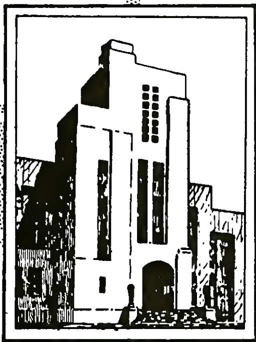
Report 1499



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THE MOTIONS OF A SPAR BUOY IN REGULAR WAVES

by

J. N. Newman

HYDROMECHANICS LABORATORY

RESEARCH AND DEVELOPMENT REPORT

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NOTATION

A	Incident wave amplitude
g	Gravitational acceleration
H	Body draft
I	Body moment of inertia in pitch about the center of gravity
i	$= \sqrt{-1}$
J_0	Bessel function of the first kind of order zero
K	Wave number, ω^2/g
k_y	Radius of gyration, $k_y^2 = I/m$
m	Body mass
n	Unit normal vector into the body
P_n	$= \int_{-H}^{0} (z - z_G)^n S(z)\,dz$

p	Pressure
$Q_n(k)$	$= \int_{-H}^{0} (z - z_G)^n S(z) e^{Kz} dz$
$R(z)$	Sectional radius of the body
$S(z)$	Sectional area of the body
r	Polar radius, $r^2 = x^2 + y^2$
t	Time
(x, y, z)	Cartesian coordinate system
z_G	Coordinate of the center of gravity
ζ	Heave displacement
ζ^*	Free surface elevation
θ	Polar coordinate
ξ	Surge displacement
ρ	Fluid density
Φ	Velocity potential
χ	Vertical prismatic coefficient
ϕ	Pitch angle
ω	Frequency of oscillations

ABSTRACT

A linearized theory is developed for the motions of a slender body of revolution, with vertical axis, which is floating in the presence of regular waves. Equations of motion are derived which are undamped to first order in the body diameter, but second-order damping forces are derived to provide solutions valid at all frequencies including resonance. Calculations made for a particular circular cylinder show extremely stable motions except for the low frequency range where very sharp maxima occur at resonance.

INTRODUCTION

The motions of a vertical body of revolution, which is floating in the presence of waves, present a problem of interest in several connections. The motions of a spar buoy, of a wave-height pole, and of floating rocket vehicles are important examples of such a problem. The same methods developed for these motions may be applied to find the forces acting on offshore radar and oil-drilling structures.

A theoretical discussion of this problem, which also treats the statistical problem of motions in irregular waves, has been presented by Barakat.[1] However, this analysis is restricted to the case of a circular cylinder and is based upon several semi-empirical concepts of applied ship-motion theory. An alternative procedure is to formulate the (inviscid) hydrodynamic problem as a boundary-value problem for the velocity potential and to employ slender-body techniques to solve this problem. The latter approach is followed in the present work, leading to linearized equations of motion which may be solved for an arbitrary slender body with a vertical axis of rotational symmetry. The particular case of a circular cylinder, whose centers of buoyancy and gravity coincide, is treated in detail and curves are presented for the amplitudes of surge, heave, and

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[1] References are listed on page 27.

pitch oscillations.

In deriving the hydrodynamic forces and moments acting on the body, we shall assume that the incident waves and the oscillations of the body are small, and thus we shall retain only terms of first order in these amplitudes. We shall also assume that the body is slender. The analysis with only first-order terms in the body's diameter leads to undamped resonance oscillations of infinite amplitude. To analyze the motions near resonance, it is necessary to introduce damping forces which are of second order with respect to the diameter-length ratio.

THE FIRST-ORDER VELOCITY POTENTIAL

We shall consider the hydrodynamic problem of a floating slender body of revolution with a vertical axis in the presence of small incident surface waves. Let (x, y, z) be a fixed Cartesian coordinate system with the z-axis positive upwards and the plane $z = 0$ situated at the undisturbed level of the free surface. The x-axis is taken to be the direction of propagation of the incident wave system, and the motion of the body is assumed to be confined to the plane $y = 0$. We shall also employ a coordinate system (x', y', z') fixed in the body, with z' the axis of the body, so that with the body at rest, $(x, y, z) = (x', y', z')$; and a circular cylindrical system (r, θ, z), where $x = r \cos \theta$ and $y = r \sin \theta$. If ξ, ζ, and ψ are the instantaneous amplitudes of surge, heave, and pitch, respectively, relative to the body's center of gravity, it follows that

$$\begin{aligned} x &= \xi + x' \cos \psi + (z' - z'_G) \sin \psi \\ y &= y' \\ z &= \zeta - x' \sin \psi + (z' - z'_G) \cos \psi + z'_G \end{aligned} \qquad [1]$$

where z'_G is the vertical coordinate of the center of gravity in the body-fixed system; see Figure 1. The displacements ξ, ζ, and ψ are assumed to be small oscillatory functions of time; we shall consistently linearize by neglecting terms of second order in these functions or their products with the incident wave amplitude A. Thus Equation [1] may be replaced by

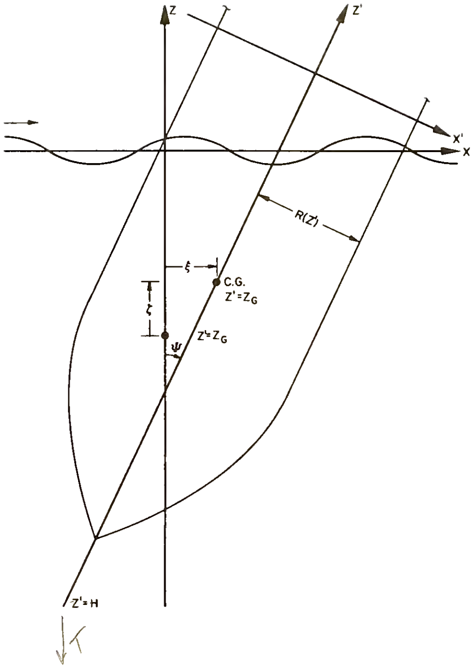

Figure 1 - The Coordinate Systems

$$x = \xi + x' + (z' - z_G)\,\psi$$

$$y = y' \qquad [2]$$

$$z = \zeta - x'\psi + z'$$

If an ideal incompressible fluid is assumed, there exists a velocity potential, $\Phi(x, y, z, t)$, satisfying Laplace's equation, such that its gradient is equal to the velocity of the fluid. This function must satisfy the following boundary conditions:

(1) On the body, the normal velocity component of the body must equal the normal derivative of Φ. For a body of revolution defined by the equation $r' = R(z')$, where $r' = \sqrt{x'^2 + y'^2}$, this boundary condition may be expressed by the equation[2]

$$\frac{D}{Dt}\left[r' - R(z')\right] \equiv \left(\frac{\partial}{\partial t} + \nabla\Phi \cdot \nabla\right)\left[r' - R(z')\right] = 0 \quad \text{on} \quad r' = R(z') \qquad [3]$$

(2) On the free surface, the normal velocity component of the free surface must equal the normal velocity component of the fluid particles in this surface, and the pressure must equal atmospheric pressure. In the linearized theory, these conditions reduce[2] to

$$\frac{\partial^2 \Phi}{\partial t^2} + g \frac{\partial \Phi}{\partial z} = 0 \quad \text{on} \quad z = 0, \qquad [4]$$

or in the case of a sinusoidal disturbance with frequency ω,

$$K\Phi - \frac{\partial \Phi}{\partial z} = 0 \quad \text{on} \quad z = 0, \qquad [5]$$

where $K = \omega^2/g$.

