Investigating the Link Between Covariance structure and Risk-Based Portfolio Performance

Nathan Potgieter[a]

[a]*Stellenbosch University, Stellenbosch, South Africa*

Abstract

This work uses Monte Carlo methods to design and simulate financial market returns for five distinctive markets types, each relating to a unique covariance structure. The equal weight, minimum variance, inverse variance, equal risk contribution and maximum diversification portfolios are each back tested in the simulated markets and the relationship between the portfolio return characteristics and the market covariance structure is evaluated. **FINDINGS**

Keywords: Monte Carlo, Risk-based Portfolios, Portfolio Selection, Copula

JEL classification L250, L100

1. Introduction

Since Harry Markovitz's (1952) seminal work on mean-variance portfolios scholars from around the globe have been aspiring to develop a robust algorithm capable of situating a portfolio on the efficient frontier *ex ante*. There are now a wide array of available alternatives portfolio optimisers raging from simple heuristic based approaches to advanced mathematical algorithms based on quadratic optimization, random matrix theory and machine learning methods; with many more are still in the making.

Unfortunately, portfolio optimisers of the mean-variance type suffer from sensitivity issues from even slight changes in their expected return forecast, which are naturally extremely difficult, if not impossible, to forecast (De Prado, 2016). Due to these issues, this work only evaluates the risk-based portfolio strategies, defined by De Carvalho *et al.* (2012a) as "systemic quantitative approaches to portfolio allocation" that solely rely on views of risk when allocating capital. These strategies therefore, do not require expected return forecasts and are said to be more robust to estimation error. Despite their

sole focus on risk mitigation risk-based portfolio's often perform surprising well, form a total return standpoint, in empirical back tests (Choueifaty *et al.*, 2013).

Furthermore, instead of opting for the standard emperical approach of evaluating said strategies through the use of historical back tests, this work opts to use Monte Carlo simulation methods to investigate the link between the markets covairiance structure and portfolio performance. Monte Carlo methods prove to be invaluable in answering this question as they allow for the creation of *ad hoc* markets with predetermined risk return characteristics, and hence the researcher is left with no uncertainty regarding the composition of the market. This creates an environment ideal for experimentation as the researcher has control over the market and can therefore adjust the independent variable, in this case the markets covariance matrix, and observe the response in the dependent variable, which in this work are the portfolio return characteristics.

The risk-based portfolios evaluated in this work include the naive equal weight (EW), minimum variance (MV), inverse variance (IV), equal risk contribution (ERC) and the maximum diversification (MD) portfolios. Section 2 lays out this works aims and objects, Section 3 provides a review of the relevant literature. This includes some general issues plaguing the field of portfolio optimization, the rational and theoretical underpinnings behind the five risk-based portfolios, their relative performance in empirical back tests and finally the importance of using Monte Carlo methods in finance. Section 4 discusses the methodology used to uncover the relationship of interest, Section 5 provides and discusses the results and finally Section 6 concludes.

2. Aims and Objectives

This work aims to use Monte Carlo Methods to uncover the relationship between a market's covariance structure and the risk return properties of various risk-based portfolio algorithms. This will be achieved through the following objectives:

1. Design and create four distinctive *ad hoc* correlation matrices and build one empirical *50 by 50* correlation matrix, each representing a market with a different risk structure. These from markets structure possessing no clusters to those exhibiting hierarchical clustering.
2. Use the R package *MCmarket* to perform Monte Carlo Simulations, using each of the five correlation matrices from step one as the primary input (Potgieter, 2020). The markets will be built to posses student t multivariate distributions, with 3 degrees of freedom. Meanwhile the individual asset returns will each be normally distributed with a random mean and standard deviation. Each market type will be simulated 10 000 times across 300 periods.
3. Use the simulated market data to calculate the returns obtained from various risk based portfolio's. The first **100** periods will be used estimate an out of sample covariance matrix, this will be used to calculate portfolio weights. These weights will remain for the next 50 periods after

which portfolios will be rebalanced by looking back *100* periods, recalculating the covariance matrix and the new portfolio weights. This process is repeated until all *300* simulated periods have been considered. Therefore, each portfolio will end up with a series of *199* returns.

4. The performance of each portfolio will then be compared and contrasted using various portfolio risk/return analytics. Portfolio optimisers will be compared with each other within market types and with themselves across markets types.

3. Litrature Review

3.1. A Review of Portfolio Optimisation Algorithms

3.1.1. Introduction

Since Harry Markovitz's (1952) seminal work on mean-variance portfolios scholars from around the globe have been aspiring to develop a robust algorithm capable of situating a portfolio on the efficient frontier *ex ante.* There are now a wide array of available alternatives portfolio optimisers raging from simple heuristic based approaches to advanced mathematical algorithms based on quadratic optimization, random matrix theory and machine learning methods; with many more are still in the making (De Prado, 2016).

