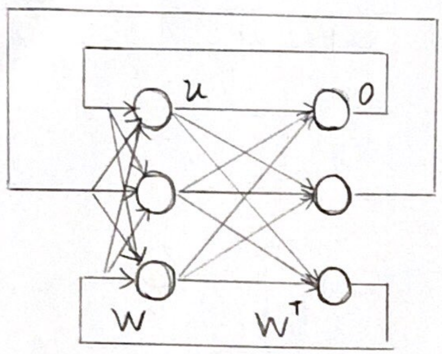
EE 7207 AY 2017 ~ 2018

1. $Q_1 = [1\ -1\ -1]^T$, $Q_2 = [-1\ 1\ 1]^T$

$Q_3 = [1\ 1\ 1]^T$, $Q_4 = [-1\ -1\ -1]^T$

(a) BAM: $Q_1 \to Q_3$, $Q_2 \to Q_4$



weight: $W = \sum_{i=1}^{N} x_i y_i^T$

for mapping $Q_1 \to Q_3$:

$$W_1 = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} [1\ 1\ 1] = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

for mapping $Q_2 \to Q_4$:

$$W_2 = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} [-1\ -1\ -1] = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

$$\Rightarrow W = W_1 + W_2 = \begin{bmatrix} 2 & 2 & 2 \\ -2 & -2 & -2 \\ -2 & -2 & -2 \end{bmatrix}$$

Test: if we have input $x = Q_1 = [1\ -1\ -1]^T$:

$$O(1) = \varphi(W^T x)$$

$$= \varphi\left(\begin{bmatrix} 2 & -2 & -2 \\ 2 & -2 & -2 \\ 2 & -2 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}\right)$$

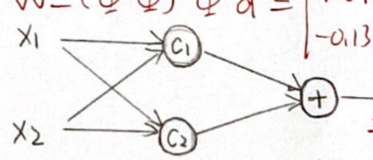
$$= \varphi([6\ 6\ 6]^T) = [1\ 1\ 1]^T = Q_3$$

$\therefore$ the above BAM works.

(b) RBF: $f(x) = \Phi w = \sum_{j=1}^{m} w_j \cdot \exp\left(-\frac{\|x - c_j\|^2}{2\sigma^2}\right)$

$$\Phi = \begin{bmatrix} \exp(0) & \exp(-2) \\ \exp(-2) & \exp(0) \end{bmatrix} = \begin{bmatrix} 1 & 0.1353 \\ 0.1353 & 1 \end{bmatrix}$$

$$W = (\Phi^T \Phi)^{-1} \Phi^T d = \begin{bmatrix} 1.0186 & -0.1378 \\ -0.1378 & 1.0186 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \end{bmatrix}$$



$$f(x) = \begin{bmatrix} 1.1564 & 1.1564 & 1.1564 \\ -1.1564 & -1.1564 & -1.1564 \end{bmatrix}$$

input	output
$Q_1 = [1\ -1\ -1]^T$	$Q_3 = [1\ 1\ 1]^T$
$Q_2 = [-1\ 1\ 1]^T$	$Q_4 = [-1\ -1\ -1]^T$

assume center vectors:

$C_1 = [1\ -1\ -1]^T$, $C_2 = [-1\ 1\ 1]^T$, $\sigma = 0.707$ $\quad \sigma = \frac{d_{max}}{\sqrt{2m}} = \frac{\sqrt{8}}{\sqrt{4}} = \sqrt{2}$

$$f(x) = \Phi w = \begin{bmatrix} 1 & 0.000006 \\ 0.000006 & 1 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}, \quad d = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = (\Phi^T \Phi)^{-1} \Phi^T d$$

$$\Phi = \begin{bmatrix} 1 & 6\times10^{-6} \\ 6\times10^{-6} & 1 \end{bmatrix} \qquad d = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = (\Phi^T\Phi)^{-1}\Phi^T d = \left(\begin{bmatrix} 1 & 6\times10^{-6} \\ 6\times10^{-6} & 1 \end{bmatrix} \begin{bmatrix} 1 & 6\times10^{-6} \\ 6\times10^{-6} & 1 \end{bmatrix} \right)^{-1} \Phi^T d$$

