

Surgical Unit -Optimal Model Selection

```
library(tidyverse)
library(cowplot)
library(ggResidpanel)
```

Read the dataset

```
surgery <- read.csv(file.choose())
```

View the data

```
head(surgery)
```

```
##   blood prog enzyme liver age gender a.mod a.heavy survival
## 1   6.7  62    81  2.59  50     0     1     0     695
## 2   5.1  59    66  1.70  39     0     0     0     403
## 3   7.4  57    83  2.16  55     0     0     0     710
## 4   6.5  73    41  2.01  48     0     0     0     349
## 5   7.8  65   115  4.30  45     0     0     1    2343
## 6   5.8  38    72  1.42  65     1     1     0     348
```

Fit the linear model

```
fit.surgery <- lm(survival ~ ., data = surgery)
fit.surgery
```

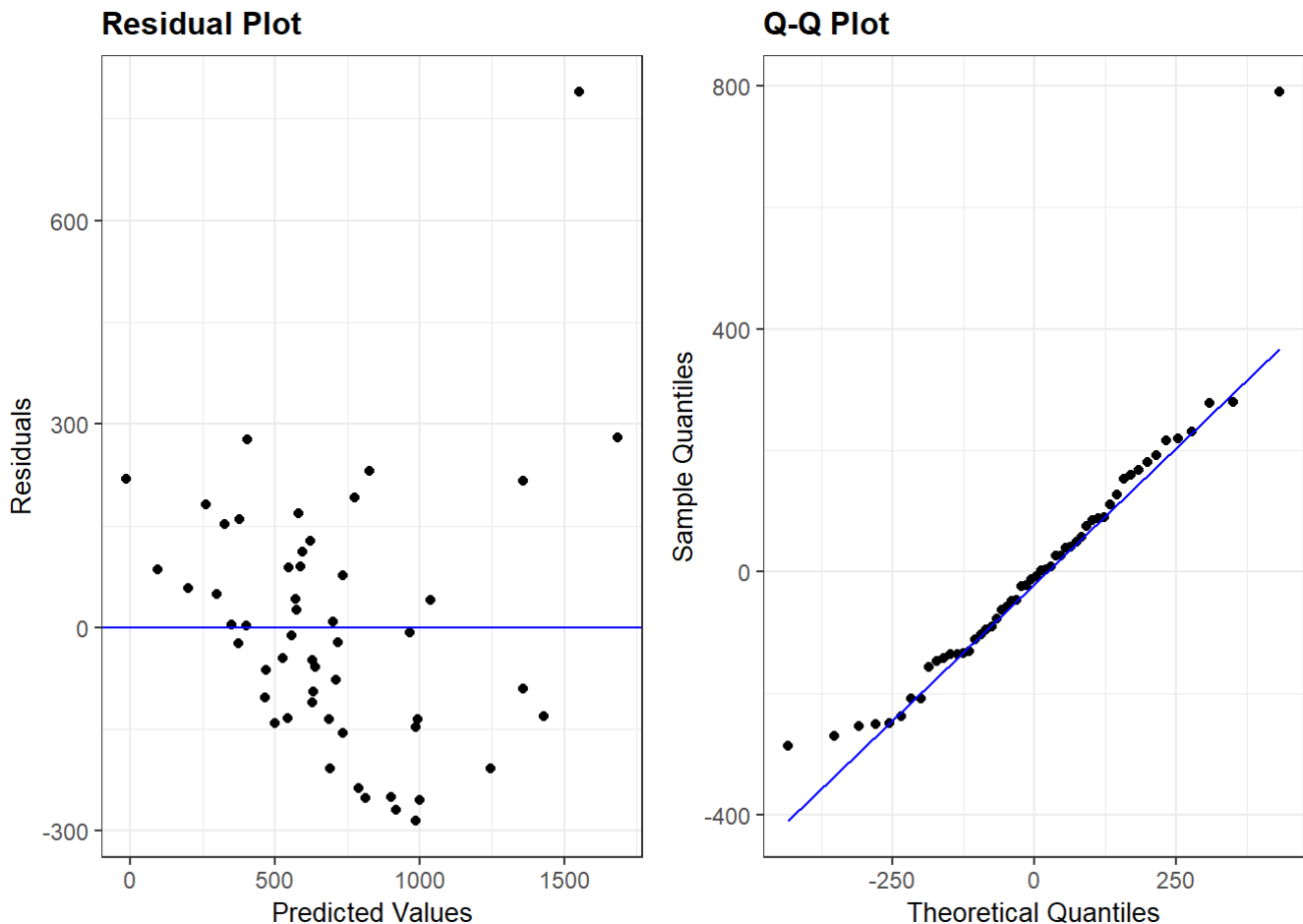
```
##
## Call:
## lm(formula = survival ~ ., data = surgery)
##
## Coefficients:
## (Intercept)      blood      prog      enzyme      liver      age
## -1148.823      62.390      8.973      9.888      50.413     -0.951
##      gender      a.mod      a.heavy
##      15.874      7.713     320.697
```

MODEL DIAGNOSTICS.