This literature review will cover some common issues discussed within the literature surrounding portfolio optimization in general, the five risk-based portfolios evaluated in this work, their respective performance in both empirical back tests and Monte Carlo studies and finally the importance of using Monte Carlo methods within finance will be discussed.

3.1.2. Common Issues Portfolio Optimizers

When working with in sample data, Portfolio optimization tends to be a perfect science, but out of sample it becomes more of an art form where it is often preferable to use heuristic over hard rules. This section highlights some general issues, highlighted within the portfolio optimization literature, that tend to worsen their performance out of sample.

Firstly, mean-variance optimisers, like those introduced by Markowitz (1952), rely heavily on the accuracy of their expected return forecasts. Small changes in their expected return input can lead to large changes in portfolio weights (De Prado, 2016). Since in practice expected returns are extremely difficult, if not impossible, to accurately estimate, this issue serves as a major hindrance to their wide spread use. Due to this issue the so-called risk based portfolio's that intentionally avoid using expected return forecasts have garnered a lot of attention (Maillard, 2010).

Unfortunately, these risk based portfolios are not void of issues. The quadratic programming methods used in many portfolio optimisers, including the mean variance and many risk-based, require the inversion of some positive-definite covariance matrix. This positive definiteness requirement can cause issues as covariance matrices estimated on empirical data are sometimes not positive definite, in which case their inverse does not exist and these portfolio's don't have solutions (De Prado, 2016). A common method to get around this issue is to simply compute the nearest positive definite matrix and use that instead (Bates & Maechler, 2019; Higham, 2002).

The covariance estimation step is particularly susceptible to error if the covariance matrix suffers from a high condition number. A condition number is defined as the absolute value of the ratio between a covariance matrix's largest and smallest eigenvalues (Bailey & Lopez De Prado, 2012; De Prado, 2016). The condition number is smallest in diagonal matrices (they have a condition number of 1) and it increases as more correlated variables are added. When working with high condition number matrices a small change in a single entry's estimated covariance can greatly alter its inverse, which in turn can effect the portfolio weights (De Prado, 2016). This is exacerbated by the fact that covariance matrices themselves are prone to estimation error (Zhou *et al.*, 2019). For a sample with a given number of periods, larger dimension covariance matrices are prone to more noise in estimation. This is essentially due to a reduction in degrees of freedom as a sample of at least $1/2N(N+1)$ independent and identically distributed (iid) observations are required to estimate an $N \times N$ covariance matrix (De Prado, 2016: 60)]. Furthermore, financial market covariance structures tend to vary over time and have been know to change rapidly during so-called regime changes (De Prado, 2016). This exacerbates the issue of requiring a large number of observations when estimating the covariance matrix, as there is no guarantee that passed data will be a good refection of the future and looking further into the passed decreases the likelihood of it being so.

". Markowitz's curse is that the more correlated investments are, the greater is the need for a diversified portfolio—and yet the greater are that portfolio's estimation errors. (De Prado, 2016)"

3.1.3. Risk Based Portfolio's

This section reviews the intuition and technical underpinnings within the litrature surrounding the so-called risk-based portfolios. These include the equal weight (EW), minimum variance (MV), inverse volatility (IV), equal risk contribution and the maximum diversification portfolios. The EW is a simple heuristic approach, the minimum variance is more akin to a Markovitz (1952) mean variance portfolio, while the inverse-variance (IV), equal risk contrition (ERC) and maximum diversification (MD) are quite similar in that they each assume that adequate diversification can be obtained by allocating equal risk to each investible security.

3.1.3.1. Naive Equal Weight (EW). Perhaps the oldest and most simple portfolio diversification heuristic constitutes holding a weight of $1/N$ of the N total assets available to the investor (DeMiguel *et al.*, 2009). Therefore, this strategy doesn't consider the data when calculating weights and doesn't involve any form of optimization (DeMiguel *et al.*, 2009). In layman's terms this strategy can be described as putting an equal number of eggs in each available basket. This portfolio is commonly called the equal weight or 1/N portfolio, but its failure to recognize the importance of both the asset variance and the covariance between assets has resulted in it also being referred to as the naive portfolio. Meanwhile its simplicity means that it has been widely used as a benchmark portfolio. From a mean variance perspective equal weighting is optimal when there is no correlation between securities and each possesses the same variance. In this scenario, the EW is equivalent to the MV portfolio.

3.1.3.2. Minimum Variance (MV). Portfolio optimisers designed to exhibit the minimum variance have more recently garnered a lot of attention, largely due their tendency to achieve surprisingly high returns in historical back tests (Clarke *et al.*, 2011). This performance has been attributed to the empirical phenomenon that low volatility stocks tend to earn returns in excess of the market, and high beta stocks tend not to be rewarded by higher returns (Clarke *et al.*, 2011; Fama & French, 1992). These findings are contrary to financial economic theory. For example, the minimum variance (MV) portfolio tends to achieve cumulative returns equal to or slightly greater than market capitalization weighted portfolio's whilst maintaining consistently lower variance and achieving a noticeable improvement in downside risk mitigation, even during times of financial crisis (Clarke *et al.*, 2011). Interestingly, the MV portfolio is the only portfolio of the efficient frontier that does not depend on expected return forecasts(De Prado, 2016).