(3) At infinite distance from the body, the waves generated by the body are outgoing (the radiation condition).

The free surface condition, Equation [5], and the radiation condition are satisfied by the potential of oscillating singularities beneath the free surface; the boundary condition on the body may be satisfied by a proper distribution of these singularities. This distribution may be found from slender-body theory but some care is required in linearizing the present problem. If $r' = R(z')$ is the equation of the body surface over its submerged length $(-H < z' < 0)$, we shall assume that R and its first derivative are continuous, that $R(-H) = 0$, and that the magnitude of the slope $|dR/dz'| << 1$. The depth H is assumed finite, and it follows that R is small of the same order, as dR/dz'. In the analysis to follow we shall also require that R be small compared to the wavelength of the incident wave system, or that $KR << 1$.

We wish to obtain the velocity potential of leading order in the small parameters of slenderness and oscillation amplitudes in order to obtain a consistent set of linearized equations of motion for the body. However, it will turn out that the potentials of different phases of the motion are of different orders of magnitude with respect to the slenderness parameter. For example, the potential due to surge or pitch is of order R as $R \to 0$, whereas the potential due to heave is $0(R^2)$. Similar differences will occur in considering the components of each potential which are in phase and out of phase with the respective velocities of the body. In order to circumvent these difficulties without unnecessary higher order perturbation analysis, we decompose the velocity potential in the following form:

$$\begin{aligned}\Phi(x,y,z;t) &= \phi_\xi(x,y,z;t) + \phi_\zeta(x,y,z;t) + \phi_\psi(x,y,z;t)\\ &\quad + A\left[g/\omega\ e^{Kz}\cos(Kx-\omega t) + \phi_A(x,y,z;t)\right]\end{aligned} \tag{6}$$

where ϕ_ξ, ϕ_ζ, and ϕ_ψ are linear in the displacements (ξ, ζ, ψ) and their time derivatives, respectively. The potential $A\ g/\omega\ e^{Kz}\cos(Kx - \omega t)$ represents the incident wave system and the potential $A\phi_A(x,y,z;t)$ represents the diffracted wave potential, corresponding to waves incident on a restrained body. Each potential ϕ in Equation [6] must satisfy the free surface boundary condition and the radiation condition; the complete potential Φ must satisfy the boundary condition on the body. This condition, Equation [3], is reduced as follows:

$$\left(\frac{\partial}{\partial t} + \nabla\Phi\cdot\nabla\right)[r' - R(z')] = \frac{\partial r'}{\partial x'}\frac{\partial x'}{\partial t} + \frac{\partial r'}{\partial y'}\frac{\partial y'}{\partial t} + \frac{\partial r'}{\partial z'}\frac{\partial z'}{\partial t} - \frac{dR}{dz'}\frac{\partial z'}{\partial t}$$

$$+ \frac{\partial\Phi}{\partial r'} - \frac{\partial\Phi}{\partial z'}\frac{dR}{dz'} = 0 \qquad \text{on} \qquad r' = R(z'),$$

or neglecting second-order terms in A, ξ, ζ, and ψ,

$$\begin{aligned}\frac{\partial\Phi}{\partial r} - \frac{\partial\Phi}{\partial z}\frac{dR}{dz} - [\dot\xi + (z - z_G)\dot\psi]\cos\theta + (\dot\zeta - x\dot\psi)\,dR/dz = 0\\ \text{on} \qquad r = R(z),\end{aligned} \tag{7}$$

where a dot denotes differentiation with respect to time. Substituting Equation [6] into Equation [7] and separating terms according to their dependence on different displacements, we obtain the following boundary conditions on the body:

$$\frac{\partial \phi_\xi}{\partial r} = \dot{\xi} \cos\theta + 0\left(R \frac{\partial \phi_\xi}{\partial z}\right) \qquad [8]$$

$$\frac{\partial \phi_\psi}{\partial r} = \dot{\psi}(z - z_G)\cos\theta + 0\left(R \frac{\partial \phi_\psi}{\partial z}\right) + 0(R^2) \qquad [9]$$

$$\frac{\partial \phi_\zeta}{\partial r} = - \frac{\dot{\zeta}\,\partial R}{\partial z} + 0\left(R \frac{\partial \phi_\zeta}{\partial z}\right) \qquad [10]$$

$$\begin{aligned}\frac{\partial \phi_A}{\partial r} &= - \omega e^{Kz}\left[\cos\theta \sin\omega t - \left(KR\cos^2\theta + \frac{dR}{dz}\right)\cos\omega t\right] \\ &\quad + 0\left(R\frac{\partial \phi_A}{\partial z}\right) + 0(R^2) \\ &= \omega e^{Kz}\left[- \cos\theta \sin\omega t + \left(\tfrac{1}{2} KR + \frac{dR}{dz} + \tfrac{1}{2} KR \cos 2\theta\right)\cos\omega t\right] \\ &\quad + 0\left(R\frac{\partial \phi_A}{\partial z}\right) + 0(R^2)\end{aligned} \qquad [11]$$

To satisfy the above boundary conditions, we employ slender-body theory.[3] For example, the potential satisfying Equation [8] is an axial line of horizontal dipoles, of moment density $\frac{1}{2}\dot{\xi}\,[R(z)]^2$ per unit length. Thus in an infinite fluid,

$$\phi_\xi = \tfrac{1}{2}\dot{\xi} \int_{-H}^{0} [R(z_1')]^2 \frac{\partial}{\partial x} \left[r^2 + (z - z_1')^2\right]^{-\frac{1}{2}} dz_1' \qquad [12]$$

