

Linear Regression:-

1) $X = [2, 3, 4, 5, 6]$

$Y = [12.8978, 17.7586, 23.3192, 28.3129, 32.1351]$

<u>X</u>	<u>Y</u>	<u>X²</u>	<u>XY</u>
2	12.8978	4	25.7956
3	17.7586	9	53.2758
4	23.3192	16	93.2768
5	28.3129	25	141.5645
6	32.1351	36	192.8106
$\Sigma X = 20$	$\Sigma Y = 114.4236$	$\Sigma X^2 = 90$	$\Sigma XY = 506.7233$
$\bar{X} = 4$	$\bar{Y} = 22.8847$		

$$a = \frac{n \Sigma XY - (\Sigma X \Sigma Y)}{n \Sigma X^2 - (\Sigma X)^2} = \frac{5(506.7233) - (20 \times 114.4236)}{5(90) - (20)^2}$$

$$a = 4.902$$

$$b = \bar{Y} - a\bar{X} = 22.8847 - (4.902 \times 4)$$

$$b = 3.276$$

\therefore The equation of straight line $y = ax + b$ is $y = 4.902x + 3.276$

<u>X</u>	<u>actual</u> <u>y</u>	<u>predicted</u> <u>\hat{y}</u>	<u>$(\hat{y} - y)^2$</u>	<u>$(y - \bar{y})^2$</u>
2	12.8978	13.080	0.0332	99.7386
3	17.7586	17.982	0.0499	26.2771
4	23.3192	22.884	0.1894	0.1888
5	28.3129	27.786	0.2776	29.4651
6	32.1351	32.688	0.3057	85.5695
			$\Sigma(\hat{y} - y)^2 =$	$\Sigma(y - \bar{y})^2 =$
			0.8558	241.2391

$$R^2 = 1 - \frac{\text{Sum Squared Regression Error}}{\text{sum Squared Total Error}}$$

$$= 1 - \frac{SSR}{SST}$$

$$= 1 - \frac{\Sigma(\hat{y} - y)^2}{\Sigma(y - \bar{y})^2}$$

$$= 1 - \frac{0.8558}{241.2391}$$

$$= 0.996$$

● Sunday 20

$$\therefore R^2 = 99.6\%$$

$$Adj R^2 = 1 - \frac{(1 - R^2)(n - 1)}{n - p - 1}$$

$$= 1 - \frac{(1 - 0.996)(5 - 1)}{5 - 1 - 1}$$

$$= 0.995$$

$$\therefore Adj R^2 = 99.5\%$$

October							2019
S	M	T	W	T	F	S	
		1	2	3	4	5	
6	7	8	9	10	11	12	
13	14	15	16	17	18	19	
20	21	22	23	24	25	26	
27	28	29	30	31			

... feel of yourself

- Charlie Chaplin

<u>X</u>	<u>Y</u>	<u>\hat{Y}</u>	<u>$\hat{Y} - Y$</u>
2	12.8978	13.080	0.1822
3	17.7586	17.982	0.2234
4	23.3192	22.884	0.4352
5	28.3129	27.786	0.5269
6	32.1351	32.688	0.5529
			$\Sigma \hat{Y} - Y = 1.92$

Mean absolute error $MAE = \frac{\Sigma |\hat{Y} - Y|}{n}$

$$= \frac{1.92}{5}$$

$$MAE = 0.384$$

Mean Squared error $MSE = \frac{\Sigma (\hat{Y} - Y)^2}{n}$

$$= \frac{0.8558}{5}$$

$$MSE = 0.1711$$

Root Mean Squared error $RMSE = \sqrt{MSE}$

$$= \sqrt{\frac{\Sigma (\hat{Y} - Y)^2}{n}}$$

$$= \sqrt{0.1711}$$

$$RMSE = 0.4137$$

X	$X - \bar{X}$	$(X - \bar{X})^2$
2	-2	4
3	-1	1
4	0	0
5	1	1
6	2	4

$$\sum (X - \bar{X})^2 = 10$$

$$\begin{aligned} \text{residual : } \hat{\sigma}^2 = \text{MSE} &= \frac{\sum_{i=1}^n e_i^2}{n-p-1} \\ &= \frac{\sum_{i=1}^n (\hat{y}_i - y_i)^2}{n-p-1} \\ &= \frac{0.8558}{5-1-1} \end{aligned}$$

$$\hat{\sigma}^2 = 0.285$$

$$\text{standard error of } x \quad SE(X) = \sqrt{\frac{\hat{\sigma}^2}{S_{XX}}}$$

$$SE(X) = \sqrt{\frac{\hat{\sigma}^2}{\sum (X - \bar{X})^2}}$$

$$= \sqrt{\frac{0.285}{10}}$$

$$SE(X) = 0.169$$

October 2019						
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Let him who would enjoy a good future waste none of his present

• standard error of Intercept

$$SE(b) = \sqrt{\hat{\sigma}^2 \left[\frac{1}{n} + \frac{\bar{x}^2}{S_{xx}} \right]}$$

$$= \sqrt{0.285 \left[\frac{1}{5} + \frac{4^2}{10} \right]}$$

$$SE(b) = 0.717$$

t-value of x $t_x = \frac{a}{SE(x)} = \frac{4.902}{0.169}$

$$t_x = 29.011$$

t-value of Intercept $t_b = \frac{b}{SE(b)} = \frac{3.276}{0.717}$

$$t_b = 4.565$$

calculating p-value using python

from scipy.stats import t

$t.sf(\text{abs}(t\text{-statistic-value}), df) * 2$

p-value for intercept:-

$$t.sf(\text{abs}(4.565), 5-1-1) * 2 = 0.02$$

p-value for x :-

$$t.sf(\text{abs}(29.011), 5-1-1) * 2 = 0.0$$

$$t_{\text{critical value}} \quad t_{\alpha/2, df} = t_{0.025, 3} = 3.182$$

$$t_{\text{ppf}}(1 - 0.025, 3) = 3.182$$

confidence interval for x

$$CI(x) = a \pm t_{\text{crit}}(SE(x))$$

$$= 4.902 \pm 3.182 \times 0.169$$

$$= [4.365, 5.440]$$

confidence interval for intercept

$$CI(b) = b \pm t_{\text{crit}}(SE(b))$$

$$= 3.276 \pm 3.182 \times 0.717$$

$$= [0.993, 5.554]$$

Regression Results:-

$$R\text{-Squared} = 0.996$$

$$\text{Adj-R-Squared} = 0.995$$

	<u>coef</u>	<u>stderr</u>	<u>t</u>	<u>P> t </u>	<u>CI</u>
Intercept	3.276	0.717	4.565	0.02	[0.993, 5.554]
x	4.902	0.169	29.011	0	[4.365, 5.440]

New Moon ☾ Sunday 27

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If we have no p-value, it is because the p-value is less than 0.001.