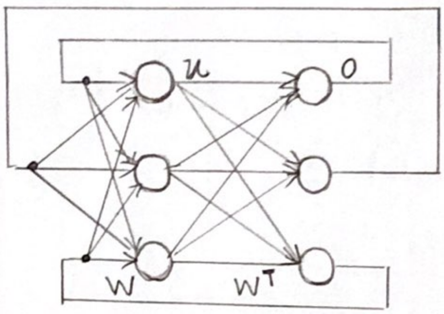
7 (a) $Q_1 = [1 \ -1 \ -1]^T$, $Q_2 = [-1 \ 1 \ 1]^T$



$$W = \sum_{i=1}^{N} x_i y_i^T = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} [-1 \ 1 \ 1]$$

$$= \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \end{bmatrix}$$

Let input $x_1 = [1 \ -1 \ -1]^T = Q_1$:

$$O(1) = \varphi(W^T x_1) = \varphi\left(\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}\right)$$

$$= \varphi([-3 \ 3 \ 3]^T) = [-1 \ 1 \ 1]^T$$

$$u(2) = \varphi(W O(1)) = \varphi\left(\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \end{bmatrix}\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}\right)$$

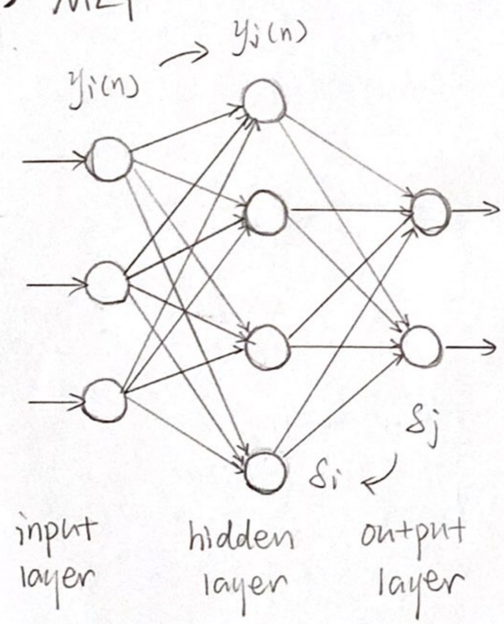
$$= \varphi([3 \ -3 \ -3]^T) = [1 \ -1 \ -1]^T$$

$$O(2) = \varphi(W^T u(2)) = \varphi\left(\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}\right)$$

$$= \varphi([-3 \ 3 \ 3]^T) = [-1 \ 1 \ 1]^T$$

The network cannot converge to a stable state.

(b) MLP



training and weight updating:

① forward pass: first, the weights remain unchanged, and the signals are computed in an neuron-by-neuro layer-by-layer manner.

② backward pass: starting from the output layer, the error signals are passed backward through the network, layer-by-layer, and recursively compute the local gradients and update the weights accordingly.

The backpropagation process:

① error signal $e_j(n) = d_j(n) - y_j(n)$ is the difference between known desired output value $d_j(n)$ and the network computed value $y_j(n) = \varphi[v_j(n)] = \varphi[W^T \cdot y_i(n) + b]$

② total squared error: $E(n) = \frac{1}{2}\sum_{j=1}^{N} e_j^2(n)$

③ gradient: $\frac{\partial E(n)}{\partial w_{ji}(n)} = \frac{\partial E(n)}{\partial e_j(n)} \frac{\partial e_j(n)}{\partial y_j(n)} \frac{\partial y_j(n)}{\partial v_j(n)} \frac{\partial v_j(n)}{\partial w_{ji}(n)}$

$$= e_j(n) \cdot (-1) \cdot \varphi'[v_j(n)] \cdot y_i(n)$$

$\Rightarrow$ $\Delta w_{ji}(n) = -\eta \frac{\partial E(n)}{\partial w_{ji}(n)} = \eta \delta_j(n) y_i(n)$

$\delta_j(n) = -\frac{\partial E(n)}{\partial e(n)} \frac{\partial e(n)}{\partial y_j(n)} \frac{\partial y_j(n)}{\partial v_j(n)} = e_j(n) \cdot \varphi'[v_j(n)]$

is the local gradient.

two cases:

I. neuron j at output layer: $\delta_j(n) = -\frac{\partial E(n)}{\partial v_j(n)} = e_j(n) \cdot \varphi'[v_j(n)]$

II. neuron j at hidden layer: no error signal $e_j(n)$.

$\delta_j(n) = -\frac{\partial E(n)}{\partial v_j(n)} = -\frac{\partial E(n)}{\partial y_j(n)} \frac{\partial y_j(n)}{\partial v_j(n)} = -\frac{\partial E(n)}{\partial y_j(n)} \cdot \varphi'[v_j(n)]$