Check for Linearity, Normality and Constant Variance

using the Residual plot (for linearity and constant variance) and Q-Q Plot (for normality)

```
plot1 <- resid_panel(fit.surgery, plots = c('resid', 'qq'))  
plot1
```



Results

The Linearity Assumption was not violated:

The residuals on the “Residual Plot” are not so far away from zero,

by standardization, since majority of our values are between 2 and -2

i.e. 200 and -200, then we are convinced there is a linear relationship

between the predictors and the response variable.

The Normality Assumption was not violated:

Majority of the residuals/errors of the model seems to lie well on the

45 degree line on the “Q-Q Plot”, although with very few outliers.

Thus, we would conclude that the assumption holds true.

The Constant Variance assumption was violated:

From the “Residual Plot”, the residuals do not have a constant variance

across the predicted values (x -axis).

Transformation of Unequal Variances

We transform the response variable.

This is because, we transform Y (response variable) when assumptions of

“constant variance” and “normality” are violated. We transform X (predictor)

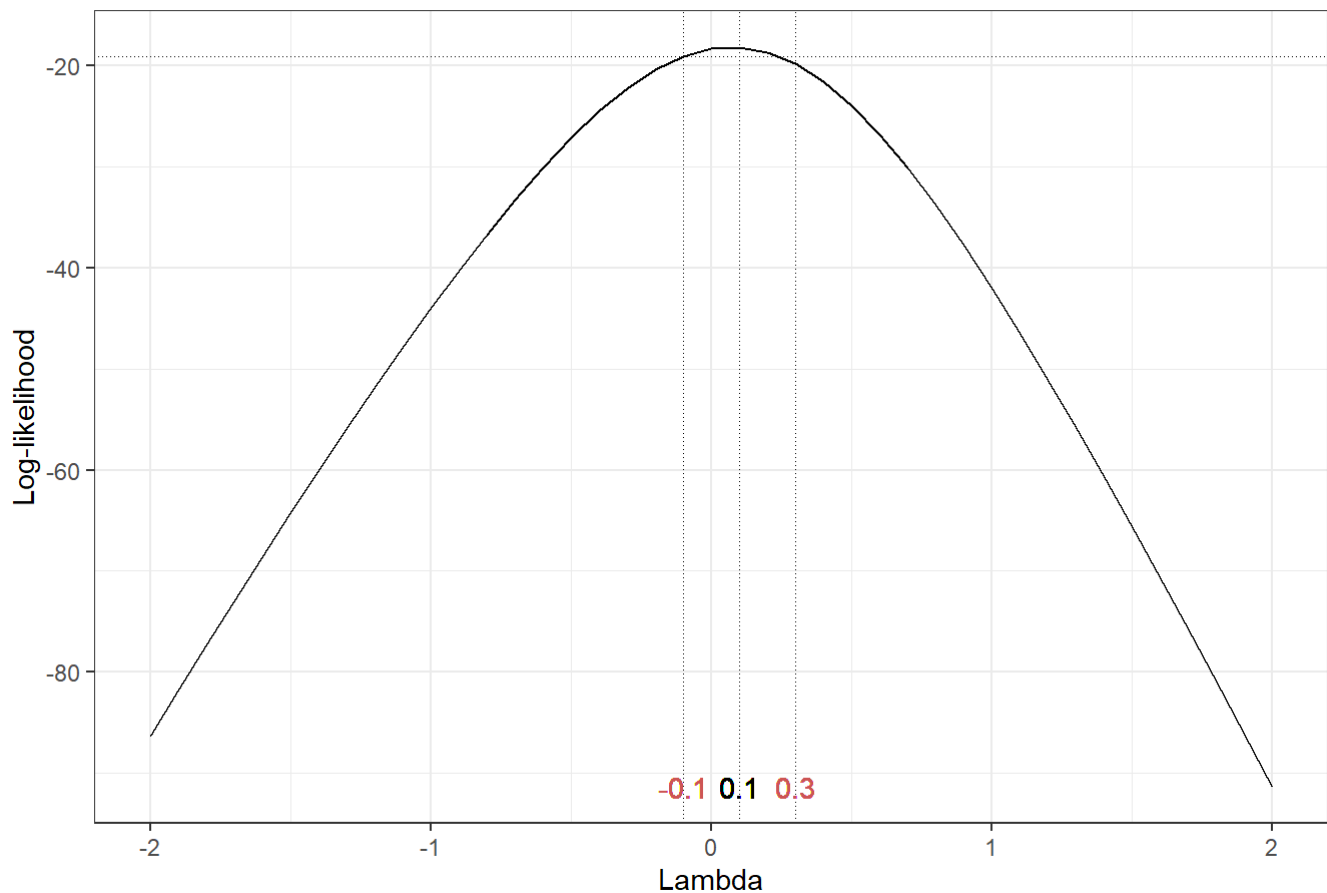
when “linearity” is violated.

