text{ and } \max\{s^b(x,y), s^c(x,y)\} = 0 \end{cases}$$

Moveover, the diameter of $E3C(r, s, t)$ is $r + s + t + 3$.

Proposition 8. *The time complexity of the algorithm is $O(n)$ where $n = r+s+t+1$.*

5. Diagnosis of the *E3C*

One typical application of parallel computing is the multiprocessor system. In multiprocessor systems, even a few malfunctions may make system service unreliable, so the reliability of each processor should be considered. It is hoped that the fault processors could be found and replaced in time in multiprocessor systems to maintain high reliability. Many scholars studied the process of identifying faulty processor (called system diagnosis) [7, 14, 20, 36, 38, 39]. The diagnosability is defined as the maximum number of faulty processors that can be identified. People proposed many diagnosis models to identify faulty processors [3, 12, 27, 28, 31]. The original diagnostic models are the PMC model introduced by Prepparata *et al.* [31] and the MM model proposed by Maeng and Malek [27]. In the PMC model, the result of the diagnosis is achieved through two linked processors testing each other. Each processor is able to test another processor if there is a link connecting the two processors. In the MM model, the result of the diagnosis is achieved through a processor sending the same task to two of its neighbors, and comparing their responses. Sengupta and Dahbura [34] proposed a further modification of the MM model, called the MM* model, in which each processor must test another two processors, if they are adjacent to it. Clearly, the diagnosability of a system is upper bounded by its minimum degree. These studies mainly consider the global faulty/fault-free status. In [16], Hsu and Tan considered some local systematic details and presented a new measure of diagnosability, called local diagnosability. They identified the diagnosability of a system by computing the local diagnosability of each individual processor. In this section, we discuss the diagnosibility of $E3C(r,s,t)$ under PMC model and MM* model with the aid of the research conclusions of the local diagnosability.

Definition 9 ([9]). A system of n processors is t-diagnosable if all faulty processors can be identified without replacement, provided that the number of faults presented does not exceed t. The diagnosability of a system G denoted as $t(G)$, is the maximum value of t such that G is t-diagnosable.

Definition 10 ([16]). Let G be a graph and v denote any one of its vertices. The graph G is locally t-diagnosable at vertex v if the fault status of the vertex v can be identified, provided that the number of faults presented does not exceed t. The local diagnosability of a vertex v in G, denoted by $\pi_G(v)$, is defined to be the maximum integer of t such that G is locally t-diagnosable at vertex v.

Hsu and Tan not only give the definition of the local diagnosability but also give the relation between the local diagnosability and the diagnosability.

Proposition 11 ([16]). *Let G denote the underlying topology of a multiprocessor system. Then $t(G) = min\{\pi_G(v)|v \in V(G)\}$.*

Proposition 12 ([16]). *Let $G = (V, E)$ be a graph and $v \in V$ be a vertex with $deg(v) = m$. The local diagnosability of the vertex v is at most m.*

Let $< n >= \{x | 0 \leq x \leq n\}$. For a vertex $u \in V(E3C(r,s,t))$, $u = u_{r+s+t}u_{r+s+t-1}\cdots u_1u_0$, let $(u)_i^+ = u'_{r+s+t}u'_{r+s+t-1}\cdots u'_1u'_0$ where $u'_j = u_j$ for every $j \in < r+s+t > -\{i\}$ and $u'_i = (u_i+1) \bmod 3$. And let $(u)_i^- = u'_{r+s+t}u'_{r+s+t-1}\cdots u'_1u'_0$ where $u'_j = u_j$ for every $j \in < r+s+t > -\{i\}$ and $u'_i = (u_i+2) \bmod 3$. Now, we discuss the local diagnosability of the $E3C$ under PMC model and MM* model, respectively.

5.1. *The diagnosability of E3C under PMC model*

In [16], the following structure is presented by Hsu *et al.* to compute the local diagnosability of any given vertex under the PMC model.