To satisfy the free surface and radiation conditions, we substitute for the source potential $[r^2 + (z - z_1)^2]^{-\frac{1}{2}}$, the potential of an oscillating source under a free surface.[2] With this substitution we obtain, in place of Equation [12]:

$$\begin{aligned}\phi_\xi &= \tfrac{1}{2}\dot{\xi} \int_{-H}^{0} [R(z_1)]^2 \frac{\partial}{\partial x} \left\{\left[r^2 + (z - z_1)^2\right]^{-\frac{1}{2}} \right. \\ &\quad \left. + \fint_0^\infty \frac{k+K}{k-K}\, e^{k(z+z_1)} J_0(kr)\,dk\right\} dz_1 \\ &\quad + \pi\omega K\xi \int_{-H}^{0} [R(z_1)]^2 e^{K(z+z_1)} \frac{\partial}{\partial x}[J_0(Kr)]\, dz_1\end{aligned} \qquad [13]$$

and, in a similar fashion,

$$\phi_\psi = \tfrac{1}{2}\dot{\psi}\int_{-H}^{0}[R(z_1)]^2(z_1 - z_G)\frac{\partial}{\partial x}\left\{[r^2 + (z - z_1)^2]^{-\frac{1}{2}}\right.$$

$$\left. + \fint_0^\infty \frac{k+K}{k-K} e^{k(z+z_1)} J_0(kr)\,dk\right\} dz_1 \qquad [14]$$

$$+ \pi\omega K\psi \int_{-H}^{0}[R(z_1)]^2(z_1 - z_G)\, e^{K(z+z_1)}\left[\frac{\partial}{\partial x} J_0(Kr)\right] dz_1$$

$$\phi_\zeta = \tfrac{1}{2}\dot{\zeta}\int_{-H}^{0} R(z_1)\frac{dR}{dz_1}\left\{[r^2 + (z - z_1)^2]^{-\frac{1}{2}}\right.$$

$$\left. + \fint_0^\infty \frac{k+K}{k-K} e^{k(z+z_1)} J_0(kr)\,dk\right\} dz_1 \qquad [15]$$

$$+ \pi\omega K\zeta \int_{-H}^{0} R(z_1)\frac{dR}{dz_1}\, e^{K(z+z_1)} J_0(Kr)\,dz_1$$

$$\phi_A = -\tfrac{1}{2}\omega\int_{-H}^{0} e^{Kz_1}\left\{\left(\tfrac{1}{2}KR + \frac{dR}{dz_1}\right) R\cos\omega t\right.$$

$$\left. + R^2 \sin\omega t \frac{\partial}{\partial x} + \tfrac{1}{4}KR^4\cos\omega t\frac{\partial^2}{\partial x^2}\right\}\left\{[r^2 + (z - z_1)^2]^{-\frac{1}{2}}\right.$$

$$\left. + \fint_0^\infty \frac{k+K}{k-K} e^{k(z+z_1)} J_0(kr)\,dk\right\} dz_1 \qquad [16]$$

$$- \pi\omega K \int_{-H}^{0} e^{K(z+2z_1)}\left\{\left(\tfrac{1}{2}KR + \frac{dR}{dz_1}\right) R\sin\omega t\right.$$

$$\left. - R^2\cos\omega t\frac{\partial}{\partial x} + \tfrac{1}{4}KR^4\sin\omega t\frac{\partial^2}{\partial x^2}\right\} J_0(Kr)\,dz_1$$

where $\fint$ denotes the Cauchy principal value. From the Appendix we see that the potentials [13] to [16] satisfy the boundary conditions [8] to [11], respectively, with a maximum fractional error of order R. Unfortunately, this error is not so small as in the classical slender-body theory for an infinite fluid, where the error is of order $R^2 \log R$; for this reason the present theory may not hold for as wide a range of slenderness as in the aerodynamic case. However, for the slender floating bodies which are envisaged at present (viz., a rocket vehicle or one support of a stable platform), this is not expected to cause practical problems.

The values of the potentials [13] to [16] on the body may be found by setting $r = R(z)$ and retaining the leading terms for small R. To leading order, only the singular term $[r^2 + (z - z_1)^2]^{-\frac{1}{2}}$ contributes to the integrals over z_1, and the integrals may be evaluated directly since for any continuous bounded function $f(z_1)$ and small values of r,

$$\int_{-H}^{0} f(z_1)[r^2 + (z - z_1)^2]^{-\frac{1}{2}} dz_1 = -2f(z)\log r + 0(1)$$

$$\int_{-H}^{0} f(z_1) \frac{\partial}{\partial x} [r^2 + (z - z_1)^2]^{-\frac{1}{2}} dz_1 = -2 \frac{f(z)}{r} \cos\theta + 0(1)$$

$$\int_{-H}^{0} f(z_1) \frac{\partial 2}{\partial x^2} [r^2 + (z - z_1)^2]^{-\frac{1}{2}} dz_1 = 2 \frac{f(z)}{r^2} \cos 2\theta + 0(1)$$

$$\text{for} \quad -H < z < 0, \quad r << H.$$

Thus on the body,

$$\phi_\xi = \dot{\xi} R(z) \cos\theta + 0(R^2) \tag{17}$$

$$\phi_\psi = -\dot{\psi} R(z)(z - z_G) \cos\theta + 0(R^2) \tag{18}$$

$$\phi_\zeta = -\dot{\zeta} R \frac{dR}{dz} \log R + 0(R^2) \tag{19}$$

$$\begin{aligned} \phi_A &= \omega e^{Kz} [(\tfrac{1}{2} KR + \frac{dR}{dz}) R \log R \cos \omega t + R \cos\theta \sin \omega t] + 0(R^2) \\ &= \omega e^{Kz} R \cos\theta \sin \omega t + 0(R^2 \log R) \end{aligned} \tag{20}$$

From Bernoulli's equation, the linearized pressure on the body is

$$p = -\rho \frac{\partial \Phi}{\partial t} - \rho g z$$

$$= -\rho g z - \rho \frac{\partial \phi_\xi}{\partial t} - \rho \frac{\partial \phi_\psi}{\partial t} - \rho \frac{\partial \phi_\zeta}{\partial t} - \rho A \left[\frac{\partial \phi_A}{\partial t} + g e^{Kz} \sin(KR \cos\theta - \omega t) \right]$$

$$= -\rho g z + \rho \ddot{\xi} R(z) \cos\theta + \rho \ddot{\psi} R(z)(z - z_G) \cos\theta - \rho A \omega^2 e^{Kz} R \cos\theta \cos\omega t$$

$$+ \rho g A e^{Kz} \sin\omega t - \rho g A K e^{Kz} R \cos\theta \cos\omega t + 0(R^2 \log R)$$

$$= -\rho g z + \rho \ddot{\xi} R(z) \cos\theta + \rho \ddot{\psi} R(z)(z - z_G) \cos\theta + \rho g A e^{Kz} \sin\omega t$$

$$- 2\rho\omega^2 A e^{Kz} R \cos\theta \cos\omega t + 0(R^2 \log R) \qquad [21]$$

The force and moment exerted on the body by the fluid are obtained by integrating the pressure over the surface. In the absence of any other external forces, the force or moment must equal the respective acceleration times the mass or moment of inertia of the body. Thus, with $\vec{n}$ the unit normal vector into the body, the equations of motion are

$$m\ddot{\xi} = \iint p \cos(n, x) dS \qquad [22]$$

$$m(\ddot{\zeta} + g) = \iint p \cos(n, z) dS \qquad [23]$$

$$I\ddot{\psi} = \iint p \left[(z - z_G)\cos(n, x) - x \cos(n, z)\right] dS \qquad [24]$$

where m is the body's mass, I its moment of inertia about the center of gravity, and the surface integrals are over the submerged surface of the body.

