CALCULATION OF GEAR DIMENSIONS

Gear dimensions are determined in accordance with their specifications, such as Module (m), Number of teeth (z), Pressureangle (α), and Profile shift coefficient (x). This section introduces the dimension calculations for spur gears, helical gears, gear rack, bevel gears, screw gears, and worm gear pairs. Calculations of external dimensions (eg. Tip diameter) are necessary for processing the gear blanks. Tooth dimensionssuch as root diameter or tooth depth are considered when gear cutting.

4.1 Spur Gears

Spur Gears are the simplest type of gear. The calculations for spur gears are also simple and they are used as the basisfor the calculations for other types of gears. This section introduces calculation methods of standard spur gears, profileshifted spur gears, and linear racks. The standard spur gear is a non-profile-shifted spur gear.

(1) Standard Spur Gear
Figure 4.1 shows the meshing of standard spur gears. The meshing of standard spur gears means the reference circlesof two gears contact and roll with each other. The calculation formulas are in Table 4.1.

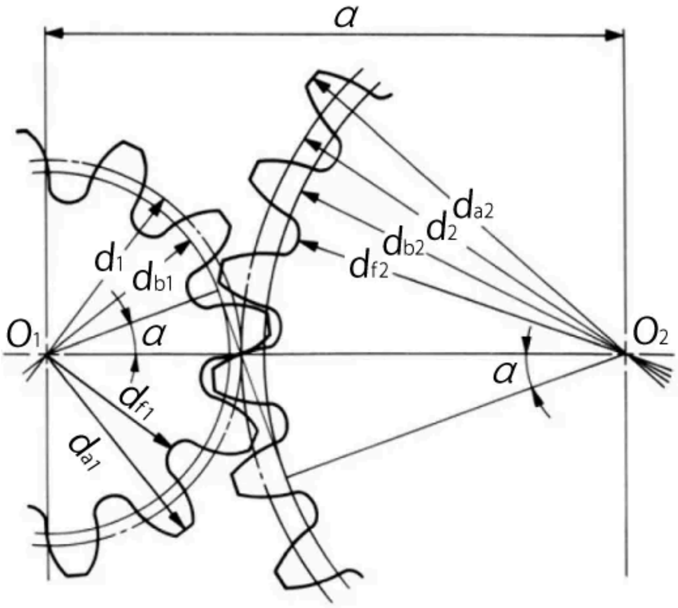


Fig. 4.1 The Meshing of Standard Spur Gears
($\alpha=20°$, $z_1=12$, $z_2=24$, $x_1=x_2=0$)

Table 4.1 Calculations for Standard Spur Gears

No.	Item	Symbol	Formula	Example	
				Pinion (1)	Gear (2)
1	Module	m	Set Value	3	
2	Reference Pressure Angle	α		20 deg	
3	Number of Teeth	z		12	24
4	Center Distance	a	$(z_1 + z_2)\,m / 2$ NOTE1	54.000	
5	Reference Diameter	d	zm	36.000	72.000
6	Base Diameter	d_b	d cos α	33.829	67.658
7	Addendum	h_a	1.00m	3.000	3.000
8	Tooth Depth	h	2.25m	6.750	6.750
9	Tip Diameter	d_a	d + 2m	42.000	78.000
10	Root Diameter	d_f	d – 2.5m	28.500	64.500

NOTE 1 : The subscripts 1 and 2 of z_1 and z_2 denote pinion and gear

All calculated values in Table 4.1 are based upon given module m and number of teeth (z_1 and z_2). If instead, the modulem, center distance a and speed ratio i are given, then the number of teeth, z_1 and z_2, would be calculated using theformulas as shown in Table 4.2.

Table 4.2 The Calculations for Number of Teeth

No.	Item	Symbol	Formula		Example	
					Pinion (1)	Gear (2)
1	Module	m	Set Value		3	
2	Center Distance	a			54.000	
3	Speed Ratio	i			1.25	
4	Sum of No. of Teeth	$z_1 + z_2$	2a / m		36	
5	Number of Teeth	z	$z_1 + z_2 / i + 1$	$i (z_1 + z_2) / i + 1$	16	20

Note, that the number of teeth will probably not be integer values when using the formulas in Table 4.2. In this case,it will be necessary to resort to profile shifting or to employ helical gears to obtain as near a transmission ratioas possible.

(2) Profile Shifted Spur Gear

Figure 4.2 shows the meshing of a pair of profile shifted gears. The key items in profile shifted gears are the operating(working) pitch diameters (d_w) and the working (operating) pressure angle (α_w). These values are obtainable from themodified center distance and the following formulas :

$$\left.\begin{aligned} d_{w1} &= 2a \frac{z_1}{z_1+z_2} \\ d_{w2} &= 2a \frac{z_2}{z_1+z_2} \\ \alpha_w &= \cos^{-1}\left(\frac{d_{b1}+d_{b2}}{2a}\right) \end{aligned}\right\} \qquad (4.1)$$

In the meshing of profile shifted gears, it is the operating pitch circle that is in contact and roll on each other thatportrays gear action. Table 4.3 presents the calculations where the profile shift coefficient has been set at x_1 and x_2 at the beginning. This calculation is based on the idea that the amount of the tip and root clearance should be 0.25m.

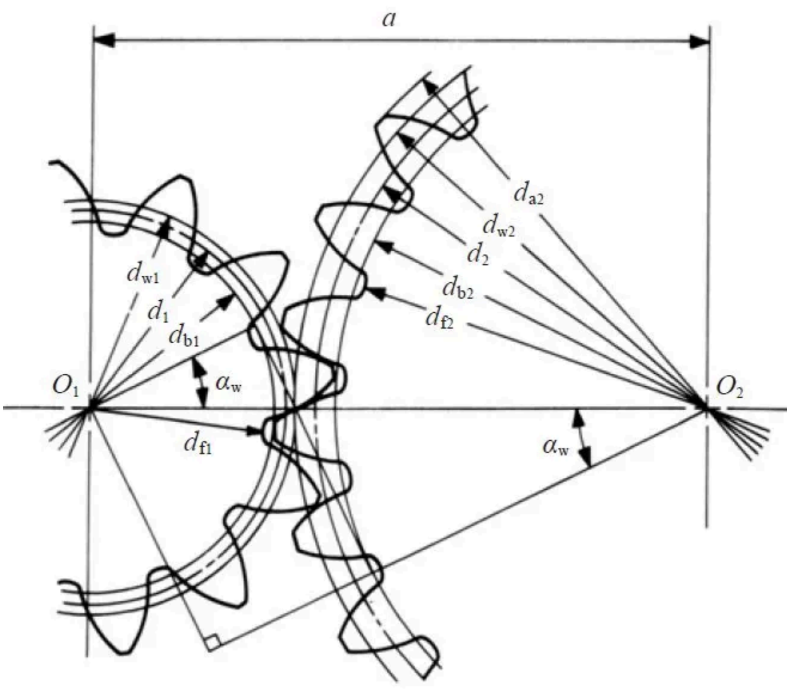


Fig. 4.2 The Meshing of Profile Shifted Gears
(α=20°, z_1=12, z_2=24, x_1=+0.6, x_2=+0.36)

Table 4.3 The Calculations for Profile Shifted Spur Gears (1)

No.	Item	Symbol	Formula	Example	
				Pinion (1)	Gear (2)
1	Module	m	Set Value	3	
2	Reference Pressure Angle	α		20 deg	
3	Number of Teeth	z		12	24

4	Profile Shift Coefficient	x		0.6	0.36
5	Involute α_w	inv α_w	$2 \tan \alpha (x_1 + x_2 / z_1 + z_2) + \text{inv } \alpha$	0.034316	
6	Working Pressure Angle	α_w	Find from Involute Function Table	26.0886 deg	
7	Center Distance Modification Coefficient	y	$z_1 + z_2 / 2 (\cos \alpha / \cos \alpha_w - 1)$	0.83329	
8	Center Distance	a	$(z_1 + z_2 / 2 + y) m$	56.4999	
9	Reference Diameter	d	zm	36.000	72.000
10	Base Diameter	d_b	$d \cos \alpha$	33.8289	67.6579
11	Working Pitch Diameter	d_w	$d_b / \cos \alpha_w$	37.667	75.333
12	Addendum	h_{a1} h_{a2}	$(1 + y – x_2) m$ $(1 + y – x_1) m$	4.420	3.700
13	Tooth Depth	h	$\{2.25 + y – (x_1 + x_2)\}m$	6.370	
14	Tip Diameter	d_a	$d + 2h_a$	44.840	79.400
15	Root Diameter	d_f	$d_a – 2h$	32.100	66.660

A standard spur gear is, according to Table 4.3, a profile shifted gear with 0 coefficient of shift; that is , $x_1=x_2=0$.

Table 4.4 is the inverse formula of items from 4 to 8 of Table 4.3.

Table 4.4 The Calculations for Profile Shifted Spur Gears (2)

No.	Item	Symbol	Formula	Example	
				Pinion (1)	Gear (2)
1	Center Distance	a	Set Value	56.4999	
2	Center Distance Modification Coefficient	y	$\frac{a}{m} - \frac{z_1 + z_2}{2}$	0.8333	
3	Working Pressure Angle	α_w	$\cos^{-1}\left(\frac{\cos \alpha}{\frac{2y}{z_1 + z_2} + 1}\right)$	26.0886 deg	
4	Sum of Profile Shift Coefficient	$x_1 + x_2$	$\frac{(z_1 + z_2)(\text{inv } \alpha_w - \text{inv } \alpha)}{2 \tan \alpha}$	0.9600	
5	Profile Shift Coefficient	x	–	0.6000	0.3600

There are several theories concerning how to distribute the sum of profile shift coefficient ($x_1 + x_2$) into pinion (x_1)and gear (x_2) separately. BSS (British) and DIN (German) standards are the most often used. In the example above, the12 tooth pinion was given sufficient correction to preventundercut, and the residual profile shift was given to the mating gear.

(3) Rack and Spur Gear
Table 4.5 presents the method for calculating the mesh of a rack and spur gear.
Figure 4.3 (1) shows the the meshing of standard gear and a rack. In this mesh, the reference circle of the gear touchesthe pitch line of the rack.

Figure 4.3 (2) shows a profile shifted spur gear, with positive correction xm, meshed with a rack. The spur gear hasa larger pitch radius than standard, by the amount xm. Also, the pitch line of the rack has shifted outward by the amount xm.
Table 4.5 presents the calculation of a meshed profile shifted spur gear and rack. If the profile shift coefficient x_1 is 0, then it is the case of a standard gear meshed with the rack.