Definition 13 ([16]). Let $G = (V, E)$ be a graph, $v \in V$ be a vertex, and k be an integer, $k \geq 1$, an extending star structure $T_G(v; k)$ of order k rooted at vertex v is defined to be the following subgraph of G: $T_G(v; k) = (V(v; k), E(v; k))$, which is composed of $2k+1$ vertices and $2k$ edges, Where $V(v; k) = \{v\} \cup \{u_{ij} | 1 \leq i \leq 2, 1 \leq j \leq k\}$ and $E(v; k) = \{(v, u_{1j}), (u_{1j}, u_{2j}) | 1 \leq j \leq k\}$. See Fig. 3 for illustration.

Based on the extending star structure, Hsu *et al.* gave a polynomial-time algorithm to determine whether any given vertex is faulty or not. From these results, we have the following proposition.

Proposition 14 ([16]). *Let $G = (V, E)$ be a graph and $u \in V$ be a vertex. Under the PMC model, if G contains an extending star structure $T_G(u; t)$ of order t at vertex u as a subgraph, G is locally t-diagnosable at vertex u.*

Now we use the extending star structure to compute the local diagnosability of the exchanged 3-ary n-cube $E3C$.

Proposition 15. *Let $u \in V(E3C(r,s,t))$ be a vertex with $deg(u) = m$. The local diagnosability of vertex u is m under the PMC model.*

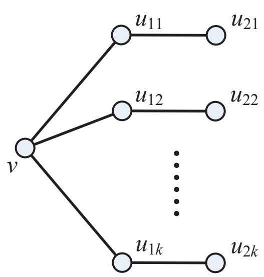


Fig. 3. An extending star structure $T_G(v; k)$ of order k rooted at vertex v.

Proof. Based on the Proposition 12, the local diagnosability of the vertex v is at most m. We only need to prove that $E3C(r,s,t)$ is locally m-diagnosable at vertex u with $deg(u)=m$. Let $u=u_{r+s+t}u_{r+s+t-1}\cdots u_1u_0$. Now we construct an extending star structure $T_{E3C(r,s,t)}(u;m)$ of order m rooted at vertex u as a subgraph in the following three cases.

Firstly, we discuss the case of $u\in V(E3C_1(r,s,t))$. Let a vertex set $A(u)=\{(u)_i^+|1\le i\le t\}\cup\{(u)_i^-|1\le i\le t\}\cup\{(u)_0^+,(u)_0^-\}$, $E(A(u))=\{(u,(u)_i^+)|1\le i\le t\}\cup\{(u,(u)_i^-)|1\le i\le t\}\cup\{(u,(u)_0^+),(u,(u)_0^-)\}$.

And let a vertex set $B(u)=\{((u)_i^+)_0^+|1\le i\le t\}\cup\{((u)_i^-)_0^+|1\le i\le t\}\cup\{((u)_0^+)_{s+t}^+,((u)_0^-)_{r+s+t}^+\}$, $E(B(u))=\{((u)_i^+,((u)_i^+)_0^+)|1\le i\le t\}\cup\{((u)_i^-,((u)_i^-)_0^+)|1\le i\le t\}\cup\{((u)_0^+,((u)_0^+)_{s+t}^+),((u)_0^-,((u)_0^-)_{r+s+t}^+)\}$.

Let a graph $G=(V_1,E_1)$, $V_1=\{u\}\cup A(u)\cup B(u)$, $E_1=E(A(u))\cup E(B(u))$. Then G is a subgraph of $E3C(r,s,t)$ and an extending star structure $T_1(u;(2t+2))$ of order $2t+2$ at vertex u. So $E3C(r,s,t)$ is locally $(2t+2)$-diagnosable at vertex u for $u\in V(E3C_1(r,s,t))$ under PMC model. Figure 4 shows an extending star structure $T_{E3C(1,1,2)}(u;6)$ of order 6 at vertex $u=00000$ in an $E3C(1,1,2)$.

Similarly, we can get that $E3C(r,s,t)$ is locally $(2s+2)$-diagnosable at vertex u for $u\in V(E3C_2(r,s,t))$ and $E3C(r,s,t)$ is locally $(2r+2)$-diagnosable at vertex u for $u\in V(E3C_3(r,s,t))$.