$$= \begin{bmatrix} 1.00034 & 0.01844 \\ 0.01844 & 1 \end{bmatrix}^{-1} \Phi^T d$$

$$= \frac{\begin{bmatrix} 1 & -0.01844 \\ -0.01844 & 1.00034 \end{bmatrix}}{0.99999} \Phi^T d$$

$$= \begin{bmatrix} 1 & -0.01844 \\ -0.01844 & 1.00034 \end{bmatrix} \begin{bmatrix} 1 & 0.01832 \\ 0.000123 & 1 \end{bmatrix} d$$

$$= \begin{bmatrix} 0.99999 & -0.00012 \\ -0.01832 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ -1.01832 & -1.01832 & -1.01832 \end{bmatrix}$$

if $x = [1 \quad -1 \quad -1]^T = Q_1$:

$$f(x) = \Phi w = \sum_{j=1}^{m} w_j \cdot \exp\left(-\frac{\|x - c_j\|^2}{2\sigma^2}\right)$$

$$= w_1 \cdot \exp(0) + w_2 \cdot \exp(-(4+4+4))$$

$$= [1 \quad 1 \quad 1]^T + [-1.01832 \quad -1.01832 \quad -1.01832]^T \times e^{-12}$$

$$= [0.99999 \quad 0.99999 \quad 0.99999]^T = Q_3$$

if $x = [-1 \quad 1 \quad 1]^T = Q_2$:

$$f(x) = \Phi w = \sum_{j=1}^{m} w_j \cdot \exp\left(-\frac{\|x - c_j\|^2}{2\sigma^2}\right) = w_1 \cdot \exp(-(4+4+4)) + w_2 \cdot \exp(0)$$

$$= [1 \quad 1 \quad 1]^T \times e^{-12} + [-1.01832 \quad -1.01832 \quad -1.01832]^T$$

$$= [-1.01831 \quad -1.01831 \quad -1.01831]^T = Q_4$$

$\therefore$ $Q_1 \to Q_3$, $Q_2 \to Q_4$. the RBF network is working.

2. (a) 3 ingredient: ① competition ② cooperation ③ weight adaptation

learning procedure:

step 1: initialization. randomly select weight vector $w_j(0)$ which $w_{ji}(0)$ are different from each other and have small magnitude.

step 2: sampling. randomly select a input sample X from training samples.

step 3: competition. find the winning neuron which satisfies: $i(x) = \arg\min \| X - w_j(n) \|$, $j = 1, 2, \dots, N$

step 4: cooperation and adaptation. update the weight of the winning neuron as:

$$w_j(n+1) = w_j(n) + \eta(n) h_{ji(x)}(n) \cdot [X - w_j(n)]$$

step 5: iteration. go back to step 2 untill convergence:

① reach the pre-defined iteration number

② no noticeable change in the map.

(b) self-organizing phase and convergence phase.

- learning rate $\eta(n) = \eta_0 \cdot \exp(-\frac{n}{T_2})$, where η_0 is the initialization learning rate, n is the iteration time, T_2 is a time constant. Without loss of generality, $\eta_0 = 0.1$, $T_2 = 1000$, and $\eta(n)$ remains above 0.01.

- neighborhood function: $h_{ji}(x) = \exp(-\frac{d_{ji}(x)}{2\sigma(n)})$.

$\sigma(n) = \sigma_0 \cdot \exp(-\frac{n}{T_1})$, where n is the iteration time, T_1 is a time constant. In 2-D, $d_{ji} = \| r_j - r_i \|$ denotes the distance between winning neuron i and excited neuron j.

(c) application 1 : density matching

The SOM map can reflect the distributions of the input space. Regions from the input space which samples are drawn with a high probability of occurrence are mapped into larger domains of the outspace.

application 2 : approximation

The feature map represented by a set of weight vectors in the output space provide a good approximation of the distribution of the input space. Such property can be used as a tool to find "prototype" for other algorithms. Such as selecting the center vectors for the RBF network.