Table 4.5 The calculations of dimensions of a profile shifted spur gear and a rack

No.	Item	Symbol	Formula	Example	
				Spur gear	Rack
1	Module	m	Set Value	3	
2	Reference pressure angle	a		20 deg	
3	Number of teeth	z		12	–
4	Profile shift coefficient	x		0.6	
5	Height of pitch line	H		–	32.000
6	Working pressure angle	α_w		20 deg	
7	Mounting distance	a	$\frac{zm}{2} + H + xm$	51.800	
8	Reference diameter	d	zm	36.000	–
9	Base diameter	d_b	$d \cos \alpha$	33.829	
10	Working pitch diameter	d_w	$\frac{d_b}{\cos \alpha_w}$	36.000	
11	Addendum	h_a	$m (1 + x)$	4.800	3.000
12	Tooth depth	h	2.25m	6.750	
13	Tip diameter	d_a	$d + 2h_a$	45.600	–
14	Root diameter	d_f	$d_a – 2h$	32.100	

One rotation of the spur gear will displace the rack l one circumferential length of the gear’s reference circle,per the formula :

$$l = \pi m z \qquad (4.2)$$

The rack displacement, l, is not changed in any way by the profile shifting. Equation (4.2) remains applicable for anyamount of profile shift.

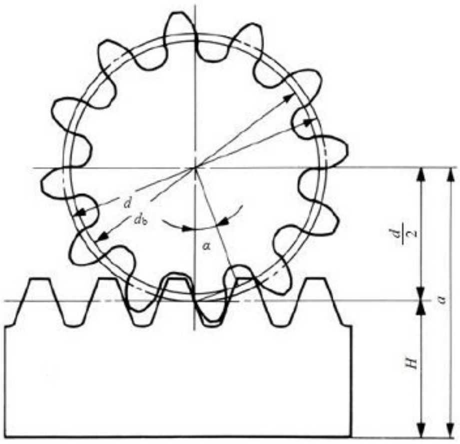


Fig. 4.3 (1) The meshing of standard spur gear and rack
(α=20°, z1=12, x1=0)

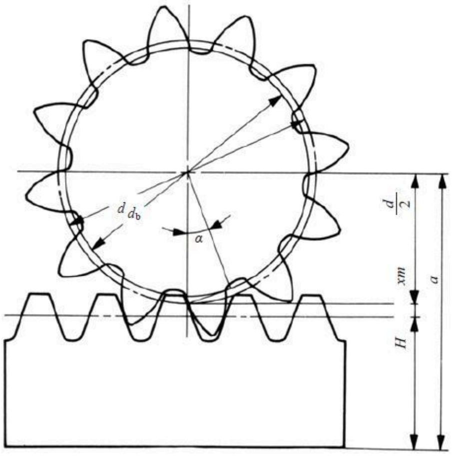


Fig. 4.3 (2) The meshing of profile shifted spur gear and rack
(α=20°, z1=12, x1=+ 0.6)

4.2 Internal Gears

Internal Gears are composed of a cylindrical shaped gear having teeth inside a circular ring. Gear teeth of the internalgear mesh with the teeth space of a spur gear. Spur gears have a convex shaped tooth profile and internal gears havereentrant shaped tooth profile; this characteristic is opposite of Internal gears. Here are the calculations for thedimensions of internal gears and their interference.

(1) Internal Gear Calculations
Figure 4.4 presents the mesh of an internal gear and external gear. Of vital importance is the working pitch diameters(dw) and working pressure angle (αw). They can be derived from center distance (a) and Equations (4.3).

$$\left.\begin{aligned} d_{w1} &= 2a\frac{z_1}{z_2 - z_1} \\ d_{w2} &= 2a\frac{z_2}{z_2 - z_1} \\ \alpha_w &= \cos^{-1}\left(\frac{d_{b2} - d_{b1}}{2a}\right) \end{aligned}\right\} \qquad (4.3)$$

Table 4.6 shows the calculation steps. It will become a standard gear calculation if x1=x2=0.

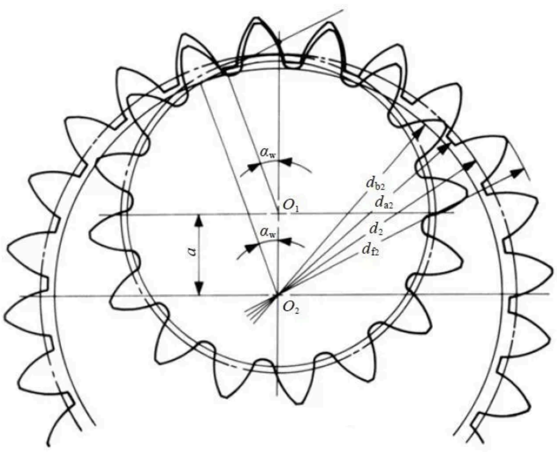


Fig.4.4 The meshing of internal gear and external gear
(α=20°, z1=16, z2=24, x1=x2=+0.5)

Table 4.6 The calculations of a profile shifted internal gear and external gear (1)

No.	Item	Symbol	Formula	Example	
				External gear (1)	Internal gear (2)
1	Module	m	Set Value	3	
2	Reference pressure angle	α		20 deg	
3	Number of teeth	z		16	24
4	Profile shift coefficient	x		0	+ 0.516
5	Involute function α_w	inv α_w	$2\tan\alpha\left(\frac{x_2 - x_1}{z_2 - z_1}\right) + \mathrm{inv}\,\alpha$	0.061857	
6	Working pressure angle	α_w	Find from involute Function Table	31.321258 deg	
7	Center distance modification coefficient	y	$\frac{z_2 - z_1}{2}\left(\frac{\cos\alpha}{\cos\alpha_w} - 1\right)$	0.4000	
8	Center distance	a	$\left(\frac{z_2 - z_1}{2} + y\right)m$	13.2	
9	Reference diameter	d	zm	48.000	72.000
10	Base diameter	d_b	d cos α	45.105	67.658
11	Working pitch diameter	d_w	$\frac{d_b}{\cos\alpha_w}$	52.7998	79.1997
12	Addendum	h_{a1} h_{a2}	$(1 + x_1)m$ $(1 - x_2)m$	3.000	1.452
13	Tooth depth	h	2.25m	6.75	
14	Tip diameter	d_{a1} d_{a2}	$d_1 + 2h_{a1}$ $d_2 - 2h_{a2}$	54.000	69.096
15	Root diameter	d_{f1} d_{f2}	$d_{a1} + 2h$ $d_{a2} + 2h$	40.500	82.596

If the center distance (a) is given, x1 and x2 would be obtained from the inverse calculation from item 4 to item 8 of Table 4.6. These inverse formulas are in Table 4.7.

Table 4.7 The calculations of profile shifted internal gear and external gear (2)

No.	Item	Symbol	Formula	Example	
				External gear (1)	Internal gear (2)
1	Center distance	a	Set Value	13.1683	
2	Center distance modification coefficient	y	$\frac{a}{m} - \frac{z_2 - z_1}{2}$	0.38943	
3	Working pressure angle	α_w	$\cos^{-1}\left(\frac{\cos\alpha}{\frac{2y}{z_2 - z_1} + 1}\right)$	31.0937 deg	
4	Difference of profile shift coefficients	$X_2 - X_1$	$\frac{(z_2 - z_1)(\mathrm{inv}\,\alpha_w - \mathrm{inv}\,\alpha)}{2\tan\alpha}$	0.5	
5	Profile shift coefficient	X	–	0	0.5

Pinion cutters are often used in cutting internal gears and external gears. The actual value of tooth depth and rootdiameter, after cutting, will be slightly different from the calculation. That is because the cutter has a profile shiftcoefficient. In order to get a correct tooth profile, the profile shift coefficient of cutter should be taken into consideration.

(2) Interference In Internal Gears
Three different types of interference can occur with internal gears: (a) Involute Interference, (b) Trochoid Interference,and (c) Trimming Interference.

(a) Involute Interference
This occurs between the dedendum of the external gear and the addendum of the internal gear. It is prevalent when thenumber of teeth of the external gear is small. Involute interference can be avoided by the conditions cited below :

$$\frac{z_1}{z_2} \geqq 1 - \frac{\tan\alpha_{a2}}{\tan\alpha_w} \qquad (4.4)$$

Where αa2 is the pressure angle at a tip of the internal gear tooth.

$$\alpha_{a2} = \cos^{-1}\left(\frac{d_{b2}}{d_{a2}}\right) \qquad (4.5)$$

αw：working pressure angle

$$\alpha_w = \cos^{-1}\left\{\frac{(z_2 - z_1)\,m\cos\alpha}{2a}\right\} \qquad (4.6)$$

Equation (4.5) is true only if the tip diameter of the internal gear is bigger than the base circle :

$$d_{a2} \geqq d_{b2} \qquad (4.7)$$

For a standard internal gear, where α=20° , Equation (4.7) is valid only if the number of teeth is z2 > 34.

(b) Trochoid Interference
This refers to an interference occurring at the addendum of the external gear and the dedendum of the internal gearduring recess tooth action. It tends to

happen when the difference between the numbers of teeth of the two gears issmall. Equation (4.8) presents the condition for avoiding trochoidal interference.

$$\theta_1 \frac{z_1}{z_2} \text{ inv } \alpha_w - \text{inv } \alpha_{a2} \geqq \theta_2 \qquad (4.8)$$

Here

$$\left.\begin{aligned} \theta_1 &= \cos^{-1}\left(\frac{r_{a2}{}^2 - r_{a1}{}^2 - a^2}{2ar_{a1}}\right) \\ &\qquad + \text{inv } \alpha_{a1} - \text{inv } \alpha_w \\ \theta_2 &= \cos^{-1}\left(\frac{a^2 + r_{a2}{}^2 - r_{a1}{}^2}{2ar_{a2}}\right) \end{aligned}\right\} \qquad (4.9)$$

where αa1 is the pressure angle of the spur gear tooth tip:

$$\alpha_{a1} = \cos^{-1}\left(\frac{d_{b1}}{d_{a1}}\right) \qquad (4.10)$$

In the meshing of an external gear and a standard internal gear α=20° , trochoid interference is avoided if the differenceof the number of teeth, z2 – z1, is larger than 9.

(c) Trimming Interference

This occurs in the radial direction in that it prevents pulling the gears apart. Thus, the mesh must be assembled bysliding the gears together with an axial motion. It tends to happen when the numbers of teeth of the two gears are veryclose. Equation (4.11) indicates how to prevent this type of interference.

$$\theta_1 + \text{inv } \alpha_{a1} - \text{inv } \alpha_w \geqq \frac{z_2}{z_1}(\theta_2 + \text{inv } \alpha_{a2} - \text{inv } \alpha_w) \qquad (4.11)$$

Here

$$\left.\begin{aligned} \theta_1 &= \sin^{-1}\sqrt{\frac{1 - (\cos \alpha_{a1}/\cos \alpha_{a2})^2}{1 - (z_1/z_2)^2}} \\ \theta_2 &= \sin^{-1}\sqrt{\frac{(\cos \alpha_{a2}/\cos \alpha_{a1})^2 - 1}{(z_2/z_1)^2 - 1}} \end{aligned}\right\} \qquad (4.12)$$

This type of interference can occur in the process of cutting an internal gear with a pinion cutter. Should that happen,there is danger of breaking the tooling. Table 4.8 (1) shows the limit for the pinion cutter to prevent trimming interferencewhen cutting a standard internal gear, with pressure angle α0=20°, and no profile shift, i.e., x0=0.

