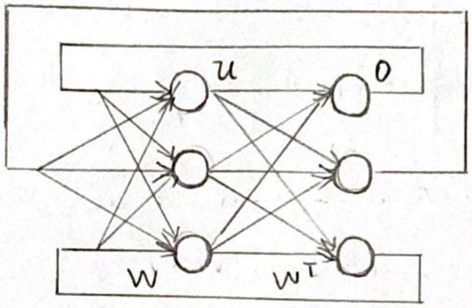
1. (a) $P_1 = [1\ -1\ 1]^T$, $P_2 = [-1\ 1\ -1]^T$

$$W = \sum_{i=1}^{N} x_i y_i^T \Rightarrow W = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} [-1\ 1\ -1] = \begin{bmatrix} -1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & -1 \end{bmatrix}$$



Let input be $x_1 = P_1 = [1\ -1\ 1]^T$:

$$o(1) = \varphi(W^T x) = \varphi\left(\begin{bmatrix} -1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}\right)$$

$$= \varphi([-3\ \ 3\ \ -3]^T) = [-1\ 1\ -1]^T = P_1$$

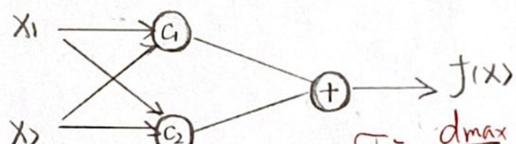
Let input be $x_2 = [1\ 1\ 1]^T \neq P_1 \neq P_2$:

$$o(1) = \varphi\left(\begin{bmatrix} -1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right)$$

$$= \varphi([-1\ \ 1\ \ -1]^T) = [-1\ 1\ -1]^T = P_2$$

(b) $Q_1 = [1\ -1\ -1]^T$; $Q_2 = [1\ 1\ 1]^T$

input	output
$[1\ -1\ -1]^T$	1
$[1\ 1\ 1]^T$	-1



$\sigma = \frac{d_{max}}{\sqrt{2m}}$

$$f(x) = \Phi w = \sum_{j=1}^{m} w_j \cdot \exp\left(-\frac{\|x - c_j\|^2}{2\sigma^2}\right)$$, assume $c_1 = [1\ -1\ -1]^T$, $c_2 = [1\ 1\ 1]^T$, $\sigma = 0.707$

$$\Phi = \begin{bmatrix} 1 & \exp(-8) \\ \exp(-8) & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0.000335 \\ 0.000335 & 1 \end{bmatrix}, \quad d = [1\ -1]^T$$

$$w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = (\Phi^T\Phi)^{-1}\Phi^T d = \left(\begin{bmatrix} 1 & 0.000335 \\ 0.000335 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0.000335 \\ 0.000335 & 1 \end{bmatrix}\right)^{-1}\Phi^T d$$

$$= \begin{bmatrix} 1 & 0.00067 \\ 0.00067 & 1 \end{bmatrix}^{-1}\Phi^T d = \begin{bmatrix} 1 & -0.00067 \\ -0.00067 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0.000335 \\ 0.000335 & 1 \end{bmatrix} d$$

$$= \begin{bmatrix} 1 & -0.000335 \\ -0.000335 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1.000335 \\ -1.000335 \end{bmatrix}$$

if $x = [1\ -1\ -1]^T$: $f(x) = (1.000335 \times \exp(0) + (-1.000335 \times \exp(-8)))$

$$= 1.000335 - 0.000335 = 1$$

(c) The cause of this problem is that the RBF network overfits the training data. To solve this problem we could use a validation set to test the trained network and adjust the weights based on the validation results, and then apply the validated network to the training set.

input	output
$Q_1 = [1\ -1\ -1]^T$	1
$Q_2 = [1\ 1\ 1]^T$	-1

$$f(x) = \Phi w = \sum_{j=1}^{N} w_{ji} \cdot \exp\left(-\frac{\|x - c_j\|^2}{2\sigma^2}\right)$$