For $u\in V(E3C_1(r,s,t))$, $deg(u)=2t+2$, for $u\in V(E3C_2(r,s,t))$, $deg(u)=2s+2$, for $u\in V(E3C_3(r,s,t))$, $deg(u)=2r+2$, thus let $u\in V(E3C(r,s,t))$ be a vertex with $deg(u)=m$, the local diagnosability of vertex u is m under PMC model. $\square$

Based on Proposition 11 and Proposition 15, we get the following proposition.

Proposition 16. *The diagnosability of* $E3C(r,s,t)$ *under PMC model is* $min\{2t+2,2s+2,2r+2\}$.

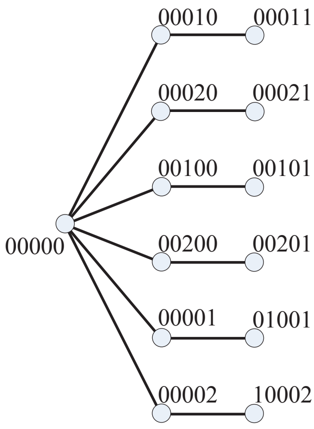


Fig. 4. An extending star structure $T_{E3C(1,1,2)}(u;6)$ of order 6 at vertex $u=00000$ in an $E3C(1,1,2)$.

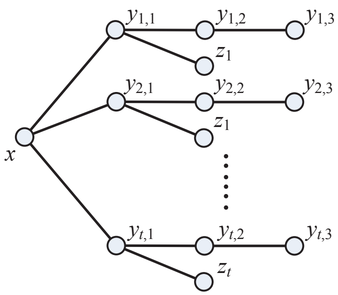


Fig. 5. A wind-bell-tree structure $WB_G(x;t)$ of order t rooted at vertex x.

5.2. *The diagnosability of the E3C under MM* model*

Lin *et al.* gave a useful structure, called a wind-bell-tree, to determine the local diagnosability under the MM* model[22]. Now we first give the definition of the structure.

Definition 17 ([22]). Let $G = (V, E)$ be a graph, and let x be a vertex in G. A wind-bell-tree of order t rooted at x is defined to be the subgraph of G, denoted by $WB_G(x;t)$, such that $V(WB_G(x;t)) = \{x\} \cup \{y_{i,j}, z_i | 1 \le i \le t, 1 \le j \le 3\}$, and $E(WB_G(x;t)) = \{(x, y_{i,1}), (y_{i,k}, y_{i,k+1}), (y_{i,1}, z_i) | 1 \le i \le t, 1 \le k \le 2\}$. Figure 5 illustrates a wind-bell-tree structure $WB_G(x;t)$ of order t rooted at vertex x.

In [22], Lin *et al.* propose a local-diagnosis algorithm to identify the faulty or fault-tree status of a vertex x in a wind-bell-tree $WB_G(x;t)$ under the MM model. They proved that the faulty or fault-free status of vertex u can be identified correctly with the algorithm if there exist a wind-bell-tree $WB_G(u;t)$ of order t rooted at u, where F is a faulty set in G with $|F| \le t$. Based on these results, we have the following proposition.

Proposition 18 ([22]). *Let $G = (V, E)$ be a graph and $u \in V$ be a vertex. Under the MM model, G is locally t-diagnosable at vertex u if G contains a wind-bell-tree $WB_G(u;t)$ of order t at vertex u as a subgraph.*

Now we use the wind-bell-tree structure to compute the local diagnosability of the exchanged 3-ary n-cube $E3C$.

Proposition 19. *Let $u \in V(E3C(r,s,t))$ be a vertex with $deg(u) = m$. The local diagnosability of vertex u is m under the MM model.*

Proof. Based on the Proposition 12, we only need to prove that $E3C(r,s,t)$ is locally m-diagnosable at vertex u. Let $u = u_{r+s+t}u_{r+s+t-1}\cdots u_1u_0$. Now we construct a wind-bell-tree $WB_{E3C(r,s,t)}(u;m)$ of order m at vertex u as a subgraph in the following cases.

Firstly, we discuss the case of $u \in V(E3C_1(r,s,t))$. Let a vertex set $A(u) = \{(u)_i^+ | 1 \le i \le t\} \cup \{(u)_i^- | 1 \le i \le t\} \cup \{(u)_0^+, (u)_0^-\}$, $E(A(u)) = \{(u,(u)_i^+) | 1 \le i \le t\} \cup \{(u,(u)_i^-) | 1 \le i \le t\} \cup \{(u,(u)_0^+),(u,(u)_0^-)\}$.