3. (a) polynomial kernel: $K(x, z) = (1 + x^T z)^p$, p is a positive integer and is specified by the user.

Gaussian kernel: $K(x, z) = \exp(-\frac{\|x - z\|^2}{2\sigma^2})$, σ denotes the width and is specified by the user.

(b) For linearly seperable problem, the SVM is defined as follows:

$$g(x) = w^T x + b = 0, \quad x = x_p + r\frac{w}{\|w\|}.$$

where x_p is the projection of x, r is the distance from x to hyperplane.

$$g(x) = w^T x + b = w^T x_p + b + r\frac{w^T w}{\|w\|} = g(x_p) + r\|w\|,$$

$$\Rightarrow g(x) = r\|w\|, \quad r = \frac{g(x)}{\|w\|}.$$

$$\Rightarrow r = \frac{g(x^{(s)})}{\|w\|} = \begin{cases} \frac{1}{\|w\|}, & \text{for } d(i) = +1 \\ \frac{-1}{\|w\|}, & \text{for } d(i) = -1 \end{cases}$$

and the margin of seperation is $\rho = \frac{2}{\|w\|}$.

In order to maximize the margin ρ, the weight w should be minimized. The optimization problem is defined as follows:

$$\min. \quad J(w) = \frac{1}{2} w^T w$$
$$\text{s.t.} \quad d(i)[w^T x(i) + b] \geq 1$$

By using the Lagrange theory, we have:

$$J(w, b, \alpha) = \frac{1}{2} w^T w - \sum_{i=1}^{N} \alpha(i) [d(i)[w^T x(i) + b] - 1]$$

where $\alpha(i) \geq 0$ is the Lagrange multiplier.

The following two conditions are obtained:

$$\frac{\partial J(w, b, \alpha)}{\partial w} = 0, \quad \frac{\partial J(w, b, \alpha)}{\partial b} = 0$$

$$J(w,b,\alpha) = \frac{1}{2}w^Tw - \sum_{i=1}^{N}\alpha(i)\left(d(i)[w^Tx(i)+b]-1\right)$$

$$\frac{\partial J}{\partial w} = w - \sum_{i=1}^{N}\alpha(i)d(i)x(i) = 0 \quad \Rightarrow \quad w = \sum_{i=1}^{N}\alpha(i)d(i)x(i)$$

$$\frac{\partial J}{\partial b} = -\sum_{i=1}^{N}\alpha(i)d(i) = 0 \quad \Rightarrow \quad \sum_{i=1}^{N}\alpha(i)d(i) = 0$$

Following the KKT condition, we know that:

$$\alpha(i)[d(i)[w^Tx(i)+b]-1] = 0$$

With non-zero $\alpha(i)$, then $(d(i)[w^Tx(i)+b]-1)$ must be zero.

Re-write J as: $J(w,b,\alpha) = \frac{1}{2}w^Tw - \sum_{i=1}^{N}\alpha(i)d(i)w^Tx(i)$

$$- \sum_{i=1}^{N}\alpha(i)d(i)b + \sum_{i=1}^{N}\alpha(i)$$

$$\Rightarrow \quad w^Tw = \sum_{i=1}^{N}\alpha(i)d(i)x^T(i)\sum_{j=1}^{N}\alpha(j)d(j)x(j)$$

$$= \sum_{i=1}^{N}\sum_{j=1}^{N}\alpha(i)\alpha(j)d(i)d(j)x^T(i)x(j)$$

$$\Rightarrow \quad J(w,b,\alpha) = \sum_{i=1}^{N}\alpha(i) - \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\alpha(i)\alpha(j)d(i)d(j)x^T(i)x(j) = Q(\alpha)$$

$Q(\alpha)$ is the dual problem, and its solutions are:

$$w^* = \sum_{i=1}^{N}\alpha(i)d(i)x(i), \quad b^* = 1 - w^*x$$

w^* and b^* can be found when $\alpha(i)$ is determined by solving QP problem $Q(\alpha)$.

