

PROBLEM 1

Factor	Fixed/Random	Nested/Crossed	Levels
State (s)	Random	Crossed w/ type	NY, PA, VT
Type (t)	Fixed	Crossed w/ state	Ag, Forest
Lake (l)	Random	Replicate	1,2,3,4

$$Y_{stl} = \mu + \alpha_s + \beta_t + \epsilon_{stl}$$

$$\epsilon_{stl} \sim N(0, \sigma^2)$$

$$\alpha_s \sim N(0, \sigma_{state}^2)$$

PROBLEM 2

Factor	Fixed/Random	Nested/Crossed	Levels
Type (t)	Fixed	Crossed with paint	Mountain, City
Road (r)	Random	Nested in Type	1, 2, 3, 4, 5, 6, 7, 8, 9,
			10
Paint (p)	Fixed	Crossed with road	Reflective, Non-
			Reflective
Year (y)	Random	Replicate	1,2,3

$$\begin{split} Y_{trpy} &= \mu + \alpha_t + \beta_{r(t)} + \gamma_p + \left(\alpha\beta\right)_{tp} + \epsilon_{trpy} \\ \epsilon_{ijt} &\sim N(0, \sigma^2) \\ \beta_{r(t)} &\sim N(0, \sigma^2_{road}) \end{split}$$

PROBLEM 3

Factor	Fixed/Random	Nested/Crossed	Levels
Store (s)	Random	Crossed with dis-	1,2,3,4
		play	
Display (r)	Fixed	Crossed with store	Yes, No
Week (w)	Random	Replicate	1,2,3,4

$$Y_{srw} = \mu + \alpha_s + \beta_r + \epsilon_{srw}$$

$$\epsilon_{srw} \sim N(0, \sigma^2)$$

$$\alpha_s \sim N(0, \sigma_{store}^2)$$

```
1 > displayModel = lmer(sales ~ (1|store) + display, data=displays)
2 > anova(displayModel)
3 Type III Analysis of Variance Table with Satterthwaite's method
4 Sum Sq Mean Sq NumDF DenDF F value Pr(>F)
5 display 266085 266085 1 27 119.17 2.095e-11 ***
7 Signif. codes: 0 Ś***Š 0.001 Ś**Š 0.01 Ś*Š 0.05 Ś.Š 0.1 Ś Š 1
8 > ranova(displayModel)
9 ANOVA-like table for random-effects: Single term deletions
10
11 Model:
12 sales ~ display + (1 | store)
        npar logLik AIC
                               LRT Df Pr(>Chisq)
13
17 Signif. codes: 0 Ś***Š 0.001 Ś**Š 0.01 Ś*Š 0.05 Ś.Š 0.1 Ś Š 1
```

Listing 1: Fixed and Random Effect ANOVA

Listing 2: Pairwise Results for Display

Looking at the ANOVA tables, we can see that both the fixed and random factors are significant. While performing a pairwise comparison on store is irrelevant, we can look at the difference using a display makes. Because we have a negative value for "No - Yes", we can conclude that the average sales while using a display are higher than average sales without a display.

CODE APPENDIX

```
2 #### Setup
4 ## Install and load libraries
5 # ipak function taken from: https://gist.github.com/stevenworthington/3178163
6 ipak = function(pkg) {
  new.pkg = pkg[!(pkg %in% installed.packages()[, "Package"])]
8
  if (length (new.pkg))
    install.packages(new.pkg, dependencies = TRUE)
9
sapply(pkg, require, character.only = TRUE)
11 }
12 packages = c("ggplot2", "ggplotify", "reshape2", "gridExtra", "TSA", "astsa",
           "orcutt", "nlme", "fGarch", "vars", "lsmeans", "multcompView",
13
           "base2grob", "lme4", "lmerTest")
14
15 ipak (packages)
18 #### Problem 3
21 #### Data Input
23 displays=read.table("displays.dat",header=TRUE)
24 displays$store=as.factor(displays$store)
25 displays$week=as.factor(displays$week)
26 displays
29 #### Model
31 displayModel = lmer(sales ~ (1|store) + display, data=displays)
32 anova (displayModel)
33 ranova(displayModel)
34
35 displayLSM = lsmeans(displayModel, ~ display)
36 contrast(displayLSM, method="pairwise")
```