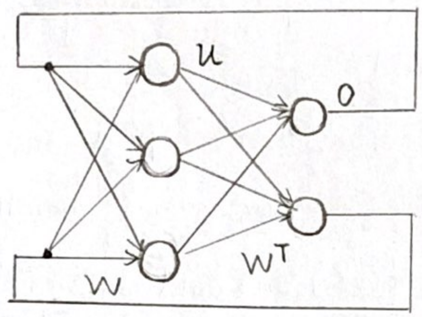
7. (a) $P_1 = [1 \ -1 \ 1]^T$, $Q_1 = [1 \ 1]^T$

$P_2 = [-1 \ 1 \ -1]^T$, $Q_2 = [-1 \ -1]^T$

$$W = \sum_{i=1}^{N} x_i y_i^T \Rightarrow W_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} [1 \ \ 1] = \begin{bmatrix} 1 & 1 \\ -1 & -1 \\ 1 & 1 \end{bmatrix}$$

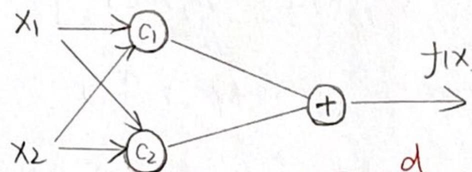
$$W_2 = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix} [-1 \ -1] = \begin{bmatrix} 1 & 1 \\ -1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$W = W_1 + W_2 = \begin{bmatrix} 2 & 2 \\ -2 & -2 \\ 2 & 2 \end{bmatrix}$$



(b)

input	output
$P_1 = [1 \ -1 \ 1]^T$	$Q_1 = [1 \ 1]^T$
$P_2 = [-1 \ 1 \ -1]^T$	$Q_2 = [-1 \ -1]^T$



$$f(x) = \Phi w = \sum_{j=1}^{m} w_j \cdot \exp\left(-\frac{\|x - c_j\|^2}{2\sigma^2}\right)$$

$\sigma = \frac{d}{\sqrt{2m}} = \frac{\sqrt{8}}{\sqrt{4}} = \sqrt{2}$

assume center vectors $c_1 = [1 \ -1 \ 1]^T$, $c_2 = [-1 \ 1 \ -1]^T$, $\underbrace{\sigma = 0.707}$,

$$\Phi = \begin{bmatrix} \exp(0) & \exp(-12) \\ \exp(-12) & \exp(0) \end{bmatrix} = \begin{bmatrix} 1 & 6.14 \times 10^{-6} \\ 6.14 \times 10^{-6} & 1 \end{bmatrix} \qquad d = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}.$$

$$w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = (\Phi^T \Phi)^{-1} \Phi^T d = \left(\begin{bmatrix} 1 & 12.28 \times 10^{-6} \\ 12.28 \times 10^{-6} & 1 \end{bmatrix} \right)^{-1} \Phi^T d$$

$$= \begin{bmatrix} 1 & -12.28 \times 10^{-6} \\ -12.28 \times 10^{-6} & 1 \end{bmatrix} \begin{bmatrix} 1 & 6.14 \times 10^{-6} \\ 6.14 \times 10^{-6} & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -6.14 \times 10^{-6} \\ -6.14 \times 10^{-6} & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$$

if $x = [1 \ -1 \ 1]^T$: $f(x) = (w_1 \cdot \exp(0) + w_2 \cdot \exp(-12))$

$$= [1 \ \ 1]^T + [-1 \ -1]^T \times 6.14 \times 10^{-6} = [1 \ \ 1]^T = Q_1$$

2. (a)

(b) ① initialization: randomly select weight $w_j(n)$ which $w_{ji}(n)$ are different from each other and have small magnitude. e.g., $0 < w_{ji}(n) < 1$.

② sampling: randomly select input sample X from training set.

③ competition: find the winning neuron $i(X)$ which satisities: $i(X) = \arg\min \| X - w_j(n) \|$.

