Evaluating Portfolio Optimisters with Simulated Markets Exhibiting Left-Tail Dependence

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Abstract

Abstract to be written here. The abstract should not be too long and should provide the reader with a good understanding what you are writing about. Academic papers are not like novels where you keep the reader in suspense. To be effective in getting others to read your paper, be as open and concise about your findings here as possible. Ideally, upon reading your abstract, the reader should feel he / she must read your paper in entirety.

Keywords: Multivariate GARCH, Kalman Filter, Copula

JEL classification L250, L100

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1. Introduction

References are to be made as follows: Fama & French (1997: 33) and Grinold & Kahn (2000) Such authors could also be referenced in brackets (Grinold & Kahn, 2000) and together Grinold & Kahn (2000). Source the reference code from scholar.google.com by clicking on "cite'' below article name. Then select BibTeX at the bottom of the Cite window, and proceed to copy and paste this code into your ref.bib file, located in the directory's Tex folder. Open this file in Rstudio for ease of management, else open it in your preferred Tex environment. Add and manage your article details here for simplicity - once saved, it will self-adjust in your paper.

> I suggest renaming the top line after @article, as done in the template ref.bib file, to something more intuitive for you to remember. Do not change the rest of the code. Also, be mindful of the fact that bib references from google scholar may at times be incorrect. Reference Latex forums for correct bibtex notation.

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Contributions:

The authors would like to thank no institution for money donated to this project. Thank you sincerely.

To reference a section, you have to set a label using "\label'' in R, and then reference it in-text as e.g. referencing a later section, Section 4.

Writing in Rmarkdown is surprizingly easy - see this website cheatsheet for a summary on writing Rmd writing tips.

2. Litrature Review

2.1. A Review of Portfolio Construction Algorithms

introduction Since Harry Markovitz's (1952) seminal work on mean variance portfolios scholars around the globe have been aspiring to develop a robust algorithm capable of **ex ante** situating a portfolio on the efficient frontier (**Markovitz?**). There are now a wide array of alternatives, with many more in the making.

These optimisers however are not without their flaws. Mean variance optimisers rely heavily on the accuracy of return forecasts, where small errors often lead to vastly different returns (**lopez?**). Furthermore, quadratic programming methods used in portfolio optimisation require the inversion of some positive-definite covariance matrix, which can be susceptible to error if the covariance matrix suffers from a high condition number[4] (**lopez?**; **lopez2012?**). Diagonal matrices have the smallest condition number which increases as more correlated variables are added. A small change in correlation within any given entry can greatly alter the inverse of a high conditional number matrix, **which in turn can greatly effect the portfolio weights** (**lopez?**). For a given sample size, larger co-variance matrices are prone to more noise in estimation. This si essentially due to a reduction in degrees of freedom as a sample of at least $1/2N(N+1)$ independent and identically distributed (iid) observations are required to estimate an $N \times N$ covariance matrix. This issues is exacerbated by the fact that a markets covariance structure tends to vary over time (**lopez?**).

Due to the sensitivity issues surrounding errors in the expected return estimation, this work will only discuss the portfolio optimisation techniques that forego this input. These include the naive equal weight, inverse variance, hierarchical risk parity, ERC and the minimum variance portfolios. The theoretical underpinnings of each will be reviewed as well as their relative performance in historical back tests.

Minimum Variance Portfolio optimisers designed to exhibit the minimum variance have garnered allot of attention for themselves, largely to their tendency to achieve surprisingly high returns in historical

[4] A condition number is defined as the absolute value of the ratio between a covariance matrix's largest and smallest eigenvalues (**lopez?**)

back tests (**clarke2011?**). This performance has been attributed to the surprising fact that both low volatility stocks tend to earn returns in excess of the market, and that high beta stocks tend not to be rewarded by higher returns (**clark2011?**; **fama1992?**). These minimum variance portfolios tend to achieve cumulative returns equal to or slightly greater than market portfolio's whilst maintaining consistently lower variance and achieving a noticeable improvement in downside risk mitigation in times of crisis (**clarke?**).

Who developed the min variance portfolio??