Table 4.8 (1) The limit to prevent an internal gear from trimming interference

α_0=20°, x_0=x_2=0

z_0	15	16	17	18	19	20	21	22	24	25	27
z_2	34	34	35	36	37	38	39	40	42	43	45

z_0	28	30	31	32	33	34	35	38	40	42
z_2	46	48	49	50	51	52	53	56	58	60

z_0	44	48	50	56	60	64	66	80	96	100
z_2	62	66	68	74	78	82	84	98	114	118

There will be an involute interference between the internal gear and the pinion cutter if the number of teeth of thepinion cutter ranges from 15 to 22 (z0=15 to 22). Table 4.8(2) shows the limit for a profile shifted pinion cutterto prevent trimming interference while cutting a standard internal gear. The correction (x0) is the magnitude of shiftwhich was assumed to be: x0=0.0075z0 + 0.05.

Table 4.8 (2) The limit to prevent an internal gear from trimming interference

α0=20°, x2=0

z0	15	16	17	18	19	20	21	22	24	25	27	28	30
x0	0.1625	0.17	0.1775	0.185	0.1925	0.2	0.2075	0.215	0.23	0.2375	0.2525	0.26	0.275
z2	36	38	39	40	41	42	43	45	47	48	50	52	54

z0	31	32	33	34	35	38	40	42	44	48	50	56	6
x0	0.2825	0.29	0.2975	0.305	0.3125	0.335	0.35	0.365	0.38	0.41	0.425	0.47	0.
z2	55	56	58	59	60	64	66	68	71	76	78	86	9

There will be an involute interference between the internal gear and the pinion cutter if the number of teeth of thepinion cutter ranges from 15 to 19 (z0=15 to 19).

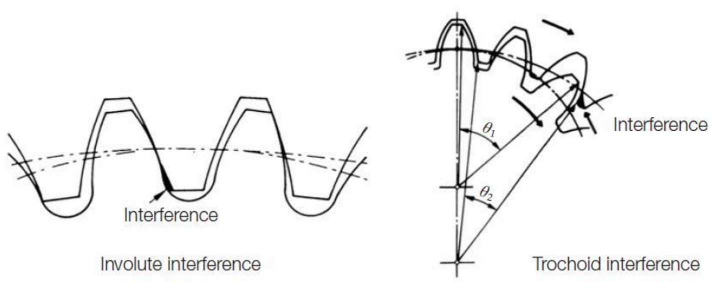


Fig.4.5 Involute interference and trochoid interference

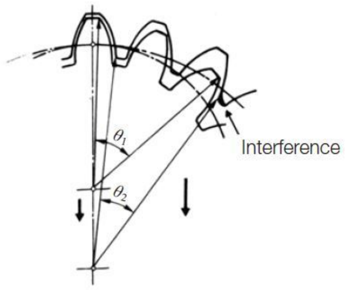


Fig.4.6 Trimming interference

4.3 Helical Gears

A helical gear such as shown in Figure 4.7 is a cylindrical gear in which the teeth flank are helicoid. The helix anglein reference cylinder is β, and the displacement of one rotation is the lead, pz.

The tooth profile of a helical gear is an involute curve from an axial view, or in the plane perpendicular to the axis. The helical gear has two kinds of tooth profiles – oneis based on a normal system, the other is based on a transverse system.

Pitch measured perpendicular to teeth is called normal pitch, pn.
And pn divided by π is then a normal module, mn.

$$m_n = \frac{p_n}{\pi} \qquad (4.13)$$

The tooth profile of a helical gear with applied normal module,
mn, and normal pressure angle αn belongs to a normal system.

In the axial view, the pitch on the reference is called the transverse pitch, pt . And pt divided by π is the transversemodule, mt.

$$m_t = \frac{p_t}{\pi} \qquad (4.14)$$

These transverse module mt and transverse pressure angle αt at are the basic configuration of transverse system helicalgear.
In the normal system, helical gears can be cut by the same gear hob if module mn and pressure angle at are constant, no matter what the value of helix angle β.

It is not that simple in the transverse system. The gear hob design must be altered in accordance with the changing ofhelix angle β, even when the module mt and the pressure angle at are the same.
Obviously, the manufacturing of helical gears is easier with the normal system than with the transverse system in theplane perpendicular to the axis.

When meshing helical gears, they must have the same helix angle but with opposite hands.

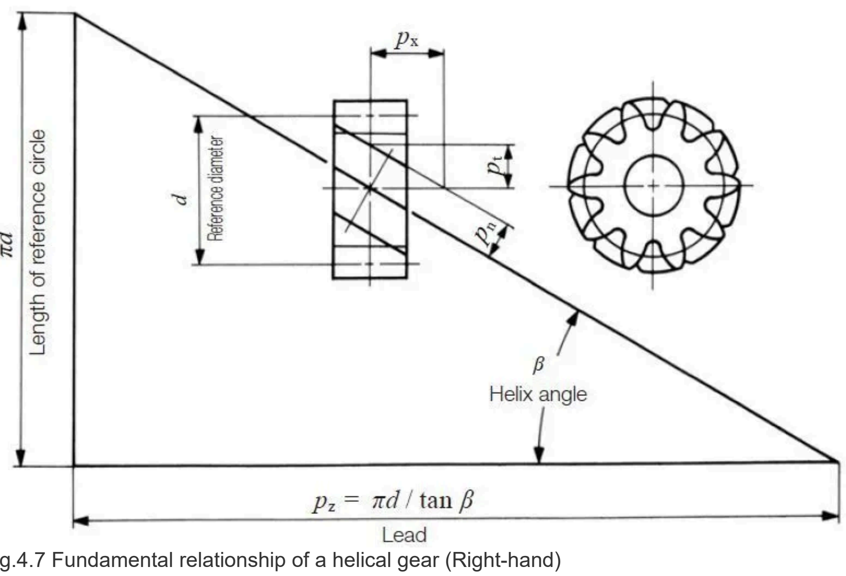


Fig.4.7 Fundamental relationship of a helical gear (Right-hand)

(1) Normal System Helical Gear
In the normal system, the calculation of a profile shifted helical gear, the working pitch diameter dw and transverseworking pressure angle αwt is done per Equations (4.15). That is because meshing of the helical gears in the transverseplane is just like spur gears and the calculation is similar.

$$\left.\begin{aligned} d_{w1} &= 2a\frac{z_1}{z_1+z_2} \\ d_{w2} &= 2a\frac{z_2}{z_1+z_2} \\ \alpha_{wt} &= \cos^{-1}\left(\frac{d_{b1}+d_{b2}}{2a}\right) \end{aligned}\right\} \quad (4.15)$$

Table 4.9 shows the calculation of profile shifted helical gears in the normal system. If normal profile shift coefficientsxn1, xn2 are zero, they become standard gears.

Table 4.9 The calculations of a profile shifted helical gear in the normal system (1)

No.	Item	Symbol	Formula	Example	
				Pinion(1)	Gear(2)
1	Normal module	m_n	Set Value	3	
2	Normal pressure angle	α_n		20 deg	
3	Reference cylinder helix angle	β		30 deg	
4	Number of teeth & helical hand	z		12 (L)	60 (R)
5	Normal coefficient of profile shift	x_n		+ 0.09809	0
6	Transverse pressure angle	α_t	$\tan^{-1}\left(\frac{\tan\alpha_n}{\cos\beta}\right)$	22.79588 deg	
7	Involute function α_{wt}	inv α_{wt}	$2\tan\alpha_n\left(\frac{x_{n1}+x_{n2}}{z_1+z_2}\right)+\text{inv}\,\alpha_t$	0.023405	
8	Transverse working pressure angle	α_{wt}	Find from involute Function Table	23.1126 deg	
9	Center distance modification coefficient	y	$\frac{z_1+z_2}{2\cos\beta}\left(\frac{\cos\alpha_t}{\cos\alpha_{wt}}-1\right)$	0.09744	
10	Center distance	a	$\left(\frac{z_1+z_2}{2\cos\beta}+y\right)m_n$	125.000	
11	Reference diameter	d	$\frac{zm_n}{\cos\beta}$	41.569	207.846
12	Base diameter	d_b	d cos α_t	38.322	191.611
13	Working pitch diameter	d_w	$\frac{d_b}{\cos\alpha_{wt}}$	41.667	208.333
14	Addendum	h_{a1} h_{a2}	$(1 + y - x_{n2})\, m_n$ $(1 + y - x_{n1})\, m_n$	3.292	2.998
15	Tooth depth	h	$\{2.25 + y - (X_{n1} + X_{n2})\}m_n$	6.748	
16	Tip diameter	d_a	$d + 2h_a$	48.153	213.842
17	Root diameter	d_f	$d_a - 2h$	34.657	200.346

If center distance, α, is given, the normal profile shift coefficients xn1 and xn2 can be calculated from Table 4.10.These are the inverse equations from items 5 to 10 of Table 4.9.

Table 4.10 The calculations for a profile shifted helical gear in the normal system (2)

No.	Item	Symbol	Formula	Example	
				Pinion (1)	Gear (2)
1	Center distance	a	Set Value	125	
2	Center distance modification coefficient	y	$\frac{a}{m_n}-\frac{z_1+z_2}{2\cos\beta}$	0.097447	
3	Transverse working pressure angle	α_{wt}	$\cos^{-1}\left(\frac{\cos\alpha_t}{\frac{2y\cos\beta}{z_1+z_2}+1}\right)$	23.1126 deg	
4	Sum of profile shift coefficient	$x_{n1} + x_{n2}$	$\frac{(z_1+z_2)(\text{inv}\,\alpha_{wt}-\text{inv}\,\alpha_t)}{2\tan\alpha_n}$	0.09809	
5	Normal profile shift coefficient	x_n	–	0.09809	0

The transformation from a normal system to a transverse system is accomplished by the following equations :

$$\left.\begin{aligned} x_t &= x_n\cos\beta \\ m_t &= \frac{m_n}{\cos\beta} \\ \alpha_t &= \tan^{-1}\left(\frac{\tan\alpha_n}{\cos\beta}\right) \end{aligned}\right\} \quad (4.16)$$

(2) Transverse System Helical Gear
Table 4.11 shows the calculation of profile shifted helical gears in a transverse system. They become standard if xt1=xt2=0.