We do this transformation of Y using the Box-Cox Transformation

```
library(lindia)

gg_boxcox(fit.surgery) +
  theme_bw() +
  theme(plot.title = element_text(face = "bold"))
```

Boxcox Plot



Results

The lambda value is 0.1 with a 95% confidence interval of -0.1 and 0.3

Being in practice, it makes more sense to take $\lambda = 0.0$

Since $\lambda = 0$, then we transform Y as " $\ln(Y)$ "

Transform Y in the dataset

```
surgery <- mutate(surgery, ln.survival = log(survival))

head(surgery)
```

##	blood	prog	enzyme	liver	age	gender	a.mod	a.heavy	survival	ln.survival
## 1	6.7	62	81	2.59	50	0	1	0	695	6.543912
## 2	5.1	59	66	1.70	39	0	0	0	403	5.998937
## 3	7.4	57	83	2.16	55	0	0	0	710	6.565265
## 4	6.5	73	41	2.01	48	0	0	0	349	5.855072
## 5	7.8	65	115	4.30	45	0	0	1	2343	7.759187
## 6	5.8	38	72	1.42	65	1	1	0	348	5.852202

Remove the original response variable

```
surgery <- surgery %>% select(-survival)

head(surgery)
```

```
##   blood prog enzyme liver age gender a.mod a.heavy ln.survival
## 1   6.7  62    81  2.59 50      0      1      0    6.543912
## 2   5.1  59    66  1.70 39      0      0      0    5.998937
## 3   7.4  57    83  2.16 55      0      0      0    6.565265
## 4   6.5  73    41  2.01 48      0      0      0    5.855072
## 5   7.8  65   115  4.30 45      0      0      1    7.759187
## 6   5.8  38    72  1.42 65      1      1      0    5.852202
```