The minimum variance portfolio selects security weights such that the resulting portfolio corresponds to that with the lowest possible in sample volatility. Therefore, it has the lowest expected volatility and is, in theory, safest/least risky portfolio (De Carvalho *et al.*, 2012b). Its primary input is a variance covariance matrix, which it uses to minimize aggregate portfolio volatility. This is accomplished by over-weighting low volatility and low correlation securities (De Carvalho *et al.*, 2012b).

Let $\sum$ indicate the markets variance covariance matrix and $w = \{w_i, ..., w_N\}$ be a vector of length N containing individual security weights. The vector containing MV portfolio weights can new be described as (De Carvalho *et al.*, 2012b):

$$w^* = arg\min(w'\sum w) \quad s.t. \ \sum_i^N w_i = 1$$

This approach often works well out of sample, but if left unrestricted is known to build highly concentrated portfolio's (De Prado, 2016). Its sole objective to minimize portfolio volatility has been cited as the primary reason for this. When near trough of its objective function it to achieves minor

reductions in *ex ante* volatility by greatly favoring a small number of low volatility/correlation securities (De Prado, 2016: 68)]. This tendency to produce highly concentrated portfolio's be costly out of sample as the portfolio has not sufficiently diversifies idiosyncratic risk, it puts too many eggs in a small number of baskets. In practice This issue can be countered by applying cleaver maximum and minimum constraints on portfolio weights.

3.1.3.3. Inverse-Varience (IV) Weighting. The IV portfolio, referred to as the equal-risk budget (ERB) portfolio in De Carvalho *et al.* (2012a), aims to allocate an equal risk budget to each investible security (De Carvalho *et al.*, 2012a). Where the risk budget is defined as the the product of a the security's weight and volatility. Therefore, if we define σ_i as security i's volatility, then marginal volatility is equally distributed across N securities by setting security weights as such:

$$w_{iv} = \left(\frac{1/\sigma_1}{\sum_{j=1}^{N} 1/\sigma}, ..., \frac{1/\sigma_N}{\sum_{j=1}^{N} 1/\sigma} \right)$$

By this definition each portfolio's weight is proportional to its the inverse of its variance (hence its name). The IV portfolio's weighting strange implies that adequate diversification is attained when allocating capital according to individual security variances and thereby implying that it ignores the role that co-variations between securities on portfolio volatility.

De Carvalho *et al.* (2012a) found that, if all securities posses the same sharp ratio and correlation coefficients between each security are equal, then the IV portfolio is efficient from a mean variance stand point and obtains the highest possible sharp ratio.

3.1.3.4. Equal Risk Contribution (ERC). The ERC portfolio is similar to the IV, but also takes the covariance between securities into account when balancing risk contributions (De Carvalho *et al.*, 2012a). The basic idea behind the ERC is to weight the portfolio such that each security contributes equally to overall portfolio risk, which in turn maximises risk diversification (Maillard, 2010). Generally speaking the ERC acts similar to a weight constrained MV portfolio, with constraints ensuring that an adequate level idiosyncratic risk is diversification. Following Maillard (2010), the weights of an ERC portfolio $x = (x_1, x_2, ..., x_n)$ consisting of n assets can be calculated as follows:

let σ_i^2 resemble asset i's variance, σ_{ij} the covariance between asset i and j and $\sum$ be the markets variance covariance matrix. Portfolio risk can now be written as $sigma(x) = \sqrt{x^T \sum x} = \sum_i \sum_{j \neq i} x_i x_j \sigma_{ij}$ and the marginal risk contribution $\partial_{x_i} \sigma(x)$ can then be defined as as such:

$$\partial_{x_i} \sigma(x) = \frac{\partial \sigma(x)}{\partial x_i} = \frac{x_i \sigma_i^2 + \sum_{j \neq i} x_j \sigma_{ij}}{\sigma(x)}$$

Therefore, $\partial_{x_i}\sigma(x)$ refers to the change in portfolio volatility resulting from a small change in asset i's weight (Maillard, 2010). ERC uses this definition to guide its algorithms central objective to equate the risk contribution for each asset in the portfolio *ex ante*. No closed form solution exists describing the weigts of the ERC portfolio, however, if we define $(\sum x)_i$ as the ith row resulting from the product of $\sum$ with x and note that $\partial_{x_i}\sigma(x) = (\sum x)_i$, then the optimal weight for the long only ERC can be described as those that satisfy the following statement:

$$x^* = \{x\ \epsilon[0,1]^n : \sum x_i = 1, x_i \times (\sum x)_i = x_j \times (\sum x)_j\ \forall\ i,j\}$$

Maillard (2010) proved mathematically that the ERC portfolio's *ex ante* volatility is always some where between those of the EW and MV portfolio's. Thereafter, De Carvalho *et al.* (2012a) found that, if all securities posses the same sharp ratio , then the ERC and ERB have identical portfolio weights. If in addition the correlation coefficients between all securities are equal, then the ERC and ERB merge into the EW portfolio and each are mean variance efficient with the maximum sharp ratio (De Carvalho *et al.*, 2012a).