$\Rightarrow$ compute $\frac{\partial E(n)}{\partial y_j(n)}$. $E(n) = \frac{1}{2}\sum_k e_k^2(n)$ $\Rightarrow$ $\frac{\partial E(n)}{\partial y_j(n)} = \sum_k e_k \cdot \frac{\partial e_k(n)}{\partial y_j(n)}$

$e_k(n) = d_k(n) - y_k(n) = d_k(n) - \varphi[v_k(n)]$

$v_k(n) = \sum w_{kj}(n) \cdot y_j(n)$

$$= \sum e_k \cdot \frac{\partial e_k(n)}{\partial v_k(n)} \cdot \frac{\partial v_k(n)}{\partial y_j(n)}$$

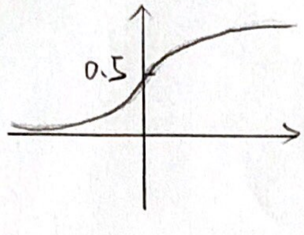
$\Rightarrow$ $\frac{\partial e_k}{\partial v_k} = -\varphi'[v_k(n)]$. $\frac{\partial v_k}{\partial y_j} = w_{kj}(n)$

$\Rightarrow$ $\frac{\partial E(n)}{\partial y_j(n)} = -\sum_k e_k(n) \varphi'[v_k(n)] w_{kj}(n) = -\sum_k \delta_k(n) w_{kj}(n)$

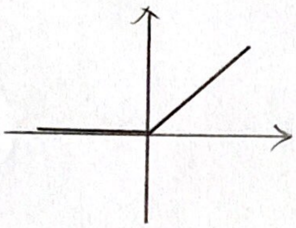
$\Rightarrow$ $\delta_j(n) = -\frac{\partial E(n)}{\partial v_j(n)} = \sum_k \delta_k(n) w_{kj}(n) \cdot \varphi'[v_j(n)]$

$\Delta w_{ji}(n) = -\eta \frac{\partial E(n)}{\partial w_{ji}(n)} = \eta \delta_j(n) y_i(n)$

(c) note the sigmoid function has the following shape:



In the early stage, the gradient becomes small and cause the "gradient vanishing" problem, which would lead to weight updating failure.



Use ReLU activation function to replace the sigmoid function can help to solve this problem.

the gradients of the early layers are the multiply of the gradients of the latter layers, hence they may become very small values.

2 (a) training procedure:

step 1: initialization. randomly select weight $w_j(0)$ which is different from each other and has Small magnitudes.

step 2: sampling. randomly select a sample x from input.

step 3: competition. find the winning neuron $i(x)$ which is:

$$i(x) = \arg\min \| x - w_j(n) \| \quad , \; j = 1, 2, \cdots, N$$

step 4: cooperation and adaptation. adjust the weight by:

$$w_j(n+1) = w_j(n) + \eta(n)\, h_{j,i(x)}(n) \cdot [x - w_j(n)]$$

step 5: iteration. go back to step 2 untill convergence:

① reached to pre-defined interation number.

② No noticeable change in the map.

(b) Step 1: $w_1(0) = [0.136 \quad 0.211 \quad 0.216]$

$w_2(0) = [2.16 \quad 1.98 \quad 2.051]$

Step 2: Select $x_1 = [0.235 \quad -0.081 \quad 0.264]$ as input:

$$d_1 = \| x_1 - w_1(0) \| = \sqrt{(0.099)^2 + (-0.292)^2 + (0.048)^2} = 0.312$$

$$d_2 = \| x_1 - w_2(0) \| = \sqrt{(-1.925)^2 + (-2.061)^2 + (-1.787)^2} = 3.339$$

=> neuron 1 wins, its weight is updated as:

$$w_1(1) = w_1(0) + \eta(0)\, h_{ji(x)}(0) \cdot [x_1 - w_1(0)]$$

$$= [0.136 \quad 0.211 \quad 0.216] + 0.01 \times [0.099 \quad -0.292 \quad 0.048]$$

$$= [0.13699 \quad 0.20808 \quad 0.21648]$$

$w_2(1) = w_2(0) = [2.16 \quad 1.98 \quad 2.051]$

Step 3: Select $x_2 = [1.994 \quad 2.195 \quad 2.083]$ as input:

$$d_1 = \| x_2 - w_1(1) \| = \sqrt{(1.857)^2 + (1.987)^2 + (1.8665)^2} = 3.2985$$

$$d_2 = \| x_2 - w_2(1) \| = \sqrt{(-0.166)^2 + (0.215)^2 + (0.032)^2} = 0.2735$$