(c)

4 (a) R1: IF x is cold THEN action is heat.

R2: IF x is warm THEN action is no-change.

R3: IF x is hot THEN action is cool.

(b) $X_1 = [0, 0.6]$, $X_2 = [0.2, 0.8]$, $Y = X_1 + X_2 = [0.2, 1.4]$

$$\mu_{X_1}(x_1) = \begin{cases} \frac{10}{3}x_1, & 0 \le x_1 \le 0.3 \\ -\frac{10}{3}x_1 + 2, & 0.3 \le x_1 \le 0.6 \end{cases} \quad \begin{array}{l} \Rightarrow \alpha = \frac{10}{3}x_1, \quad x_{11} = \frac{3}{10}\alpha \\ \Rightarrow \alpha = -\frac{10}{3}x_1 + 2, \quad x_{12} = -\frac{3}{10}\alpha + \frac{6}{10} \end{array}$$

$$\Rightarrow (X_1)_\alpha = [\frac{3}{10}\alpha, -\frac{3}{10}\alpha + \frac{6}{10}]$$

$$\mu_{X_2}(x_2) = \begin{cases} 5x_2 - 1, & 0.2 \le x_2 \le 0.4 \\ -\frac{5}{2}x_2 + 2, & 0.4 \le x_2 \le 0.8 \end{cases} \quad \begin{array}{l} \Rightarrow \alpha = 5x_2 - 1, \quad x_{21} = \frac{1}{5}\alpha + \frac{1}{5} \\ \Rightarrow \alpha = -\frac{5}{2}x_2 + 2, \quad x_{22} = -\frac{2}{5}\alpha + \frac{4}{5} \end{array}$$

$$\Rightarrow (X_2)_\alpha = [\frac{1}{5}\alpha + \frac{1}{5}, -\frac{2}{5}\alpha + \frac{4}{5}]$$

$$(Y)_\alpha = (X_1)_\alpha + (X_2)_\alpha = [\frac{1}{2}\alpha + \frac{1}{5}, -\frac{7}{10}\alpha + \frac{7}{5}]$$

$$\begin{array}{l} \frac{1}{2}\alpha + \frac{1}{5} = y_1 \Rightarrow \alpha = 2y - \frac{2}{5} \\ -\frac{7}{10}\alpha + \frac{7}{5} = y_2 \Rightarrow \alpha = -\frac{10}{7}y + 2 \end{array} \Rightarrow \mu_{Y = X_1 + X_2}(y) = \begin{cases} 2y - \frac{2}{5}, & 0.2 \le y \le 0.7 \\ -\frac{10}{7}y + 2, & 0.7 \le y \le 1.4 \end{cases}$$

(c) Strong points: ① No need for mathematical model

② Less sensitive to system fluctuations;

③ Simple, fast and adaptable.

weak points:

① need expert knowledges;

② hard to evaluate system's reliability;

③ hard to verify system stability.

5 (a) for input x:

$$\mu_N(x)=\begin{cases}2x+6, & -3.0\le x\le -2.5\\ 1, & -2.5\le x\le -0.5\\ -x+0.5, & -0.5\le x\le 0.5\end{cases}$$

$$\mu_{ZE}(x)=\begin{cases}\frac{4}{3}x+\frac{4}{3}, & -1.0\le x\le -0.25\\ 1, & -0.25\le x\le 0.25\\ -4x+2, & 0.25\le x\le 1.0\end{cases}$$

$$\mu_P(x)=\begin{cases}x+0.5, & -0.5\le x\le 0.5\\ 1, & 0.5\le x\le 2.5\\ -2x+6, & 2.5\le x\le 3.0\end{cases}$$

for input y:

$$\mu_N(y)=\begin{cases}2y+6, & -3.0\le y\le -2.5\\ 1, & -2.5\le y\le -0.5\\ -y+0.5, & -0.5\le y\le 0.5\end{cases}$$

$$\mu_{ZE}(y)=\begin{cases}\frac{4}{3}y+\frac{4}{3}, & -1.0\le y\le -0.25\\ 1, & -0.25\le y\le 0.25\\ -4y+2, & 0.25\le y\le 1.0\end{cases}$$

