UNIVERSITY OF HONG KONG, DEPARTMENT OF COMPUTER SCIENCE

Comp 7405: Mercury

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1 BLACK SCHOLES

$$\begin{aligned} C(S,t) &= Se^{-q(T-t)}N(d_1) - Ke^{-r(T-t)}N(d_2) \\ P(S,t) &= Ke^{-r(T-t)}N(-d_2) - Se^{-q(T-t)}N(-d_1) \\ d_1 &= \frac{\ln(S/K)+(r-q)(T-t)}{\sigma\sqrt{T-t}} + \frac{1}{2}\sigma\sqrt{T-t} \\ d_2 &= \frac{\ln(S/K)+(r-q)(T-t)}{\sigma\sqrt{T-t}} - \frac{1}{2}\sigma\sqrt{T-t} \end{aligned} \tag{1.1}$$

2 NORMAL DISTRIBUTION

$$\begin{aligned} \phi(x) &= \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}x^2} \\ \Phi(x) &= \frac{1}{2} + \frac{1}{\sqrt{2\pi}}\cdot e^{-x^2/2}\left[x + \frac{x^3}{3} + \frac{x^5}{3\cdot 5} + \cdots \frac{x^{2n+1}}{(2n+1)!!} + \cdots\right] \end{aligned} \tag{2.1}$$

3 GEOMETRIC MEAN BASKET EUROPEAN OPTION

$$B_g(t) = \Big(\prod_{i=1}^{n} S_i(t)\Big)^{\frac{1}{n}} \tag{3.1}$$

$$\begin{aligned} \sigma_{B_g} &= \frac{\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_i\sigma_j\rho_{i,j}}}{n} \\ \mu_{B_g} &= r - \frac{1}{2}\frac{\sum_{i=1}^{n}\sigma_i^2}{n} + \frac{1}{2}\sigma_{B_g}^2 \end{aligned} \tag{3.2}$$

$$\begin{aligned} C_{B_g} &= e^{-rT}\big(B_g(0)e^{\mu_{B_g}T}N(\hat{d}_1) - KN(\hat{d}_2)\big) \\ P_{B_g} &= e^{-rT}\big(KN(-\hat{d}_2) - B_g(0)e^{\mu_{B_g}T}N(-\hat{d}_1)\big) \end{aligned} \tag{3.3}$$

$$\hat{d}_1 = \hat{d}_2 + \sigma_{B_g}\sqrt{T} = \frac{\ln(B_g(0)/K) + (\mu_{B_g} + \frac{1}{2}\sigma_{B_g}^2)T}{\sigma_{B_g}\sqrt{T-t}} \tag{3.4}$$

4 GEOMETRIC ASIAN OPTION

$$\hat{S}(T) = \Big(\prod_{i=1}^{n} S(t_i)\Big)^{\frac{1}{n}} \tag{4.1}$$

$$\begin{aligned} \hat{\sigma} &= \sigma\sqrt{\frac{(n+1)(2n+1)}{6n^2}} \\ \hat{\mu} &= (r - \frac{1}{2}\sigma^2)\frac{n+1}{2n} + \frac{1}{2}\hat{\sigma}^2 \end{aligned} \tag{4.2}$$

$$\begin{aligned} C_(\hat{S},T) &= e^{-rT}\big(S_0e^{\hat{\mu}T}N(\hat{d}_1) - KN(\hat{d}_2)\big) \\ P_(\hat{S},T) &= e^{-rT}\big(KN(-\hat{d}_2) - S_0e^{\hat{\mu}T}N(-\hat{d}_1)\big) \end{aligned} \tag{4.3}$$

$$\hat{d}_1 = \hat{d}_2 + \hat{\sigma}\sqrt{T} = \frac{\ln(S_0/K) + (\hat{\mu} + \frac{1}{2}\hat{\sigma}^2)T}{\hat{\sigma}\sqrt{T}} \tag{4.4}$$