In computing the pressure integrals over the body surface, it is expedient to employ the (x', y', z') system, fixed in the body. The direction cosines are

$$\cos(n, x') = -\cos\theta + 0(R^2)$$

$$\cos(n, z') = \frac{dR}{dz'} + 0(R^2)$$

and the forces along the (x, z) axis are related to the forces along the (x', z') axis by

$$F_x = F_{x'} \cos\psi + F_{z'} \sin\psi = F_{x'} + \psi F_{z'} + 0(\psi^2)$$

$$F_z = F_{z'} \cos\psi - F_{x'} \sin\psi = F_{z'} - \psi F_{x'} + 0(\psi^2)$$

Thus the equations of motion may be written in the form

$$m\ddot{\xi} = \int_0^{2\pi} \int_{-H}^{\zeta^* - \zeta + x'\psi} \left(-\cos\theta + \psi \frac{dR}{dz'} \right) p R \, dz' \, d\theta'$$

$$m(\ddot{\zeta} + g) = \int_0^{2\pi} \int_{-H}^{\zeta^* - \zeta + x'\psi} \left(\frac{dR}{dz'} + \psi \cos\theta \right) p R \, dz' \, d\theta'$$

$$I\ddot{\psi} = \int_0^{2\pi} \int_{-H}^{\zeta^* - \zeta + x'\psi} [(z' - z_G) \cos(n, x') - x' \cos(n, z')] \, p R \, dz' \, d\theta'$$

$$= - \int_0^{2\pi} \int_0^{\zeta^* - \zeta + x'\psi} (z' - z_G) \cos\theta \, p R \, dz' \, d\theta' + 0(R^3)$$

where ζ^* is the free surface elevation at the body. Substituting Equation [21] for the pressure and neglecting second-order terms in the oscillatory displacements ξ, ζ, ψ, and A, we obtain

$$m\ddot{\xi} = -\rho g \int_0^{2\pi} \int_{-H}^{0} \left(-\cos\theta' + \psi \frac{dR}{dz'} \right) (z' + \zeta - \psi R \cos\theta') R \, dz' \, d\theta'$$

$$- \rho \int_0^{2\pi} \int_{-H}^{0} \cos\theta' \, [\ddot{\xi} R \cos\theta' + \ddot{\psi} R (z - z_G) \cos\theta'$$

$$+ g A e^{Kz'} \sin\omega t - 2\omega^2 A e^{Kz'} R \cos\theta' \cos\omega t] \, R \, dz' \, d\theta'$$

$$= -\pi\rho g \int_{-H}^{0} \left(\psi R + 2\psi z' \frac{dR}{dz'}\right) R\, dz'$$

$$- \rho\pi \int_{-H}^{0} [\ddot{\xi} + \ddot{\psi}(z' - z_G) - 2\omega^2 A e^{Kz'} \cos \omega t]\, R^2\, dz'$$

or, since

$$\int_{-H}^{0} \left(\psi R + 2\psi z' \frac{dR}{dz'}\right) R\, dz' = \psi \int_{-H}^{0} \frac{d}{dz'} (R^2 z')\, dz' = 0$$

it follows that

$$m\ddot{\xi} = -\rho \int_{-H}^{0} [\ddot{\xi} + \ddot{\psi}(z - z_G) - 2\omega^2 A e^{Kz} \cos \omega t]\, S(z)\, dz + 0(R^3 \log R) \qquad [25]$$

where

$$S(z) = \pi\, [R(z)]^2$$

is the sectional area function.

In a similar manner we obtain

$$m(\ddot{\zeta} + g) = -\rho g \zeta S(0) + \rho g \int_{-H}^{0} S(z)\, dz + \rho g A \sin \omega t \int_{-H}^{0} e^{Kz} \frac{dS}{dz}\, dz + 0(R^4 \log R) \qquad [26]$$

$$I\ddot{\psi} = -\rho g \psi \int_{-H}^{0} (z - z_G)\, S(z)\, dz - \rho \int_{-H}^{0} [\ddot{\xi} + \ddot{\psi}(z - z_G) - 2\omega^2 A e^{Kz} \cos \omega t](z - z_G)\, S(z)\, dz + 0(R^3 \log R) \qquad [27]$$

From Archimedes' principle, or equivalently, satisfying Equation [26] to zero order in ζ,

$$gm = \rho g \int_{-H}^{0} S(z)\,dz \qquad [28]$$

and thus

$$m\ddot{\zeta} = -\rho g \zeta S(0) + \rho g A \sin \omega t \int_{-H}^{0} e^{Kz} \frac{dS}{dz}\,dz + 0(R^4 \log R) \qquad [29]$$

while, from Equations [28] and [25],

$$2m\ddot{\xi} = -\rho \int_{-H}^{0} [\ddot{\psi}(z - z_G) - 2\omega^2 A e^{Kz} \cos \omega t]\, S(z)\,dz + 0(R^3 \log R) \qquad [30]$$

Let us denote:

$$I = mk_y^2$$

$$\chi = \frac{m}{\rho H S(0)} = \text{Vertical Prismatic Coefficient}$$

$$P_n = \frac{\rho}{m} \int_{-H}^{0} (z - z_G)^n S(z)\,dz \qquad (n = 1, 2)$$

$$Q_n(K) = \frac{\rho}{m} \int_{-H}^{0} e^{Kz} (z - z_G)^n S(z)\,dz \qquad (n = 0, 1)$$

and note that ξ, ζ, and ψ must be sinusoidal with frequency ω. The equations of motion then become

$$2\xi + P_1 \psi = -2AQ_0 \cos \omega t \qquad [31]$$

$$(1 - \chi KH)\zeta = A(1 - \chi KH Q_0) \sin \omega t \qquad [32]$$

$$\left(P_2 + k_y^2 - \frac{P_1}{K}\right)\psi + P_1 \xi = -2AQ_1 \cos \omega t \qquad [33]$$

Note also that surge and pitch are coupled, unless $P_1 = 0$ or unless the centers of gravity and buoyancy coincide.

The above equations of motion are not unexpected. The restoring forces on the left-hand side consist of hydrostatic and inertial forces plus entrained mass terms which double the inertial force at each section. This might have been deduced as a consequence of slender-body theory and the fact that the entrained mass of a circular cylinder in an infinite fluid is just equal to the displaced mass. In other words, the hydrodynamic forces on the left-hand side of Equations [31] to [33] could have been obtained by neglecting the presence of the free surface. Moreover, the exciting forces on the right-hand side of these equations are those which follow from the "Froude-Krylov" hypothesis that the pressure in the wave system is not affected by the presence of the body. These results are, of course, a consequence of the fact that the body is slender.