3.1.3.5. Maximum Diversification (MD). Choueifaty & Coignard (2008) originally designed the MD portfolio to maximize some diversification ratio (DR), hwich he defigned as the sum of each securities risk bucket divided by portfolio volatility (De Carvalho *et al.*, 2012a). If we define $w = (w_1,...w_N)^T$ as a vector of portfolio weights, V as a vector of asset volatilities and $\sum$ as the covariance matrix. Then the DR can be expresses as:

$$DR = \frac{w'.V}{\sqrt{w'Vw}}$$

Therefore, much like the IV and ERC portfolio's, the MD portfolio attempts to diversify the portfolio by allocating equal risk to each security (Choueifaty & Coignard, 2008). The MD portfolio accomplishes this by over-weighting low volatility securities and those that are less correlated with other stocks (De Carvalho *et al.*, 2012a). For further detail regarding the theoretical results and properties of the MD portfolio see Choueifaty & Coignard (2008: 33–35).

3.2. Empirical Backtests and Monte Carlo Findings

Choueifaty *et al.* (2013) conducted an empirical back test comparing the relative performance if numerous portfolio optimisers between 1999 and 2010. They used historical data from the MSCI World world index and considered the largest 50% of assets at each semi-annual rebalance date. To reduce the noise in estimation, at each rebalance date covariance matrices were estimated using the

previous years worth of data (Choueifaty *et al.*, 2013). These were then used as the primary inputs in estimating the respective portfolio weights, which were restricted to long only. The MV portfolio achieved an annual return of 6.7% and outperformed the ERC and EW portfolio's who returned 6.3% and 5.8% respectively. Unsurprisingly, the MV portfolio possessed the lowest daily volatility (10%) followed by the ERC and then the EW portfolio's (with 12.9% and 16.4% respectively). Accordingly the MV portfolio scored the highest sharp ratio (0.36) followed by the ERC and EW portfolio's (0.24 and 0.16 respectively). According to De Carvalho *et al.* (2012a) the performance of the EW portfolio primarily depends on the premium on small-capitalization stocks, thereby suggesting that the relatively poor performance of the EW portfolio in Choueifaty *et al.* (2013), can be attributed to the relatively poor returns achieved by the smaller stocks in the MSCI world index.

Despite the simplistic nature of the EW portfolio empirical studies, like those by DeMiguel *et al.* (2009), tend to find a statistically insignificant difference in Sharp ratio between the naive portfolio and more advanced portfolio optimisers. DeMiguel *et al.* (2009) the EW ro the the mean-variance, MV and Bayes-Stein portfolios, where the EW also performed surprisingly well from a total return perspective.

Due to the aforementioned issues surrounding estimation error in a market's covariance matrix Ardia *et al.* (2017) set out to evaluate the impact of covariance matrix misspecification on the properties of risk based portfolio's. They used Monte Carlo methods to build six distinctive investment universes, each with a unique, variance/correlation structure and a varying number of assets. Numerous covariance matrix estimation techniques were then estimated on the simulated data, one of which serving as the benchmark. They then used the simulated data and the various covariance matrices to access the impact of alternative covariance specifications on the performance of the MV, IV, ERC and MD portfolio's. The ERC and IV portfolios were found to be "relatively robust to covariance misspecification", the MV was found to be sensitive to misspecification in both the variance and covarienve and the MD portfolio was found to be robust to misspecification in the variances but sensitive to misspecification in the covariances (Ardia *et al.*, 2017: 1).

3.3. Monte Carlo Methods in Portfolio Optimisation

Ever since the pioneering age of computers people have shown a keen interest in using their ability to perform rapid calculations to conduct randomized experiments (Kroese *et al.*, 2014: 1). The core of Monte Carlo simulation is in the creation of random objects or processes using a computer. There are a number of reasons for doing this, but the primary one used in this work and thereby discussed in this review is of the sampling kind (Kroese *et al.*, 2014). This typically involves the modeling of some stochastic object or process, followed by sampling from some probability distribution and the manipulating said sample through some deterministic process such that the result mimics the true underlying process. The primary idea behind Monte Carlo simulation is to repeat this simulation

process many times so that interesting properties can be uncovered by invoking the law of large numbers and central limit theorem.