Table 4.11 The calculations for a profile shifted helical gear in the transverse system (1)

No.	Item	Symbol	Formula	Example	
				Pinion(1)	Gear(2)
1	Transverse module	m_t	Set Value	3	
2	Transverse pressure angle	α_t		20 deg	
3	Reference cylinder helix angle	β		30 deg	
4	Number of teeth & helical hand	z		12 (L)	60 (R)
5	Transverse profile shift coefficient	x_t		0.34462	0
6	Involute function α_{wt}	inv α_{wt}	$2\tan\alpha_t\left(\frac{x_{t1}+x_{t2}}{z_1+z_2}\right)+\mathrm{inv}\,\alpha_t$	0.0183886	
7	Transverse working pressure angle	α_{wt}	Find from Involute Function Table	21.3975 deg	
8	Center distance modification coefficient	y	$\frac{z_1+z_2}{2}\left(\frac{\cos\alpha_t}{\cos\alpha_{wt}}-1\right)$	0.33333	
9	Center distance	a	$\left(\frac{z_1+z_2}{2}+y\right)m_t$	109.0000	
10	Reference diameter	d	zm_t	36.000	180.000
11	Base diameter	d_b	$d\cos\alpha_t$	33.8289	169.1447
12	Working pitch diameter	d_w	$\frac{d_b}{\cos\alpha_{wt}}$	36.3333	181.6667
13	Addendum	h_{a1} h_{a2}	$(1+y-x_{t2})m_t$ $(1+y-x_{t1})m_t$	4.000	2.966
14	Tooth depth	h	$\{2.25+y-(X_{t1}+X_{t2})\}m_t$	6.716	
15	Tip diameter	d_a	$d+2h_a$	44.000	185.932
16	Root diameter	d_f	d_a-2h	30.568	172.500

Table 4.12 presents the inverse calculation of item 5 to 9 of Table 4.11.

Table 4.12 The calculations for a profile shifted helical gear in the transverse system (2)

No.	Item	Symbol	Formula	Example	
				Pinion (1)	Gear (2)
1	Center distance	a	Set Value	109	
2	Center distance modification coefficient	y	$\frac{a}{m_t}-\frac{z_1+z_2}{2}$	0.33333	
3	Transverse working pressure angle	α_{wt}	$\cos^{-1}\left(\frac{\cos\alpha_t}{\frac{2y}{z_1+z_2}+1}\right)$	21.39752 deg	
4	Sum of profile shift coefficient	$x_{t1}+x_{t2}$	$\frac{(z_1+z_2)(\mathrm{inv}\,\alpha_{wt}-\mathrm{inv}\,\alpha_t)}{2\tan\alpha_t}$	0.34462	
5	Transverse profile shift coefficient	x_t	–	0.34462	0

The transformation from a transverse to a normal system is described by the following equations :

$$\left.\begin{aligned} x_n &= \frac{x_t}{\cos\beta} \\ m_n &= m_t\cos\beta \\ \alpha_n &= \tan^{-1}(\tan\alpha_t\cos\beta) \end{aligned}\right\} \qquad (4.17)$$

(3) Helical Rack
Viewed in the transverse plane, the meshing of a helical rack and gear is the same as a spur gear and rack. Table 4.13presents the calculation examples for a mated helical rack with normal module and normal pressure angle. Similarily,Table 4.14 presents examples for a helical rack in the transverse system (i.e., perpendicular to gear axis).

Table 4.13 The calculations for a helical rack in the normal system

No.	Item	Symbol	Formula	Example	
				Pinion	Rack
1	Normal module	m_n	Set Value	2.5	
2	Normal pressure angle	α_n		20 deg	
3	Reference cylinder helix angle	β		10 deg 57'49"	
4	Number of teeth & helical hand	z		20 (R)	– (L)
5	Normal profile shift coefficient	x_n		0	–
6	Pitch line height	H		–	27.5
7	Transverse pressure angle	α_t	$\tan^{-1}\left(\frac{\tan\alpha_n}{\cos\beta}\right)$	20.34160 deg	
8	Mounting distance	a	$\frac{zm_n}{2\cos\beta}+H+x_n m_n$	52.965	
9	Reference diameter	d	$\frac{zm_n}{\cos\beta}$	50.92956	–
10	Base diameter	d_b	$d\cos\alpha_t$	47.75343	

11	Addendum	h_a	m_n (1 + X_n)	2.500	2.500
12	Tooth depth	h	2.25m_n	5.625	
13	Tip diameter	d_a	d + 2h_a	55.929	–
14	Root diameter	d_f	d_a – 2h	44.679	

The formulas of a standard helical rack are similar to those of Table 4.14 with only the normal profile shift coefficientxn=0.
To mesh a helical gear to a helical rack, they must have the same helix angle but with opposite hands.

The displacement of the helical rack, l, for one rotation of the mating gear is the product of the transverse pitch andnumber of teeth.

$$l = \frac{\pi m_n}{\cos\beta} z \qquad (4.18)$$

According to the equations of Table 4.13, let transverse pitch pt=8 mm and displacement l=160 mm. The transversepitch and the displacement could be resolved into integers, if the helix angle were chosen properly.

Table 4.14 The calculations for a helical rack in the transverse system

No.	Item	Symbol	Formula	Example	
				Pinion	Rack
1	Transverse module	m_t	Set Value	2.5	
2	Transverse pressure angle	α_t		20 deg	
3	Reference cylinder helix angle	β		10 deg 57'49"	
4	Number of teeth & helical hand	z		20 (R)	– (L)
5	Transverse profile shift coefficient	x_t		0	–
6	Pitch line height	H		–	27.5
7	Mounting distance	a	$\frac{zm_t}{2} + H + x_t m_t$	52.500	
8	Reference diameter	d	zm_t	50.000	–
9	Base diameter	d_b	d cos α_t	46.98463	
10	Addendum	h_a	m_t (1 + X_t)	2.500	2.500
11	Tooth depth	h	2.25m_t	5.625	
12	Tip diameter	d_a	d + 2h_a	55.000	–
13	Root diameter	d_f	d_a – 2h	43.750	

In the meshing of transverse system helical rack and helical gear, the movement, l, for one turn of the helical gearis the transverse pitch multiplied by the number of teeth.

$$l = \pi m_t z \qquad (4.19)$$

4.4 Bevel Gears

Bevel gears, whose pitch surfaces are cones, are used to drive intersecting axes. Bevel gears are classified accordingto their type of the tooth forms into Straight Bevel Gear, Spiral Bevel Gear, Zerol Bevel Gear, Skew Bevel Gear etc.The meshing of bevel gears means the pitch cone of two gears contact and roll with each other. Let z1 and z2 be pinionand gear tooth numbers; shaft angle Σ ; and reference cone angles δ1 and δ2 ; then:

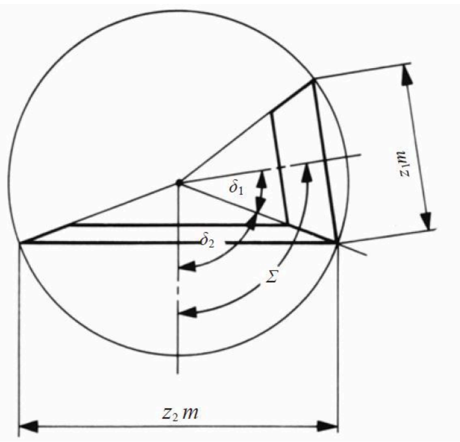


Fig. 4.8 The reference cone angle of bevel gear

$$\left.\begin{aligned} \tan\delta_1 &= \frac{\sin\Sigma}{\frac{z_2}{z_1} + \cos\Sigma} \\ \tan\delta_2 &= \frac{\sin\Sigma}{\frac{z_1}{z_2} + \cos\Sigma} \end{aligned}\right\} \qquad (4.20)$$

Generally, a shaft angle Σ=90° is most used. Other angles (Figure 4.8) are sometimes used. Then, it is called “bevelgear in nonright angle drive”. The 90° case is called “bevel gear in right angle drive”. When Σ=90°, Equation (4.20)becomes :

$$\left.\begin{aligned}\delta_1 &= \tan^{-1}\left(\frac{z_1}{z_2}\right)\\ \delta_2 &= \tan^{-1}\left(\frac{z_2}{z_1}\right)\end{aligned}\right\} \qquad (4.21)$$

Miter gears are bevel gears with Σ=90° and z1=z2. Their transmission ratio z2 / z1=1.

Figure 4.9 depicts the meshing of bevel gears. The meshing must be considered in pairs. It is because the reference coneangles δ1 and δ2 are restricted by the gear ratio z2 / z1. In the facial view, which is normal to the contact line ofpitch cones, the meshing of bevel gears appears to be similar to the meshing of spur gears.

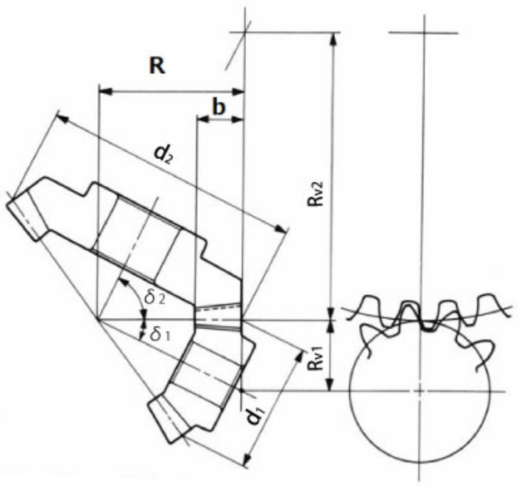


Fig. 4.9 The meshing of bevel gears

(1) Gleason Straight Bevel Gears
A straight bevel gear is a simple form of bevel gear having straight teeth which, if extended inward, would come togetherat the intersection of the shaft axes. Straight bevel gears can be grouped into the Gleason type and the standard type.
In this section, we discuss the Gleason straight bevel gear. The Gleason Company defines the tooth profile as: toothdepth h=2.188m; tip and root clearance c=0.188m; and working depth hw=2.000m.

The characteristics are :

** Design specified profile shifted gears
In the Gleason system, the pinion is positive shifted and the gear is negative shifted. The reason is to distributethe proper strength between the two gears. Miter gears, thus, do not need any shift.
** The tip and root clearance is designed to be parallel
The face cone of the blank is turned parallel to the root cone of the mate in order to eliminate possible fillet interferenceat the small end of the teeth.