The weight attributed to an given security in the long-only minimum-variance portfolio is given in Eq. 2.1 (**clarke?**).

$$w_i = \sigma^2_{MV}/\sigma^2_{\epsilon i}(1 - \beta_i/\beta_L) \;\; for \;\; \beta_i < \beta_L \;\; else \; w_i = 0 \tag{2.1}$$

where

- σ^2_{LMV} is the ex ante return variance of the long-only minimum variance portfolio.
- $\sigma^2_{\epsilon i}$ is the ex ante idiosyncratic return variance for security i.
- β_i ex ante market beta for security i.
- β_L is the long-only threshold beta.

The simplicity of Eq. 2.1 can be attributed to the simplifying assumption of single index risk and the expected returns being absent from the objective function. Eq. 2.1 indicates that asset weights depend on two portfolio specific risk parameters, the ex ante variance of the long-only portfolio and the long only threshold beta, and two security specific risk parameters, the market beta and the idiosyncratic return variance (**clarke?**). High market beta and idiosyncratic variance reduce an assets optimal weight towards zero, while a market beta greater than long-only threshold results in a optimal weight of zero (**clark?**). Interestingly empirical results indicate that the long-only minimum variance portfolio tends to apply a zero weight to approximately 80% of investable securities.

mention first rule of investing??

Hierarchical Risk Parity (HRP) Due to the multitude of robustness issues related to traditional portfolio optimisers, (**lopez?**) developed a new approach incorporating machine-learning methods and graph theory (**arevalo?**). (**lopez?**) argues that the "lack of hierarchical structure in a correlation matrix allows weights to vary freely in unintended way" and that this contributes to the instability issues. His HRP algorithm requires only a singular covariance matrix and can utilize the information within without the need for the positive definite property (**lopex?**). This procedure works in three stages:

1) Tree clustering: This involves utilizing a hierarchical tree clustering method such that security weights can be determined downstream through a tree graph (**lopez?**).
2) Quasi-diagonalization: this stage reorganizes the covariances matrix so that similar securities are placed near one another while dissimilar securities are far apart. This is done through an algorithm that situates the largest covariances near the diagonal and smaller covariances further away (**lopez?**).
3) Recursive bisection: Portfolio funds are then allocated top down through the tree structure such that riskier clusters receive fewer funds. This can be accomplished through the inverse-variance allocation (**lopez?**).

(**lopez?**) carried out an in sample simulation study comparing the respective allocations of the long-only minimum variance, IVP and HRP portfolios using a covariance matrix with a condition number that is "not unfavourable" to the minimum variance portfolio. The simulated data consisted of 10000 observations across 10 variables. The following findings were made: The minimum variance portfolio concentrated 92.66% of funds in the top 5 holdings and assigned a zero weight to 3 assets. Conversly, HRP only assigned 62.5% of its funds to the top 5 holdings (**lopez?**). The minimum variance portfolio's objective function causes it to build highly concentrated portfolio's in favour of a small reduction in volatility; the HRP portfolio had only a slightly higher volatility (**lopez?**). This apparent diversification advantage achieved by the minimum variance portfolio is rather deceptive as the portfolio remains highly susceptible to idiosyncratic risk incidents within its top holdings (**lopez?**). This claim was further validated by the finding that HRP achieved significantly lower out of sample variance compared to the minimum variance portfolio.

Inverse Variance Portfolio (IVP)

Data

Notice how I used the curly brackets and dash to remove the numbering of the data section.

Discussion of data should be thorough with a table of statistics and ideally a figure.

In your tempalte folder, you will find a Data and a Code folder. In order to keep your data files neat, store all of them in your Data folder. Also, I strongly suggest keeping this Rmd file for writing and executing commands, not writing out long pieces of data-wrangling. In the example below, I simply create a ggplot template for scatter plot consistency. I suggest keeping all your data in a data folder.