$\sigma = 0.707,\ 2\sigma^2 = 1$

assume $c_1 = Q_1 = [1\ -1\ -1]^T,\ c_2 = Q_2 = [1\ 1\ 1]^T$

$$\Phi = \begin{bmatrix} 1 & \exp(-8) \\ \exp(-8) & 1 \end{bmatrix},\quad d = [1\ -1]^T$$

$$W = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = (\Phi^T\Phi)^{-1}\Phi^T d = \begin{bmatrix} 1 & -3.3546\times10^{-4} \\ -3.3\times10^{-4} & 1 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

for $x = [1\ -1\ -1]^T$: $f(x) = \Phi w = \sum_{j=1}^{N} w_{ji} \cdot \exp\left(\frac{-\|x - w_j\|^2}{2\sigma^2}\right)$

$$= 1 \cdot \exp(0) + (-1) \cdot \exp(-8)$$

$$= 1$$

$\sigma = \frac{d_{max}}{\sqrt{2m}} = \frac{\sqrt{8}}{\sqrt{4}} = \sqrt{2}$. $f(x) = \Phi w = \sum_{i=1}^{N} w_{ji} \cdot \exp\left(-\frac{\|x - c_j\|^2}{2\sigma^2}\right)$

$$\Phi = \begin{bmatrix} \exp(0) & \exp(-\frac{8}{4}) \\ \exp(-2) & \exp(0) \end{bmatrix} = \begin{bmatrix} 1 & 0.1353 \\ 0.1353 & 1 \end{bmatrix}$$

$$W = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = (\Phi^T\Phi)^{-1}\Phi^T d = \begin{bmatrix} 1.0186 & -0.1378 \\ -0.1378 & 1.0186 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1.1564 \\ -1.1564 \end{bmatrix}$$

for input $x = [1\ -1\ -1] = Q_1$:

$$f(x) = \Phi w = \sum_{j=1}^{N} w_{ji} \exp\left(-\frac{\|x - c_j\|^2}{\sigma^2}\right) = 1.1564 \times \exp(0) + (-1.1564 \times \exp(-2))$$

$$= 0.9999$$ ~

2. (a) 3 ingredients: ① competition ② cooperation ③ weight adaptation

(b) Step 1: initialization. Randomly select weight vector $w_j(0)$ which satifies $w_j(0)$ are different from each other and have small magnitudes.

Step 2: sampling. Randomly select a input sample x from training samples.

Step 3: competition. Find a winning neuron which satisfies: $i(x) = \arg\min \| x - w_j(n) \|$, $j = 1, 2, \cdots, N$

Step 4: cooperation and adaptation. Adjust the weight of the winning neuron as:

$$w_j(n+1) = w_j(n) + \eta(n) \cdot h_{ji(x)}(n) \cdot [x - w_j(n)]$$

where $h_{ji(x)} = \exp(-\frac{d_{ji}^2}{2\sigma^2(n)})$ is the neighborhood function, $d_{ji} = \| x_j - x_i \|$ is the distance between winning neuron i and activated neuron j. $\sigma(n) = \sigma_0 \cdot \exp(-\frac{n}{\tau_1})$, n is the iteration time, τ_1 is the time constant, σ_0 is the initialization value. learning rate $\eta(n) = \eta_0 \cdot \exp(-\frac{n}{\tau_2})$

Step 5: iteration. Go back to step 2 untill the stopping criterion is satisified:

① reach the pre-defined iteration number

② no noticeable change in the network

(c) SOM can find the prototype of the input patterns

- self-organizing phase: $\eta(n) = 0.1 \exp(-\frac{n}{1000})$, $h_{ji} = \exp(-\frac{d_{ji}^2}{2\sigma^2(n)})$

$\sigma(n) = \sigma_0 \exp(-\frac{n}{\tau_1})$

$\tau_1 = \frac{1000}{\ln \sigma_0}$.

$\sigma_0 =$ radius of two lattice.

- convergence phase:

$\eta(n) = 0.01$

$h_{ji} = \begin{cases} 1, & \text{neuron } j \text{ is the winning neuron} \\ 0, & \text{otherwise} \end{cases}$

3. (a) for the linearly separable 2-class pattern,

the decision boundary is given by:

$$g(x) = w^T x + b$$

where w^T is the weight, $x = x_p + r\frac{w}{\|w\|}$, x_p is the projection of x to the decision boundary, r is the distance, b is the bias.

$\Rightarrow$ $g(x) = w^T x + b = w^T x_p + b + r\frac{w^T w}{\|w\|} = g(x_p) + r\|w\| = r\|w\|$

$\therefore$ $r = \frac{g(x)}{\|w\|}$, to maximize the margin is equal to minimize the weight w.