④ cooperation and weight adaptation: update the weight of the winning neuron by:

$$w_j(n+1) = w_j(n) + \eta(n)\, h_{ji(X)}(n)\, (X - w_j(n))$$

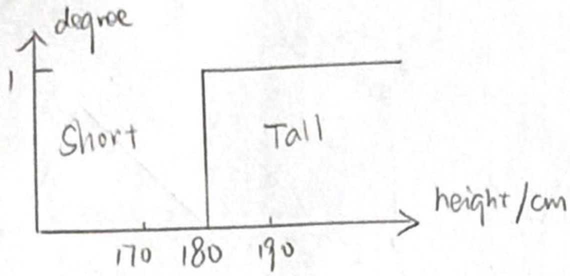
$$\eta(n) = \eta_0 \cdot \exp(-\frac{n}{T_1}), \quad h_{ji(X)}(n) = \exp(-\frac{d_{ji}}{2\sigma^2}), \quad d_{ji} = \| r_j - r_i \|$$

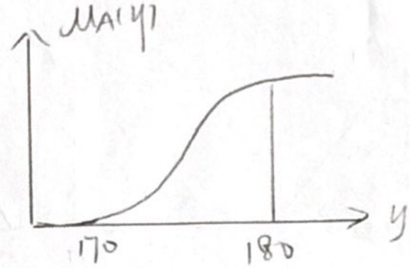
the weights of other neurons remain unchanged.

⑤ iteration: go back to ② until the stop criterion is satisitied: (i) reached pre-defined iteration number.
(ii) no noticeable change in the network.

(c)

3. (a)

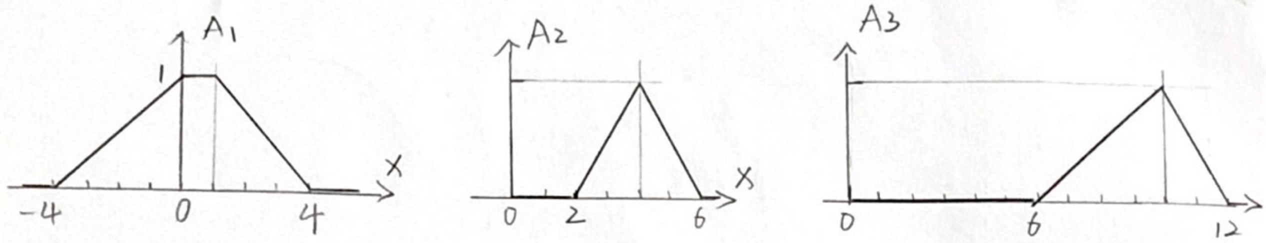


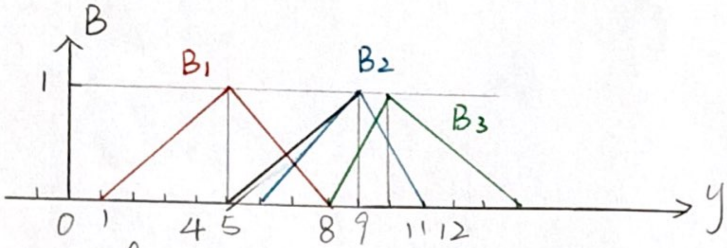


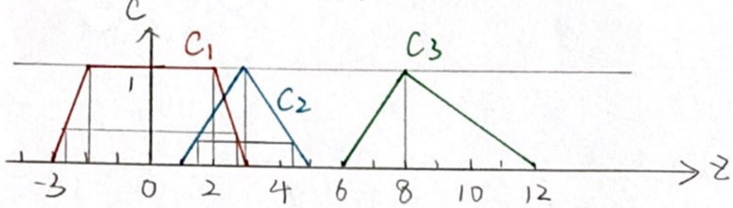
$$\mu_A(x) = \begin{cases} 1, & x \geq 180 \\ 0, & x < 180 \end{cases}$$

$$\mu_A(y) = \begin{cases} 1, & y \geq 180 \\ (0,1) & 170 < y < 180 \\ 0, & y \leq 170 \end{cases}$$

4. (a)







(b) $x_0 = 3$, $y_0 = 6$

$\mu_{A1}(x_0) = \frac{1}{3}$, $\mu_{A2}(x_0) = \frac{1}{2}$, $\mu_{A3}(x_0) = 0$

