Assaignment of 5 marks

MAT - 103 - Ordinary and partial differential equation and coordinate geometry.

P-1. Form partial diff. equation by eliminating orbitra... constant from $z = (x+a)(y+b)$

Solⁿ:

Given that

$$z = (x+a)(y+b)$$

Diffe. partially with respect to x'

$$\frac{\partial z}{\partial x} = 1 \cdot y + b \quad \text{or } p = y + b \text{ --- (i)}$$

$$\frac{\partial z}{\partial y} = (x + a) \cdot 1 \quad \text{or } q = x + a \text{ --- (ii)}$$

$$\therefore z = q \cdot p$$

Ans

P-2

Given that, $z = f(x^2 + y^2)$ --- (i)

Differentiating (i) partially with respect to x and y,

we get, $\frac{\partial z}{\partial x} = f'(x^2 + y^2) \cdot 2x$

$\Rightarrow \frac{\partial z}{\partial x} = 2x f'(x^2 + y^2)$

$\Rightarrow p = 2x f'(x^2 + y^2)$ --- (ii)

and $\frac{\partial z}{\partial y} = f'(x^2 + y^2) \cdot 2y$

$\Rightarrow q = 2y f'(x^2 + y^2)$ --- (iii)

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Dividing (2) by (3)

We get, $$p/q = \frac{2xf'(x^2+y^2)}{2yf'(x^2+y^2)}$$

$$\Rightarrow p/q = x/y$$

$$\Rightarrow yp = xq$$

$$\therefore \quad yp - xq = 0$$

Ans

P-3

Solve:

(i) $$\frac{\partial^2 z}{\partial x^2} + 4\frac{\partial^2 z}{\partial x \partial y} - \frac{5\partial^2 z}{\partial y^2} = 0$$

(ii) $$\frac{\partial^2 z}{\partial x^2} + 5\frac{\partial^2 z}{\partial x \partial y} + 4\frac{\partial^2 z}{\partial y^2} = 0$$

Solution of (i)

$$\frac{\partial^2 z}{\partial x^2} + 4\frac{\partial^2 z}{\partial x \partial y} - 5\frac{\partial^2 z}{\partial y^2} = 0$$

$\Rightarrow (D^2 + 4DD' - 5D'^2)z = 0$

$\therefore$ The auxillary equation is,

$m^2 + 4m - 5 = 0$

$\Rightarrow m^2 + 5m - m - 5 = 0$

$\Rightarrow m(m+5) - 1(m+5) = 0$

$\Rightarrow (m+5)(m-1) = 0 \quad [M = 1 \text{ or } m = 5]$

The required solution is

$z = f_1(y+x) + f_2(y-5x)$

~~Ans~~

Solution of (ii)

given that, $\dfrac{\partial^2 t}{\partial x^2} + 5\dfrac{\partial^2 z}{\partial x \partial y} + 4\dfrac{\partial^2 z}{\partial y^2} = 0$

$\Rightarrow (D^2 + 5DD' + 4D'^2)t = 0$

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The auxillary equation is

$m^2 + 5m + 4 = 0$

$\Rightarrow m^2 + 4m + m + 4 = 0$

$\Rightarrow m(m+4) + 1(m+4)$

$\Rightarrow (m+1)(m+4) = 0$

$\therefore m = -1$ OR $m = -4$

The required solution is

$z = f_1(y - x) + f_2(y - 4x)$