=> neuron 2 wins, its weight is updated as:

$$W_2(2) = W_2(1) + \eta(1)\, h_{ji}(x)(1) \cdot [\, x_2 - W_2(1) \,]$$
$$= [2.16 \quad 1.98 \quad 2.051] + 0.01 \times [-0.166 \quad 0.215 \quad 0.032]$$
$$= [2.15834 \quad 1.98215 \quad 2.05132]$$

$$W_2(1) = W_1(1) = [0.13699 \quad 0.20808 \quad 0.21648]$$

(C) the setting of the number of neurons:

the neurons are the prototypes which can represent the distribution of the input space. So we can select the most representitive samples as the neurons based on the demand of clustering. for example, generate how many different types.

3. (a) $\mu_Z(z) = \bigvee_{z=x+y} \{\mu_X(x) \wedge \mu_Y(y)\}$

$X = [-10, 5]$, $Y = [0, 10]$ $\Rightarrow$ $Z = X + Y = [-10, 15]$

for μ_X : $\alpha = \frac{x}{4} + \frac{5}{4} \Rightarrow x_1 = 4\alpha - 5$

$\alpha = -\frac{x}{6} + \frac{5}{6} \Rightarrow x_2 = -6\alpha + 5$

$(X)_\alpha = [4\alpha - 5, -6\alpha + 5]$

for μ_Y : $\alpha = \frac{y}{6} + \frac{1}{6} \Rightarrow y_1 = 6\alpha - 1$

$\alpha = -\frac{y}{7} + \frac{12}{7} \Rightarrow y_2 = -7\alpha + 12$

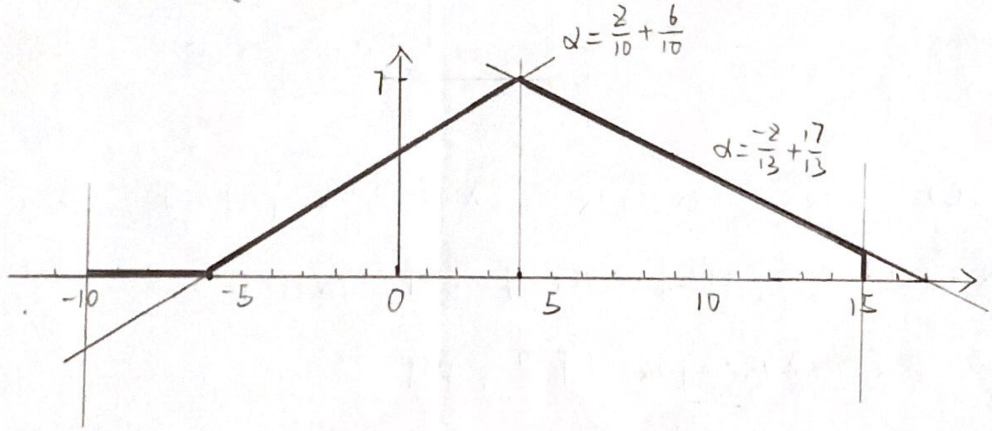
$(Y)_\alpha = [6\alpha - 1, -7\alpha + 12]$

$\Rightarrow$ $(Z)_\alpha = (X)_\alpha + (Y)_\alpha = [10\alpha - 6, -13\alpha + 17]$

$z_1 = 10\alpha - 6 \Rightarrow \alpha = \frac{z_1}{10} + \frac{6}{10}$

$z_2 = -13\alpha + 17 \Rightarrow \alpha = -\frac{z_2}{13} + \frac{17}{13}$

$$\Rightarrow \mu_Z(z) = \begin{cases} 0 , & -10 \le z \le -6 \\ \frac{z}{10} + \frac{6}{10} , & -6 \le z \le 4 \\ -\frac{z}{13} + \frac{17}{13} , & 4 \le z \le 15 \end{cases}$$



(b) $\mu_Z(z) = \bigvee_{z=x-y} \{\mu_X(x) \wedge \mu_Y(y)\}$

$X = [-10, 5]$, $Y = [0, 10]$, $Z = X - Y = [-20, 5]$

for μ_X : $\alpha = \frac{x}{4} + \frac{5}{4} \Rightarrow x_1 = 4\alpha - 5$
$\alpha = -\frac{x}{6} + \frac{5}{6} \Rightarrow x_2 = -6\alpha + 5$
$\Rightarrow (X)_\alpha = [4\alpha - 5, -6\alpha + 5]$