Re-fit the model using the transformed Y

```
# use the new response variable to fit the model

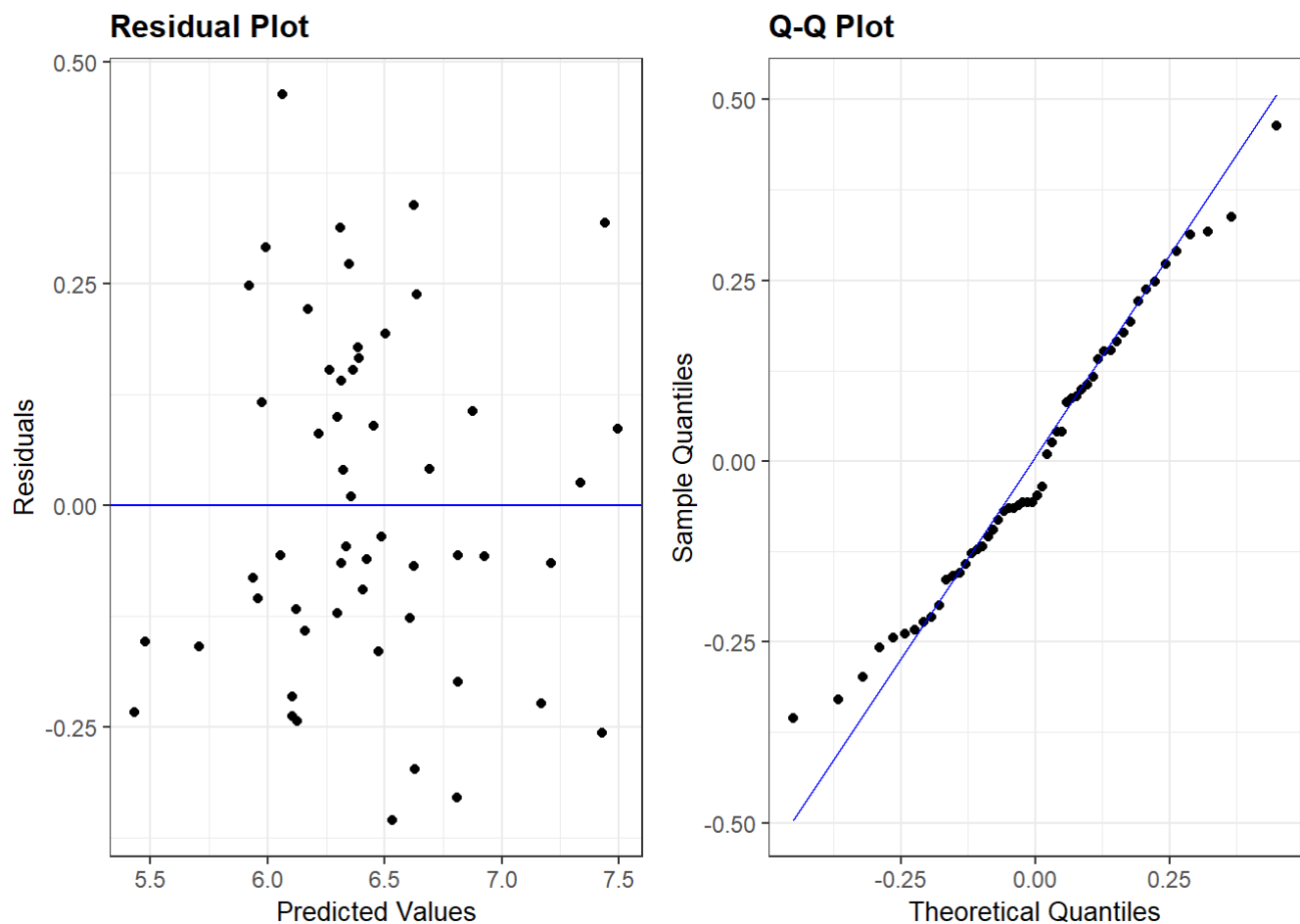
fit2.surgery <- lm(ln.survival ~., surgery)
fit2.surgery
```

```
##
## Call:
## lm(formula = ln.survival ~ ., data = surgery)
##
## Coefficients:
## (Intercept)      blood      prog      enzyme      liver      age
##   4.050949    0.068551    0.013459    0.014948    0.007931   -0.003567
##   gender      a.mod      a.heavy
##   0.084151    0.057313    0.388190
```