A fiancial application of this can be found in Wang *et al.* (2012) who designed a Monte Calro procedure that (1) models both the time-series and cross-section properties of financial market returns, this involves the estimation of a random term's probability distribution function (pdf) using extreme value theory. And (2) sampling from the modeled process to produce an ensemble of market returns, with each exerting the same risk properties. The simulated data can then be used in risk management and/or the pricing of financial securities (Kroese *et al.*, 2014; Wang *et al.*, 2012). This unique ability to generate a large number of counterfactuals for an asset market with a known risk structure has made it a uniquely powerful tool in accessing the properties of portfolio optimization algorithms (Bailey & Lopez De Prado, 2012).

Glasserman (2013) is a useful source for understanding the methods and applications of Monte Carlo methods in finance.

4. Methadology

This work used Monte Carlo simulation methods to investigate the link between a markets correlation structure and the relative performance of the EW, MV, IV, ERC and MD portfolios. To avoid possible confusion note the following terminology: the term market refers to the daily returns for a number of assets, but since this is a Monte Carlo study, a market can be thought of as a single observation. The term market type refers to an set or ensemble of markets each with the same specified same risk characteristics.

The R package MCmarket was used to simulate five distinctive market types, each corresponding to a unique correlation structure (Potgieter, 2020). Four of the correlation matrices were designed *ad hoc*, to posses a unique correlation structure, while the fifth was estimated using empirical data from the constitutes of the S&P 500. These correlation matrices range from one exhibiting no correlation (i.e. a diagonal matrix) to one with hierarchical clustering (see 4.1).

The long only EW, MV, IV, ERC and MD portfolios were then backtested on the simulated markets and portfolio analytics were performed for each portfolio type in each market within the five market types. These portfolio analytics include the standard deviation (sd) of daily returns, downside deviation, value at risk (VaR), conditional VAR (CVaR), Sharp ratio, average drawdown and maximum drawdown. The mean and median of the portfolio metrics are then calculated for each market type.

Finally, the portfolio metrics are compared within portfolio types across market types and within market types across portfolio types.

4.1. Correlation Structures

This section first describes the composition and attributes behind the four *ad hoc* correlation matrices used in this study (section 4.1.1). Secondly, the methodology behind the estimation of the empirical correlation matrix (section \ref{emp) is discussed and finally each of their top 10 eigenvalues are listed in Table 4.1.

4.1.1. Ad Hoc

This section describes the four *ad hoc* 50 by 50 correlation matrices used as the key inputs in their respective Monte Carlo simulations. See Figure 4.1for a graphical representation of each correlation matrix. Note that the *gen_corr* function from the R package *MCmarket* was used in the construction of the four *ad hoc* matrices (Potgieter, 2020).

The first and most simplistic of the four matrices is a diagonal matrix (see Diagonal Matrix in Figure 4.1). It describes a market with a zero correlation coefficient between each asset. Each of its 50 eigenvalues are 1, it has no risk clusters and is therefore plenty scope for diversification.

The second matrix (labeled No Clusters in Figure 4.1) has no risk clusters but describes a market with significant correlation between its constituents. Each asset has a correlation of 0.9 with is closest neighbor (i.e. Asset 1 and 2, 5 and 6 and 11 and 12 each have a pairwise correlation coefficient of 0.9). Correlations then diminish exponentially by the absolute distance between the two assets (i.e. the correlation between Asset 1 and 5 is $0.9^{|1-5|} = 0.6561$). Its has a large first eigenvalue of 15.93, but they quickly diminish such that its 9th largest eigenvalue is less than 1 at 0.79. **COMMENTS**

The third matrix (labeled Five Clusters in Figure 4.1) contains five distinctive non-overlapping risk clusters. Assets within the same cluster have a pairwise correlation coefficient of 0.6 while those that are not in the same cluster are uncorrelated. Its first five eigenvalues are 6.5, with the remaining 45 equal to 0.4.

The final *ad hoc* correlation matrix has three layers of overlapping risk clusters. The first layer has 10 distinctive clusters, within which assets have a correlation coefficient of 0.7. The second layer has four clusters where assets that are not in same first layer cluster have a correlation coefficient of 0.5. Assets that are in the same third layer cluster but not clustered in layers one and two have a correlation coefficient of 0.3. Finally, those who do not share any cluster have a correlation coefficient of 0.05. Its largest eigenvalue is 14.36, but they diminish fairly quickly as its third largest is only 3.3.