The solutions of Equations [31] to [33] are

$$\zeta = A \sin \omega t \left(\frac{1 - \chi Q_0 KH}{1 - \chi KH} \right) \qquad [34]$$

$$\xi = 2A \cos \omega t \left[\frac{P_1 Q_1 - Q_0 (P_2 + k_y^2 - P_1/K)}{2(P_2 + k_y^2 - P_1/K) - P_1^2} \right] \qquad [35]$$

$$\psi = 2A \cos \omega t \left[\frac{P_1 Q_0 - 2Q_1}{2(P_2 + k_y^2 - P_1/K) - P_1^2} \right] \qquad [36]$$

We note that when

$$K = \frac{1}{\chi H} \qquad [37]$$

there is resonance in heave, and when

$$K = \frac{P_1}{P_2 + k_y^2 - \frac{1}{2} P_1^2} \qquad [38]$$

there is resonance in pitch and surge. To determine the oscillation amplitudes in the vicinity of these resonance frequencies, it is necessary to consider the damping mechanism due to energy dissipation in outgoing waves. Thus, for these frequencies, we must consider the free-surface effects on the restoring forces. For this purpose we must retain some terms which are of second order in the radius of the body.

THE DAMPING FORCES

The damping forces will follow by considering the last terms in Equations [13] to [16] and will consequently be of higher order in R than those terms which we retained in the previous analysis. This procedure is nevertheless consistent, since at resonance the lower order restoring forces vanish. In other words, we are retaining the lowest order force or moment of each phase separately. For a further discussion of this point, see Reference 4.

We proceed, therefore, to study the damping forces, or the forces in phase with each velocity. The only contribution from Equations [13] to [16] is the potential

$$\phi^* = \pi \omega K e^{Kz} \int_{-H}^{0} \left\{ \zeta R \frac{dR}{dz_1} + [\xi + \psi (z_1 - z_G)] R^2 \frac{\partial}{\partial x} \right\} e^{Kz_1} J_0(Kr) dz_1 \tag{39}$$

Since $J_0(Kr) = 1 - \frac{1}{4}(Kr)^2 + \ldots$, it follows that on the surface $r = R(z)$, the leading terms are

$$J_0(Kr) \cong 1$$

and

$$\frac{\partial}{\partial x} J_0(Kr) \cong -\frac{1}{2} K^2 x = -\frac{1}{2} K^2 R \cos \theta$$

Thus, to second order in R the damping potential on the body is

$$\phi^* = \pi \omega K e^{Kz} \int_{-H}^{0} \Big\{ \zeta R \frac{dR}{dz_1}$$

$$- \tfrac{1}{2} [\xi + \psi (z_1 - z_G)] K^2 R(z) R^2(z_1) \cos\theta \Big\} e^{Kz_1} dz_1$$

$$= \tfrac{1}{2} \omega K e^{Kz} \zeta \int_{-H}^{0} e^{Kz_1} \frac{dS}{dz_1} dz_1 \qquad [40]$$

$$- \tfrac{1}{2} \omega K^3 R(z) \cos\theta \, e^{Kz} \int_{-H}^{0} [\xi + \psi(z_1 - z_G)] e^{Kz_1} S(z_1) dz_1$$

The damping pressure on the body is

$$p^* = - \rho \frac{\partial \phi^*}{\partial t} = - \tfrac{1}{2} \omega\rho K e^{Kz} \dot{\zeta} \int_{-H}^{0} e^{Kz_1} \frac{dS}{dz_1} dz_1$$

$$[41]$$

$$+ \tfrac{1}{2} \omega\rho K^3 R(z) \cos\theta \, e^{Kz} \int_{-H}^{0} [\dot{\xi} + \dot{\psi}(z_1 - z_G)] e^{Kz_1} S(z_1) dz_1$$

Then the heave damping force is

$$F_z^* = \int_{-H}^{0} \int_{0}^{2\pi} p^* R \frac{dR}{dz} d\theta \, dz = - \tfrac{1}{2} \omega\rho K \dot{\zeta} \left(\int_{-H}^{0} e^{Kz} \frac{dS}{dz} dz \right)^2 \qquad [42]$$

Similarly, the surge damping force and the pitch damping moment are

$$F_x^* = - \int_{-H}^{0} \int_{0}^{2\pi} p^* \cos\theta \, R \, d\theta \, dz$$

$$[43]$$

$$= - \tfrac{1}{2} \omega\rho K^3 \left(\int_{-H}^{0} S(z) e^{Kz} dz \right) \left(\int_{-H}^{0} [\dot{\xi} + \dot{\psi}(z_1 - z_G)] e^{Kz_1} S(z_1) dz_1 \right)$$

and

$$M_y^* = - \int_{-H}^{0} \int_{0}^{2\pi} p^*(z - z_G) \cos\theta \, R \, d\theta \, dz$$

$$= - \tfrac{1}{2} \omega\rho K^3 \left(\int_{-H}^{0} (z - z_G) S(z) e^{Kz} dz \right) \left(\int_{-H}^{0} [\dot{\xi} + \dot{\psi}(z_1 - z_G)] e^{Kz_1} S(z_1) dz_1 \right) \quad [44]$$

or in terms of the integrals P_1, P_2, Q_0, and Q_1:

$$F_z^* = - \tfrac{1}{2} \omega\rho K \dot{\zeta} [K \tfrac{m}{\rho} Q_0(K) - S(0)]^2 = - \tfrac{1}{2} \frac{\omega m^2 K}{\rho \chi^2 H^2} \dot{\zeta} [1 - Q_0(K) \chi KH]^2 \quad [45]$$

$$F_x^* = - \tfrac{1}{2} \frac{\omega m^2}{\rho} K^3 Q_0(K) [\dot{\xi} Q_0(K) + \dot{\psi} Q_1(K)] \quad [46]$$

$$M_y^* = - \tfrac{1}{2} \frac{\omega m^2}{\rho} K^3 Q_1(K) [\dot{\xi} Q_0(K) + \dot{\psi} Q_1(K)] \quad [47]$$

In place of Equations [31], [32], and [33], we obtain the damped equations of motion

$$2\xi + P_1 \psi = - 2A Q_0(K) \cos\omega t + \tfrac{1}{2} \frac{m}{\omega\rho} K^3 Q_0(K) [\dot{\xi} Q_0(K) + \dot{\psi} Q_1(K)] \quad [48]$$

$$(1 - \chi KH)\zeta = A \sin\omega t [1 - \chi KH \, Q_0(K)] + \tfrac{1}{2} \frac{m}{\omega\rho} \frac{K\dot{\zeta}}{\chi^2 H^2} [1 - Q_0(K) \chi KH]^2 \quad [49]$$

$$\psi(P_2 - P_1/K + k_y^2) + P_1 \xi = - 2A \, Q_1(K) \cos\omega t + \tfrac{1}{2} \frac{m}{\omega\rho} K^3 Q_1(K) [\dot{\xi} Q_0(K) + \dot{\psi} Q_1(K)] \quad [50]$$

The damping terms of these equations of motion are given by the terms linear in the velocities $\dot{\xi}$, $\dot{\zeta}$, and $\dot{\psi}$. It should be noted that for a slender body $m \to 0$, and thus the damping coefficients will be small, as was to be expected. To solve these equations for the three unknown displacements and their phases is a straightforward but tedious matter. For applications in ranges not including a resonance frequency, it is much simpler to employ the undamped equations of motion, [31] to [33], and the resulting displacements, [34] to [36].