$\mu_{B1}(y_0) = \frac{2}{3}$, $\mu_{B2}(y_0) = \frac{1}{4}$, $\mu_{B3}(y_0) = 0$

firing strength: $w_1 = \min(\mu_{A1}(x_0), \mu_{B1}(y_0)) = \frac{1}{3}$

$w_2 = \min(\mu_{A2}(x_0), \mu_{B2}(y_0)) = \frac{1}{4}$

$w_3 = 0$

$z+3 = \frac{1}{3} \Rightarrow z_1 = -\frac{8}{3}$ $\quad a_1 = (6-\frac{2}{3}) \times \frac{1}{3} + \frac{1}{2} \times \frac{2}{3} \times \frac{1}{3} = \frac{17}{9}$

$-z+3 = \frac{1}{3} \Rightarrow z_2 = \frac{8}{3}$

$\frac{z-1}{2} = \frac{1}{4} \Rightarrow z_1 = \frac{3}{2}$ $\quad a_2 = (4-1) \times \frac{1}{4} + \frac{1}{2} \times 1 \times \frac{1}{4} = \frac{7}{8}$

$\frac{-z+5}{2} = \frac{1}{4} \Rightarrow z_2 = \frac{9}{2}$

(c) $$C_{COG} = \frac{0 \times \frac{17}{9} + 3 \times \frac{7}{8} + 0}{\frac{17}{9} + \frac{7}{8} + 0} = 0.9497$$

5 (a)

$$\mu_A(x) = \begin{cases} 0, & x \le -5 \\ \frac{x+5}{5}, & -5 < x \le 0 \\ \frac{-x+5}{5}, & 0 < x \le 5 \\ 0, & x > 5 \end{cases}$$

$\alpha = \frac{x+5}{5} \Rightarrow x_1 = 5\alpha - 5$

$\alpha = \frac{-x+5}{5} \Rightarrow x_2 = -5\alpha + 5$

$\Rightarrow (A)_\alpha = [5\alpha - 5, -5\alpha + 5]$, $A = [-5, 5]$

$$\mu_B(x) = \begin{cases} 0, & x \le 1 \\ \frac{x-1}{5}, & 1 < x \le 6 \\ \frac{-x+8}{2}, & 6 < x \le 8 \\ 0, & x > 8 \end{cases}$$

$\alpha = \frac{x-1}{5} \Rightarrow x_1 = 5\alpha + 1$

$\alpha = \frac{-x+8}{2} \Rightarrow x_2 = -2\alpha + 8$

$\Rightarrow (B)_\alpha = [5\alpha + 1, -2\alpha + 8]$, $B = [1, 8]$

for A+B: $z_1 = A + B = [-5, 5] + [1, 8] = [-4, 13]$

$(z_1)_\alpha = (A)_\alpha + (B)_\alpha = [10\alpha - 4, -7\alpha + 13]$

$10\alpha - 4 = z_1 \Rightarrow \alpha = \frac{z}{10} + \frac{4}{10}$, $-7\alpha + 13 = z_2 \Rightarrow \alpha = -\frac{z}{7} + \frac{13}{7}$

$$\Rightarrow \mu_{z_1 = A+B}(z) = \begin{cases} \frac{z}{10} + \frac{4}{10}, & -4 \le z \le 6 \\ -\frac{z}{7} + \frac{13}{7}, & 6 \le z \le 13 \end{cases}$$

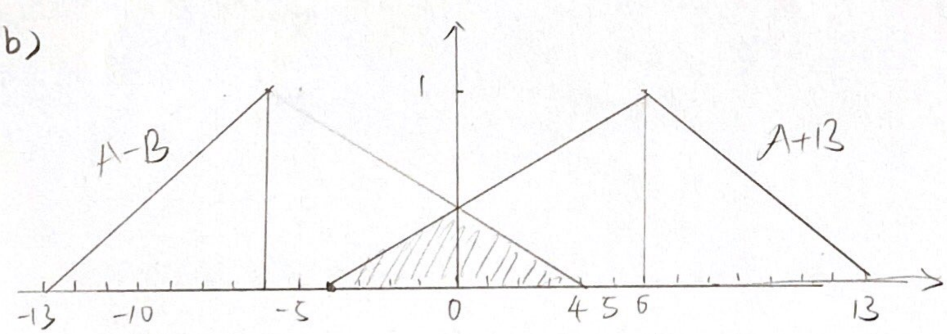
for A-B: $z_2 = A - B = [-5, 5] - [1, 8] = [-13, 4]$