$$\mu_P(y)=\begin{cases}y+0.5, & -0.5\le y\le 0.5\\ 1, & 0.5\le y\le 2.5\\ -2y+6, & 2.5\le y\le 3.0\end{cases}$$

for output z:

$$\mu_{NC}(z)=\begin{cases}\frac{2}{5}z+2, & -5.0\le z\le -2.5\\ -\frac{2}{5}z, & -2.5\le z\le 0\end{cases}$$

$$\mu_{BC}(z)=\begin{cases}\frac{2}{5}z+1, & -2.5\le z\le 0\\ -\frac{2}{5}z+1, & 0\le z\le 2.5\end{cases}$$

$$\mu_{PC}(z)=\begin{cases}\frac{2}{5}z, & 0\le z\le 2.5\\ -\frac{2}{5}z+2 & 2.5\le z\le 5.0\end{cases}$$

$\mu_N(y_0)=0.85$

$\mu_{ZE}(y_0)=\frac{13}{15}$

$\mu_P(y_0)=0.15$

(b) $x_0=0.2$, $y_0=-0.35$, $\mu_N(x_0)=0.3$, $\mu_{ZE}(x_0)=1$, $\mu_P(x_0)=0.7$

R_1: $w_1=\min(\mu_N(x),\mu_N(y))=0.3$

R_2: $w_2=\min(\mu_{ZE}(x),\mu_N(y))=0.85$

R_3: $w_3=\min(\mu_P(x),\mu_N(y))=0.7$

0.2356

… …

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2017-2018

EE7207 – NEURAL AND FUZZY SYSTEMS

November/December 2017 Time Allowed: 3 hours

INSTRUCTIONS

1. This paper contains 5 questions and comprises 5 pages.
2. Answer all 5 questions.
3. All questions carry equal marks.
4. This is a closed-book examination.

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1. There are four vectors:

$$Q_1 = [1 \quad -1 \quad -1]^T$$
$$Q_2 = [-1 \quad 1 \quad 1]^T$$
$$Q_3 = [1 \quad 1 \quad 1]^T$$
$$Q_4 = [-1 \quad -1 \quad -1]^T$$

(a) Design a bi-directional associative memory (BAM) neural network to map Q_1 and Q_2 to Q_3 and Q_4, respectively. Sketch the architecture of the BAM neural network designed, compute the weights on the links between neurons, and test whether the designed BAM works.

(7 Marks)

(b) Design a Gaussian radial basis function (RBF) neural network to map Q_1 and Q_2 to Q_3 and Q_4, respectively. Sketch the RBF neural network architecture, determine centre vectors of hidden layer neurons and the weights on the links between neurons, and test whether the designed RBF neural network works.

(9 Marks)

(c) Through an example, discuss the fault tolerating capability of the BAM and RBF neural networks designed in parts 1(a) and 1(b), respectively.

(4 Marks)

$w_j(n+1) = w_j(n) + \eta(n) h_{ji(x)}(n) (x(n) - w_j(n))$

$\eta(n) = \eta_0 \cdot exp(-\frac{n}{\tau_1})$ $h_{ji}(n)(x) = exp(-\frac{d_{ji}(x)}{2\sigma(n)})$ $d_{ji}(x) = \|v_i - \eta\|$

2. Self-organizing map (SOM) neural network is a very popular tool in data analytics.

(a) List the 3 ingredients of SOM neural network, and describe the learning procedure of SOM neural network. competition, cooperation, adaption

(7 Marks)

(b) List the two learning phases of SOM neural network, and discuss settings of learning rate and neighbourhood function in the two phases.