CALCULATIONS FOR THE CIRCULAR CYLINDER

As a special case, we shall consider the circular cylinder $R(z) = R$ = constant. Then

$$\chi = 1.0$$

$$P_1 = \frac{1}{H}\int_{-H}^{0} (z - z_G)\,dz = -\tfrac{1}{2}H - z_G$$

$$P_2 = \frac{1}{H}\int_{-H}^{0} (z - z_G)^2\,dz = \tfrac{1}{3}H^2 + Hz_G + z_G^2$$

$$Q_0(K) = \frac{1}{H}\int_{-H}^{0} e^{Kz}\,dz = \frac{1}{KH}(1 - e^{-KH})$$

$$Q_1(K) = \frac{1}{H}\int_{-H}^{0} e^{Kz}(z - z_G)\,dz = \frac{1}{K}e^{-KH} - \frac{1}{K^2H}(1 - e^{-KH})(1 + Kz_G)$$

We shall assume, moreover, that the centers of buoyancy and gravity coincide, or $z_G = -H/2$, so that the equations of motion are uncoupled and there is no resonance in pitch or surge.

Then

$$P_1 = 0; \qquad P_2 = \frac{H^2}{12}; \qquad Q_1(K) = \frac{1 + e^{-KH}}{2K} - \frac{1 - e^{-KH}}{K^2H};$$

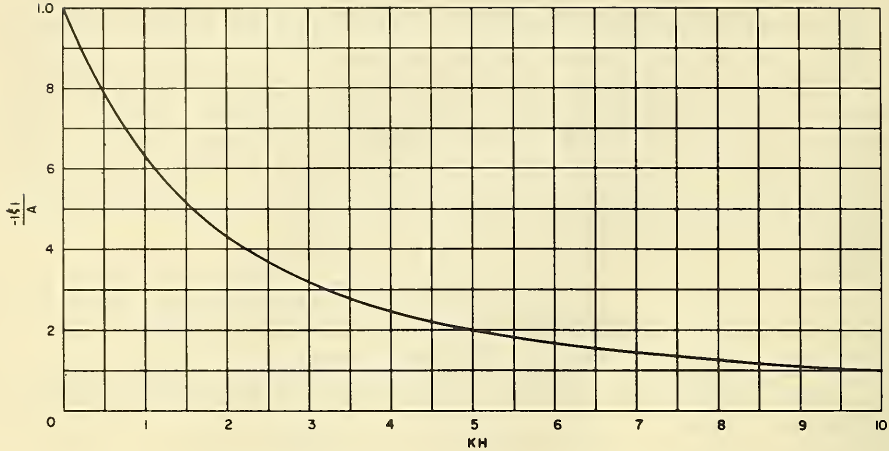

Figure 2 - Plot of the Surge Amplitude-Wave Amplitude Ratio for the Circular Cylinder

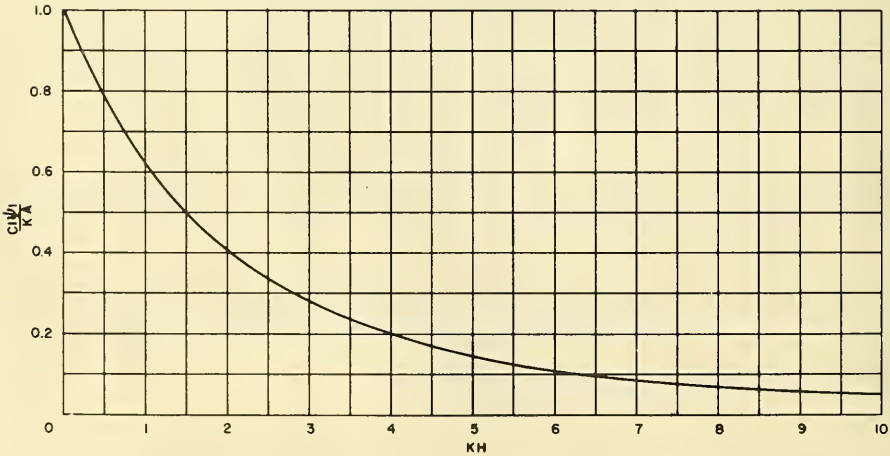

Figure 3 - Plot of the Pitch Amplitude-Wave Slope Ratio for the Circular Cylinder

and it follows that

$$\xi = -\frac{A}{2KH}(1 - e^{-KH}) \cos \omega t \tag{51}$$

$$\psi = -\frac{2A}{H} \frac{\cos \omega t}{\left[\left(\frac{k_y}{H}\right)^2 + \frac{1}{12}\right]} \left[\frac{1 + e^{-KH}}{2KH} - \frac{1 - e^{-KH}}{(KH)^2}\right] \tag{52}$$

$$\zeta = \frac{2A\, e^{-KH}}{(1 - KH)^2 + \left[\frac{\pi}{2} KH \left(\frac{R}{H}\right)^2 e^{-2KH}\right]^2} \left[(1 - KH) \sin \omega t + \frac{\pi}{2} KH \left(\frac{R}{H}\right)^2 e^{-2KH} \cos \omega t\right] \tag{53}$$

Plots of the above amplitudes and the heave phase angle are shown in Figures 2 to 6 as functions of KH. Figure 2 shows the ratio of surge amplitude to wave amplitude. For zero frequency this ratio is one and for increasing frequencies it decreases monotonically to zero. Figure 3 shows the ratio of pitch angle to the maximum wave slope KA, multiplied by the coefficient $C = \frac{1}{2} + 6(k_y/H)^2$. This coefficient is equal to one if the mass in the cylinder is uniformly distributed throughout its submerged length. The ratio starts at one for zero frequency and decreases monotonically to zero. Thus the pitch amplitude is always less than the wave slope. Figure 4 shows the ratio of heave amplitude to wave height for frequencies away from the vicinity of resonance. Near resonance, the amplitude is shown in Figure 5 and the phase angle in Figure 6 for the particular case $R/H = 0.1$. The ratio of heave amplitude to wave amplitude is unity for zero frequency, rises to a maximum of

$$\frac{e}{\pi}\left(\frac{H}{R}\right)^2 \cong 0.865\left(\frac{H}{R}\right)^2$$

at the resonance frequency $KH = 1$, and then decreases monotonically to zero. The phase angle is similar to conventional one-degree-of-freedom

