Beijing-Dublin International College

SEMESTER II FINAL EXAMINATION – 2021/2022

BDIC1030J & BDIC1026J Maths 2 (Advanced Mathematics)

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Time Allowed: 90 minutes

Instructions for Candidates

Answer ALL questions. The marks that each question carry is written as shown.

BJUT Student ID: _____ **UCD Student ID: _____**

I have read and clearly understand the Examination Rules of both Beijing University of Technology and University College Dublin. I am aware of the Punishment for Violating the Rules of Beijing University of Technology and/or University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I accept the punishment thereof.

Honesty Pledge：_____ (Signature)

Instructions for Invigilators

Non-programmable calculators are permitted. NO dictionaries are permitted.
No rough-work paper is to be provided for candidates.

NOTE: Answer **ALL** questions.

Time allowed is **90** minutes.

The exam paper has **2** sections on **6** pages, with a full score of 100 marks.

You are required to use only the provided **Examination Book** for answers.

SECTION A — Brief Answer Questions

This section is worth a total of **70** marks, with each question worth **5** marks.

1. Compute the indefinite integral

$$\int \frac{x^2}{x^3+1} dx = \_\_\_\_\_.$$

2. Compute the indefinite integral

$$\int x^2 \ln(x)\, dx = \_\_\_\_\_.$$

3. Compute the indefinite integral

$$\int \sin^3(x)\cos^2(x)\, dx = \_\_\_\_\_.$$

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4. Compute the indefinite integral

$$\int \frac{1}{x(x-1)^2} dx = \_\_\_\_\_.$$

5. Compute the indefinite integral

$$\int \frac{x^2}{[\sqrt{1-x^2}]^3} dx = \_\_\_\_\_.$$

6. Compute the indefinite integral

$$\int \ln(x) dx = \_\_\_\_\_.$$

7. Evaluate the definite integral

$$\int_0^2 x^x (1 + \ln(x)) dx = \_\_\_\_\_.$$

8. Evaluate the definite integral

$$\int_{-1}^1 \sin(x) \cos(x) \log(x^2) e^{x^2} dx = \_\_\_\_\_.$$

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9. Evaluate the definite integral

$$\int_0^2 x \cdot \sqrt{2-x}dx = \_\_\_\_\_.$$

10. Consider an ODE

$$\frac{dy}{dx} = ky^2\,, \quad y(0) = 1\,, \quad y(1) = 2\,.$$

$k = \_\_\_\_\_$.

11. Consider an ODE

$$(x + \cos(y))\,\frac{dy}{dx} = \cot(y)\,.$$

Its general solution is given by $x(y) = \_\_\_\_\_$.

12. Consider an ODE

$$x\frac{dy}{dx} - 2y = x^4y^2.$$

Its general solution is given by $y(x) = \_\_\_\_\_$.

13. Suppose $y^* = x\cos(x)e^{2x}$ is a particular solution of a second-order linear non-homogeneous ODE with constant coefficients. The corresponding characteristic roots are then _____.

14. Suppose $y = C_1e^{-x} + C_2xe^{-x}$ is a general solution of a second-order linear ODE with constant coefficients, C_1 and C_2 being two constants. Then a possible expression for this ODE is _____.

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SECTION B — Extended Answer Questions

Write your answers on the **Examination Book** provided.

This section is worth a total of **30** marks. The marks of each question are as shown.

15. (**4 marks**) Write the following limit as a definite integral:

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(3\left(\frac{i}{n}\right)^2 + 2\left(\frac{i}{n}\right) + 1 \right) \left(\frac{1}{n}\right)$$

16. (**10 marks**) Find the particular solution of the following ODE:

$$y'' - y' - 2y = (3x^2 + x + 1)e^{-x}\,, \quad y(0) = 0\,, \quad y'(0) = 0$$

17. (**16 marks**) In the two-dimensional xy-plane, consider the curve specified by $\{x = x, y = 1 - x^2\}$ for $x \in [0, 1]$

(a) Write down a mathematical expression for length of this curve using definite integration. (4 marks)

(b) By performing the integration, calculate the length of the curve. (4 marks)

(c) Calculate the area of the region specified by this curve and the lines $x = 0, y = 0$. (4 marks)

(d) Find the volume of the solid body obtained by rotating this region about the x-axis. (4 marks)

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Useful formulae

$$\int \frac{1}{\sqrt{x^2+1}}dx = \ln(x+\sqrt{x^2+1})+c$$

$$\int \frac{1}{\cos^2 x}dx = \tan x + c$$

$$\int \frac{1}{\sin^2 x}dx = -\cot x + c$$

$$\int \tan x dx = -\ln|\cos x| + c$$

$$\int \cot x dx = \ln|\sin x| + c$$

$$\int \frac{1}{1+x^2}dx = \arctan x + c$$

$$\int \frac{1}{x}dx = \ln x + c$$

$$\int \frac{1}{\sqrt{1-x^2}}dx = \arcsin x + c$$