See Table 4.1 for a list of each correlation matrices largest ten eigenvalues.
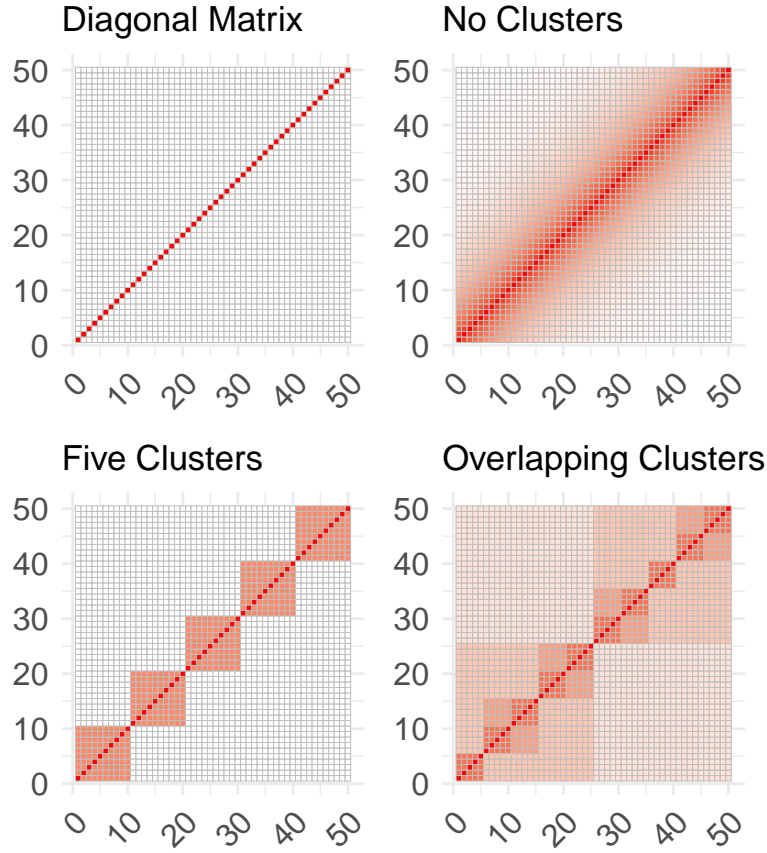
ad hoc Correlation Matrices

Figure 4.1: Correlation Matricies

4.1.2. Emperical

The empirical correlation matrix used in this study was estimated from the daily returns of a random subset of 50 of the largest (by market capitalization) 100 S&P 500 stocks between 1 January 2016 and 1 January 2021. The market capitalizations were measured as of 12 January 2020. The covariance matrix was then calculated using the *fit_mvt* function from the R package *fitHeavyTail*. This function uses maximum likelihood estimation and generalized expectation maximization method to fit a multivariate t-distribution to a matrix of asset returns (Liu & Rubin, 1995).

The estimated multivariate t distribution was found to have 4.43 degrees of freedom and a correlation matrix shown in Figure 4.2. Note that the assets were ordered by hierarchical clustering so the reader can easily visualize the risk clusters. The correlation matrix's largest eigenvalue is 18.6 and they quickly diminish to below zero by its 8th largest eigenvalue.
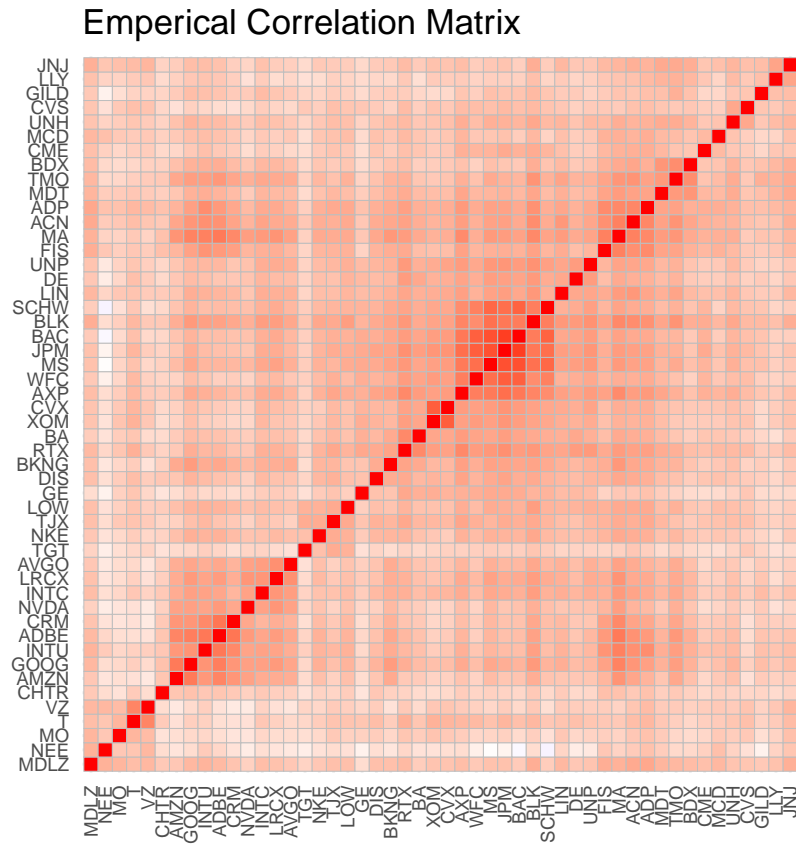
Figure 4.2: Emperical Correlation Matrix

Table 4.1: Eigenvalues

Diagonal	No Clusters	Five Clusters	Overlapping Clusters	Emperical
1	15.93	6.4	14.36	18.6
1	10.38	6.4	6.89	3.09
1	6.28	6.4	3.3	2.52
1	3.92	6.4	3.3	1.4
1	2.59	6.4	2.49	1.28
1	1.81	0.4	2.46	1.21
1	1.32	0.4	1.3	1.03
1	1.01	0.4	1.3	0.92
1	0.79	0.4	1.3	0.88
1	0.64	0.4	1.3	0.83

4.2. Monte Carlo

This section outlines the methodology behind the Monte Carlo simulation performed as part of this study.