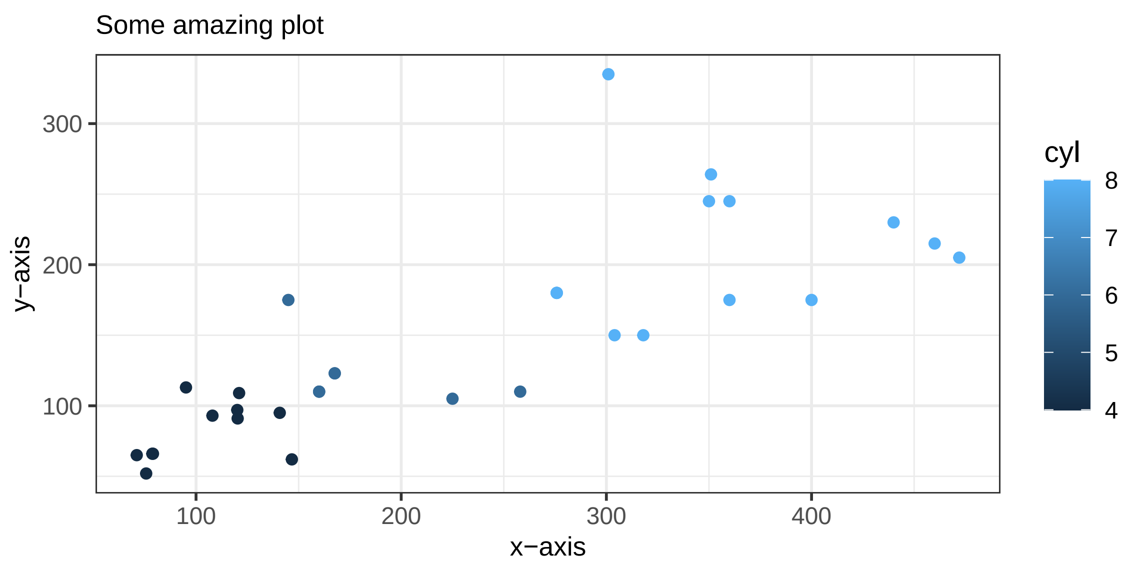


Figure 2.1: Caption Here

To make your graphs look extra nice in latex world, you could use Tikz device. Replace dev - 'png' with 'tikz' in the chunk below. Notice this makes the build time longer and produces extra tex files - so if you are comfortable with this, set your device to Tikz and try it out:

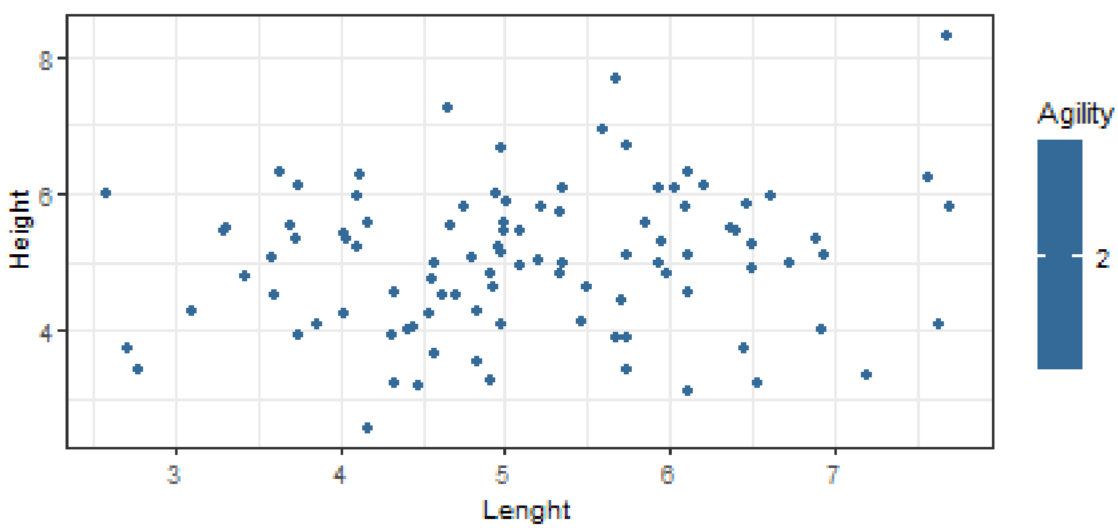


Figure 2.2: Caption Here

To reference the plot above, add a "\label'' after the caption in the chunk heading, as done above. Then reference the plot as such: As can be seen, Figures 2.1 and 2.2 are excellent, with Figure 2.2