MODEL DIAGNOSTICS 2

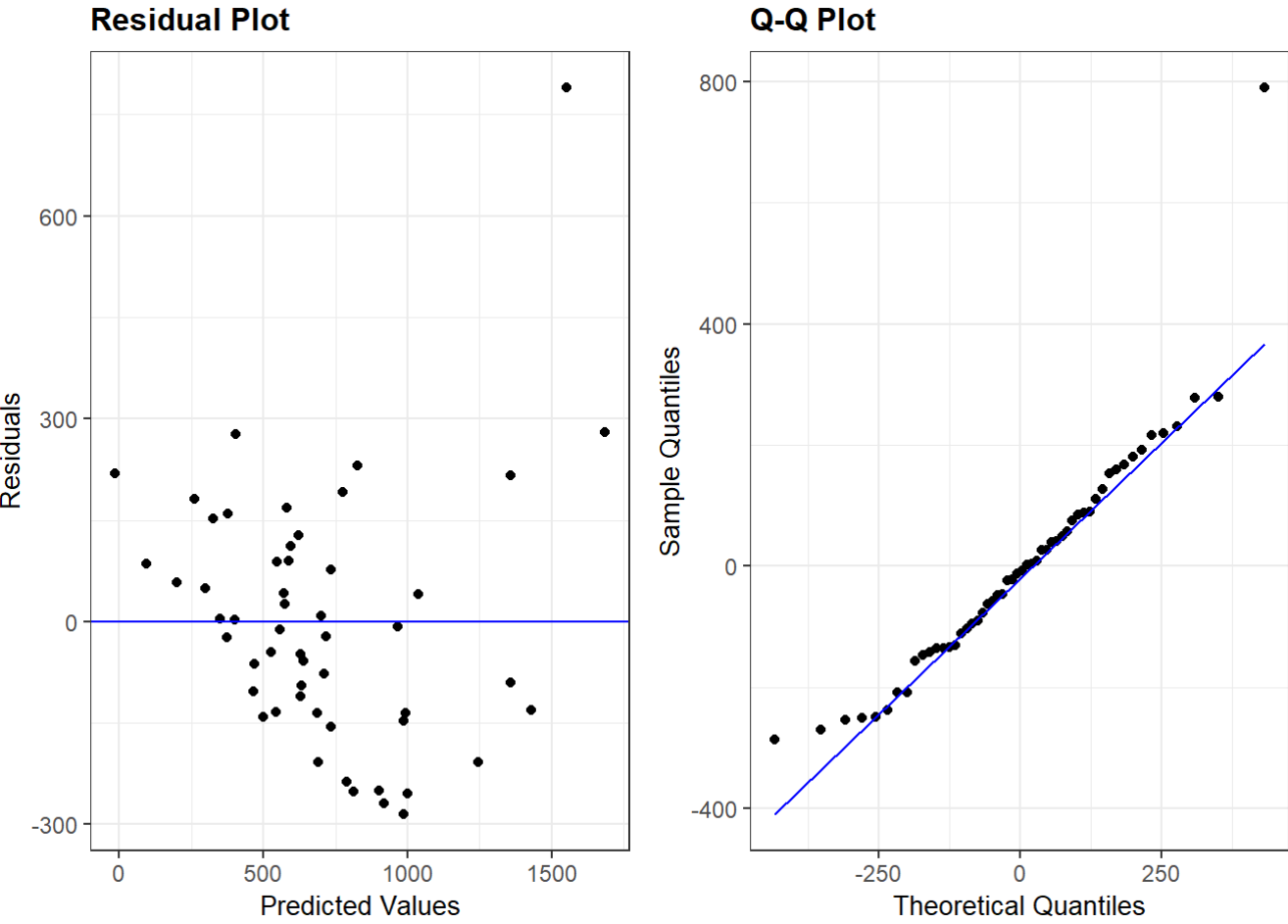
Recheck for Linearity, Normality and Constant Variance assumptions

```
plot2 <- resid_panel(fit2.surgery, plots = c('resid', 'qq'))
plot2
```