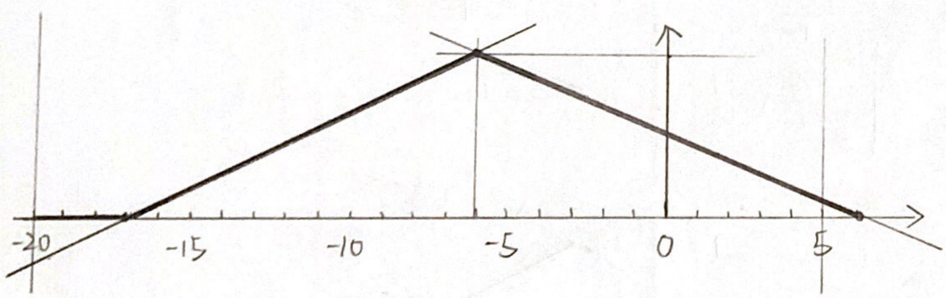
for μ_Y : $\alpha = \frac{y}{6} + \frac{1}{6} \Rightarrow y_1 = 6\alpha - 1$
$\alpha = -\frac{y}{7} + \frac{12}{7} \Rightarrow y_2 = -7\alpha + 12$
$\Rightarrow (Y)_\alpha = [6\alpha - 1, -7\alpha + 12]$

$\Rightarrow (Z)_\alpha = (X)_\alpha - (Y)_\alpha = [11\alpha - 17, -12\alpha + 6]$

$z_1 = 11\alpha - 17$, $\alpha = \frac{z}{11} + \frac{17}{11}$

$z_2 = -12\alpha + 6$, $\alpha = -\frac{z}{12} + \frac{6}{12}$

$$\Rightarrow \mu_Z(z) = \begin{cases} 0 , & -20 \le z \le -17 \\ \frac{z}{11} + \frac{17}{11} , & -17 \le z \le -6 \\ -\frac{z}{12} + \frac{6}{12} , & -6 \le z \le 5 \end{cases}$$



(c) $(Z)_\alpha = (X)_\alpha \cdot (Y)_\alpha$, $X = [-10, 5]$, $Y = [0, 10] \Rightarrow Z = [-100, 50]$

$(X)_\alpha = [4\alpha - 5, -6\alpha + 5]$, $(Y)_\alpha = [6\alpha - 1, -7\alpha + 12]$

$\Rightarrow (Z)_\alpha = (X)_\alpha \cdot (Y)_\alpha = [\underline{P}, \overline{P}]$

$\underline{P} = \min\{24\alpha^2 - 34\alpha + 5, -28\alpha^2 + 83\alpha - 60, -36\alpha^2 + 36\alpha - 5, 42\alpha^2 - 107\alpha + 60\}$

$\overline{P} = \max\{24\alpha^2 - 34\alpha + 5, -28\alpha^2 + 83\alpha - 60, -36\alpha^2 + 36\alpha - 5, 42\alpha^2 - 107\alpha + 60\}$

4. (a)

$$\mu_R(x,y) = \mu_A(x) \wedge \mu_B(y)$$

$$= \begin{bmatrix} 0 \\ 0.6 \\ 0.4 \\ 0.1 \\ 0 \\ 0.5 \\ 0.1 \end{bmatrix} \begin{bmatrix} 0.6 & 0.2 & 0 & 0.5 & 0 & 0.5 & 0 \end{bmatrix}$$

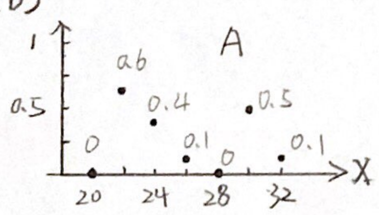
$$= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.6 & 0.2 & 0 & 0.5 & 0 & 0.5 & 0 \\ 0.4 & 0.2 & 0 & 0.4 & 0 & 0.4 & 0 \\ 0.1 & 0.1 & 0 & 0.1 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.5 & 0.2 & 0 & 0.5 & 0 & 0.5 & 0 \\ 0.1 & 0.1 & 0 & 0.1 & 0 & 0.1 & 0 \end{bmatrix}, \quad \mu_{A_1}(x) = \begin{bmatrix} 0 \\ 0 \\ 0.1 \\ 0.5 \\ 0.2 \\ 0.6 \\ 0.1 \end{bmatrix}$$

