

Undergraduate Topology
Robert H. Kasriel (Dover Publication)
Solutions to exercises
Part I
Chapters I to IV

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Figure 1

Remarks and warnings

You're welcome to use these notes, but they may contain errors, so proceed with caution : I graduated in 1979, went straight in the industry (where I didn't have to use fancy maths), and picked mathematics and physics again after I retired, so my mathematics got rusty for sure. If you do find an error, typo's , I'd be happy to receive bug reports, suggestions, and the like, through Github.

Contents

1	Sets, Functions, and Relations	4
1.1	Sets and Membership	5
1.1.1	5
1.1.2	5
1.1.3	5
1.2	Some remarks on the use of the connectives <i>and</i> , <i>or</i> , <i>implies</i>	6
1.2.1	6
1.2.2	6
1.2.3	7
1.2.4	7
1.2.5	7
1.2.6	8
1.2.7	8
1.2.8	8
1.2.9	9
1.3	Subsets	10
1.4	Union and Intersection of sets	10
1.4.1	10

List of Figures

1	1
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Sets, Functions, and Relations

1.1 Sets and Membership

1.1.1

List explicitly the elements of the set

$$\{x : x < 0 \text{ and } (x-1)(x+2)(x+3) = 0\}$$

$$\{-3, -2\}$$



1.1.2

List the elements of the set

$$\{x : 3x - 1 \text{ is a multiple of } 3\}$$

$$\{x : x = k + \frac{1}{3}, k \in \mathbb{Z}\}$$



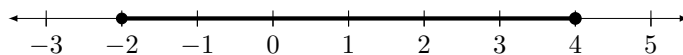
1.1.3

Sketch on a number line each of the following sets.

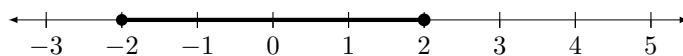
(a) $\{x : |x - 1| \leq 3\}$

(b) $\{x : |x - 1| \leq 3 \text{ and } |x| \leq 2\}$

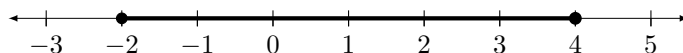
(c) $\{x : |x - 1| \leq 3 \text{ or } |x| \leq 2\}$



(a)



(b)



(c)



1.2 Some remarks on the use of the connectives *and*, *or*, *implies*

1.2.1

Demonstrate by means of a table showing truth values that the following is a true statement for any choice of p and q . Thus show that it is a tautology.

$$(\neg q \Rightarrow \neg p) \Rightarrow (p \Rightarrow q)$$

p	q	$\neg q$	$\neg p$	$\neg q \Rightarrow \neg p$	$p \Rightarrow q$	$(\neg q \Rightarrow \neg p) \Rightarrow (p \Rightarrow q)$
T	T	F	F	T	T	T
T	F	T	F	F	F	T
F	T	F	T	T	T	T
F	F	T	T	T	T	T



1.2.2

Show by means of a truth table that the statement

$$((p \Rightarrow q) \wedge (q \Rightarrow r)) \Rightarrow (p \Rightarrow r)$$

is a tautology.

p	q	r	$p \Rightarrow q$	$q \Rightarrow r$	$(p \Rightarrow q) \wedge (q \Rightarrow r)$	$p \Rightarrow r$	$((p \Rightarrow q) \wedge (q \Rightarrow r)) \Rightarrow (p \Rightarrow r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T



1.2.3

Show by means of a truth table that

$$(p \wedge q) \Rightarrow (p \vee q)$$

is a tautology.

p	q	$p \wedge q$	$p \vee q$	$(p \wedge q) \Rightarrow (p \vee q)$
T	T	T	T	T
T	F	F	F	T
F	T	F	T	T
F	F	F	F	T



1.2.4

Suppose that p and q are statements such that $(p \wedge q)$ is a false statement. Does it follow that the statement

$$(p \text{ is false}) \vee (q \text{ is false})$$

is a true statement?

p	q	$p \wedge q$	$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	F	F	F	T	T
F	T	F	T	F	T
F	F	F	T	T	T

The answer is Yes.



1.2.5

Negate the following statement: *If two angles of a triangle have equal measure, then the length of two sides of that triangle are equal.*

First we note that $\neg(p \Rightarrow q) \Leftrightarrow (p \wedge \neg q)$. Indeed,

p	q	$p \Rightarrow q$	$\neg(p \Rightarrow q)$	$\neg q$	$p \wedge \neg q$	$\neg(p \Rightarrow q) \Leftrightarrow (p \wedge \neg q)$
T	T	T	F	F	F	T
T	F	F	T	T	T	T
F	T	T	F	F	F	T
F	F	T	F	T	F	T

Putting p as *two angles of a triangle have equal measure* and $\neg q$ as *no two sides of that triangle have equal length* we get the true 'false' statement:

Two angles of a triangle have equal measure \wedge no two sides of that triangle have equal length.



1.2.6

Write the contrapositive of the statement in Exercise 5.

The contrapositive of $p \Rightarrow q$ is $\neg q \Rightarrow \neg p$. Putting $\neg p$ as *no two angles of a triangle have equal measure* and $\neg q$ as *no two sides of that triangle have equal length* we get

If no two sides of that triangle have equal length then no two angles of a triangle have equal measure.



1.2.7

Write the converse of the statement in Exercise 5.

The converse of $p \Rightarrow q$ is $q \Rightarrow p$, giving

If two sides of a triangle have equal length then two angles of a that triangle have equal measure.



1.2.8

Write the contrapositive of the following statement

If a person belongs to Committee A, then he must be a member of Committee B and he must be a member of Committee C.

Lets put

$p \equiv$ a person belongs to Committee A

$q \equiv$ a person belongs to Committee B

$r \equiv$ a person belongs to Committee C

then the given statement translates as

$$p \Rightarrow (q \wedge r)$$

and the contrapositive

$$\neg(q \wedge r) \Rightarrow \neg p$$

This last statement is equivalent with

$$(\neg q \vee \neg r) \Rightarrow \neg p$$

or in plain text:

If a person does not belong to Committee B or C , then he is not a member of Committee A.



1.2.9

Write the contrapositive of the following statement

$$\text{If } x \in A \text{ and } x \in B, \text{ then } x \in C$$

Lets put

$$p \equiv x \in A$$

$$q \equiv x \in B$$

$$r \equiv x \in C$$

then the given statement translates as

$$p \wedge q \Rightarrow r$$

and the contrapositive

$$\neg(r) \Rightarrow \neg(p \wedge q)$$

This last statement is equivalent with

$$\neg(r) \Rightarrow (\neg p \vee \neg q)$$

i.e:

$$x \notin C \Rightarrow (x \notin A \vee x \notin B)$$



1.3 Subsets

1.4 Union and Intersection of sets

1.4.1