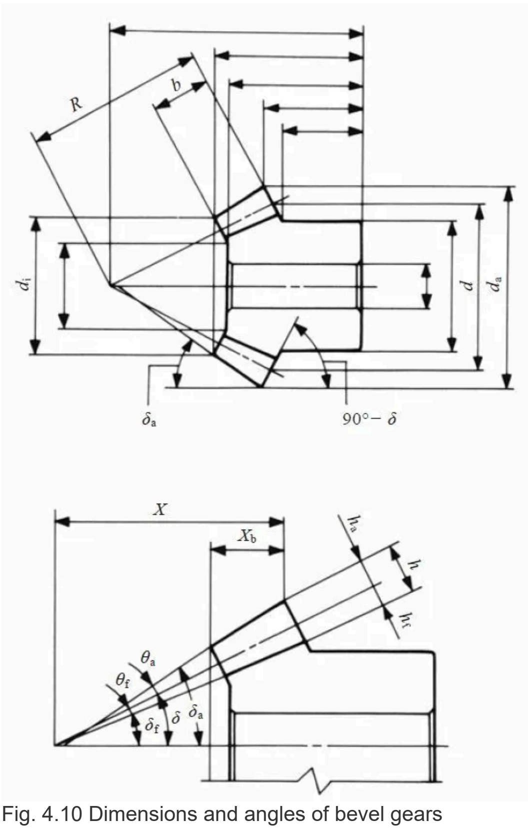


Fig. 4.10 Dimensions and angles of bevel gears

Table 4.15 shows the minimum number of the teeth to prevent undercut in the Gleason system at the shaft angle Σ=90.°

Table 4.15 The minimum numbers of teeth to prevent undercut

Pressure angle	Combination of number of teeth z_1/z_2					
(14.5°)	29/29 and higher	28/29 and higher	27/31 and higher	26/35 and higher	25/40 and higher	24/57 and higher
20°	16/16 and higher	15/17 and higher	14/20 and higher	13/30 and higher		
(25°)	13/13 and higher					

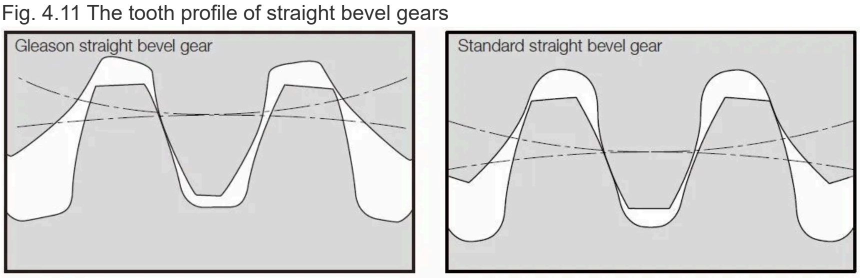
Table 4.16 presents equations for designing straight bevel gears in the Gleason system. The meanings of the dimensionsand angles are shown in Figure 4.10 above. All the equations in Table 4.16 can also be applied to bevel gears with anyshaft angle.
The straight bevel gear with crowning in the Gleason system is called a Coniflex gear. It is manufactured by a specialGleason “Coniflex” machine. It can successfully eliminate poor tooth contact due to improper mounting and assembly.

Tale 4.16 The calculations of straight bevel gears of the Gleason system

No.	Item	Symbol	Formula	Example	
				Pinion(1)	Gear(2)
1	Shaft angle	Σ	Set Value	90 deg	
2	Module	m		3	
3	Reference pressure angle	α		20 deg	
4	Number of teeth	z		20	40
5	Reference diameter	d	zm	60	120
6	Reference cone angle	δ_1 δ_2	$\tan^{-1}\left(\frac{\sin\Sigma}{\frac{z_2}{z_1}+\cos\Sigma}\right)$ $\Sigma-\delta_1$	26.56505 deg	63.43495 deg
7	Cone distance	R	$\frac{d_2}{2\sin\delta_2}$	67.08204	
8	Facewidth	b	It should not exceed R / 3	22	
9	Addendum	h_{a1} h_{a2}	$2.000m - h_{a2}$ $0.540m + \frac{0.460m}{\left(\frac{z_2\cos\delta_1}{z_1\cos\delta_2}\right)}$	4.035	1.965
10	Dedendum	h_f	$2.188m - h_a$	2.529	4.599
11	Dedendum angle	θ_f	$\tan^{-1}(h_f / R)$	2.15903 deg	3.92194 deg
12	Addendum angle	θ_{a1} θ_{a2}	θ_{f2} θ_{f1}	3.92194 deg	2.15903 deg
13	Tip angle	δ_a	$\sigma + \theta_a$	30.48699 deg	65.59398 deg
14	Root angle	δ_f	$\sigma - \theta_f$	24.40602 deg	59.51301 deg
15	Tip diameter	d_a	$d + 2h_a \cos\sigma$	67.2180	121.7575
16	Pitch apex to crown	X	$R\cos\sigma - h_a\sin\sigma$	58.1955	28.2425
17	Axial facewidth	X_b	$\frac{b\cos\delta_a}{\cos\theta_a}$	19.0029	9.0969
18	Inner tip diameter	d_i	$d_a - \frac{2b\sin\delta_a}{\cos\theta_a}$	44.8425	81.6609

The first characteristic of a Gleason Straight Bevel Gear that it is a profile shifted tooth. From Figure 4.11, we cansee the tooth profile of Gleason Straight Bevel Gear and the same of Standard Straight Bevel Gear.

Fig. 4.11 The tooth profile of straight bevel gears



(2) Standard Straight Bevel Gears
A bevel gear with no profile shifted tooth is a standard straight bevel gear. The are also referred to as Klingelnbergbevel gears. The applicable equations are in Table 4.17.

Table 4.17 The calculations for a standard straight bevel gears

No.	Item	Symbol	Formula	Example	
				Pinion(1)	Gear(2)

1	Shaft angle	Σ	Set Value	90 deg	
2	Module	m		3	
3	Reference pressure angle	α		20 deg	
4	Number of teeth	z		20	40
5	Reference diameter	d	zm	60	120
6	Reference cone angle	δ_1 δ_2	$\tan^{-1}\left(\frac{\sin\Sigma}{\frac{z_2}{z_1}+\cos\Sigma}\right)$ $\Sigma-\delta_1$	26.56505 deg	63.43495 deg
7	Cone distance	R	$\frac{d_2}{2\sin\delta_2}$	67.08204	
8	Facewidth	b	It should not exceed R / 3	22	
9	Addendum	h_{a1} h_{a2}	$2.000m - h_{a2}$ $0.540m + \frac{0.460m}{\left(\frac{z_2\cos\delta_1}{z_1\cos\delta_2}\right)}$	4.035	1.965
10	Dedendum	h_f	$2.188m - h_a$	2.529	4.599
11	Dedendum angle	θ_f	$\tan^{-1}(h_f / R)$	2.15903 deg	3.92194 deg
12	Addendum angle	θ_{a1} θ_{a2}	θ_{f2} θ_{f1}	3.92194 deg	2.15903 deg
13	Tip angle	δ_a	$\sigma + \theta_a$	30.48699 deg	65.59398 deg
14	Root angle	δ_f	$\sigma - \theta_f$	24.40602 deg	59.51301 deg
15	Tip diameter	d_a	$d + 2h_a \cos\sigma$	67.2180	121.7575
16	Pitch apex to crown	X	$R\cos\sigma - h_a\sin\sigma$	58.1955	28.2425
17	Axial facewidth	X_b	$\frac{b\cos\delta_a}{\cos\theta_a}$	19.0029	9.0969
18	Inner tip diameter	d_i	$d_a - \frac{2b\sin\delta_a}{\cos\theta_a}$	44.8425	81.6609

These equations can also be applied to bevel gear sets with other than 90° shaft angles.

(3) Gleason Spiral Bevel Gears
A spiral bevel gear is one with a spiral tooth flank as in Figure 4.12. The spiral is generally consistent with thecurve of a cutter with the diameter dc. The spiral angle β is the angle between a generatrix element of the pitch coneand the tooth flank. The spiral angle just at the tooth flank center is called the mean spiral angle βm. In practice,the term spiral angle refers to the mean spiral angle.

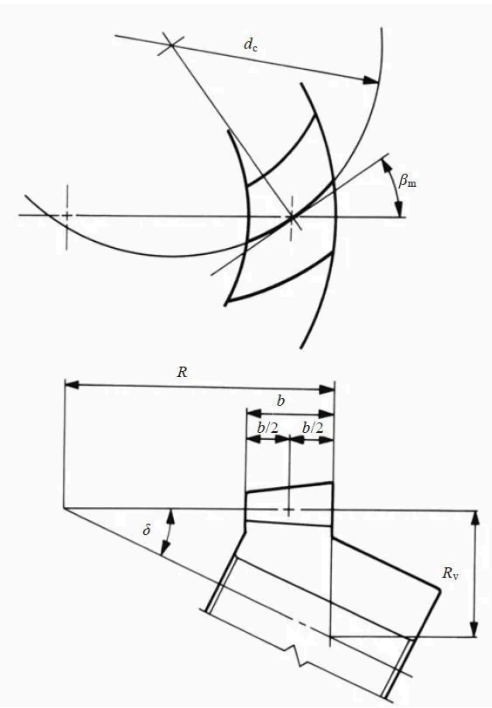


Fig.4.12 Spiral Bevel Gear (Left-hand)

All equations in Table 4.20 are specific to the manufacturing method of Spread Blade or of Single Side from Gleason.If a gear is not cut per the Gleason system, the equations will be different from these.

The tooth profile of a Gleason spiral bevel gear shown here has the tooth depth h=1.888m; tip and root clearance c=0.188m; and working depth hw=1.700m. These Gleason spiral bevel gears belong to a stub gear system. This is applicableto gears with modules m > 2.1.

Table 4.18 shows the minimum number of teeth to avoid undercut in the Gleason system with shaft angle Σ=90° and pressureangle αn=20°.

Table 4.18 The minimum numbers of teeth to prevent undercut β=35°

Pressure angle	Combination of numbers of teeth z_1 / z_2					
20°	17/17 and higher	16/18 and higher	15/19 and higher	14/20 and higher	13/22 and higher	12/26 and higher

If the number of teeth is less than 12, Table 4.19 is used to determine the gear sizes.

Table 4.19 Dimentions for pinions with number of teeth less than 12

Number of teeth in pinion z1		6	7	8	9	10	11
Number of teeth in gear z2		34 and higher	33 and higher	32 and higher	31 and higher	30 and higher	29 and higher
Working depth hw		1.500	1.560	1.610	1.650	1.680	1.695
Tooth depth h		1.666	1.733	1.788	1.832	1.865	1.882
Gear addendum ha2		0.215	0.270	0.325	0.380	0.435	0.490
Pinion addendum ha1		1.285	1.290	1.285	1.270	1.245	1.205
Tooth thickness of gear s2	30	0.911	0.957	0.975	0.997	1.023	1.053
	40	0.803	0.818	0.837	0.860	0.888	0.948
	50	-	0.757	0.777	0.828	0.884	0.946
	60	-	-	0.777	0.828	0.883	0.945
Normal pressure angle αn		20°					
Spiral angle β		35° - 40°					
Shaft angle ∑		90°					

NOTE: All values in the table are based on m=1

Table 4.20 shows the calculations for spiral bevel gears in the Gleason system

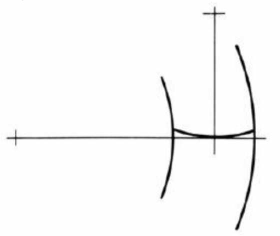
Table 4.20 The calculations for spiral bevel gears in the Gleason system