The distance from support vector $x^{(s)}$ to the hyperplane is:

$$r = \begin{cases} \frac{g(x^{(s)})}{\|w\|} = \frac{1}{\|w\|}, & \text{for } d^{(s)} = +1 \\ \frac{g(x^{(s)})}{\|w\|} = \frac{-1}{\|w\|}, & \text{for } d^{(s)} = -1 \end{cases}$$

$\Rightarrow$ The optimization can be formulated as: given samples $\{x(i), d(i)\}$, $i = 1, 2, \cdots, N$, find the optimal w and b such that:

$$\min. \quad J = \frac{1}{2} w^T w$$
$$s.t. \quad d(i) \times [w^T x(i) + b] \geq 1$$

By using Lagrange multiplier theory, we have:

$$J(w, b, \alpha) = \frac{1}{2} w^T w - \sum_{i=1}^{N} \alpha(i)[d(i) \times [w^T x(i) + b] - 1]$$

where $\alpha(i)$ is the Lagrange multiplier and $\alpha(i) \geq 0$.

$$\frac{\partial J(w, b, \alpha)}{\partial w} = w - \sum_{i=1}^{N} \alpha(i) d(i) x(i) = 0$$

$$\frac{\partial J(w, b, \alpha)}{\partial b} = -\sum_{i=1}^{N} \alpha(i) d(i) = 0$$

$$\Rightarrow \begin{cases} w = \sum_{i=1}^{N} \alpha(i) d(i) x(i) \\ \sum_{i=1}^{N} \alpha(i) d(i) = 0 \end{cases}$$

With KKT conditions we know that: $\alpha(i)[d(i)\times[w^T x(i)+b]-1]=0$
for non-zero $\alpha(i)$, $(d(i)\times[w^T x(i)+b]-1)$ must be 0.

$\Rightarrow$ Rewrite $J(w,b,\alpha)$ as follows:

$$J(w,b,\alpha)=\frac{1}{2}w^T w-\sum_{i=1}^{N}\alpha(i)d(i)w^T x(i)-\sum_{i=1}^{N}\alpha(i)d(i)b+\sum_{i=1}^{N}\alpha(i)$$

with $w=\sum_{i=1}^{N}\alpha(i)d(i)x(i)$, we have

$$w^T w=\sum_{i=1}^{N}\alpha(i)d(i)x(i)\sum_{j=1}^{N}\alpha(j)d(j)x(j)=\sum_{i=1}^{N}\sum_{j=1}^{N}\alpha(i)\alpha(j)d(i)d(j)x^T(i)x(j)$$

$$\Rightarrow J(w,b,\alpha)=\sum_{i=1}^{N}\alpha(i)-\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\alpha(i)\alpha(j)d(i)d(j)x^T(i)x(j)=Q(\alpha)$$

where $Q(\alpha)$ is the dual problem of the above optimization problem and can be solved when $\alpha(i)$ is found by QP solvers.

$$\Rightarrow \quad \begin{aligned}\min.\ & Q(\alpha)\\ \text{s.t.}\ & \sum_{i=1}^{N}\alpha(i)d(i)=0\\ & \alpha(i)\geq 0\end{aligned} \qquad \therefore\ \begin{aligned} w^* &= \sum_{i=1}^{N}\alpha(i)d(i)x(i)\\ b^* &= 1-w^* x^{(s)}(i)\end{aligned}$$

(b) Simillar with (a), for linearly non-seperable 2-class patterns, we can formulate the optimization problem as:

$$\begin{aligned}\min.\ & J=\frac{1}{2}w^T w+C\sum_{i=1}^{N}\xi(i)\\ \text{s.t.}\ & d(i)\times[w^T x(i)+b]\geq 1-\xi(i)\end{aligned}$$

where $\xi(i)$ denotes the slack variable.

$\Rightarrow$ dual problem:

$$\begin{aligned}\min.\ & Q(\alpha)=\sum_{i=1}^{N}\alpha(i)-\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\alpha(i)\alpha(j)d(i)d(j)x^T(i)x(j)\\ \text{s.t.}\ & \sum_{i=1}^{N}\alpha(i)d(i)=0\\ & 0\leq\alpha(i)\leq C\end{aligned}$$

$$\therefore\ w^*=\sum_{i=1}^{N}\alpha(i)d(i)x(i),\quad b^*=1-w^* x^{(s)}(i)$$

(c) We can use the one-v.s.-all method to solve multi-class classification problem.

