Untitled

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Equally Risk Contributions (ERC)

The ERC portfolio is another optimiser that intentionally avoids using expected returns and is therefore said to be more robust to estimation error [@maillard2010]. The basic idea behind the ERC is to weight the portfolio such that each security contributes equally to risk, which in turn maximises risk diversification [@maillard2010]. Generally speaking the ERC acts similar to a weight constrained minimum variance portfolio, with constraints ensuring that adequate diversification is maintained. The weights of the ERC portfolio $x = (x_1, x_2, ..., x_n)$ consisting of n assets is calculated as follows.

let σ_i^2 resemble asset i's variance, σ_{ij} the covariance between asset i and j and $\sum$ be the markets variance covariance matrix. Portfolio risk can now be written as $sigma(x) = \sqrt{x^T \sum x} = \sum_i \sum_{j \neq i} x_i x_j \sigma_{ij}$ [@maillard2010]. The marginal risk contribution $\partial_{x_i} \sigma(x)$ can then be defined as follows [@maillard2010]:

$\partial_{x_i} \sigma(x) = \frac{\partial \sigma(x)}{\partial x_i} = \frac{x_i \sigma_i^2 + \sum_{j \neq i} x_j \sigma_{ij}}{\sigma(x)}$

Therefore, $\partial_{x_i} \sigma(x)$ refers to the change in portfolio volatility resulting from a small change in asset i's weight. ERC uses this definition to guide its algorithms central objective to equate the risk contribution for each asset in the portfolio *ex ante*.

If we define $(\sum x)_i$ as the i^{th} row resulting from the product of $\sum$ with x and note that $\partial_{x_i} \sigma(x) = (\sum x)_i$, then the optimal ERC weight can be written as [@maillard2010]:

$x^* = \{x \, \epsilon [0, 1]^n : \sum x_i = 1, x_i \times (\sum x)_i = x_j \times (\sum x)_j \ \forall \ i, j\}$