Self-organizing: $\eta(n) = 0.1\, exp(-\frac{n}{1000})$, $h_{ji}(n) = exp(-\frac{d_{ji}^2}{2\sigma^2(n)})$, $\sigma(n) = \sigma_0 exp(-\frac{n}{\tau_1})$

(7 Marks)

$\tau_1 = \frac{1000}{\ln \sigma_0}$, σ_0 = radius of 2-D lattice

convergence phase: $\eta(n) = 0.01$

$d_{ji}(n) = \begin{cases} 1, & \text{if neuron } j \text{ wins} \\ 0, & \text{otherwise} \end{cases}$

(c) Give two application examples of SOM neural network, and explain the rationale of adopting SOM neural network in these applications.

density matching

approximation of input space

(6 Marks)

3. Support vector machine (SVM) is widely used in pattern classification.

(a) Give two kernel functions used in kernel support vector machines, and briefly describe the two functions.

$K_1(x, z) = (1 + x^T z)^p$, $K_2(x, z) = exp(-\frac{\|x - z\|^2}{2\sigma^2})$

(6 Marks)

(b) Based on linearly separable 2-class pattern classification problem, describe the optimal separating hyperplane of linear SVM and the primal optimization problem for finding this optimal hyperplane.

(8 Marks)

(c) Discuss how Kernel SVM fundamentally differs from RBF neural network, and discuss how to solve multi-class pattern classification problems using SVM.

(6 Marks)

$g(x) = w^T x + b$, $x = x_p + r\frac{w}{\|w\|}$. $g(x) = w^T x_p + b + r\frac{w^T w}{\|w\|} = r\|w\|$

$\Rightarrow r = \frac{g(x)}{\|w\|} = \begin{cases} \frac{+1}{\|w\|}, & \text{for } d(i) = +1 \\ \frac{-1}{\|w\|}, & \text{for } d(i) = -1 \end{cases}$ $\rho = \frac{2}{\|w\|}$

$\Rightarrow \begin{cases} \text{min. } J(w) = \frac{1}{2} w^T w \\ \text{s.t. } d(i) \times [w^T x(i) + b] \geq 1 \end{cases} \Rightarrow J(w, b, \alpha) = \frac{1}{2} w^T w - \alpha(i)[d(i)[w^T x(i) + b] - 1]$

$\alpha(i) \geq 0$ is the lagrange multiplier.

$\frac{J(w,b,\alpha)}{\partial w} = w - \sum_{i=1}^{N} \alpha(i) d(i) x(i) = 0 \Rightarrow w = \sum_{i=1}^{N} \alpha(i) d(i) x(i)$

$\frac{J(w,b,\alpha)}{\partial b} = -\sum_{i=1}^{N} \alpha(i) d(i) = 0$

$J(w, b, \alpha) = \frac{1}{2} w^T w - \sum_{i=1}^{N} \alpha(i) d(i) w^T x(i) - \sum_{i=1}^{N} \alpha(i) d(i) b + \sum_{i=1}^{N} \alpha(i)$

$w^T w = \sum_{i=1}^{N} \sum_{j=1}^{N} \alpha(i)\alpha(j) d(i) d(j) x^T(i) x(j)$

$\Rightarrow J(w, b, \alpha) = \sum_{i=1}^{N} \alpha(i) - \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \alpha(i)\alpha(j) d(i) d(j) x^T(i) x(j) = Q(\alpha)$

$\Rightarrow \begin{cases} \text{min. } Q(\alpha) \\ \text{s.t. } \sum_{i=1}^{N} \alpha(i) d(i) = 0 \end{cases} \Rightarrow \alpha(i) \Rightarrow w^* = \sum_{i=1}^{N} \alpha(i) d(i) x(i)$, $b^* = 1 - w^{*} x(i)$

4. (a) Assume that x represents the input of room temperature for a temperature feedback control system. The linguistic terms can be defined as:

Temperature x = {very-cold, cold, warm, very-warm, hot}.