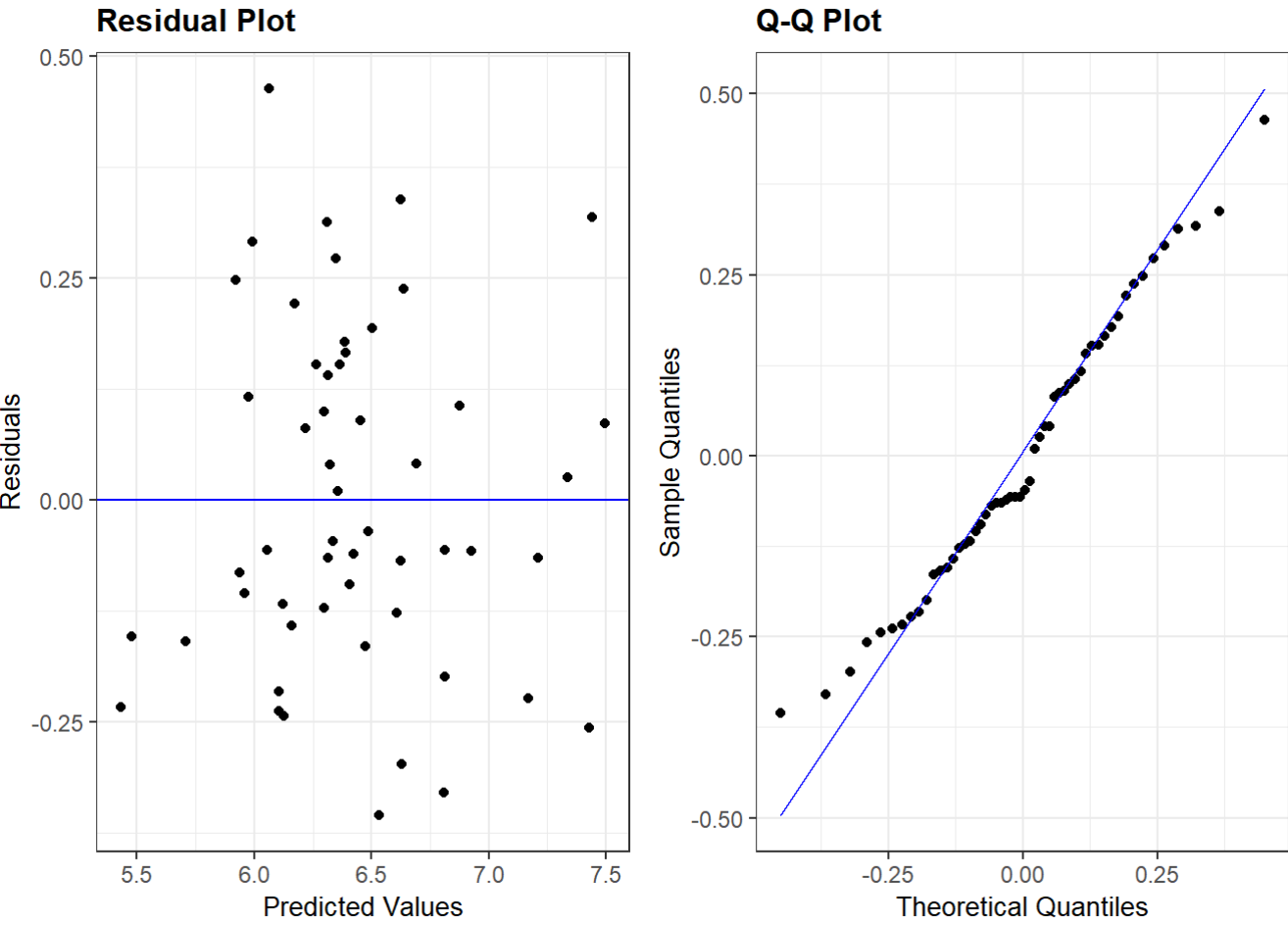


Compare old (before transformation) and new (after transformation)

```
old.par <- par(mfrow=c(2, 1))  
plot1
```



plot2



```
par(old.par)
```

Results

1) With this transformation, the variance of the predictor is a little better

i.e. more constant than before the transformation

2) The main outliers on the Q-Q plot has been taken care of.

Our final model is “fit2.surgery”

BEST MODEL SELECTION

Using the “Best Subset Algorithms” method

We have “p-1” predictor variables. So the possible number of models will be

$$2^{(p-1)} = 2^8 = 256.$$

* where “p” is the number of predictor variables and p-1 is 8 (intercept excluded)

As “p” increases, so does the multiple-coefficient-of-determination (R^2_p)

thus, we should not pick the model with the largest R^2_p .

We need the model with the least R^2_p .

We seek a levelling off point where adding more variables provides little

increase in R^2_p .

```
library(leaps)

select_model <- regsubsets(ln.survival ~ .,
                          data = surgery)

search.models = summary(select_model)
search.models
```



```
## Subset selection object
## Call: regsubsets.formula(ln.survival ~ ., data = surgery)
## 8 Variables (and intercept)
##           Forced in Forced out
## blood      FALSE      FALSE
## prog       FALSE      FALSE
## enzyme     FALSE      FALSE
## liver      FALSE      FALSE
## age        FALSE      FALSE
## gender     FALSE      FALSE
## a.mod      FALSE      FALSE
## a.heavy    FALSE      FALSE
## 1 subsets of each size up to 8
## Selection Algorithm: exhaustive
##           blood prog enzyme liver age gender a.mod a.heavy
## 1 ( 1 ) " " " " "*" " " " " " " " " " "
## 2 ( 1 ) " " "*" "*" " " " " " " " " " "
## 3 ( 1 ) " " "*" "*" " " " " " " " " "*"
## 4 ( 1 ) "*" "*" "*" " " " " " " " " "*"
## 5 ( 1 ) "*" "*" "*" " " " " "*" " " " "*"
## 6 ( 1 ) "*" "*" "*" " " "*" "*" " " " "*"
## 7 ( 1 ) "*" "*" "*" " " "*" "*" "*" " " "*"
## 8 ( 1 ) "*" "*" "*" "*" "*" "*" "*" "*" *
```

Results

We already established that there are 256 possible models and from these 256 models:

- * the best model with 1 predictor variable has “enzyme”
- * the best model with 2 predictor variables has “prog” and “enzyme”
- * the best model with 3 predictor variables has “blood”, “prog” and “enzyme”
- * ...
- * ...
- * ...
- * the best model with 7 predictor variables has all predictors except “liver”
- * the best model with 8 predictor variables has all predictors

Which is the Best Model to use?

i.e. we have 8 best model, which of them is the best, since we must not use

all the predictor variables.

* We use the adjusted R-squared approach ($R^2_{a,p}$), since it adjust for us more

paramters to the regression model.