$(z_2)_\alpha = (A)_\alpha - (B)_\alpha = [7\alpha - 13, -10\alpha + 4]$

$7\alpha - 13 = z_1 \Rightarrow \alpha = \frac{z}{7} + \frac{13}{7}$, $-10\alpha + 4 = z_2 \Rightarrow \alpha = -\frac{z}{10} + \frac{4}{10}$

$$\Rightarrow \mu_{z_2 = A-B}(z) = \begin{cases} \frac{z}{7} + \frac{13}{7}, & -13 \le z \le -6 \\ -\frac{z}{10} + \frac{4}{10}, & -6 \le z \le 4 \end{cases}$$

(b)



NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2015-2016

EE7207 - NEURAL AND FUZZY SYSTEMS

November/December 2015 Time Allowed: 3 hours

INSTRUCTIONS

1. This paper contains 5 questions and comprises 4 pages.
2. Answer all 5 questions.
3. All questions carry equal marks.
4. This is a closed-book examination.

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1. There are four vectors:

$$P_1 = [1 \quad -1 \quad 1]^T$$

$$P_2 = [-1 \quad 1 \quad -1]^T$$

$$Q_1 = [1 \quad 1]^T$$

$$Q_2 = [-1 \quad -1]^T$$

(a) Design a Bi-directional Associative Memory (BAM) neural network to map P_1 and P_2 to Q_1 and Q_2 respectively: sketch the architecture of the BAM neural network designed, and compute the weights on the links between neurons.

(6 Marks)

(b) Design a Gaussian radial basis function (RBF) neural network to map P_1 and P_2 to Q_1 and Q_2, respectively. Sketch the RBF neural network architecture, select centre vectors of hidden layer neurons and determine the weights on the links between neurons.

(10 Marks)

Note: Question No. 1 continues on page 2

(c) Discuss the mechanisms of vector mapping of the BAM and RBF neural networks.

(4 Marks)

2. In a data analytic task, a Self-organizing Map (SOM) neural network is used to select 100 representative samples from a total of *N* samples.

(a) Explain the rationale of using the SOM neural network for representative sample selection.

(4 Marks)

(b) Describe how you would design and train the SOM neural network for the sample selection task.

(10 Marks)

(c) Discuss how you would use SOM neural networks to reduce dimensionality of data.

(6 Marks)

3. (a) Use an example to illustrate the differences between the membership function of a classic crisp set and of a fuzzy set.

(4 Marks)

(b) A t-conorm *S* is a binary operation on the unit interval that satisfies the following axioms for all $a, b, c \in [0,1]$:

(i) $S(a,0)= a$;

(ii) $b \leq c$ implies that $S(a,b) \leq S(a,c)$;

(iii) $S(a,b)= S(b,a)$;

(iv) $S(a, S(b,c))=S(S(a,b),c)$

Show that the Łukasiewicz union $S(a, b) = \min(1, a + b)$ is a t-conorm.

(10 Marks)

(c) Consider the following fuzzy sets defined on $R^+ = [0,+\infty)$:

$$\mu_A(x) = \frac{1}{1+x} \text{ and } \mu_B(x) = \begin{cases} -x+1 & 0 \leq x \leq 1 \\ 0 & x > 1 \end{cases}$$

Determine the membership function of C=A∨B using the Łukasiewicz union operation.

$\mu_{A \vee B}(x) = max(\mu_A(x)+\mu_B(x), 1)$ (6 Marks)

$\frac{1}{1+x} - x+1 = 1$

$x = \frac{1}{1+x}$

$x+x^2=1$

4. A fuzzy system has two inputs x and y and one output z with the following three rules:

Rule 1: IF X is A_1 and Y is B_1 THEN Z is C_1
Rule 2: IF X is A_2 and Y is B_2 THEN Z is C_2
Rule 1: IF X is A_3 and Y is B_3 THEN Z is C_3