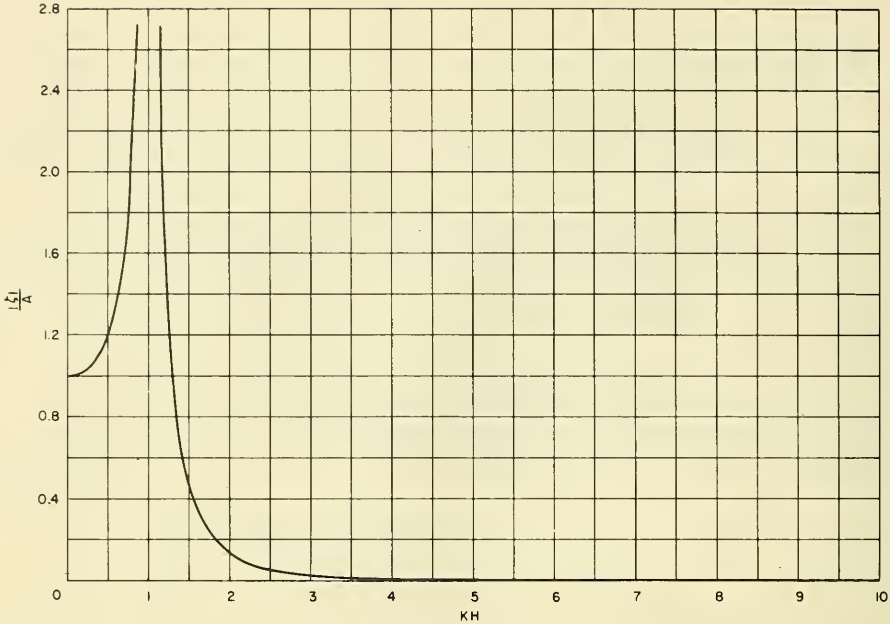

Figure 4 - Plot of the Heave Amplitude-Wave Amplitude Ratio for the Undamped Circular Cylinder

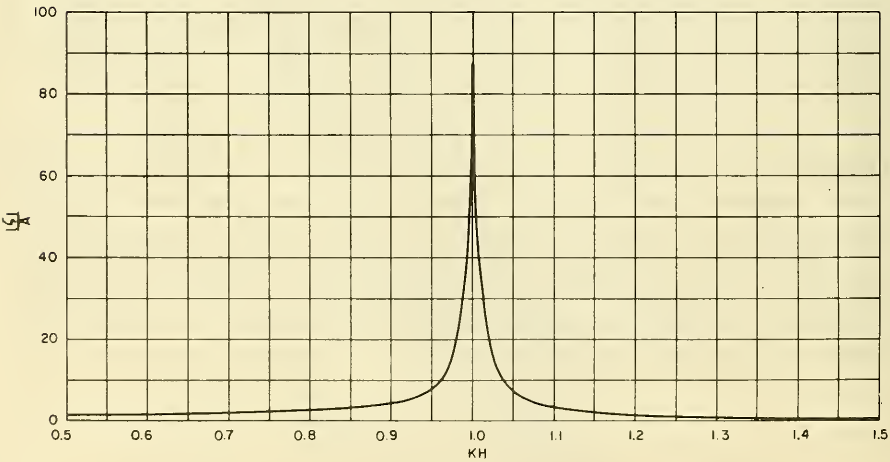

Figure 5 - Plot of the Heave Amplitude-Wave Amplitude Ratio for the Damped Circular Cylinder with R/H = 0.1

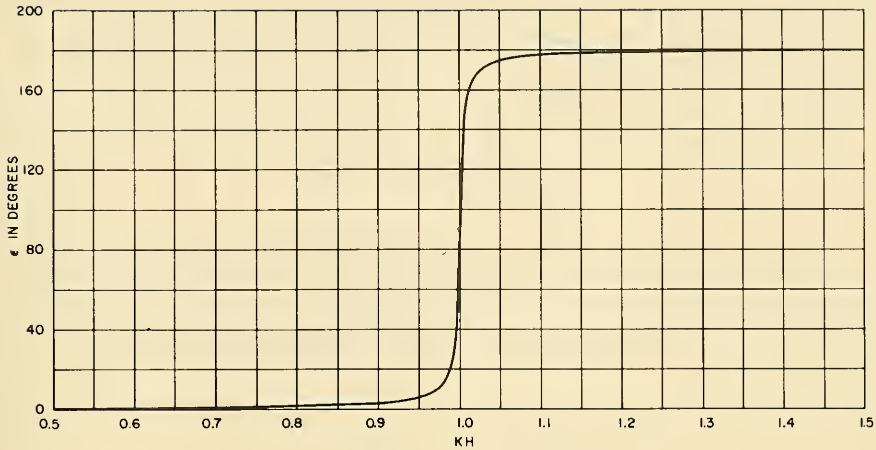

Figure 6 - Plot of the Heave Phase Lag for the Damped Circular Cylinder with R/H = 0.1

harmonic oscillators with linear damping; for low frequencies the heave displacement and wave height are in phase, at resonance they are in quadrature, and at high frequencies they are 180 deg out of phase.

DISCUSSION AND CONCLUSIONS

The damped equations of motion as given by Equations [48] to [50] may be solved for an arbitrary body of revolution to obtain the oscillation amplitudes and phases. Except in the vicinity of the resonance frequencies defined by Equations [37] and [38], it should be sufficient to use the simpler undamped equations; the resulting oscillations are given by Equations [34] to [36]. Plots of these oscillations are shown in Figures 2 to 6 for a circular cylinder, with the important restriction that the centers of buoyancy and gravity coincide. If this restriction is relaxed, a resonance will be introduced into the equations for pitch and surge, but the frequency of this resonance may be kept small by ballasting. The amplitudes at resonance are extreme, but the resonance frequency for heave is quite small and can be kept out of the practical range of ocean waves by making the draft sufficiently large. It would seem wise to do this in practice and to provide appropriate ballast so that the pitch resonance occurs at or below the heave resonance frequency. From Equations [37] and [38] this requires that

$$\frac{P_1}{P_2 + k_y^2 - \frac{1}{2} P_1^2} \overline{<} \frac{1}{\chi H}$$

The advantage of spar-buoy-type bodies lies in their very small motions in the higher frequency range. By proper design this advantage may be utilized; thus very calm motions can be expected in waves.

ACKNOWLEDGMENT

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APPENDIX

Here the potentials ϕ_ξ, ϕ_ζ, ϕ_ψ, and ϕ_A, defined by Equations [13] to [16], are shown to satisfy the boundary conditions [8] to [11], respectively, to leading order in R. For this purpose, let us consider the potential

$$\psi = \frac{1}{2} \int_{-H}^{0} \frac{\partial f(z_1, t)}{\partial t} \left\{ \left[r^2 + (z - z_1)^2 \right]^{-\frac{1}{2}} + \fint_0^\infty \frac{k + K}{k - K}\, e^{k(z + z_1)} J_0(kr)\, dk \right\} dz_1 + \pi\omega K \int_{-H}^{0} f(z_1, t)\, e^{K(z + z_1)} J_0(Kr)\, dz_1 \qquad [54]$$

where $f(z_1, t)$ has sinusoidal time dependence with circular frequency ω. By appropriate choice of the function f, the potentials ϕ_ξ, ϕ_ζ, ϕ_ψ, and ϕ_A can all be obtained from ψ and $\partial\psi/\partial x$. Thus it is sufficient to establish that the following conditions are satisfied on the body surface $r = R$:

$$\frac{\partial \psi}{\partial r} \cong - \frac{1}{R} \frac{\partial}{\partial t} f(z, t) \qquad [55]$$

$$\frac{\partial^2 \psi}{\partial r\, \partial x} \cong \frac{\cos\theta}{R^2} \frac{\partial}{\partial t} f(z, t) \qquad [56]$$