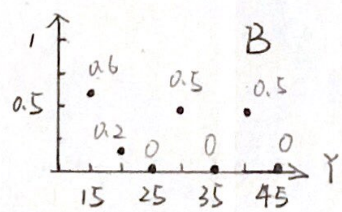
$$\mu_{B_1}(y) = A_1 \circ R$$

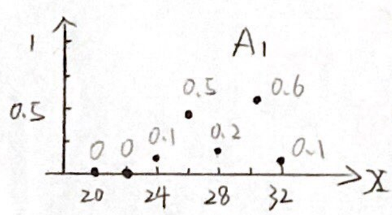
$$\sup \min(\mu_{A_1}(x) \wedge \mu_R(x,y)) = \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.1 & 0.1 & 0 & 0.1 & 0 & 0.1 & 0 \\ 0.1 & 0.1 & 0 & 0.1 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.5 & 0.2 & 0 & 0.5 & 0 & 0.5 & 0 \\ 0.1 & 0.1 & 0 & 0.1 & 0 & 0.1 & 0 \end{bmatrix}}_{\max} \left.\vphantom{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}}\right\} \min$$

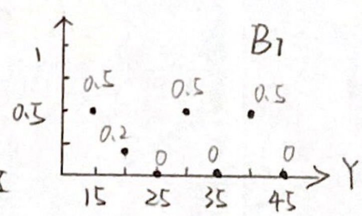
$$= \begin{bmatrix} 0.5 & 0.2 & 0 & 0.5 & 0 & 0.5 & 0 \end{bmatrix}$$

(b)









(c) $C_1 = (A_1 \times B_1) \circ (A \times B \to C)$

$$\mu_{C1} = \vee_{x,y} [\mu_{A1}(x) \wedge \mu_{B1}(y)] \wedge [\mu_A(x) \wedge \mu_B(y) \wedge \mu_C(z)]$$

$$= \vee_{x,y} \{[\mu_{A1}(x) \wedge \mu_{B1}(y) \wedge \mu_A(x) \wedge \mu_B(y)]\} \wedge \mu_C(z)$$

$$= \{\vee_x [\mu_{A1}(x) \wedge \mu_A(x)]\} \wedge \{\vee_y [\mu_{B1}(y) \wedge \mu_B(y)]\} \wedge \mu_C(z)$$

$$= \underbrace{(w_1 \wedge w_2)}_{\text{firing strength}} \wedge \mu_C(z)$$

$C_1 = (A_1 \times B_1) \circ (A \times B \to C)$

$$\mu_{C1} = \vee_{x,y} [\mu_{A1}(x) \wedge \mu_{B1}(y)] \wedge [\mu_A(x) \wedge \mu_B(y) \wedge \mu_C(z)]$$

$$= \vee_{x,y} \{[\mu_{A1}(x) \wedge \mu_A(x) \wedge \mu_{B1}(y) \wedge \mu_B(y)]\} \wedge \mu_C(z)$$

$$= \vee_x [\mu_{A1}(x) \wedge \mu_A(x)] \wedge \vee_y [\mu_{B1}(y) \wedge \mu_B(y)] \wedge \mu_C(z)$$

$$= (w_1 \wedge w_2) \wedge \mu_C(z)$$

5. (a) R1: IF θ is ZP AND $\dot{\theta}$ is ZS THEN θ_c is No Change

R2: IF θ is ZP AND $\dot{\theta}$ is RS THEN θ_c is Turn Left

R3: IF θ is RP AND $\dot{\theta}$ is ZS THEN θ_c is Turn Left

R4: IF θ is RP AND $\dot{\theta}$ is RS THEN θ_c is Turn Left

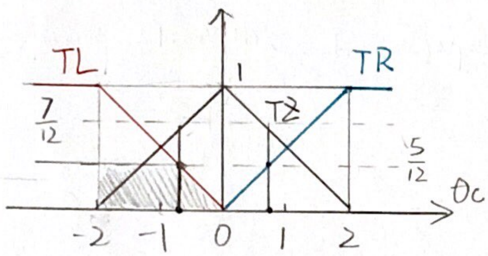
(b)

$$\mu_{ZP}(\theta) = \begin{cases} \frac{1}{4}\theta + 1, & -4 \le \theta \le 0 \\ -\frac{1}{4}\theta + 1, & 0 \le \theta \le 4 \end{cases} \qquad \mu_{ZS}(\dot{\theta}) = \begin{cases} \frac{1}{6}\dot{\theta} + 1, & -6 \le \dot{\theta} \le 0 \\ -\frac{1}{6}\dot{\theta} + 1, & 0 \le \dot{\theta} \le 6 \end{cases}$$

$$\mu_{RP}(\theta) = \begin{cases} \frac{1}{4}\theta, & 0 \le \theta \le 4 \\ 1, & \theta \ge 4 \end{cases} \qquad \mu_{RS}(\dot{\theta}) = \begin{cases} \frac{1}{6}\dot{\theta}, & 0 \le \dot{\theta} \le 6 \\ 1, & \dot{\theta} \ge 6 \end{cases}$$