4. (a)

$$\mu_{X_1}(x) = \begin{cases} 1, & 0 \le x \le 0.3 \\ -\frac{10}{3}x + 2, & 0.3 \le x \le 0.6 \\ 0, & x \ge 0.6 \end{cases}$$

$$\mu_{X_2}(x) = \begin{cases} 0, & 0 \le x \le 0.4 \\ \frac{10}{3}x - \frac{4}{3}, & 0.4 \le x \le 0.7 \\ 1, & x \ge 0.7 \end{cases}$$

(b)

5. (a)

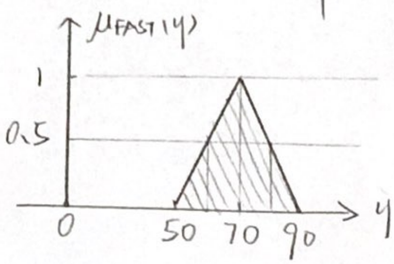
$$B = A \circ R = \begin{bmatrix} 0.1 \\ 0.8 \\ 0.4 \end{bmatrix} \begin{bmatrix} 0.7 & 0.3 & 0.1 \\ 0.4 & 0.8 & 0.2 \\ 0.1 & 0.2 & 0.9 \end{bmatrix} = \underbrace{\left.\begin{bmatrix} 0.1 & 0.1 & 0.1 \\ 0.4 & 0.8 & 0.2 \\ 0.1 & 0.2 & 0.4 \end{bmatrix}\right\} \text{min}}_{\text{max}}$$

$$= [\, 0.4 \quad 0.8 \quad 0.4 \,]$$

(b) $x_0 = 22.5$, $\mu_{WARM}(x_0) = 5.5 - 0.2 \times 22.5 = 1$

Rule 4: IF TEMP is WARM THEN SPEED is FAST

$$\Rightarrow \mu_{FAST}(y) = \begin{cases} 0.05y - 2.5, & 50 \le y < 70 \\ 4.5 - 0.05y, & 70 \le y < 90 \\ 0, & \text{otherwise} \end{cases}$$



$$\text{speed}_{CoG} = \frac{(90-50) \times \frac{1}{2} \times 1 \times (70)}{\frac{1}{2} \times (90-50) \times 1}$$

$$= \frac{20 \times 70}{20} = 70$$

(c) $x_1 = 20$, $\mu_{WARM}(x_1) = 0.2 \times 20 - 3.5 = 0.5$.

$$\text{speed}_{CoG} = \frac{15 \times 70}{15} = 70$$

$0.05y - 2.5 = 0.5$

$y = 60$

$4.5 - 0.05y = 0.5$

$y = 80$

$a_1 = 20 \times 0.5 + \frac{1}{2} \times 20 \times 0.5$

$= 10 + 5 = 15$

$40 \times (0.5 - \frac{0.5^2}{2})$

$= 1.5$

$\frac{1}{2} \times 20 \times 0.5 = 5$

$+ \; 20 \times 0.5 = 10$

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2016-2017

EE7207 – NEURAL AND FUZZY SYSTEMS

November/December 2016 Time Allowed: 3 hours

INSTRUCTIONS

1. This paper contains 5 questions and comprises 5 pages.
2. Answer all 5 questions.
3. All questions carry equal marks.
4. This is a closed-book examination.

$$W=\sum_{i=1}^{N} x_i y_i^T = \begin{bmatrix}1\\-1\\1\end{bmatrix}[-1\ \ 1\ \ -1] = \begin{bmatrix}-1&1&-1\\1&-1&1\\-1&1&-1\end{bmatrix}$$

let input be $x = P_1 = [1\ \ -1\ \ 1]^T$

$$O(1)=\varphi(W^T x)=\varphi\left(\begin{bmatrix}-1&1&-1\\1&-1&1\\-1&1&-1\end{bmatrix}\begin{bmatrix}1\\-1\\1\end{bmatrix}\right)=[-1\ \ 1\ \ -1]^T = P_2$$

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1. (a) There are two vectors:

$$P_1 = [1 \quad -1 \quad 1]^T \text{ and } P_2 = [-1 \quad 1 \quad -1]^T.$$

Design a bi-directional associative memory (BAM) neural network to store P_1 and P_2. Sketch the architecture of the BAM neural network designed, compute the weights on the links between neurons, and test whether the designed BAM works.