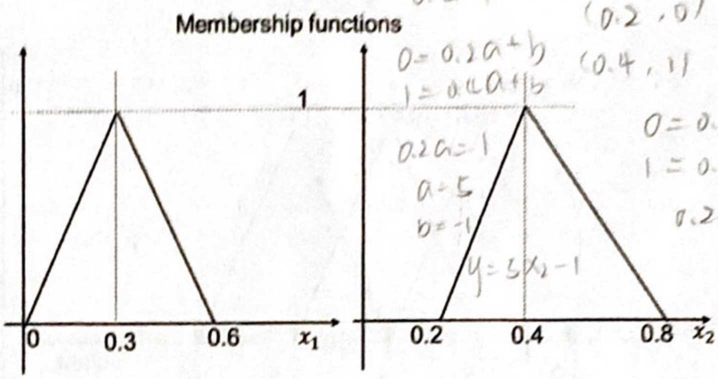
Build a set of three fuzzy rules, based on three-control action variables: heat, cool and no-change, into the fuzzy knowledge database in the form of if-then fuzzy logic rules.

Create a simple fuzzy associative memory matrix (FAMM) of room temperature values versus target temperature values (measured by a feedback sensor), corresponding to the five given linguistic terms, that an air conditioner control system is expected to provide.

(5 Marks)

(b) For the two fuzzy membership functions defined in Figure 1, determine the range of interval and membership function of the combined system with an addition operation.

0.2 0
0.4 1
(0.2 , 0)
$0 = 0.2a + b$
$1 = 0.4a + b$
(0.4 , 1)
$0.2a = 1$
$a = 5$
$b = -1$
$y = 5x_2 - 1$



(0.8 , 0)
$0 = 0.2a + b$
$1 = 0.4a + b$
$0.2a = 1$ $a = 5$
$b = -1$
$1 = 0.4a + b$
$0 = 0.8a + b$
$-0.4a = 1$
$a = -\frac{5}{2}$
$b = 2$

0.3 1
0.6 0
$1 = 0.3a + b$
$0 = 0.6a + b$
$-0.3a = 1$
$a = -\frac{10}{3}$
$b = 2$

Figure 1

(0.3 , 1)
$1 = 0.3a + b$, $b = 0$ $a = \frac{10}{3}$
$y = \cancel{0.3a}$

(10 Marks)

(c) List and discuss two main strong points and two weak points of fuzzy systems, respectively.

(5 Marks)

$-\frac{10}{7}y + 2 = 1$
$-\frac{10}{7}y = -1$
$y = \frac{7}{10}$
$-\frac{10}{7}y + 2 = 0$
$-\frac{10}{7}y = -2$
$y = \frac{14}{10}$

$-3, 0$ $\quad 0 = -3a + b$ $\quad b = 6$

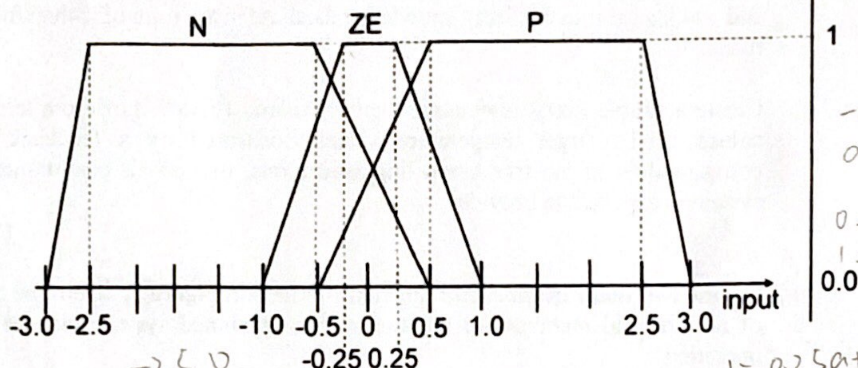
$-2.5\ 1$ $\quad 1 = -2.5a + b$

$0.5a = 1 \quad a = 2$

5. A fuzzy temperature control system has two inputs x and y and one output z. The membership functions of fuzzy sets for both x and y are given in Figure 2 and the fuzzy membership functions of output z are given in Figure 3.