$\theta_0 = 1.2$, $\dot{\theta}_0 = 2.5 \Rightarrow \mu_{ZP}(\theta_0) = 0.7$, $\mu_{RP} = 0.3$

$\mu_{ZS}(\dot{\theta}_0) = \frac{7}{12}$, $\mu_{RS}(\dot{\theta}_0) = \frac{5}{12}$



$$\mu_{TZ}(\theta_c) = \begin{cases} \frac{1}{2}\theta_c + 1, & -2 \le \theta_c \le 0 \\ -\frac{1}{2}\theta_c + 1, & 0 \le \theta_c \le 2 \end{cases}$$

$$\mu_{TL}(\theta_c) = \begin{cases} -\frac{1}{2}\theta_c, & 0 \le \theta_c \le 2 \\ 1, & \theta_c \ge 2 \end{cases}$$

firing strength: R1: $w_1 = \min(\mu_{ZP}(\theta), \mu_{ZS}(\dot{\theta})) = \frac{7}{12}$, $\quad \frac{1}{2}\theta_c + 1 = \frac{7}{12}$, $\theta_{c1} = -\frac{5}{6}$

$-\frac{1}{2}\theta_c + 1 = \frac{7}{12}$, $\theta_{c2} = \frac{5}{6}$

$\Rightarrow S_1 = \frac{5}{3} \times \frac{7}{12} + \frac{1}{2} \times (\frac{7}{3} \times \frac{7}{12}) = 1.6528$

R2: $w_2 = \min(\mu_{ZP}(\theta), \mu_{RS}(\dot{\theta})) = \frac{5}{12}$, $\quad -\frac{1}{2}\theta_c = \frac{5}{12}$, $\theta_c = -\frac{5}{6}$

$\Rightarrow S_2 = \frac{1}{2} \times \frac{5}{6} \times \frac{5}{12} + \frac{7}{6} \times \frac{5}{12} = 0.6597$

R3: $w_3 = \min(\mu_{RP}(\theta), \mu_{ZS}(\dot{\theta})) = \frac{3}{10}$, $\quad -\frac{1}{2}\theta_c = \frac{3}{10}$, $\theta_c = -\frac{3}{5}$

$\Rightarrow S_3 = \frac{1}{2} \times \frac{3}{5} \times \frac{3}{10} + \frac{7}{5} \times \frac{3}{10} = 0.51$

R4: $w_4 = \min(\mu_{RP}(\theta), \mu_{RS}(\dot{\theta})) = \frac{3}{10}$, $\quad S_4 = 0.51$

$$\Rightarrow \theta_{c\,COG} = \frac{0 \times 1.6528 + (-2) \times 0.6597 + (-2) \times 0.51 + (-2) \times 0.51}{1.6528 + 0.6597 + 0.51 + 0.51}$$

$= -\cancel{1.4403}$

-1.0081

NANYANG TECHNOLOGICAL UNIVERSITY

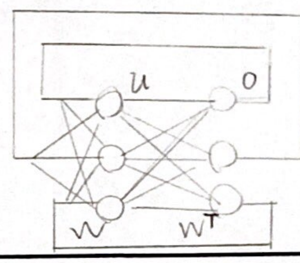
SEMESTER 1 EXAMINATION 2018-2019

EE7207 – NEURAL AND FUZZY SYSTEMS

November/December 2018 Time Allowed: 3 hours

INSTRUCTIONS

1. This paper contains 5 questions and comprises 5 pages.
2. Answer all 5 questions.
3. All questions carry equal marks.
4. This is a closed-book examination.



$o = \varphi(W^T u)$

$u = \varphi(W o)$

1. (a) Design a bi-directional associative memory (BAM) neural network to store the following two vectors:

$$Q_1 = [1 \quad -1 \quad -1]^T$$

$$Q_2 = [-1 \quad 1 \quad 1]^T$$

Sketch the architecture of the BAM neural network designed, compute the weights on the links between neurons, and test whether the designed BAM works.

(6 Marks)

(b) Train a multilayer perceptron (MLP) neural network with one hidden layer to map the following two input vectors to 1 and 0, respectively:

$$Q_3 = [0.8 \quad 0.9 \quad 0.8]^T$$

$$Q_4 = [-0.9 \quad -0.7 \quad -0.95]^T$$

Assume that the logistic sigmoid activation function is used in both the hidden layer and the output layer. Sketch the MLP neural network architecture, and describe the training process and weight updating rules.