And let a vertex set $B(u) = \{((u)_i^+)_0^+ | 1 \le i \le t\} \cup \{((u)_i^-)_0^+ | 1 \le i \le t\} \cup \{((u)_0^+)_{s+t}^+, ((u)_0^-)_{r+s+t}^+\}$, $E(B(u)) = \{((u)_i^+, ((u)_i^+)_0^+) | 1 \le i \le t\} \cup \{((u)_i^-, ((u)_i^-)_0^+) | 1 \le i \le t\} \cup \{((u)_0^+, ((u)_0^+)_{s+t}^+), ((u)_0^-, ((u)_0^-)_{r+s+t}^+)\}$.

And let a vertex set $C(u) = \{(((u)_i^+)_0^+)_{s+t}^+ | 1 \le i \le t\} \cup \{(((u)_i^-)_0^+)_{s+t}^+ | 1 \le i \le t\} \cup \{(((u)_0^+)_{s+t}^+)_0^-, (((u)_0^-)_{r+s+t}^+)_0^+\}$, $E(C(u)) = \{(((u)_i^+)_0^+, (((u)_i^+)_0^+)_{s+t}^+) | 1 \le i \le t\} \cup \{(((u)_i^-)_0^+, (((u)_i^-)_0^+)_{s+t}^+) | 1 \le i \le t\} \cup \{(((u)_0^+)_{s+t}^+, (((u)_0^+)_{s+t}^+)_0^-), (((u)_0^-)_{r+s+t}^+, (((u)_0^-)_{r+s+t}^+)_0^+)\}$.

And let a vertex set $D(u) = \{((u)_i^+)_0^- | 1 \le i \le t\} \cup \{((u)_i^-)_0^- | 1 \le i \le t\} \cup \{((u)_0^+)_{s+t}^-, ((u)_0^-)_{r+s+t}^-\}$, $E(D(u)) = \{((u)_i^+, ((u)_i^+)_0^-) | 1 \le i \le t\} \cup \{((u)_i^-, ((u)_i^-)_0^-) | 1 \le i \le t\} \cup \{((u)_0^+, ((u)_0^+)_{s+t}^-), ((u)_0^-, ((u)_0^-)_{r+s+t}^-)\}$.

We list these set symbols in Table 2.

Let a graph $G = (V_1, E_1)$, $V_1 = \{u\} \cup A(u) \cup B(u) \cup C(u) \cup D(u)$, $E_1 = E(A(u)) \cup E(B(u)) \cup E(C(u)) \cup E(D(u))$. Then G is a subgraph of $E3C(r,s,t)$ and a wind-bell-tree $WB_{E3C(r,s,t)}(u; 2t+2)$ of order $2t+2$ at vertex u. So $E3C(r,s,t)$ is locally $(2t+2)$-diagnosable at vertex u for $u \in V(E3C_1(r,s,t))$ under the MM model. Figure 6 shows a wind-bell-tree $WB_{E3C(1,1,2)}(u; 6)$ of order 6 at vertex $u = 00000$ in an $E3C(1,1,2)$.

Similarly, we can get that $E3C(r,s,t)$ is locally $(2s+2)$-diagnosable at vertex u for $u \in V(E3C_2(r,s,t))$ and $E3C(r,s,t)$ is locally $(2r+2)$-diagnosable at vertex u for $u \in V(E3C_3(r,s,t))$.

For $u \in V(E3C_1(r,s,t))$, $deg(u) = 2t+2$, for $u \in V(E3C_2(r,s,t))$, $deg(u) = 2s+2$, for $u \in V(E3C_3(r,s,t))$, $deg(u) = 2r+2$, thus let $u \in V(E3C(r,s,t))$ be a vertex with $deg(u) = m$, the local diagnosability of vertex u is m under the MM model. □

Table 2. Symbols.

