Beijing-Dublin International College

SEMESTER II FINAL EXAMINATION – 2020/2021

School of Mathematics and Statistics
BDIC1026J & BDIC1030J Maths 2 (Advanced Mathematics)

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MODULE COORDINATOR: Yanru PING
OTHER EXAMINER: Miaomiao NIU

Time Allowed: 90 minutes

Instructions for Candidates

Answer ALL questions. The marks that each question carry is written as shown.

BJUT Student ID: _____ UCD Student ID: _____

I have read and clearly understand the Examination Rules of both Beijing University of Technology and University College Dublin. I am aware of the Punishment for Violating the Rules of Beijing University of Technology and/or University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I accept the punishment thereof.

Honesty Pledge：_____ (Signature)

Instructions for Invigilators

Non-programmable calculators are permitted. NO dictionaries are permitted.
No rough-work paper is to be provided for candidates.

NOTE: Answer **ALL** questions.

Time allowed is **90** minutes.

The exam paper has **2** sections on **4** pages, with a full score of 100 marks.

You are required to use only the provided **Examination Book** for answers.

SECTION A — Gap-Filling Questions

This section is worth a total of **70** marks, with each question worth **5** marks.

1. Compute

$$\int \frac{1}{1-\sin x} dx = \_\_\_\_\_.$$

2. Compute

$$\int x \ \cot^2 x \ dx = \_\_\_\_\_.$$

3. Compute

$$\int \frac{1}{x(1+x^{2020})} dx = \_\_\_\_\_.$$

4. Compute

$$\int \sqrt{\frac{x}{x-1}} dx = \_\_\_\_\_.$$

5. Compute

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{\sin^3 x}{\sqrt{1+x^2}} - \cos^2 x \right) dx = \_\_\_\_\_.$$

6. Compute

$$\int_0^{\frac{\pi}{2}} \sqrt{1-\sin 2x} dx = \_\_\_\_\_.$$

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7. Evaluate the definite integral

$$\int_0^1 x \sqrt[2020]{1-x}\,dx = \_\_\_\_\_.$$

8. Determine if the following improper integral is convergent or not:

$$\int_0^{+\infty} xe^{-x}dx.$$

If this integral is convergent, find its value: _____.

9. Evaluate the definite integral

$$\int_0^1 \frac{x}{\sqrt{1+x^2}}dx = \_\_\_\_\_.$$

10. Try to estimate the following limit by making use of the definition of definite integral:

$$\lim_{n\to\infty} \frac{1+\sqrt{2}+\sqrt{3}+\cdots+\sqrt{n}}{\sqrt{n^3}} = \_\_\_\_\_.$$

11. Suppose $y = C_1e^x + C_2e^{-2x}$ is a general solution of a second-order linear ODE with constant coefficients, C_1 and C_2 being two constants. Then a possible expression for this ODE is _____.

12. Consider an ODE

$$y' + \frac{y}{x} = \frac{\tan x}{x}.$$

Its general solution is given by $y(x) =$ _____.

13. Given an ODE

$$y'' - 6y' + 9y = 0,$$

its characteristic equation can be expressed as _____.

14. Classify the type of the following ODE:

$$\left(x^2 + y^2\right) dx - \left(y^2 - xy + y^2\right) dy = 0.$$

It is a _____ (first-order/second-order), _____ (linear/non-linear) _____ (homogeneous/non-homogeneous) ordinary differential equation.

SECTION B — Extended Answer Questions

Write your answers on the **Examination Book** provided.
This section is worth a total of **30** marks. The marks of each question are as shown.

15. (**8 marks**) Find the general solution of the following ODE:

$$y'' - 3y' + 2y = (x+1)e^x.$$

16. (**16 marks**) In the two-dimensional xy-plane, let D be the region enclosed by the x-axis and a parabola $y = 2x - x^2$.

(a) Find the area of the region D. (4 marks)

(b) Extend the study to the three-dimensional xyz coordinate system. Let Ω be the solid obtained by revolving D about a line $\begin{cases} x = -1 \\ z = 0 \end{cases}$. Try to find the volume of Ω. (4 marks)

(c) Express the length of the circumference of D by means of definite integral. (4 marks)

(d) Let Ω' be the solid obtained by revolving the region D about the x-axis. Try to compute the volume of Ω'. (4 marks)

17. (**6 marks**) Suppose $y = x$ and $y = e^x$ are two particular solutions of an ODE

$$y'' + p(x)y' + q(x)y = 0,$$

try to find $p(x)$ and $q(x)$, and the general solution of the ODE.

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Useful formulae

$$\int x^{\alpha} dx = \frac{1}{\alpha+1} x^{\alpha+1} + C \ , \qquad \alpha \neq -1$$

$$\int \frac{1}{x} dx = \ln x + C$$

$$\int \frac{1}{\sqrt{x^2 \pm 1}} dx = \ln \left| (x + \sqrt{x^2 \pm 1}) \right| + C$$

$$\int \frac{1}{\cos^2 x} dx = \tan x + C$$

$$\int \frac{1}{\sin^2 x} dx = -\cot x + C$$

$$\int \sin x dx = -\cos x + C$$

$$\int \cos x dx = \sin x + C$$

$$\int \tan x dx = -\ln |\cos x| + C$$

$$\int \cot x dx = \ln |\sin x]| + C$$

$$\int \sec x dx = \ln |\sec x + \tan x| + C$$

$$\int \csc x dx = -\ln |\csc x + \cot x| + C$$

$$\int a^x dx = \frac{1}{\ln a} a^x + C$$

$$\int \frac{1}{1+x^2} dx = \arctan x + C$$

$$\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + C$$

Glossary

Antiderivative	原函数
Area	面积
Boundary	边界
Characteristic Equation	特征方程
Circumference	周长
Coefficient	系数
Convergent	收敛的
Curve	曲线
Definite integral	定积分
Definition	定义
Even function	偶函数
General solution	通解
Homogeneous	齐次的
Improper integral	广义积分
Odd function	奇函数
ODE (Ordinary differential equation)	常微分方程
Region	区域
Volume	体积