(9 Marks)

Note: Question No. 1 continues on page 2

(c) The back propagation (BP) algorithm is a commonly used gradient-based algorithm for MLP neural network training. It is discovered that the gradients of the network's output with respect to the weights in the early layers become extremely small when multiple hidden layers are used. Discuss the cause of the problem, the impacts, and solution to the problem.

(5 Marks)

2. (a) Describe the self-organizing phase training procedure of the self-organizing map (SOM) neural network.

(5 Marks)

(b) Assume that the following samples are used to train an SOM neural network with two neurons:

$w_j(n+1) = w_j(n) + \eta(n) \cdot h_{ji(x)}(n) \cdot [X - w_j(n)]$

$$x_1 = [0.235 \quad -0.081 \quad 0.264]$$

$$x_2 = [1.994 \quad 2.195 \quad 2.083]$$

$$x_3 = [0.169 \quad 0.156 \quad 0.301]$$

$$x_4 = [2.211 \quad 1.909 \quad 2.161]$$

$$x_5 = [-0.006 \quad 0.297 \quad 0.083]$$

$W_1(0) = [0.136 \quad 0.211 \quad 0.216]$

$$x_6 = [2.096 \quad 2.052 \quad 2.101]$$

$d_1 = \| X_1 - W_1(0) \| = \sqrt{(0.099)^2 + (-0.292)^2 + (0.048)^2} = 0.321$

At the end of the self-organizing phase, the weight vectors of the two neurons are obtained as follows:

$$w_1 = [0.136 \quad 0.211 \quad 0.216]$$

$W(1) = W(0) + \eta(0)\, h_{ji(x)}(n) \left(X - W_1(0)\right)$

$$w_2 = [2.16 \quad 1.98 \quad 2.051]$$

If x_1 and x_2 are used in the first and the second iterations respectively, what are the weight vectors of the two neurons after 2 iterations of the convergence phase? Assume that the learning rate is fixed to 0.01, and only the weight vector of the winning neuron is updated at each step of the convergence phase.

(9 Marks)

(c) Discuss the setting of the number of neurons in SOM neural networks.

(6 Marks)

3. Let two fuzzy variables x and y have the sets $X=[-10,5]$ and $Y=[0,10]$ with the membership functions:

$$\mu_X = \begin{cases} 0 & -10 \le x \le -5 \\ \frac{x}{4}+\frac{5}{4} & -5 \le x \le -1 \\ -\frac{x}{6}+\frac{5}{6} & -1 \le x \le 5 \end{cases}$$

$\alpha = \frac{x}{4}+\frac{5}{4}$, $x_1 = 4\alpha - 5$

$\alpha = -\frac{x}{6}+\frac{5}{6}$, $x_2 = -6\alpha + 5$

$$\mu_Y = \begin{cases} 0 & -2 \le y \le -1 \\ \frac{y}{6}+\frac{1}{6} & -1 \le y \le 5 \\ -\frac{y}{7}+\frac{12}{7} & 5 \le x \le 10 \end{cases}$$

$\alpha = \frac{y}{6}+\frac{1}{6}$, $y_1 = 6\alpha - 1$

$\alpha = -\frac{y}{7}+\frac{12}{7}$, $y_2 = -7\alpha + 12$

Use the α cut method to solve the following problems:

$X = [4\alpha - 5, -6\alpha + 5]$

$Y = [6\alpha - 1, -7\alpha + 12]$

(a) Derive and sketch $\mu_Z(z) = \vee_{Z=X+Y} \{\mu_X(x) \wedge \mu_Y(y)\}$.

(7 Marks)

(b) Derive and sketch $\mu_Z(z) = \vee_{Z=X-Y} \{\mu_X(x) \wedge \mu_Y(y)\}$.

(7 Marks)

(c) Derive a mathematical representation of $(Z)_\alpha = (X)_\alpha \cdot (Y)_\alpha$.