$(-1, 0)$ $\quad 0 = -a + b$ $\quad 0.75a = 1 \quad a = \frac{4}{3}$

$(-0.25\ 1)$ $\quad 1 = -0.25a + b$ $\quad b = \frac{4}{3}$



$-3,\ 0$
$-2.5\ 1$
$0 = -3a + b$
$1 = -2.5a + b$
$0.5a = 1 \quad a = 2$
$b = 6$

$-0.5\ 0$
$0.5\ 1$
$0 = -0.5a + b$
$1 = 0.5a + b$
$b = 0.5 \quad a = 1$

$(-0.5, 1)$
$(0.5\ 0)$
$1 = -0.5a + b$
$0 = 0.5a + b$
$2b = 1 \quad a = -2$
$b = 0.5 \quad a = -1$

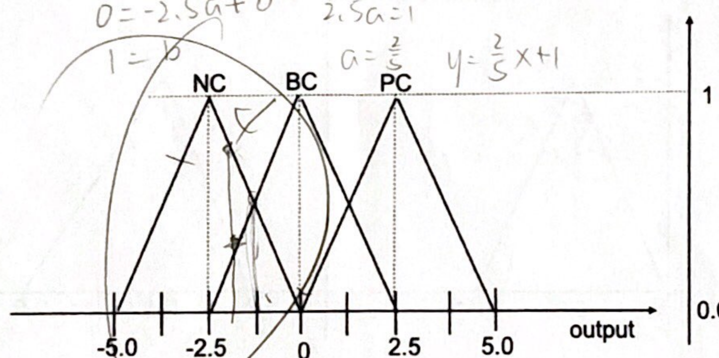
$-2.5\ 0$
$0\ \ 1$

$0.25, 1$
$0.5, 0$

$1 = 0.25a + b$
$0 = 0.5a + b$
$0.25a = -1$
$a = -4$
$b = 2$

Figure 2

$0 = -2.5a + b$
$1 = b$
$2.5a = 1$
$a = \frac{2}{5} \quad y = \frac{2}{5}x + 1$



$-5, 0$
$-2.5\ 1$
$0 = -5a + b$
$1 = -2.5a + b$
$2.5a = 1 \quad a = \frac{2}{5} \quad b = 2$

$2.5\ \ 1$
$3.0\ \ 0$
$1 = 2.5a + b$
$0 = 3a + b$
$0.5a = -1$
$a = -2$
$b = 6$

$-2.5\ \ 0$
$0\ \ 1$

Figure 3

(a) Calculate given fuzzy trapezoidal membership functions for x and y and the triangular membership functions for z, respectively.

(5 Marks)

$1 = -2.5a + b$
$0 = b$
$-\frac{2}{5}$

$0 = -2.5a + b$
$1 = b$

Note: Question No. 5 continues on page 5

(b) Given crisp inputs $x = 0.2$ and $y = -0.35$, if the inputs are applied to a set of if-then fuzzy logical rules, which are summarised in Table 1 (row corresponds to the fuzzy term set of x and column corresponds to the fuzzy term set of y). Assuming that we are using the conjunction operator (FUZZY AND) in the antecedents of the rules based on Mamdani's minimum, calculate the corresponding rule firing levels.

$\mu_N(x_0) = 0.3$
$\mu_{ZE}(x_0) = 1$
$\mu_P(x_0) = 0.7$

$\mu_N(y_0) = 0.85$
$\mu_{ZE}(y_0) = 1$
$\mu_P(y_0) = 0.15$

Table 1

	N	ZE	P
N	If x is N and y is N Then z is PC 0.3 1	If x is ZE and y is N Then z is PC 0.85 2	If x is P and y is N Then z is BC 0.7 3
ZE	If x is N and y is ZE Then z is PC 0.3 4	If x is ZE and y is ZE Then z is BC 5	If x is P and y is ZE Then z is NC 6
P	If x is N and y is P Then z is BC 7	If x is ZE and y is P Then z is NC 8	If x is P and y is P Then z is NC 9

(10 Marks)

(c) Assuming that we are using the center of gravity of defuzzification method, calculate the crisp output.

(5 Marks)

$S = w(h - \frac{h^2}{2})$, $S_1 = 5 \times (0.3 - \frac{0.09}{2}) = 1.275$

END OF PAPER