* We want the model with the largest value of adjusted R-square

Note.

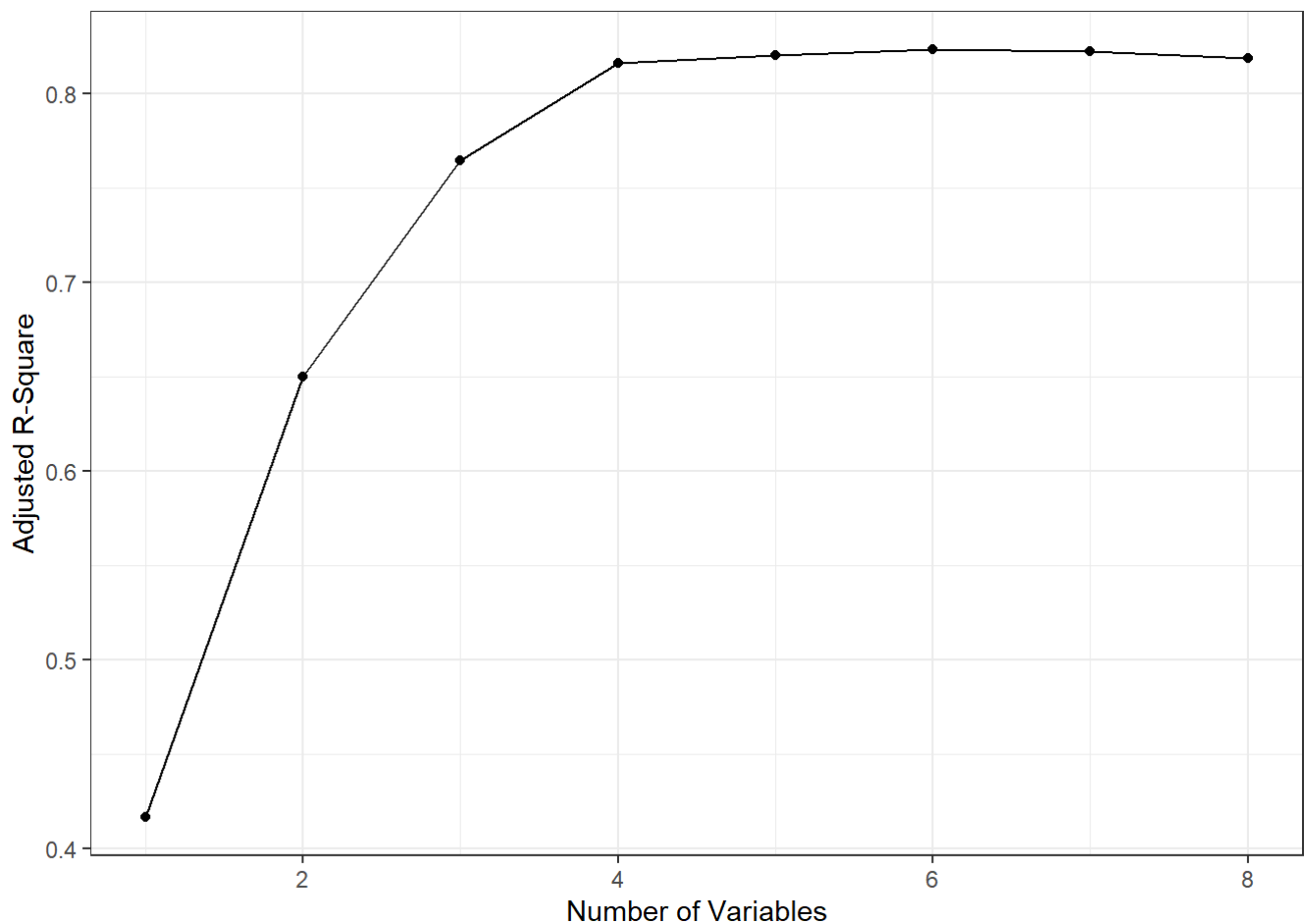
What is “R-square”? This is called the coefficient of determination. It is

the percentage of observed variability in the Y-variable that is explained by the model (predictors).

What is “Adjusted R-square”? Adjusted R-squared adjusts the statistic based

on the number of independent variables in the model.

```
tibble(predictors = seq(1,8),
        y = search.models$adjr2) %>%
  ggplot(aes(x = predictors, y = y)) +
  geom_point() +
  geom_line() +
  labs(x = "Number of Variables",
       y = "Adjusted R-Square") +
  theme_bw()
```



Check the number of variables that has the highest Adjusted R-Square in the graph

```
which.max(search.models$adjr2)
```

```
## [1] 6
```

RESULTS

1) Both with the graph and extra check, we see that the number of variables

as our best model is 6 (without intercept).