(6 Marks)

$Z = X + Y = [10\alpha - 6, -13\alpha + 17]$

$z_1 = 10\alpha - 6$, $\alpha = \frac{z_1}{10} + \frac{6}{10}$

$z_2 = -13\alpha + 17$, $\alpha = -\frac{1}{13} z_2 + \frac{17}{13}$

4. Two discrete variables are given as:

$$X = \{20 \quad 22 \quad 24 \quad 26 \quad 28 \quad 30 \quad 32\}; \; Y = \{15 \quad 20 \quad 25 \quad 30 \quad 35 \quad 40\} \quad 45$$

with the respective fuzzy membership functions:

$$\mu_A(x) = [0 \quad 0.6 \quad 0.4 \quad 0.1 \quad 0 \quad 0.5 \quad 0.1]; \; \mu_B(y) = [0.6 \quad 0.2 \quad 0 \quad 0.5 \quad 0 \quad 0.5 \quad 0].$$

Assume that when the fuzzy reasoning is "if x in A then y is B", then a generalized membership function is given as $\mu_{A1}(x) = [0 \quad 0 \quad 0.1 \quad 0.5 \quad 0.2 \quad 0.6 \quad 0.1]$.

(a) If x is A1, what is membership function of y in B1?

(7 Marks)

(b) Give a sample graphic representation of the fuzzy set relationship among A, A1, B and B1 based on Question 4(a).

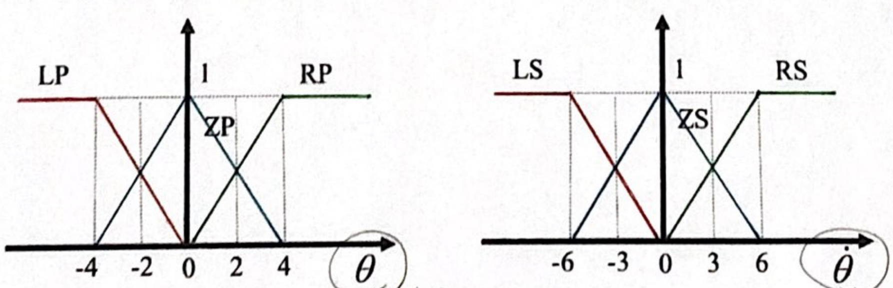
(7 Marks)

(c) Based on Mamdani's fuzzy implication function $C1 = (A1 \times B1) \circ (A \times B \to C)$, show the fuzzy membership function $\mu_{C1}(z)$.

$\mu_{C1}(z) = \vee_{x,y} [\mu_{A1}(x) \wedge \mu_{B1}(y)] \wedge (\mu_A(x) \wedge \mu_B(y)) \wedge \mu_C(z))$

(6 Marks)

5. A fuzzy car control system has two inputs: steering wheel angle θ and steering wheel angular speed $\dot{\theta}$ and one output: car directional angle θ_c. The membership functions of fuzzy sets for both θ and $\dot{\theta}$ are given in Figure 1 and the fuzzy membership functions of output θ_c are given in Figure 2 on page 5.



1.2

2.5

Figure 1

$y = ax + b$

$0 = -4a + b$

$1 = b$

$(0, 1)$

$(4, 0)$

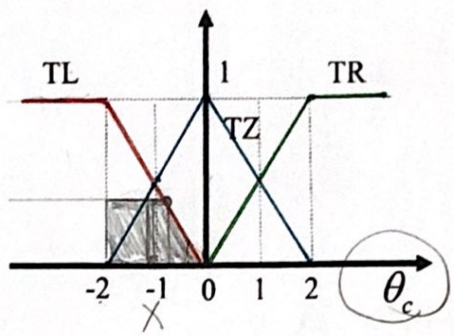
$1 = b$

$0 = 4a + b$

$a = -\frac{1}{4}$

$y = -\frac{1}{4}\theta + 1$

Note: Question No. 5 continues on page 5



0.1736+

Figure 2

(a) Write a set of linguistic rules according to Table 1 in which LP, ZP and RP represent fuzzy inputs of steering angles; SL, ZS and RS represent steering angular speeds.

(5 Marks)

(b) The control inputs $\theta = 1.2$ and $\dot{\theta} = 2.5$ are applied to the linguistic rules derived in Question 5(a). Assuming that we are using the conjunction operator (FUZZY AND) in the antecedents of the rules based on Mamdani's minimum, calculate the corresponding rule firing levels.

Table 1

	LP	ZP	RP
LS	Turn Right	Turn Right	Turn Left
ZS	Turn Right	No Change	Turn Left
RS	Turn Right	Turn Left	Turn Left

(10 Marks)

(c) Use the center of gravity defuzzification method to calculate the car directional angle θ_c with the given crisp inputs in Question 5(b).

(5 Marks)

$S = w(h - \frac{h^2}{2})$

$= 4 \times (\frac{7}{12} - \frac{1}{2} \times (\frac{7}{12})^2)$

$= 1.6528$

$(\frac{10}{6} + 4) \times \frac{7}{12} \times \frac{1}{2}$

END OF PAPER