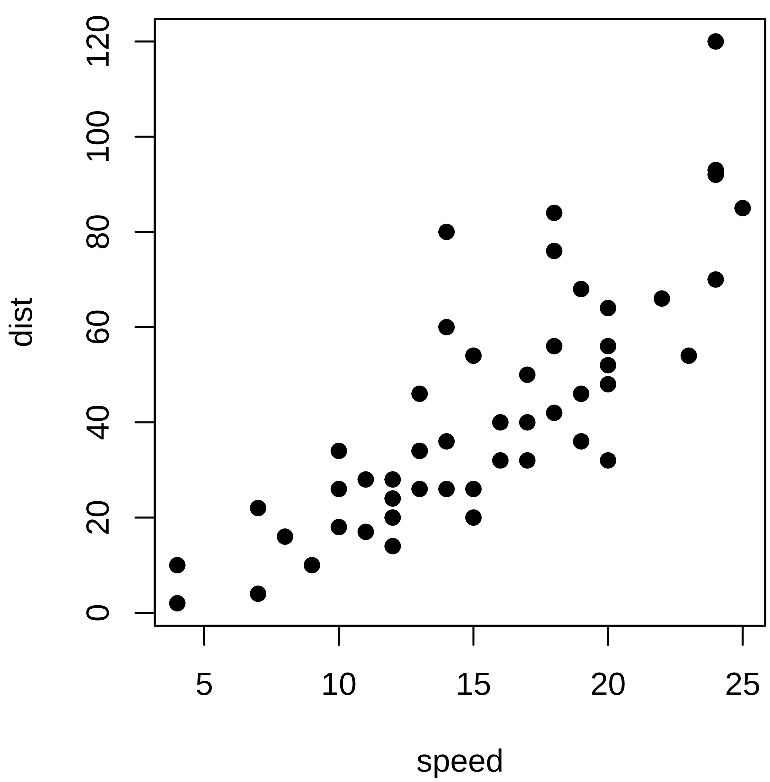
being particularly aesthetically pleasing due to its device setting of Tikz. The nice thing now is that it correctly numbers all your figures (and sections or tables) and will update if it moves. The links are also dynamic.

I very strongly suggest using ggplot2 (ideally in combination with dplyr) using the ggtheme package to change the themes of your figures.

Also note the information that I have placed above the chunks in the code chunks for the figures. You can edit any of these easily - visit the Rmarkdown webpage for more information.

3. Splitting a page

You can also very easily split a page using the format below:



Data

The figure on the left-hand side shows the `cars` data.
Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed do eiusmod tempor incididunt ut labore et dolore magna aliqua. Ut enim ad minim veniam, quis nostrud exercitation ullamco laboris nisi ut aliquip ex ea commodo consequat.

4. Methodology

4.1. Subsection

Ideally do not overuse subsections. It equates to bad writing.[5]

4.2. Math section

Equations should be written as such:

$$
\begin{aligned}
\beta &= \sum_{i=1}^{\infty}\frac{\alpha^2}{\sigma_{t-1}^2} \\
\int_{x=1}^{\infty} x_i &= 1
\end{aligned}
\tag{4.1}
$$

If you would like to see the equations as you type in Rmarkdown, use $ symbols instead (see this for yourself by adjusted the equation):

$$\beta = \sum_{i=1}^{\infty}\frac{\alpha^2}{\sigma_{t-1}^2} \int_{x=1}^{\infty} x_i = 1$$