This work uses a generalized version of the Monte Carlo procedure developed in Wang *et al.* (2012). This framework was build into the R package *MCmarket* which was used to carry out the simulations in this project (Potgieter, 2020). The following briefly outlines the construction of the simulated markets:

An Elliptical t copula with 4.5 degrees of freedom is used, in conjunction with one of each of the five previously discussed correlation matrices, to specify the each market types multivariate distribution. 300 random uniformly distributed draws, corresponding to 300 trading days, were then taken from the specified multivariate distributions described by the t copula. The random uniformly distributed observations are then transformed, via the inverted normal cumulative distribution function, into normally distributed observations (Wang *et al.*, 2012: 3). **The expected returns are set by randomly selecting 50 observations from the normal distribution with a mean of 0.02 and sd of 0.01. While the asset sd's are set by**

4.3. Back Tests

This section describes the back testing procedure used to calculate the returns obtained by the respective EW, MV, IV, ERC and MD portfolios.

To remain consistent with the literature, a long-only weight constraint was applied to all portfolio's. An additional constraint limiting the maximum weight allocation for a single security to *20%* is also applied to prevent some portfolios from building unreasonably highly concentrated holdings, while remaining flexible enough to punish those who do so. Therefore, these constraints act to provide a fair playing ground for the portfolio's to compete. The back testing procedure works as follows:

The first 100 periods are used to estimate the covariance matrix using the maximum likelihood methodology described in Liu & Rubin (1995). Interestingly, the assumption regarding the multivariate t distribution are correct by definition as the distribution used to for the monte Carlo simulations. This covariance matrix is then used as the sole input when calculating the respective portfolio weights. These weights are then used to calculate portfolio returns over the next 50 periods, when each portfolio is replaced by looking back another 100 periods to recalculated the covariance matrix. This process is repeated until the data is exhausted. Since 300 periods were simulated for each market each portfolio is weighted four times and 200 periods of returns are calculated for each portfolio within each market.

4.4. Portfolio Analytics

This section describes the portfolio performance metrics used to evaluate and compare the respective portfolio's.

SD simple and widely accepted method of measuring portfolio volatility.

The 95% Value at risk (VaR) is another risk metric used to evaluate the portfolio risk. It is one of the financial industry standards for measuring the downside risk (Peterson & Carl, 2020). It can be interpreted as the maximum return expected from in the worst 5% of scenarios, that is, in such a scenario, one should expect to loose at least their 95% value at risk. The Gaussian VaR is calculated by assuming that returns are normally $N(\mu, \sigma)$ distributed, where μ and σ are estimated using historical data. In this study this assumption is correct by definition and will therefore result in an accurate estimate of downside risk. However, one must be careful when extrapolating these results as fairly contentious in real world returns.

The 95% conditional value at risk (CVaR) or expected shortfall (ES) is another downside risk measure used to compare portfolio's. It is interpreted as the expected loss given that the portfolio is in the work 5% of scenarios.

The sharp ratio is calculated by dividing the portfolio return by some risk measure and is therefore interpreted as the return per unit of risk. In this work risk is measured as the sd of portfolio returns.

The maximum drawdown is calculated by first, calculating portfolio cumulative returns and the maximum cumulative return achieved. The maximum drawdown is then the maximum amount that the cumulative return dips below its maximum, it is measured as a percentage of the maximum cumulative return (Peterson & Carl, 2020).

5. Results and Discussion

5.1. Diag

With such a market structure it is reasonable to expect that, depending if the asset variance allocation is fairly evenly distribution then the EW, MV, IV, ERC and MD will produce fairly similar.

6. Conclusion

I hope you find this template useful. Remember, stackoverflow is your friend - use it to find answers to questions. Feel free to write me a mail if you have any questions regarding the use of this package.

To cite this package, simply type citation("Texevier") in Rstudio to get the citation for Katzke (2017) (Note that united references in your bibtex file will not be included in References).

References

Ardia, D., Bolliger, G., Boudt, K. & Gagnon-Fleury, J.-P. 2017. The impact of covariance misspecification in risk-based portfolios. *Annals of Operations Research*. 254(1-2):1–16.

Bailey, D.H. & Lopez De Prado, M. 2012. Balanced baskets: A new approach to trading and hedging risks. *Journal of Investment Strategies (Risk Journals)*. 1(4).