Symbol	Definition
$A(u)$	$\{(u)_i^+ \mid 1 \le i \le t\} \cup \{(u)_i^- \mid 1 \le i \le t\} \cup \{(u)_0^+, (u)_0^-\}$
$E(A(u))$	$\{(u,(u)_i^+) \mid 1 \le i \le t\} \cup \{(u,(u)_i^-) \mid 1 \le i \le t\} \cup \{(u,(u)_0^+),(u,(u)_0^-)\}$
$B(u)$	$\{((u)_i^+)_0^+ \mid 1 \le i \le t\} \cup \{((u)_i^-)_0^+ \mid 1 \le i \le t\} \cup \{((u)_0^+)_{s+t}^+, ((u)_0^-)_{r+s+t}^+\}$
$E(B(u))$	$\{((u)_i^+, ((u)_i^+)_0^+) \mid 1 \le i \le t\} \cup \{((u)_i^-, ((u)_i^-)_0^+) \mid 1 \le i \le t\}$ $\cup \{((u)_0^+, ((u)_0^+)_{s+t}^+), ((u)_0^-, ((u)_0^-)_{r+s+t}^+)\}$
$C(u)$	$\{(((u)_i^+)_0^+)_{s+t}^+ \mid 1 \le i \le t\} \cup \{(((u)_i^-)_0^+)_{s+t}^+ \mid 1 \le i \le t\}$ $\cup \{(((u)_0^+)_{s+t}^+)_0^-, (((u)_0^-)_{r+s+t}^+)_0^+\}$
$E(C(u))$	$\{(((u)_i^+)_0^+, (((u)_i^+)_0^+)_{s+t}^+) \mid 1 \le i \le t\} \cup \{(((u)_i^-)_0^+, (((u)_i^-)_0^+)_{s+t}^+) \mid 1 \le i \le t\}$ $\cup \{(((u)_0^+)_{s+t}^+, (((u)_0^+)_{s+t}^+)_0^-), (((u)_0^-)_{r+s+t}^+, (((u)_0^-)_{r+s+t}^+)_0^+)\}$
$D(u)$	$\{((u)_i^+)_0^- \mid 1 \le i \le t\} \cup \{((u)_i^-)_0^- \mid 1 \le i \le t\} \cup \{((u)_0^+)_{s+t}^-, ((u)_0^-)_{r+s+t}^-\}$
$E(D(u))$	$\{((u)_i^+, ((u)_i^+)_0^-) \mid 1 \le i \le t\} \cup \{((u)_i^-, ((u)_i^-)_0^-) \mid 1 \le i \le t\}$ $\cup \{((u)_0^+, ((u)_0^+)_{s+t}^-), ((u)_0^-, ((u)_0^-)_{r+s+t}^-)\}$

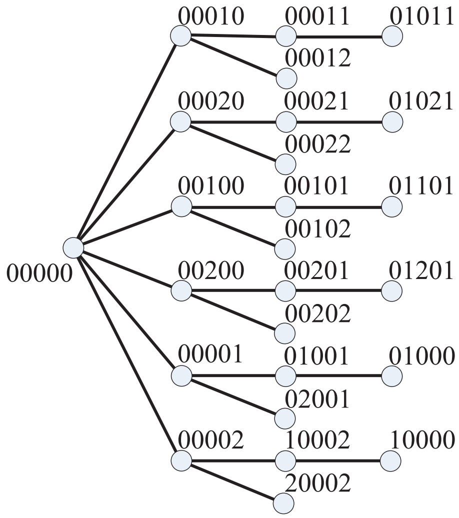


Fig. 6. A wind-bell-tree $WB_{E3C(1,1,2)}(u;6)$ of order 6 at vertex $u = 00000$ in an $E3C(1,1,2)$.

According the definition of MM model and MM* model, based on Propositions 11 and 19, we get the following proposition.

Proposition 20. *The diagnosability of* $E3C(r,s,t)$ *under MM* model is* $min\{2t+2, 2s+2, 2r+2\}$.

6. Conclusions

This paper has presented a novel interconnection topology called exchanged 3-ary n-cube. A major advantage is that it has fewer links and scales upward with lower edge costs than the 3-ary n-cube. This new topology also has many desirable properties such as regularity, expandability, isomorphism and decomposition. The diameter and vertex degree of an $E3C$ have low values. An optimal routing algorithm that guarantees the shortest path and the diagnosibility under PMC model and MM* model are given. The attractive properties of the $E3C$ make it applicable to large-scale parallel computing systems very well. Our following work is to study the exchanged k-ary n-cube by improving our current ideas. It is also worth to discuss the class of generalization graphs and study their routing algorithm, diagnosability and other properties.