Note again the reference to equation 4.1. Writing nice math requires practice. Note I used a forward slashes to make a space in the equations. I can also align equations using **&**, and set to numbering only the first line. Now I will have to type "begin equation' ' which is a native LaTeXcommand. Here follows a more complicated equation:

$$
\begin{aligned}
y_t &= c + B(L)y_{t-1} + e_t \\
e_t &= H_t^{1/2} z_t; \quad z_t \sim N(0, I_N) \quad \& \quad H_t = D_t R_t D_t \\
D_t^2 &= \sigma_{1,t}, \dots, \sigma_{N,t} \\
\sigma_{i,t}^2 &= \gamma_i + \kappa_{i,t} v_{i,t-1}^2 + \eta_i \sigma_{i,t-1}^2, \quad \forall i \\
R_{t,i,j} &= diag(Q_{t,i,j}{}^{-1}).Q_{t,i,j}.diag(Q_{t,i,j}^{-1}) \\
Q_{t,i,j} &= (1-\alpha-\beta)\bar{Q} + \alpha z_t z_t' + \beta Q_{t,i,j}
\end{aligned}
\tag{4.2}
$$

[5] This is an example of a footnote by the way. Something that should also not be overused.

Note that in 4.2 I have aligned the equations by the equal signs. I also want only one tag, and I create spaces using “quads’ ’.

See if you can figure out how to do complex math using the two examples provided in 4.1 and 4.2.

5. Results

Tables can be included as follows. Use the xtable (or kable) package for tables. Table placement = H implies Latex tries to place the table Here, and not on a new page (there are, however, very many ways to skin this cat. Luckily there are many forums online!).

	mpg	cyl	disp	hp	drat	wt	qsec	vs	am	gear	carb
1	21.00	6.00	160.00	110.00	3.90	2.62	16.46	0.00	1.00	4.00	4.00
2	21.00	6.00	160.00	110.00	3.90	2.88	17.02	0.00	1.00	4.00	4.00
3	22.80	4.00	108.00	93.00	3.85	2.32	18.61	1.00	1.00	4.00	1.00
4	21.40	6.00	258.00	110.00	3.08	3.21	19.44	1.00	0.00	3.00	1.00
5	18.70	8.00	360.00	175.00	3.15	3.44	17.02	0.00	0.00	3.00	2.00

Table 5.1: Short Table Example

To reference calculations **in text**, do this: From table 5.1 we see the average value of mpg is 20.98.

Including tables that span across pages, use the following (note that I add below the table: “continue on the next page’ ’). This is a neat way of splitting your table across a page.

Use the following default settings to build your own possibly long tables. Note that the following will fit on one page if it can, but cleanly spreads over multiple pages:

Table 5.2: Long Table Example

	mpg	cyl	disp	hp	drat	wt	qsec	vs	am	gear	carb
	21.00	6.00	160.00	110.00	3.90	2.62	16.46	0.00	1.00	4.00	4.00
	21.00	6.00	160.00	110.00	3.90	2.88	17.02	0.00	1.00	4.00	4.00
	22.80	4.00	108.00	93.00	3.85	2.32	18.61	1.00	1.00	4.00	1.00
	21.40	6.00	258.00	110.00	3.08	3.21	19.44	1.00	0.00	3.00	1.00
	18.70	8.00	360.00	175.00	3.15	3.44	17.02	0.00	0.00	3.00	2.00
	18.10	6.00	225.00	105.00	2.76	3.46	20.22	1.00	0.00	3.00	1.00
	14.30	8.00	360.00	245.00	3.21	3.57	15.84	0.00	0.00	3.00	4.00
	24.40	4.00	146.70	62.00	3.69	3.19	20.00	1.00	0.00	4.00	2.00