No.	Item	Symbol	Formula	Example	
				Pinion (1)	Gesr (2)
1	Shaft angle	∑	Set Value	90 deg	
2	Module	m		3	
3	Normal pressure angle	α_n		20 deg	
4	Mean spiral angle	β_m		35 deg	
5	Number of teeth and spiral hand	z		20 (L)	40 (R)
6	Transverse pressure angle	α_t	$\tan^{-1}\left(\dfrac{\tan\alpha_n}{\cos\beta_m}\right)$	23.95680	
7	Reference diameter	d	zm	60	120
8	Reference cone angle	σ_1 σ_2	$\tan^{-1}\left(\dfrac{\sin\Sigma}{\dfrac{z_2}{z_1}+\cos\Sigma}\right)$ $\Sigma-\delta_1$	26.56505 deg	63.43495 deg
9	Cone distance	R	$\dfrac{d_2}{2\sin\delta_2}$	67.08204	
10	Facewidth	b	It should be less than 0.3R or 10m	20	
11	Addendum	h_{a1} h_{a2}	$1.700m-h_{a2}$ $0.460m+\dfrac{0.390m}{\left(\dfrac{z_2\cos\delta_1}{z_1\cos\delta_2}\right)}$	3.4275	1.6725
12	Dedendum	h_f	$1.888m - h_a$	2.2365	3.9915
13	Dedendum angle	θ_f	$\tan^{-1}(h_f / R)$	1.90952 deg	3.40519 deg
14	Addendum angle	θ_{a1} θ_{a2}	θ_{f2} θ_{f1}	29.97024 deg	1.90952 deg
15	Tip angle	σ_a	$\sigma + \theta_a$	29.97024 deg	65.34447 deg
16	Root angle	σ_f	$\sigma - \theta_f$	24.65553 deg	60.02976 deg
17	Tip diameter	d_a	$d + 2h_a \cos\sigma$	66.1313	121.4959
18	Pitch apex to crown	X	$R\cos\sigma - h_a\sin\sigma$	58.4672	28.5041
19	Axial facewidth	X_b	$\dfrac{b\cos\delta_a}{\cos\theta_a}$	17.3565	8.3479
20	Inner tip diameter	d_i	$d_a-\dfrac{2b\sin\delta_a}{\cos\theta_a}$	46.1140	85.1224

All equations in Table 4.20 are also applicable to Gleason bevel gears with any shaft angle. A spiral bevel gear setrequires matching of hands; left-hand and right-hand as a pair.

(4) Gleason Zerol Bevel Gears
When the spiral angle bm=0, the bevel gear is called a Zerol bevel gear. The calculation equations of Table 4.16for Gleason straight bevel gears are applicable. They also should take care again of the rule of hands; left and rightof a pair must be matched. Figure 4.13 is a left-hand Zerol bevel gear.

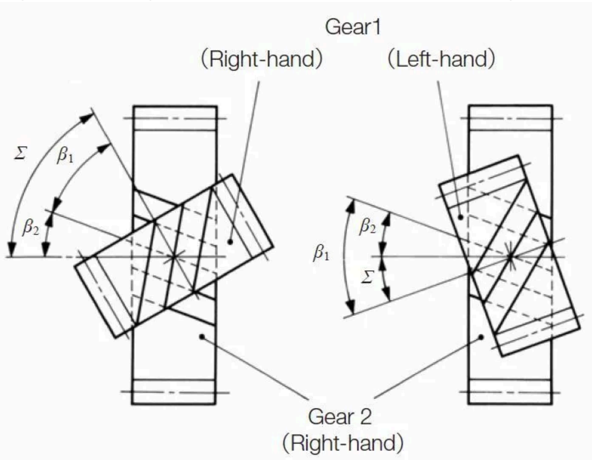
Fig. 4.13 Left-hand zerol bevel gear



4.5 Screw Gears

Screw gearing includes various types of gears used to drive nonparallel and nonintersecting shafts where the teeth ofone or both members of the pair are of screw form. Figure 4.14 shows the meshing of screw gears. Two screw gears canonly mesh together under the conditions that normal modules (mn1) and (mn2) and normal pressure angles (αn1, αn2) arethe same.

Fig.4.14 Screw gears of nonparallel and nonintersecting axes



Let a pair of screw gears have the shaft angle Σ and helix angles β1 and β2 :

$$\left.\begin{array}{l}\text{If they have the same hands, then:}\\ \quad \Sigma = \beta_1 + \beta_2 \\ \text{If they have the opposite hands, then:}\\ \quad \Sigma = \beta_1 - \beta_2 \text{ or } \Sigma = \beta_2 - \beta_1 \end{array}\right\} \quad (4.22)$$

If the screw gears were profile shifted, the meshing would become a little more complex. Let βw1, βw2 represent the working pitch cylinder ;

$$\left.\begin{array}{l}\text{If they have the same hands, then:}\\ \quad \Sigma = \beta_{w1} + \beta_{w2} \\ \text{If they have the opposite hands, then:}\\ \quad \Sigma = \beta_{w1} - \beta_{w2} \text{ or } \Sigma = \beta_{w2} - \beta_{w1} \end{array}\right\} \quad (4.23)$$

Table 4.21 presents equations for a profile shifted screw gear pair. When the normal profile shift coefficients xn1=xn2=0, the equations and calculations are the same as for standard gears.

Table 4.21 The equations for a screw gear pair on nonparallel and Nonintersecting axes in the normal system

No.	Item	Symbol	Formula	Example	
				Pinion (1)	Gear (2)
1	Normal module	m_n		3	
2	Normal pressure angle	α_n		20 deg	
3	Reference cylinder helix angle	β	Set Value	20 deg	30 deg
4	Number of teeth & helical hand	z		15 (R)	24 (R)
5	Normal profile shift coefficient	x_n		0.4	0.2
6	Number of teeth of an Equivalent spur gear	z_v	$\dfrac{z}{\cos^3\beta}$	18.0773	36.9504
7	Transverse pressure angle	α_t	$\tan^{-1}\left(\dfrac{\tan\alpha_n}{\cos\beta}\right)$	21.1728 deg	22.7959 deg

8	Involute function α_{wn}	inv α_{wn}	$2\tan\alpha_n\left(\frac{x_{n1}+x_{n2}}{z_{v1}+z_{v2}}\right)+\text{inv }\alpha_n$	0.0228415	
9	Normal working pressure angle	α_{wn}	Find from involute function table	22.9338 deg	
10	Transverse working pressure angle	α_{wn}	$\tan^{-1}\left(\frac{\tan\alpha_{wn}}{\cos\beta}\right)$	24.2404 deg	26.0386 deg
11	Center distance modification coefficient	y	$\frac{1}{2}(z_{v1}+z_{v2})\left(\frac{\cos\alpha_n}{\cos\alpha_{wn}}-1\right)$	0.55977	
12	Center distance	a	$\left(\frac{z_1}{2\cos\beta_1}+\frac{z_2}{2\cos\beta_2}+y\right)m_n$	67.1925	
13	Reference diameter	d	$\frac{zm_n}{\cos\beta}$	47.8880	83.1384
14	Base diameter	d_b	$d\cos\alpha_t$	44.6553	76.6445
15	Working pitch diameter	d_{w1} d_{w2}	$2a\,\frac{d_1}{d_1+d_2}$ $2a\,\frac{d_2}{d_1+d_2}$	49.1155	85.2695
16	Working helix angle	β_w	$\tan^{-1}\left(\frac{d_w}{d}\tan\beta\right)$	20.4706 deg	30.6319 deg
17	Shaft angle	∑	$\beta_{w1}+\beta_{w2}$ or $\beta_{w1}-\beta_{w2}$	51.1025 deg	
18	Addendum	h_{a1} h_{a2}	$(1+y-x_{n2})m_n$ $(1+y-x_{n1})m_n$	4.0793	3.4793
19	Tooth depth	h	$\{2.25+y-(x_{n1}+x_{n2})\}m_n$	6.6293	
20	Tip diameter	d_a	$d+2h_a$	56.0466	90.0970
21	Root diameter	d_f	d_a-2h	42.7880	76.8384

Standard screw gears have relations as follows:

dw1=d1
dw2=d2
βw1=β1
βw2=β2
(4.24)

Appendix – What is screw gear ?

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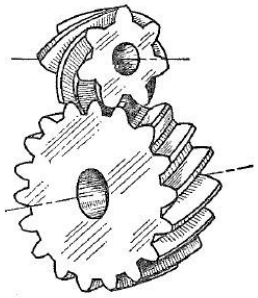
Masao Kubota,*Haguruma Nyumon*, Tokyo : Ohmsha, Ltd., 1963.

The screw gear (or crossed helical gear) in pic 5.1 is a type of gear whose two axes are neither parallel nor crossed(skew gears), and whose pitch surface consists of two cylindrical surfaces circumscribing at one point on the shortestdistance between the two axes. The screw gear is a point contact gear which consists of obliquely meshed helical gearswhose sum or difference of torsion angle of tooth traces is equal to the included angle of the two axes.

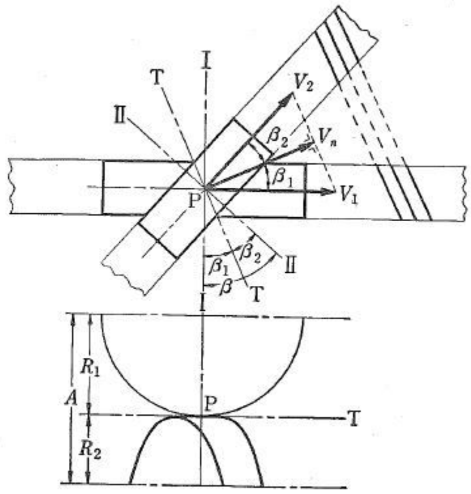


Pic 5.1 Screw gear

Background of screw gear

In Pic 5.2, the point P at one point on the shortest distance between two axes is called pitch point, where two cylinders with radius R_1 or R_2 whose axes I and II constitute the center distance A and included angle circumscribe at the point P.

Assuming that the two cylinders are reference curved surfaces for making gear teeth, and the gears mesh at the pitchpoint P and its neighborhood. In order that both tooth flanks make contact at the point P to transmit motion, theyneed to share the normal line, and the velocity component of the both gears in the direction of normal line of thetooth flanks need to be equal. Therefore, at the point P, the direction of the tooth traces should be same, and thevelocity component of the both gears at right angle to the tooth traces should be equal. More specifically, as in Pic5.2, the direction of the vertical line from the point P toward the directions of the vectors of gear speed V_1 and V_2 at the point P equals to the velocity component of both gears (V_n), and the right angle (TT) to this direction at the point P becomes the tooth trace's direction at the pointP. The velocity components of V_1 and V_2 are not equal in the direction TT. That is to say, there is a slide in the direction of the tooth trace.