Employing an alternative form of the source potential,[2] we write ψ in the form

$$\psi = \frac{1}{2} \int_{-H}^{0} \frac{\partial f}{\partial t} \left\{ \left[r^2 + (z - z_1)^2 \right]^{-\frac{1}{2}} + \left[r^2 + (z + z_1)^2 \right]^{-\frac{1}{2}} + 2K \fint_0^\infty \frac{1}{k - K}\, e^{k(z + z_1)} J_0(kr)\, dk \right\} dz_1 \qquad [57]$$

$$+ \pi\omega K \int_{-H}^{0} f(z_1, t)\, e^{K(z+z_1)} J_0(Kr)\, dz_1 \qquad [57] \text{ continued}$$

$$\equiv \psi_1 + \psi_2$$

where

$$\psi_1 = \tfrac{1}{2} \int_{-H}^{0} \frac{\partial f}{\partial t} \left\{ [r^2 + (z_1 - z)^2]^{-\frac{1}{2}} + [r^2 + (z_1 + z)^2]^{-\frac{1}{2}} \right\} dz_1$$

$$\psi_2 = K \int_{-H}^{0} \frac{\partial f}{\partial t} \fint_{0}^{\infty} \frac{1}{k - K}\, e^{k(z+z_1)} J_0(kr)\, dk\, dz_1$$

$$+ \pi\omega K \int_{-H}^{0} f(z_1, t)\, e^{K(z+z_1)} J_0(Kr)\, dz_1$$

The potential ψ_1 corresponds to an axial distribution of simple sources together with an image distribution above the free surface $z = 0$. To emphasize this fact we write ψ_1 in the form

$$\psi_1 = \tfrac{1}{2} \int_{-H}^{H} \frac{\partial}{\partial t} f(-|z_1|, t) [r^2 + (z - z_1)^2]^{-\frac{1}{2}} dz_1 \qquad [58]$$

From the conventional slender-body theory of aerodynamics, we may expect this potential to satisfy the boundary conditions [55] and [56] on the body to leading order in R. In fact, differentiating with aspect to r and neglecting terms which are of order R^2 or $R \cos\theta$ in the neighborhood of the body $r = R$, we have

$$\frac{\partial \psi_1}{\partial r} = -\tfrac{1}{2} \int_{-H}^{H} \frac{\partial}{\partial t} f(-|z_1|, t)\, r\, [r^2 + (z - z_1)^2]^{-\frac{3}{2}}\, dz_1$$

$$\cong -\tfrac{1}{2} \frac{\partial}{\partial t} f(-|z|, t) \int_{-H}^{H} r\, [r^2 + (z - z_1)^2]^{-\frac{3}{2}}\, dz_1 \qquad [59]$$

$$= \frac{1}{2} \frac{\partial f}{\partial t} \left[\frac{z - z_1}{r \sqrt{r^2 + (z - z_1)^2}} \right]_{-H}^{H} \qquad [59] \text{ continued}$$

$$\cong - \frac{1}{r} \frac{\partial f}{\partial t}$$

and similarly

$$\frac{\partial^2 \psi_1}{\partial r \, \partial x} \cong \frac{\cos \theta}{r^2} \frac{\partial f}{\partial t} \qquad [60]$$

Thus on the body the potential ψ satisfies the conditions [55] and [56] to leading order in R. To establish that the same is true of ψ, we now show that the contributions from ψ_2 and $\partial\psi_2/\partial x$ are of higher order in R. Since

$$\frac{\partial}{\partial r} J_0(kr) = -k J_1(kr)$$

it follows that

$$\begin{aligned} \frac{\partial \psi_2}{\partial r} &= - K \int_{-H}^{0} \frac{\partial f}{\partial t} \fint_0^{\infty} \frac{k}{k - K} e^{k(z + z_1)} J_1(kr) \, dk \, dz_1 \\ &\quad + \pi \omega K^2 \int_{-H}^{0} f(z_1, t) \, e^{K(z + z_1)} J_1(Kr) \, dz_1 \end{aligned} \qquad [61]$$

We wish to show that

$$\frac{\partial \psi_2}{\partial r} = 0(f)$$

and

$$\frac{\partial^2 \psi_2}{\partial r \, \partial x} = 0\left(\frac{f}{R}\right)$$

as $R \to 0$, and thus that

$$\frac{\partial \psi_2}{\partial r} << \frac{\partial \psi_1}{\partial r} \quad \text{and} \quad \frac{\partial^2 \psi_2}{\partial r\, \partial x} << \frac{\partial^2 \psi_1}{\partial r\, \partial x}$$

for $R/H << 1$. From the series expansion of the Bessel function,

$$J_1(kr) = \tfrac{1}{2} kr + 0(k^3 r^3)$$

and thus, where this expansion is permissible in Equation [61], the resulting terms are clearly of order fK. However, in the neighborhood of $z = 0$, the power series expansion is not permissible in the integral over k. It follows that, in the neighborhood of $r = R$,

$$\frac{\partial \psi_2}{\partial r} = - K \frac{\partial f(0,t)}{\partial t} \int_{-H}^{0} \fint_0^\infty \frac{k}{k - K} e^{kz_1} J_1(kr)\, dk\, dz_1$$

$$+ 0(fR)$$

$$\cong - K \frac{\partial f(0,t)}{\partial t} \fint_0^\infty \frac{1}{k - K} J_1(kr)\, dk$$

$$\cong - K \frac{\partial f(0,t)}{\partial t} \int_0^\infty \frac{J_1(kr)}{k}\, dk$$

$$= - K \frac{\partial f(0,t)}{\partial t} = 0(f)$$

Similarly,

$$\frac{\partial^2 \psi_2}{\partial r\, \partial x} = 0\left(\frac{f}{R}\right)$$

Thus, on the body,

$$\frac{\partial \psi_2}{\partial r} = 0\left(R \frac{\partial \psi_1}{\partial r}\right)$$

and

$$\frac{\partial^2 \psi_2}{\partial r \, \partial x} = 0 \left(R \, \frac{\partial^2 \psi_1}{\partial r \, \partial x} \right)$$

Therefore, the potential ψ satisfies the conditions [55] and [56] with a fractional error of order R.

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David Taylor Model Basin. Report 1499.
THE MOTIONS OF A SPAR BUOY IN REGULAR WAVES, by J.N. Newman. May 1963. iv, 30p. illus., refs. UNCLASSIFIED

A linearized theory is developed for the motions of a slender body of revolution, with vertical axis, which is floating in the presence of regular waves. Equations of motion are derived which are undamped to first order in the body diameter, but second-order damping forces are derived to provide solutions valid at all frequencies including resonance. Calculations made for a particular circular cylinder show extremely stable motions except for the low frequency range where very sharp maxima occur at resonance.

1. Spar buoys--Motion--Mathematical analysis
2. Bodies of revolution (Vertical)--Motion--Theory
I. Newman, J. Nicholas