$$f(x)=\Phi w=\sum_{j=1}^{N} w_{ji}\exp\left(-\frac{\|x-c_j\|^2}{2\sigma^2}\right)$$

(8 Marks)

(b) There are two vectors:

$$f(x)=\Phi w=\sum_{j=1}^{N} w_{ji}\left(\|x-c_j\|+\sigma^2\right)^{\frac{1}{2}}$$

$$Q_1 = [1 \quad -1 \quad -1]^T \text{ and } Q_2 = [1 \quad 1 \quad 1]^T.$$

Design a Gaussian radial basis function (RBF) neural network to map Q_1 and Q_2 to 1 and -1, respectively. Sketch the RBF neural network architecture, determine centre vectors of hidden layer neurons and the weights on the links between neurons, and test whether the designed RBF neural network works.

(8 Marks)

$$f(x)=\Phi w=\sum_{j=1}^{m} w_{ji}\cdot\exp\left(-\frac{\|x-c_j\|^2}{2\sigma^2}\right)$$

Note: Question No. 1 continues on page 2

(c) In an application, it was found that the trained RBF neural network fitted the training data perfectly, but generalized badly on the unseen testing data. Discuss the causes of the above problem and ways to deal with the problem.

(4 Marks)

2. Self-organizing map (SOM) neural network is a powerful tool in data analysis.

(a) Describe the 3 ingredients of the SOM neural network learning.

competition cooperation adaptation (6 Marks)

(b) Describe SOM neural network learning procedure, and discuss parameter settings in different learning phases of the SOM neural network.

(9 Marks)

(c) Discuss the potentials of SOM neural networks in data visualization.

(5 Marks)

3. Support vector machine (SVM) is widely used in pattern classification.

(a) Describe the optimal separating hyperplane of linear SVM for linearly separable 2-class patterns, and formulate the solution of the optimal hyperplane as an optimization problem.

(7 Marks)

(b) Describe the optimal separating hyperplane of linear SVM for linearly non-separable 2-class patterns, and formulate the solution of the optimal hyperplane as an optimization problem.

(7 Marks)

(c) SVM is inherently formulated for solving 2-class pattern classification problems. Discuss how to extend SVM to solve multi-class pattern classification problems.

(6 Marks)

$\mu_{X_1}(x) = \{ -\frac{10}{3}x_1 + 2$

4. (a) Consider the membership functions $\mu_{X_1}(x)$ and $\mu_{X_2}(x)$ in Figure 1, where $X_1 = [0, \infty)$ and $X_2 = [0, \infty)$.

$1 = 0.3a + b$ $\quad 0.3a = -1$

$0 = 0.6a + b$ $\quad a = -\frac{10}{3}$

$b = 2$

(0.3,1)

(0.6,0)

(0.3,1)

(0.6,0)

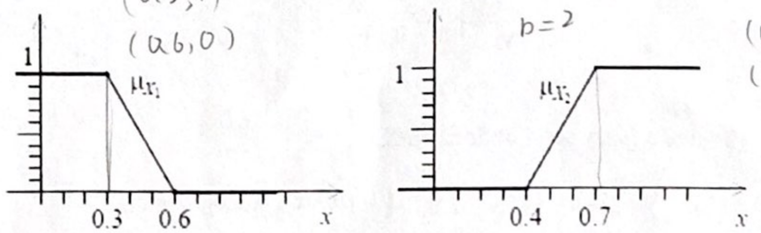
$y = ax + b$

$1 = 0.3a + b$

$0 = 0.6a + b$

$0.3a = -1$

$a = -\frac{10}{3}$ $\quad b = 2$



(0.4, 0)

(0.7, 1)

$y = ax + b$

$0 = 0.4a + b$

$1 = 0.7a + b$

$0.3a = 1$

$a = \frac{10}{3}$

$b = -\frac{4}{3}$

Figure 1

Determine their analytical expressions.

(4 Marks)

(b) Suppose that an unknown system with input x and output y is modelled by the following fuzzy rules:

R1: IF x is X_1 THEN $y = a_{11} + a_{12}x$
R2: IF x is X_2 THEN $y = a_{21} + a_{22}x$

where X_1 and X_2 are fuzzy sets with membership functions $\mu_{X_1}(x)$ and $\mu_{X_2}(x)$ in Figure 1, a_{11}, a_{12}, a_{21} and a_{22} are unknown parameters.

Some experiments are carried out on the unknown system and N pairs of input output data in Table 1 are obtained.