2) Hence, by “Best Subset Algorithm” approach, our best model will only need:

blood, prog, enzyme, age, gender, and a.heavy(heavy alcohol users) as predictors

The best model will become:

```
best_model = lm(formula = ln.survival ~ blood + prog + enzyme + age + gender + a.heavy,
                data = surgery)
```

```
best_model
```

```
##
## Call:
## lm(formula = ln.survival ~ blood + prog + enzyme + age + gender +
##      a.heavy, data = surgery)
##
## Coefficients:
## (Intercept)      blood      prog      enzyme      age      gender
##  4.054389      0.071524      0.013760      0.015109     -0.003452      0.087227
##      a.heavy
##  0.351069
```

Although our predictor variables here are less than 30, we would like to

also use a method that will suit cases when the variables are 30 or more.

Using “Stepwise Regression Method”

- * When the number of variables is 30 or more then “best” subset algorithms
- * become very computationally expensive.

We will employ the Backward Stepwise Search (or simply backward elimination):

- * This elimination starts with the full model (all predictors) and then
- * removes one variable at a time in the model.
- * It compares models using numerical criteria such as “AIC” (Akaike Information Criteria)

Source: Applied Linear Statistical Methods by Michael H. Kutner et al. 5th edition

Backward Elimination:

```
backward_elim <- step(object      = fit2.surgery,
                      direction = "backward")
```

```
## Start:  AIC=-160.78
## ln.survival ~ blood + prog + enzyme + liver + age + gender +
##   a.mod + a.heavy
##
##           Df Sum of Sq   RSS   AIC
## - liver    1   0.00126 1.9718 -162.74
## - a.mod    1   0.03159 2.0021 -161.92
## - age      1   0.07359 2.0441 -160.80
## <none>                        1.9705 -160.78
## - gender   1   0.08403 2.0545 -160.52
## - blood    1   0.31845 2.2890 -154.69
## - a.heavy  1   0.84489 2.8154 -143.51
## - prog     1   2.09285 4.0634 -123.70
## - enzyme   1   2.98863 4.9591 -112.94
##
## Step:  AIC=-162.74
## ln.survival ~ blood + prog + enzyme + age + gender + a.mod +
##   a.heavy
##
##           Df Sum of Sq   RSS   AIC
## - a.mod    1   0.0326 2.0043 -163.858
## <none>                        1.9718 -162.743
## - age      1   0.0876 2.0593 -162.396
## - gender   1   0.0969 2.0687 -162.152
## - blood    1   0.6269 2.5987 -149.835
## - a.heavy  1   0.8438 2.8156 -145.506
## - prog     1   2.6755 4.6473 -118.446
## - enzyme   1   5.0934 7.0652 -95.825
##
## Step:  AIC=-163.86
## ln.survival ~ blood + prog + enzyme + age + gender + a.heavy
##
##           Df Sum of Sq   RSS   AIC
## <none>                        2.0043 -163.858
## - age      1   0.0769 2.0812 -163.826
## - gender   1   0.0975 2.1018 -163.293
## - blood    1   0.6284 2.6327 -151.133
## - a.heavy  1   0.9011 2.9054 -145.810
## - prog     1   2.7644 4.7688 -119.052
## - enzyme   1   5.0752 7.0795 -97.716
```

RESULT

- 1) The reduction stopped at 6 variables.
- 2) This implies that our BEST model is with the following predictors:

blood, prog, enzyme, age, gender and a.heavy

3) These predictors are the same the predictors we got when we used the

Adjusted R-squared criteria. Hence, the same final and best model.

DIAGNOSIS OF BEST MODEL

Check for Multicollinearity of of our final and best model.

Using the Variance Inflation Factor (VIF) = $1/(1 - R_squared)$

- * VIF is between 1 and infinity.

- * VIF > 10 indicated “severe multicollinearity” but as a rule of thumb.

- * VIF > 5 is often regarded as “severe multicollinearity”.

```
library(car)

vif(best_model)
```

```
##      blood      prog  enzyme      age  gender  a.heavy
## 1.108821 1.037099 1.076909 1.016471 1.047745 1.114506
```

RESULT

We see that there exist no significant MULTICOLLINEARITY amongst

our predictor variables in our model.