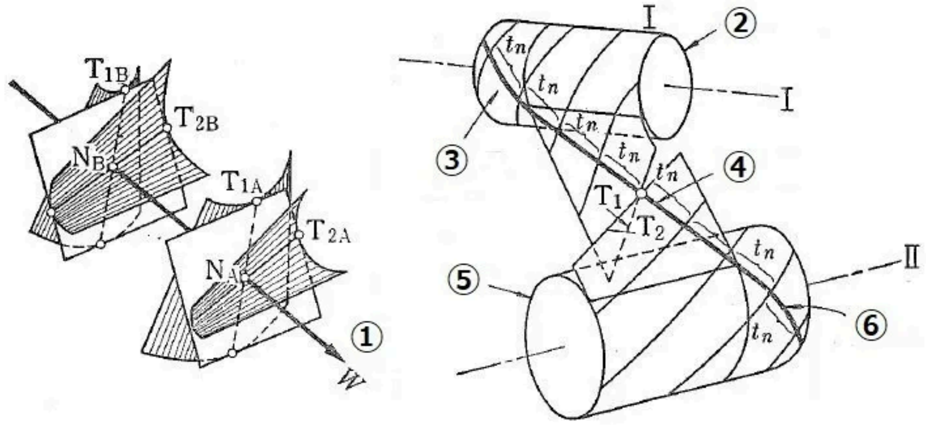


Pic 5.2 Screw gear’s background

Assuming that there is a helical rack, which has the tooth trace in the direction TT and its tangential plane of both pitch cylinders at the point P is the pitch plane. When it moves with a velocity of V_n, the curve formed on each gear as an envelope surface of the rack tooth flank becomes the tooth flank of bothgears. When the tooth flank of the helical rack is plane, the tooth flank of both gears becomes an involute helicoid.It is an involute screw gear, and its normal section is an involute tooth profile.

The simultaneous contact line of the tooth flank of each gear and rack is the trace of a foot of a perpendicular fromthe arbitrary point on each pitch cylinder’s bus to the rack tooth surface through the pitch point P (it becomes astraight line for involute screw gear). Both traces cross at the foot of a perpendicular from the pitch point P tothe rack tooth profile. (See Pic 5.3 (a) N_A and N_B) Therefore, both tooth profiles point-contact at that point.

The trace of the contact point is generally the curve through the pitch point P. As for the involute screw gear, thetrace of the contact point becomes a straight line W which passes through the pitch point P, because the plane of therack tooth profile moves parallel. The line is called action line (see Pic 5.3), the crossing line of tangential planes of base cylinders of gears, and it is alsothe fixed line contacts with both base cylinders. Same as normal gears, the angular velocity ratio is equal to thereciprocal ratio of the number of teeth, and the normal plane module should be equal for both gears.



Pic 5.3 Mesh of involute screw gear
Left Picture – Contact of screw gear’s flank
(1) Action line
Right Picture – Relation of base cylinders, action line, tangential plane, tooth trace of screw gear
(2) Base cylinder of gear I
(3) Screw line orthogonal to tooth trace
(4) Action line
(5) Base cylinder of gear II
(6) Screw line orthogonal to tooth trace

Suppose that the helical angle of the tooth trace is β1 and β2, the normal plane module of helical rack is m_n, and the number of teeth of each gear is z_1 and z_2, the radius of pitch cylinders R_1 and R_2 are :

$R_1=z_1m_n / 2\cos\beta1$, $R_2=z_2m_n / 2\cos\beta2$

Then, $R_1 + R_2=A$, $\beta1 + \beta2=\beta$

Therefore, $2A / m_n=z_1 / \cos\beta1 + z_2 / \cos(\beta - \beta1)$

For example, when A, β, z_1, z_2 and m_n are given, β1 and β2 are defined by the preceding formula. However, β1>0, β2>0 in the preceding picture.β1 and β2could be 0 or negative number. In fact, β=90° in many cases. When β=90°, to minimize center distance, setdA / dβ1=0 and obtain

$$\cot\beta_1 = \sqrt[3]{z_1/z_2}$$

Application of screw gear

As screw gears are point-contact, the contact stress at the contact point is large and lubricant film is easy to becomethinner and as a result, the gears easily wear out. Therefore, the screw gears are not suitable for transmitting largepower. On the other hand, the gears mesh smoothly and easy to do cut adjustment, so frequently used for transmissionmechanism between skew shafts whose center distance is in the middle. In addition, it is well known that the meshingrelation of cutter and machined gear at gear shaving is similar to screw gears. The meshing relation of hob and gearsto be cut is also similar to screw gears.

When one of screw gears (driven gear) is a rack gear, they can line-contact and transmit heavy load. They may be usedfor the table drive of a planning machine. The rack type shaving cutter can be used, too.

Only the curve which goes on each tooth flank diagonally through the pitch point is useful for meshing of tooth flankof screw gears, and therefore the working face width is limited. However, enlarging the face width a little and enablingthe gears to move toward the axis will avoid excessive local wear, and lengthens the life of the entire gear.

4.6 Cylindrical Worm Gear Pair

Cylindrical worms may be considered cylindrical type gears with screw threads. Generally, the mesh has a 90° shaft angle.The number of threads in the worm is equivalent to the number of teeth in a gear of a screw type gear mesh. Thus, aonethread worm is equivalent to a one-tooth gear; and two-threads equivalent to two-teeth, etc. Referring to Figure4.15, for a reference cylinder lead angle γ, measured on the pitch cylinder, each rotation of the worm makes the threadadvance one lead pz.

There are four worm tooth profiles in JIS B 1723-1977, as defined below.
Type I ： The tooth profile is trapezoidal on the axial plane.
Type II ： The tooth profile is trapezoid on the plane normal to the space.
Type III ： The tooth profile which is obtained by inclining the axis of the milling or grinding, of which cutter shapeis trapezoidal on the cutter axis, by the lead angle to the worm axis.
Type IV ： The tooth profile is of involute curve on the plane of rotation.

KHK stock worm gear products are all Type III. Worm profiles (Fig 4.15). The cutting tool used to process worm gearsis called a single-cutter that has a single-edged blade. The cutting of worm gears is done with worm cutting machine.Because the worm mesh couples nonparallel and nonintersecting axes, the axial plane of worm does not correspond withthe axial plane of worm wheel. The axial plane of worm corresponds with the transverse plane of worm wheel. The transverseplane of worm corresponds with the axial plane of worm wheel. The common plane of the worm and worm wheel is the normalplane. Using the normal module, mn, is most popular. Then, an ordinary hob can be used to cut the worm wheel.

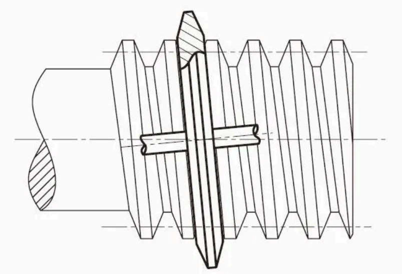


Fig. 4.15 Cutting – Grinding for Type III Worm

Table 4.22 presents the relationships among worm and worm wheel with regard to axial plane, transverse plane, normalplane, module, pressure angle, pitch and lead.

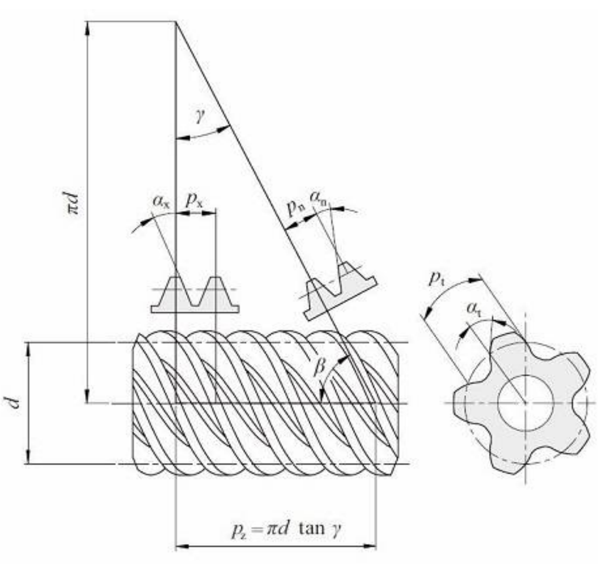


Fig. 4.16 Cylindrical worm (Right-hand)

Table 4.22 The relations of cross sections of worm gear pairs

Worm		
Axial plane	Normal plane	Transverse plane

$m_x = \dfrac{m_n}{\cos\gamma}$	m_n	$m_t = \dfrac{m_n}{\sin\gamma}$
$\alpha_x = \tan^{-1}\left(\dfrac{\tan\alpha_n}{\cos\gamma}\right)$	α_n	$\alpha_t = \tan^{-1}\left(\dfrac{\tan\alpha_n}{\sin\gamma}\right)$
$p_x = \pi m_x$	$p_n = \pi m_n$	$p_t = \pi m_t$
$p_z = \pi m_x z$	$p_z = \dfrac{\pi m_n z}{\cos\gamma}$	$p_z = \pi m_t z \tan\gamma$
Transverse plane	Normal plane	Axial plane
Worm wheel		

Reference to Figure 4.16 can help the understanding of the relationships in Table 4.22. They are similar to the relationsin Formulas (4.16) and (4.17) in that the helix angle β be substituted by (90 deg – γ). We can consider thata worm with lead angle γ is almost the same as a helical gear with helix angle (90 deg – γ).

(1) Axial Module Worm Gear Pair
Table 4.23 presents the equations, for dimensions shown in Figure 4.16, for worm gears with axial module, mx, andnormal pressure angle αn=20°.

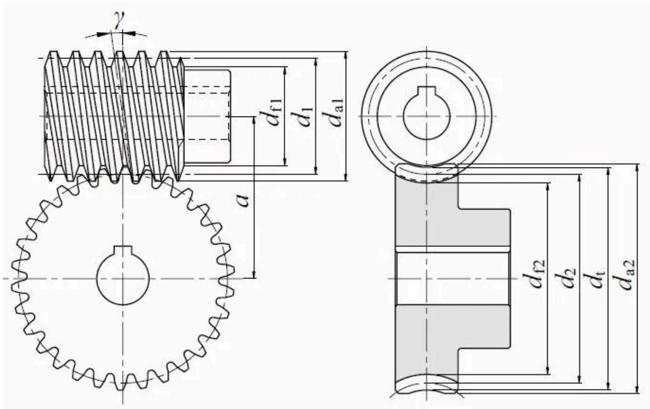


Fig. 4.17 Dimentions of cylindrical worm gear pair

Table 4.23 The calculations for an axial module system worm gear pair

No.	Item	Symbol	Formula	Example	
				Worm (1)	Wheel (2)
1	Axial module	m_x	Set Value	3	
2	Normal pressure angle	(α_n)		(20 deg)	
3	No. of threads, no. of teeth	z		Double Thread (R)	30 (R)
4	Coefficient of Profile shift	Z_{t2}		–	0
5	Reference diameter	d_1 d_2	(Qm_x) NOTE1 $z_2 m_x$	44.000	90.000
6	Reference cylinder lead angle	γ	$\tan^{-1}\left(\dfrac{m_x z_1}{d_1}\right)$	7.76517 deg	
7	Center distance	a	$\dfrac{d_1+d_2}{2} + x_{t2} m_x$	67.000	
8	Addendum	h_{a1} h_{a2}	1.00 m_x (1.00 + x_{t2}) m_x	3.000	3.000
9	Tooth depth	h	2.25 m_x	6.750	
10	Tip diameter	d_{a1} d_{a2}	$d_1 + 2h_{a1}$ $d_2 + 2h_{a2} + m_x$ NOTE2	50.000	99.000
11	Throat diameter	d_t	$d_2 + 2h_{a2}$	–	96.000
12	Throat surface radius	r_i	$\dfrac{d_1}{2} - h_{a1}$	–	19.000
13	Root diameter	d_{f1} d_{f2}	d_{a1} – 2h d_t – 2h	36.500	82.500

NOTE 1.
Diameter factor, Q, means reference diameter of worm, d1, over axial module, mx.
Q=d1 / mx
NOTE 2.
There are several calculation methods of worm wheel tip diameter da2 besides those in Table 4.25.
NOTE 3.
The facewidth of worm, b1, would be sufficient if: b1=πmx (4.5 + 0.02z2)
NOTE 4.
Effective facewidth of worm wheel bw= $2m_x\sqrt{Q+1}$.
So the actual facewidth of b2 ≧ bw + 1.5mx would be enough.