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References

[1] N. R. Adiga, M. A. Blumrich, D. Chen, P. Coteus, A. Gara, M. E. Giampapa, P. Heidelberger, S. Singh, B. D. Steinmacher-Burow, T. Takken, M. Tsao and P. Vranas, Blue gene/l torus interconnection network, *IBM Journal of Research and Development* **49**(2–3) (2005) 265–276.

[2] E. Anderson, J. Brooks, C. Grassl and S. Scott, Performance of the CRAY T3E multiprocessor, in *Proceedings of the 1997 ACM/IEEE Conference on Supercomputing, Association for Computing Machinery* (New York, USA, San Jose, CA, United states, 1997), pp. 1–17.

[3] F. Barsi, F. Grandoni and P. Maestrini, A theory of diagnosability of digital systems, *IEEE Trans. Comput.* **25**(6) (1976) 585–593.

[4] S. Bettayeb, On the k-ary hypercube, *Theor. Comput. Sci.* **140** (1995) 333–339.

[5] L. Bhuyan, Q. Yang and D. Agrawal, Performance of multiprocessor intercommection networks, *Computer* **22**(2) (1989) 25–37.

[6] B. Bose, B. Broeg, Y. Kwon and Y. Ashir, Lee distance and topological properties of k-ary n-cubes, *IEEE Trans. Comput.* **44**(8) (1995) 1021–1030.

[7] N.-W. Chang and S.-Y. Hsieh, Conditional diagnosability of augmented cubes under the pmc model, *IEEE Transactions on Dependable and Secure Computing* **9**(1) (2010) 46–60.

[8] N.-W. Chang, C.-Y. Tsai and S.-Y. Hsieh, On 3-extra connectivity and 3-extra edge connectivity of folded hypercubes, *IEEE Trans. Comput.* **63**(6) (2014) 1594–1600.

[9] A. T. Dahbura and G. M. Masson, An $o(n^{2.5})$ fault identification algorithm for diagnosable systems, *IEEE Trans. Comput.* **33**(6) (1984) 486–492.

[10] K. Day and A. E. Al-Ayyoub, Fault diameter of k-ary n-cube networks, *IEEE Transactions on Parallel and Distributed Systems* **8**(9) (1997) 903–907.

[11] A. EI-Amawy and S. Latifi, Properties and performance of folded hypercubes, *IEEE Transactions on Parallel Distributed Systems* **2**(1) (1991) 31–42.

[12] A. D. Friedman and L. Simoncini, System-level fault diagnosis, *The Computer Journal* **13**(3) (1980) 47–53.

[13] S.-Y. Hsieh and Y.-H. Chang, Extraconnectivity of k-ary n-cube networks, *Theor. Comput. Sci.* **443** (2012) 63–69.

[14] S.-Y. Hsieh and C.-Y. Kao, The conditional diagnosability of k-ary n-cubes under the comparison diagnosis model, *IEEE Trans. Comput.* **62**(4) (2013) 839–843.

[15] S.-Y. Hsieh, T.-J. Lin and H.-L. Huang, Panconnectivity and edge-pancyclicity of 3-ary n-cubes, *J. Supercomput.* **42**(2) (2007) 225–233.

[16] G.-H. Hsu and J. J. Tan, A local diagnosiability measure for multiprocessor systems, *IEEE Transactions on Parallel and Distributed Systems* **18**(5) (2007) 598–607.

[17] L.-H. Hsu and C.-K. Lin, *Graph Theory and Interconnection Networks* (CRC Press, Boca Raton, FL, USA, 2008).

[18] W. Hsu, Fibonacci cubes-a new interconnection topology, *IEEE Transactions on Parallel and Distributed Systems* **4**(1) (1993) 3–12.

[19] R. E. Kessler and J. L. Schwarzmeier, Cray T3D: A new dimension for Cray research, in: *38th Annual IEEE Computer Society International Computer Conference — COMPCON SPRING '93* (Publ by IEEE, Piscataway, NJ, United States, San Francisco, CA, USA, 1993), pp. 176–182.

