Beijing-Dublin International College

SEMESTER II FINAL EXAMINATION – 2018/2019

School of Mathematics and Statistics

BDIC1031J Maths 3 (Advanced Mathematics; Engineering)

HEAD OF SCHOOL: Wenying Wu

MODULE LECTURERS: Yanru Ping, Wenqing Xu

Time Allowed: 90 minutes

Instructions for Candidates

Answer ALL questions. The marks that each question carry is written as shown.

BJUT Student ID: ________________ **UCD Student ID:** ________________

I have read and clearly understand the Examination Rules of both Beijing University of Technology and University College Dublin. I am aware of the Punishment for Violating the Rules of Beijing University of Technology and/or University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I accept the punishment thereof.

Honesty Pledge： ____ **(Signature)**

Instructions for Invigilators

Non-programmable calculators are permitted. NO dictionaries are permitted.

No rough-work paper is to be provided for candidates.

NOTE: Answer **ALL** questions.

Time allowed is **90** minutes.

The exam paper has **2** sections on **5** pages, with a full score of 100 marks.

You are required to use the provided **Examination Book** only for answers.

Section A: Fill-in-the-blank Questions

This section is worth a total of **76** marks.

1. (5 marks) Determining convergence or divergence for the series $\sum_{n=1}^{\infty} \frac{2n}{3n-1}$ ___________. Select convergence or divergences and then give the reason.

2. (5 marks) For what value of a, if any, does the series $\sum_{n=1}^{\infty} \left[\frac{a}{n+2} - \frac{1}{n+4}\right]$ converge? ___________.

3. (5 marks) Let $a_n = \begin{cases} \frac{n}{2^n}, & \text{if } n \text{ is a prime number} \\ \frac{1}{2^n}, & \text{otherwise} \end{cases}$ Does the series $\sum_{n=1}^{\infty} a_n$ converge? Give reasons for your answer. ___________.

4. (5 marks) Determine the domain of p when the series $\sum_{n=1}^{\infty} \frac{\cos n\pi}{n \ln^p n}$ is absolutely convergent, conditionally convergent or divergent. Justify your conclusions.

That is,

when $p \in$ ___________, the series is absolutely convergent.

when $p \in$ ___________, the series is conditionally convergent.

when $p \in$ ___________, the series is divergent, if any.

5. (5 marks) Suppose that the series $\sum_{n=1}^{\infty} a_n(x-1)^n$ is conditionally convergent at $x=-3$, then the radius of convergence with respect to the power series $\sum_{n=1}^{\infty} a_n x^n$ is $R =$ __________

6. (5 marks) Find the sum of the series

$$1 - \frac{\pi^2}{4^2 \cdot 2!} + \frac{\pi^4}{4^4 \cdot 4!} - \frac{\pi^6}{4^6 \cdot 6!} + \cdots + \frac{(-1)^k \pi^{2k}}{4^{2k} \cdot (2k)!} + \cdots$$

__________.

7. (8 marks) Express $f(x) = x\ln(1+2x)$ as Maclaurin series, i.e.in the form of $f(x) = \sum_{n=0}^{\infty} a_n x^n$ and then determine $f^{(2019)}(0) =$ __________.

8. (5 marks) Express $\sqrt{4-x}$ as Maclaurin series, and specify the radius of the convergence with respect to the expansion.

9. (5 marks)Evaluate $\sum_{n=0}^{\infty} \int_n^{n+1} \frac{1}{1+x^2} dx$ __________.

10. (8 marks) Given $f(x) = \begin{cases} 1, & \text{if } 0 \le x \le \pi \\ 2, & \text{if } \pi < x \le 2\pi \end{cases}$, and $f(x)$ is periodic with period 2π. Then its corresponding Fourier series

$$S(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos nx + b_n \sin nx],$$

where $a_n =$ __________, $b_n =$ __________ $n = 1, 2, \cdots$, and $S(3\pi) =$ __________.

11. (5 marks) Express the complex number $1+i$ in the form of polar exponential form (i.e. $re^{i\theta}$) __________ , and then calculate $(1+i)^{2019}$ in the form of Cartesian form (i.e. $a+bi$, $a, b$are real numbers).

12. (5 marks) Write down an equality which contains five important constant:$e, \pi, i, 0, 1$__________

13. (5 marks) The sphere is given by the equation $x^2+y^2+z^2+4x+6y-2z=0$, and its center is at point __________ and the length of the radius is __________

14. (5 marks) Match each equation with the surface it defines. Furthermore, identify each surface by its type.

a.	$z^2+4y^2-4x^2=4$,	Paraboloid of revolution
b.	$x^2+z^2=y^2$,	Cylinder
c.	$x=-y^2-z^2$,	Hyperboloid of one sheet
d.	$x^2-y^2-z^2=1$,	Hyperboloid of two sheets
e.	$9y^2+z^2=16$.	Cone

Section B: Extended Answer Questions

This section is worth a total of **24** marks.

15. (**12** marks) Given a power series

$$\sum_{n=1}^{\infty}(n+1)nx^n$$

(a) Determine its interval of convergence.

(b) Find its sum function $S(x) = \sum_{n=1}^{\infty}(n+1)nx^n$

(c) Find the value of the sum of the series $\sum_{n=1}^{\infty}\frac{n(n+1)}{2^n}$

16. (**6** marks) Given $z = -1 + i$ is a root for a polynomial equation

$$z^4 + 6z^3 + 15z^2 + 18z + 10 = 0.$$

Find the other three roots of the equation.

17. (**6** marks) Prove that the series $\sum_{n=1}^{\infty}(-1)^n\left[\sin\frac{1}{2n} - \sin\frac{1}{2n+1}\right]$ is absolutely convergent.

Useful formulae

$$e^x = \quad 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots \qquad x \in (-\infty, +\infty)$$

$$\sin x = \quad x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots + \frac{(-1)^n x^{2n+1}}{(2n+1)!} + \cdots \qquad x \in (-\infty, +\infty)$$

$$\cos x = \quad 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots + \frac{(-1)^n x^{2n}}{(2n)!} + \cdots \qquad x \in (-\infty, +\infty)$$

$$\frac{1}{1-x} = \quad 1 + x + x^2 + \cdots + x^n + \cdots, \qquad x \in (-1, 1)$$

$$\ln(1+x) = \quad x - \frac{x^2}{2} + \frac{x^3}{3} + \cdots + \frac{(-1)^{n-1} x^n}{n} + \cdots, \qquad x \in (-1, 1]$$

$$(1+x)^\alpha = \quad 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!} x^2 + \cdots + \frac{\alpha(\alpha-1)\cdots(\alpha-n+1)}{n!} x^n + \cdots, \quad x \in (-1, 1)$$

Glossary

Absolutely convergent	绝对收敛
Cartesian form	坐标形式
Conditionally convergent	条件收敛
Cone	锥面
Convergence	收敛
Cylinder	柱面
Directrix	准线
Divergence	发散
Domain	定义域
Ellipsoid	椭球面
Expansion	展开式
Exponential form	指数形式
Fourier series	傅里叶级数
Generating line	母线
Hyperboloid of one sheet	单叶双曲面
Hyperboloid of two sheets	双叶双曲面
Identify	识别
Interval of convergence	收敛区间
Maclaurin series	马克老林级数
Modulus	模长
Paraboloid of revolution	旋转抛物面
Polar exponential form	极指数形式
Polar form	极坐标形式
Power series	幂级数
Principal argument	辐角主值
Radius of convergence	收敛半径
Sphere	球面
Sum function	和函数
Surface of revolution	旋转曲面