(2) Normal Module System Worm Gear Pair
The equations for normal module system worm gears are based on a normal module, mn, and normal pressure angle, αn=20°. See Table 4.24.

Table 4.24 The calculations for a normal module system worm gear pair

No.	Item	Symbol	Formula	Example	
				Worm (1)	Wheel (2)
1	Normal module	m_n	Set Value	3	
2	Normal pressure angle	α_n		(20 deg)	
3	No. of threads, No. of teeth	z		Double (R)	30 (R)
4	Reference diameter of worm	d_1		44.000	–
5	Normal profile shift coefficient	x_{n2}		–	– 0.1414
6	Reference cylinder lead angle	γ	$\sin^{-1}\left(\frac{m_n z_1}{d_1}\right)$	7.83748 deg	
7	Reference diameter of worm wheel	d_2	$\frac{z_2 m_n}{\cos\gamma}$	–	90.8486
8	Center distance	a	$\frac{d_1 + d_2}{2} + x_{n2} m_n$	67.000	
9	Addendum	h_{a1} h_{a2}	$1.00\ m_n$ $(1.00 + x_{n2})\ m_n$	3.000	2.5758
10	Tooth depth	h	$2.25\ m_n$	6.75	
11	Tip diameter	d_{a1} d_{a2}	$d_1 + 2h_{a1}$ $d_{t2} + 2h_{a1} + m_n$	50.000	99.000
12	Throat diameter	d_t	$d_2 + 2h_{a2}$	–	96.000
13	Throat surface radius	r_i	$\frac{d_1}{2} - h_{a1}$	–	19.000
14	Root diameter	d_{f1} d_{f2}	$d_{a1} - 2h$ $d_t - 2h$	36.500	82.500

NOTE : All notes are the same as those of Table 4.23.

(3) Crowning of the Tooth
Crowning is critically important to worm gears. Not only can it eliminate abnormal tooth contact due to incorrectassembly, but it also provides for the forming of an oil film, which enhances the lubrication effect of the mesh. Thiscan favorably impact endurance and transmission efficiency of the worm mesh. There are four methods of crowning wormgear pair :

(a) Cut Worm Wheel with a Hob Cutter of Greater Reference Diameter than the Worm.
A crownless worm wheel results when it is made by using a hob that has an identical pitch diameter as that of theworm. This crownless worm wheel is very difficult to assemble correctly. Proper tooth contact and a complete oil filmare usually not possible.
However, it is relatively easy to obtain a crowned worm wheel by cutting it with a hob whose reference diameter isslightly larger than that of the worm.

This is shown in Figure 4.18. This creates teeth contact in the center region with space for oil film formation.

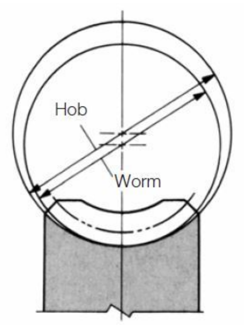


Fig.4.18 The method of using a greater diameter hob

(b) Recut With Hob Center Position Adjustment.
The first step is to cut the worm wheel at standard center distance. This results in no crowning. Then the worm wheelis finished with the same hob by recutting with the hob axis shifted parallel to the worm wheel axis by ±Δh. This resultsin a crowning effect, shown in Figure 4.19.

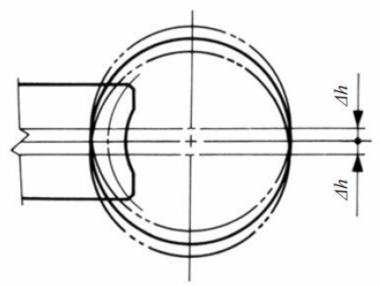


Fig.4.19 Offsetting up or down

(c) Hob Axis Inclining Δθ From Standard Position.
In standard cutting, the hob axis is oriented at the proper angle to the worm wheel axis. After that, the hob axisis shifted slightly left and then right, Δθ, in a plane parallel to the worm wheel axis, to cut a crown effect on theworm wheel tooth.

This is shown in Figure 4.20. Only method (a) is popular. Methods (b) and (c) are seldom used.

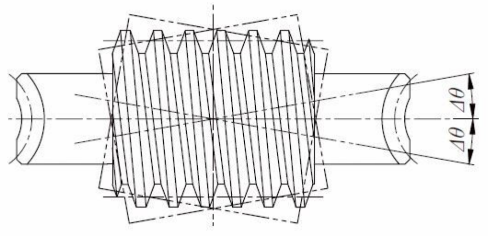


Fig. 4.20 Inclining right or left

(d) Use a Worm with a Larger Pressure Angle than the Worm Wheel.
This is a very complex method, both theoretically and practically. Usually, the crowning is done to the worm wheel,but in this method the modification is on the worm. That is, to change the pressure angle and pitch of the worm withoutchanging base pitch, in accordance with the relationships shown in Equations 4.25 :

$$p_x \cos \alpha_x = p_{wx} \cos \alpha_{wx} \qquad (4.25)$$

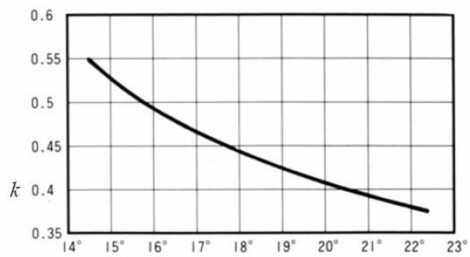
In order to raise the pressure angle from before change, αwx, to after change, αx , it is necessary to increase theaxial pitch, pwx, to a new value, px, per Equation (4.25). The amount of crowning is represented as the space betweenthe worm and worm wheel at the meshing point A in Figure 4.22. This amount may be approximated by the following equation:

$$\fallingdotseq k \frac{p_x - p_{wx}}{p_{wx}} \frac{d_1}{2} \qquad (4.26)$$

Where
d1 : Reference diameter of worm
k : Factor from Table 4.25 and Figure 4.21

Table 4.25 The value of factor k

α_x	14.5°	17.5°	20°	22.5°
k	0.55	0.46	0.41	0.375



Axial pressure angle αx
Fig. 4.21 The value of factor (k)

Table 4.26 shows an example of calculating worm crowning.

Table 4.26 The calculations for worm crowning

No.	Item	Symbol	Formula	Example
1	Axial module	m_{wx}	NOTE: This is the data before crowning.	3
2	Normal pressure angle	α_{wn}		20 deg
3	Number of threads of worm	z_1		2
4	Reference diameter of worm	d_1		44.000
5	Reference cylinder lead angle	γ_w	$\tan^{-1}\left(\frac{m_{wx} z_1}{d_1}\right)$	7.765166 deg
6	Axial pressure angle	α_{wx}	$\tan^{-1}\left(\frac{\tan \alpha_{wn}}{\cos \gamma_w}\right)$	20.170236 deg
7	Axial pitch	P_{wx}	πm_{wx}	9.424778
8	Lead	P_{wz}	$\pi m_{wx} z_1$	18.849556
9	Amount of crowning	C_R	It should be determined by considering the size of tooth contact.	0.04
10	Factor	k	From Table 4.26	0.41
* After crowning				
11	Axial pitch	P_x	$p_{wx}\left(\frac{2C_R}{kd_1} + 1\right)$	9.466573
12	Axial pressure angle	α_x	$\cos^{-1}\left(\frac{p_{wx}}{p_x}\cos \alpha_{wx}\right)$	20.847973 deg
13	Axial module	m_x	$\frac{p_x}{\pi}$	3.013304
14	Reference cylinder lead angle	γ	$\tan^{-1}\left(\frac{m_x z_1}{d_1}\right)$	7.799179 deg
15	Normal pressure angle	α_n	$\tan^{\wedge-1}(\tan\alpha_x \cos \gamma)$	20.671494 deg

16	Lead	P_z	$\pi m_x z_1$	18.933146 deg

(4) Self-Locking Of Worm Gear Pairs
Self-locking is a unique characteristic of worm meshes that can be put to advantage. It is the feature that a wormcannot be driven by the worm wheel. It is very useful in the design of some equipment, such as lifting, in that thedrive can stop at any position without concern that it can slip in reverse. However, in some situations it can be detrimentalif the system requires reverse sensitivity, such as a servomechanism.

Self-locking does not occur in all worm meshes, since it requires special conditions as outlined here. In this analysis,only the driving force acting upon the tooth surfaces is considered without any regard to losses due to bearing friction,lubricant agitation, etc. The governing conditions are as follows :

Let Ft1=tangential driving force of worm

Then,
Ft1=Fn (cos αn sin γ – μ cos γ) (4.27)
If Ft1 > 0 then there is no self-locking effect at all. Therefore,
Ft1 ≤ 0 is the critical limit of self-locking.

Let αn in Equation (4.27) be 20°, then the condition:
Ft1 ≤ 0 will become :
(cos 20° sing – mcosg) ≤ 0

Figure 4.22 shows the critical limit of self-locking for lead angle g and coefficient of friction m. Practically, itis very hard to assess the exact value of coefficient of friction μ. Further, the bearing loss, lubricant agitationloss, etc. can add many side effects. Therefore, it is not easy to establish precise self-locking conditions.
However, it is true that the smaller the lead angle γ, the more likely the self-locking condition will occur.

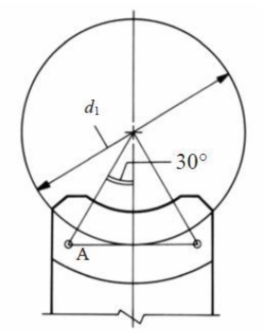


Fig.4.22 Position A is the point of determining crowning amount

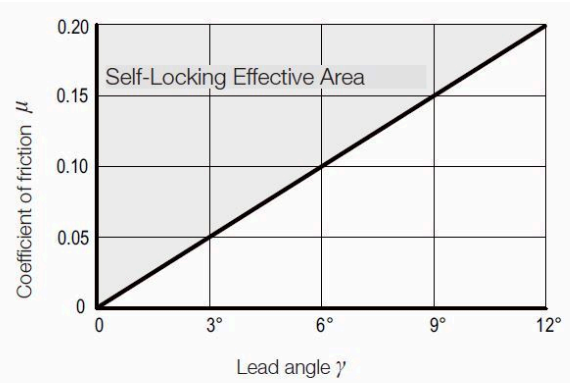


Fig. 4.23 The critical limit of self-locking of lead angle g and coefficient of friction m

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