[20] D. Li and M. Lu, The g-good-neighbor conditional diagnosability of star graphs under the pmc and mm* model, *Theor. Comput. Sci.* **674** (2017) 53–59.

[21] K. Li, Y. Mu, K. Li and G. Min, Exchanged crossed cube: A nobel interconnection network for parallel computation, *IEEE Transactions on Parallel and Distributed Systems* **24**(11) (2013) 2211–2219.

[22] C.-K. Lin, Y.-H. Teng, J. J. Tan and L.-H. Hsu, Local diagnosis algorithms for multiprocessor systems under the comparison diagnosis model, *IEEE Transactions on Reliability* **62**(4) (2013) 800–810.

[23] P. K. Loh, W. J. Hsu and Y. Pan, The exchanged hypercube, *IEEE Transactions on Parallel and Distributed Systems* **16**(9) (2005) 866–874.

[24] Y. Lv, J. Fan, D. F. Hsu and C.-K. Lin, Structure connectivity and substructure connectivity of k-ary n-cube networks, *Inf. Sci.* **433**(10) (2018) 115–124.

[25] Y. Lv, C.-K. Lin and J. Fan, Hamiltonian cycle and path embeddings in k-ary n-cubes based on structure faults, *The Computer Journal* **60**(12) (2017) 159–179.

[26] Y. Lv, C.-K. Lin, J. Fan and X. Jia, Hamiltonian cycle and path embeddings in 3-ary n-cubes based on $k_{1,3}$-structure faults, *Journal of Parallel and Distributed Computing* **120** (2018) 148–158.

[27] J. Maeng and M. Malek, A comparison connection assignment for self-diagnosis of multiprocessor systems, in: *Proceedings of 11th International Symposium on Fault-Tolerant Computing* (1981), pp. 173–175.

[28] S. Mallela and G. M. Masson, Diagnosable system for intermittent faults, *IEEE Trans. Comput.* **27**(6) (1978) 461–470.

[29] P. Messina, D. Culler, W. Pfeiffer, W. Martin, J. Oden and G. Smith, Architecture, *Comm. ACM* **41**(11) (1998) 36–44.

[30] M. D. Noakes, D. A. Wallach and W. J. Dally, The J-machine multicomputer: An architectural evaluation, *Computer Architecture News* **21**(2) (1993) 224–235.

[31] F. Preparata, G. Metze and R. Chien, On the connection assignment problem of diagosis systems, *IEEE Trans. Electron. Comput.* **16**(6) (1967) 848–854.

[32] H. Qi, Y. Li, K. Li and M. Stojmenovic, An exchanged folded hypercube-based topology structure for interconnection networks, *Concurrency and Computation: Practice and Experience* **27** (2015) 4194–4210.

[33] Y. Saad and M. Schultz, Topological properties of hypercubes, *IEEE Trans. Comput.* **37**(7) (1988) 867–872.

[34] A. Sengupta and A. Dahbura, On self-diagnosable multiprocessor system diagnosis by the comparison approach, *IEEE Trans. Comput.* **41**(11) (1992) 1386–1396.

[35] S. Wang, J. Li and R. Wang, Hamiltonian paths and cycles with prescribed edges in the 3-ary n-cube, *Inf. Sci.* **181** (2011) 3054–3065.

[36] S. Wang, Z. Wang, M. Wang and W. Han, g-good-neighbor conditional diagnosability of star graph networks under pmc model and mm* model, *Frontiers of Mathematics in China* **12** (2017) 1221–1234.

[37] D. B. West, Introduction to graph theory, second ed. Prentice-Hall, 2001.

[38] J. Yuan, A. Liu, X. Ma, X. Liu, X. Qin and J. Zhang, The g-good-neighbor conditional diagnosability of k-ary n-cubes under the pmc model and mm model, *IEEE Transactions on Parallel and Distributed Systems* **26**(4) (2015) 1165–1177.

[39] J. Yuan, A. Liu, X. Qin, J. Zhang and J. Li, g-good-neighbor conditional diagnosability measures for 3-ary n-cube networks, *Theor. Comput. Sci.* 626 (2016) 144–162.