Table 1 Input-Output Data

Input x	x_1	x_2	……	x_N
Output y	y_1	y_2		y_N

Based on the data, a system of linear equations is obtained as follows:

$$A\theta = b.$$

What is θ? Determine A and b using the given data.

(10 Marks)

(c) Suppose that the optimal estimate of θ in part 4(b) is θ^* based on the least-squares parameter identification scheme. Find θ^* in terms of A and b.

(6 Marks)

5. (a) Consider the fuzzy relation $R: X \times Y \rightarrow [0,1]$.

$$R = \begin{array}{c|ccc} & y_1 & y_2 & y_3 \\ \hline x_1 & 0.7 & 0.3 & 0.1 \\ x_2 & 0.4 & 0.8 & 0.2 \\ x_3 & 0.1 & 0.2 & 0.9 \end{array}$$

and a fuzzy set A defined on X:

$$A = \{0.1/x_1,\ 0.8/x_2,\ 0.4/x_3\}.$$

Compute fuzzy set $B = A \text{ o } R$, where o is the sup-min composition operator.

(6 Marks)

(b) Consider an air-conditioner with five control switches: ***COLD, COOL, PLEASANT, WARM*** and ***HOT*** with input temperature x in degrees Celsius (°C). The corresponding speeds of the motor controlling the fan speed y revolutions per minute (rpm) on the air-conditioner are ***MINIMUM, SLOW, MEDIUM, FAST*** and ***BLAST.***

The rules governing the air-conditioner are as follows:

- RULE 1:

IF TEMP	is	COLD
THEN SPEED	is	MINIMUM

- RULE 2:

IF TEMP	is	COOL
THEN SPEED	is	SLOW

- RULE 3:

IF TEMP	is	PLEASANT
THEN SPEED	is	MEDIUM

- RULE 4:

IF TEMP	is	WARM
THEN SPEED	is	FAST

- RULE 5:

IF TEMP	is	HOT
THEN SPEED	is	BLAST

Note: Question No. 5 continues on page 5

The membership functions for the fuzzy sets are respectively given as follows:

$\mu_{COLD}(x) = \begin{cases} -0.1x + 1 & 0 \le x \le 10 \\ 0 & otherwise \end{cases}$	$\mu_{COOL}(x) = \begin{cases} 0.08x & 0 \le x < 12.5 \\ 3.5 - 0.2x & 12.5 \le x < 17.5 \\ 0 & otherwise \end{cases}$
$\mu_{PLESANT}(x) = \begin{cases} 0.4x - 6 & 15 \le x < 17.5 \\ 8 - 0.4x & 17.5 \le x < 20 \\ 0 & otherwise \end{cases}$	$\mu_{WARM}(x) = \begin{cases} 0.2x - 3.5 & 17.5 \le x < 22.5 \\ 5.5 - 0.2x & 22.5 \le x < 27.5 \\ 0 & otherwise \end{cases}$
$\mu_{HOT}(x) = \begin{cases} 0.2x - 5 & 25 \le x < 30 \\ 1 & 30 \le x \\ 0 & otherwise \end{cases}$	$\mu_{MINIMUM}(y) = \begin{cases} -\frac{1}{30}y + 1 & 0 \le y \le 30 \\ 0 & otherwise \end{cases}$
$\mu_{SLOW}(y) = \begin{cases} 0.05y - 0.5 & 10 \le y < 30 \\ 2.5 - 0.05y & 30 \le y < 50 \\ 0 & otherwise \end{cases}$	$\mu_{MEDIUM}(y) = \begin{cases} 0.1y - 4 & 40 \le y < 50 \\ 6 - 0.1y & 50 \le y < 60 \\ 0 & otherwise \end{cases}$
$\mu_{FAST}(y) = \begin{cases} 0.05y - 2.5 & 50 \le y < 70 \\ 4.5 - 0.05y & 70 \le y < 90 \\ 0 & otherwise \end{cases}$	$\mu_{BLAST}(y) = \begin{cases} \frac{1}{30}y - \frac{7}{3} & 70 \le y \le 100 \\ 0 & otherwise \end{cases}$

For a temperature of 22.5°C, determine the fan speed using the centre of gravity (COG) method.

(7 Marks)

(c) If the temperature is 20°C in part 5(b), what is the fan speed determined by using the mean-of-maxima (MOM) method?

(7 Marks)

END OF PAPER