Bates, D. & Maechler, M. 2019. *Matrix: Sparse and dense matrix classes and methods.* ed. [Online], Available: https://CRAN.R-project.org/package=Matrix.

Choueifaty, Y. & Coignard, Y. 2008. Toward maximum diversification. *The Journal of Portfolio Management*. 35(1):40–51.

Choueifaty, Y., Froidure, T. & Reynier, J. 2013. Properties of the most diversified portfolio. *Journal of investment strategies*. 2(2):49–70.

Clarke, R., De Silva, H. & Thorley, S. 2011. Minimum-variance portfolio composition. *The Journal of Portfolio Management*. 37(2):31–45.

De Carvalho, R.L., Lu, X. & Moulin, P. 2012a. Demystifying equity risk–based strategies: A simple alpha plus beta description. *The Journal of Portfolio Management*. 38(3):56–70.

De Carvalho, R.L., Lu, X. & Moulin, P. 2012b. Demystifying equity risk–based strategies: A simple alpha plus beta description. *The Journal of Portfolio Management*. 38(3):56–70.

DeMiguel, V., Garlappi, L. & Uppal, R. 2009. Optimal versus naive diversification: How inefficient is the 1/n portfolio strategy? *The review of Financial studies*. 22(5):1915–1953.

De Prado, M.L. 2016. Building diversified portfolios that outperform out of sample. *The Journal of Portfolio Management*. 42(4):59–69.

Fama, E.F. & French, K.R. 1992. The cross-section of expected stock returns. *the Journal of Finance*. 47(2):427–465.

Glasserman, P. 2013. *Monte carlo methods in financial engineering.* ed. Vol. 53. Springer Science & Business Media.

Higham, N.J. 2002. Computing the nearest correlation matrix—a problem from finance. *IMA journal of Numerical Analysis*. 22(3):329–343.

Katzke, N.F. 2017. *Texevier: Package to create elsevier templates for rmarkdown.* ed. Stellenbosch,

South Africa: Bureau for Economic Research.

Kroese, D.P., Brereton, T., Taimre, T. & Botev, Z.I. 2014. Why the monte carlo method is so important today. *Wiley Interdisciplinary Reviews: Computational Statistics.* 6(6):386–392.

Liu, C. & Rubin, D.B. 1995. ML estimation of the t distribution using em and its extensions, ecm and ecme. *Statistica Sinica.* 19–39.

Maillard, T., Roncalli. 2010. The properties of equally weighted risk contribution portfolios. *The Journal of Portfolio Management.* 36(4):60–70.

Markowitz, H. 1952. Portfolio selection. *The Journal of Finance.* 7(1):77–91.

Peterson, B.G. & Carl, P. 2020. *PerformanceAnalytics: Econometric tools for performance and risk analysis.* ed. [Online], Available: https://CRAN.R-project.org/package=PerformanceAnalytics.

Potgieter, N. 2020. *MCmarket: Package to simplify the monte carlo simulation of financial markets.* ed. Stellenbosch, South Africa: Stellenbosch University.

Wang, P., Sullivan, R.N. & Ge, Y. 2012. Risk-based dynamic asset allocation withExtreme tails and correlations. *The Journal of Portfolio Management.* 38(4):26–42.

Zhou, R., Liu, J., Kumar, S. & Palomar, D.P. 2019. Robust factor analysis parameter estimation. In ed. Springer *International conference on computer aided systems theory.* 3–11.

Appendix

6.0.1. Hierarchical Risk Parity (HRP)

Due to the multitude of robustness issues related to traditional portfolio optimisers, De Prado (2016) developed a new approach incorporating machine-learning methods and graph theory (**???**). De Prado (2016) argues that the "lack of hierarchical structure in a correlation matrix allows weights to vary freely in unintended ways" and that this contributes to the instability issues. His HRP algorithm requires only a singular co-variance matrix and can utilize the information within without the need for the positive definite property (De Prado, 2016). This procedure works in three stages:

De Prado (2016) carried out an in sample simulation study comparing the respective allocations of the long-only minimum variance, IVP and HRP portfolios using a co-variance matrix using a condition number that is "not unfavourable" to the minimum variance portfolio. The simulated data consisted of 10000 observations across 10 variables. The following findings were made: The minimum variance portfolio concentrated 92.66% of funds in the top 5 holdings and assigned a zero weight to 3 assets. *Conversly*, HRP only assigned 62.5% of its funds to the top 5 holdings (De Prado, 2016). The minimum variance portfolio's objective function causes it to build highly concentrated portfolio's in favor of a small reduction in volatility; the HRP portfolio had only a slightly higher volatility (De Prado, 2016). This apparent diversification advantage achieved by the minimum variance portfolio is rather deceptive as the portfolio remains highly susceptible to idiosyncratic risk incidents within its top holdings (De Prado, 2016). This claim was further validated by the finding that HRP achieved significantly lower out of sample variance compared to the minimum variance portfolio.

Appendix A

Some appendix information here

Appendix B