The respective membership functions are given as follows:

$$\mu_{A_1}(x) = \begin{cases} 0, & x \le -4 \\ \frac{x+4}{4} & -4 < x \le 0 \\ 1 & 0 < x \le 1 \\ \frac{-x+4}{3} & 1 < x \le 4 \\ 0 & x > 4 \end{cases}, \mu_{A_2}(x) = \begin{cases} 0 & x \le 2 \\ \frac{x-2}{2} & 2 < x \le 4 \\ \frac{-x+6}{2} & 4 < x \le 6 \\ 0 & x > 6 \end{cases}, \mu_{A_3}(x) = \begin{cases} 0 & x \le 6 \\ \frac{x-6}{4} & 6 < x \le 10 \\ \frac{-x+12}{2} & 10 < x \le 12 \\ 0 & x > 12 \end{cases},$$

$$\mu_{B_1}(y) = \begin{cases} 0 & y \le 1 \\ \frac{y-1}{4} & 1 < y \le 5 \\ \frac{-y+8}{3} & 5 < y \le 8 \\ 0 & y > 8 \end{cases}, \mu_{B_2}(y) = \begin{cases} 0 & y \le 5 \\ \frac{y-5}{4} & 5 < y \le 9 \\ \frac{-y+11}{2} & 9 < y \le 11 \\ 0 & y > 11 \end{cases}, \mu_{B_3}(y) = \begin{cases} 0 & y \le 8 \\ \frac{y-8}{2} & 8 < y \le 10 \\ \frac{-y+14}{4} & 10 < y \le 14 \\ 0 & y > 14 \end{cases},$$

$$\mu_{C_1}(z) = \begin{cases} 0, & z \le -3 \\ z+3 & -3 < z \le -2 \\ 1 & -2 < z \le 2 \\ -z+3 & 2 < z \le 3 \\ 0 & z > 3 \end{cases}, \mu_{C_2}(z) = \begin{cases} 0 & z \le 1 \\ \frac{z-1}{2} & 1 < z \le 3 \\ \frac{-z+5}{2} & 3 < z \le 5 \\ 0 & z > 5 \end{cases}, \mu_{C_3}(z) = \begin{cases} 0 & z \le 6 \\ \frac{z-6}{2} & 6 < z \le 8 \\ \frac{-z+12}{4} & 8 < z \le 12 \\ 0 & z > 12 \end{cases},$$

Suppose that there is a pair of inputs $x_0 = 3$ and $y_0 = 6$.

(a) Sketch all the membership functions.

(6 Marks)

(b) Use the max-min composition rule of inference to determine the aggregated fuzzy output under the given inputs.

(10 Marks)

(c) Determine the crisp output by using Centre of Average (COA) and Mean-of-Maxima (MOM) defuzzification methods, respectively.

(4 Marks)

5. Consider the two fuzzy numbers A and B defined as follows:

$$\mu_A(x) = \begin{cases} 0, & x \le -5 \\ \dfrac{x+5}{5} & -5 < x \le 0 \\ \dfrac{-x+5}{5} & 0 < x \le 5 \\ 0 & x > 5 \end{cases}$$

$\alpha = \frac{x+5}{5}$, $x_1 = 5\alpha - 5$

$\alpha = \frac{-x+5}{5}$, $x_2 = -5\alpha + 5$

$(X_A)_\alpha = [5\alpha - 5, -5\alpha + 5]$

$$\mu_B(x) = \begin{cases} 0, & x \le 1 \\ \dfrac{x-1}{5} & 1 < x \le 6 \\ \dfrac{-x+8}{2} & 6 < x \le 8 \\ 0 & x > 8 \end{cases}$$

$\alpha = \frac{x-1}{5}$, $x_1 = 5\alpha + 1$

$\alpha = \frac{-x+8}{2}$, $x_2 = -2\alpha + 8$

$(X_B)_\alpha = [5\alpha + 1, -2\alpha + 8]$

(a) Calculate $A+B$ and $A-B$.

(12 Marks)

(b) Find the intersection between the fuzzy set with membership function $A+B$ and the fuzzy set with membership function $A-B$ using the standard intersection $T(a, b) = \min(a, b)$. Determine whether the intersection is convex by sketching the membership function of the intersection.

(8 Marks)

$(A-B)_\alpha = [7\alpha - 13, -10\alpha + 4]$

$z_1 = 7\alpha - 13$, $\alpha = \frac{z_1}{7} + \frac{13}{7}$

$z_2 = -10\alpha + 4$, $\alpha = \frac{z_1}{10} + \frac{4}{10}$

END OF PAPER