Continued on next page

Table 5.2: Long Table Example

	mpg	cyl	disp	hp	drat	wt	qsec	vs	am	gear	carb
	22.80	4.00	140.80	95.00	3.92	3.15	22.90	1.00	0.00	4.00	2.00
	19.20	6.00	167.60	123.00	3.92	3.44	18.30	1.00	0.00	4.00	4.00
	17.80	6.00	167.60	123.00	3.92	3.44	18.90	1.00	0.00	4.00	4.00
	16.40	8.00	275.80	180.00	3.07	4.07	17.40	0.00	0.00	3.00	3.00
	17.30	8.00	275.80	180.00	3.07	3.73	17.60	0.00	0.00	3.00	3.00
	15.20	8.00	275.80	180.00	3.07	3.78	18.00	0.00	0.00	3.00	3.00
	10.40	8.00	472.00	205.00	2.93	5.25	17.98	0.00	0.00	3.00	4.00
	10.40	8.00	460.00	215.00	3.00	5.42	17.82	0.00	0.00	3.00	4.00
	14.70	8.00	440.00	230.00	3.23	5.34	17.42	0.00	0.00	3.00	4.00
	32.40	4.00	78.70	66.00	4.08	2.20	19.47	1.00	1.00	4.00	1.00
	30.40	4.00	75.70	52.00	4.93	1.61	18.52	1.00	1.00	4.00	2.00
	33.90	4.00	71.10	65.00	4.22	1.83	19.90	1.00	1.00	4.00	1.00
	21.50	4.00	120.10	97.00	3.70	2.46	20.01	1.00	0.00	3.00	1.00
	15.50	8.00	318.00	150.00	2.76	3.52	16.87	0.00	0.00	3.00	2.00
	15.20	8.00	304.00	150.00	3.15	3.44	17.30	0.00	0.00	3.00	2.00
	13.30	8.00	350.00	245.00	3.73	3.84	15.41	0.00	0.00	3.00	4.00
	19.20	8.00	400.00	175.00	3.08	3.85	17.05	0.00	0.00	3.00	2.00
	27.30	4.00	79.00	66.00	4.08	1.94	18.90	1.00	1.00	4.00	1.00
	26.00	4.00	120.30	91.00	4.43	2.14	16.70	0.00	1.00	5.00	2.00
	30.40	4.00	95.10	113.00	3.77	1.51	16.90	1.00	1.00	5.00	2.00
	15.80	8.00	351.00	264.00	4.22	3.17	14.50	0.00	1.00	5.00	4.00
	19.70	6.00	145.00	175.00	3.62	2.77	15.50	0.00	1.00	5.00	6.00
	15.00	8.00	301.00	335.00	3.54	3.57	14.60	0.00	1.00	5.00	8.00
	21.40	4.00	121.00	109.00	4.11	2.78	18.60	1.00	1.00	4.00	2.00

5.1. Huxtable

Huxtable is a very nice package for making working with tables between Rmarkdown and Tex easier.

This cost some adjustment to the Tex templates to make it work, but it now works nicely.

See documentation for this package here. A particularly nice addition of this package is for making the printing of regression results a joy (see here). Here follows an example:

If you are eager to use huxtable, comment out the Huxtable table in the Rmd template, and uncomment the colortbl package in your Rmd's root.

Note that I do not include this in the ordinary template, as some latex users have complained it breaks when they build their Rmds (especially those using tidytex - I don't have this problem as I have the full Miktex installed on mine). Up to you, but I strongly recommend installing the package manually and using huxtable. To make this work, uncomment the Adding additional latex packages part in yaml at the top of the Rmd file. Then comment out the huxtable example in the template below this line. Reknit, and enjoy.

Table 5.3: Regression Output

	Reg1	Reg2	Reg3
(Intercept)	-2256.361 ***	5763.668 ***	4045.333 ***
	(13.055)	(740.556)	(286.205)
carat	7756.426 ***		7765.141 ***
	(14.067)		(14.009)
depth		-29.650 *	-102.165 ***
		(11.990)	(4.635)
N	53940	53940	53940
R2	0.849	0.000	0.851

*** $p < 0.001$; ** $p < 0.01$; * $p < 0.05$.

FYI - R also recently introduced the gt package, which is worthwhile exploring too.

6. Lists

To add lists, simply using the following notation

- This is really simple
  - Just note the spaces here - writing in R you have to sometimes be pedantic about spaces...
- Note that Rmarkdown notation removes the pain of defining LaTeXenvironments!

7. Conclusion

I hope you find this template useful. Remember, stackoverflow is your friend - use it to find answers to questions. Feel free to write me a mail if you have any questions regarding the use of this package. To cite this package, simply type citation("Texevier") in Rstudio to get the citation for Katzke (2017) (Note that uncited references in your bibtex file will not be included in References).

References

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Appendix

Appendix A

Some appendix information here

Appendix B