



Beijing-Dublin International College



SEMESTER I FINAL EXAMINATION – 2016/2017

School of Mathematics and Statistics
BDIC1030J & BDIC1026J Maths 2 (Advanced Mathematics)

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MODULE COORDINATOR: Yanru Ping
OTHER EXAMINER: Yuehong Feng

Time Allowed: 90 minutes

Instructions for Candidates

Answer ALL questions. The marks that each question carry is written as shown.

BJUT Student ID: _____

UCD Student ID: _____

I have read and clearly understand the Examination Rules of both Beijing University of Technology and University College Dublin. I am aware of the Punishment for Violating the Rules of Beijing University of Technology and/or University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I accept the punishment thereof.

Honesty Pledge: _____ **(Signature)**

Instructions for Invigilators

Non-programmable calculators are permitted. NO dictionaries are permitted.
No rough-work paper is to be provided for candidates.

SECTION A — Gap-Filling & Brief Answer Questions

This section is worth a total of **70** marks, with each question worth **5** marks.

1. Determine whether the following statement is correct, and explain your answer:

If $f(x)$ is a continuous even function, then its antiderivative is an odd function.

2. Compute $\int \frac{\cos \sqrt{x}}{\sqrt{x}} dx =$ _____.

3. Compute $\int_0^{\frac{\pi}{2}} \sqrt{1 - \sin 2x} dx =$ _____.

4. Compute $\int x^3 \arctan x dx =$ _____.

5. Compute $\int \tan^3 x dx =$ _____.

6. Evaluate the definite integral

$$\int_0^{\frac{\pi}{2}} \frac{\cos^3 x}{\cos^3 x + \sin^3 x} dx = \text{_____}.$$

7. Evaluate the definite integral

$$\int_0^1 x \sqrt{1-x} dx = \text{_____}.$$

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8. Evaluate the improper integral $\int_1^{+\infty} \frac{\ln x}{x^2} dx = \underline{\hspace{2cm}}$.

9. Given that

$$f(x) = \frac{1}{1+x^2} + \sin x \int_0^1 f(x) dx,$$

try to find $f(x)$.

10. Compute $\int \sqrt{a^2 - x^2} dx$, where $a > 0$.

11. Let $f(x)$ be a continuous function, and

$$\int_0^x f(t) dt - \ln(x + \sqrt{1+x^2}) = 1.$$

Try to evaluate $f(x)$.

12. Find the limit

$$\lim_{x \rightarrow 0} \frac{\int_0^x \sin(t - \sin t) dt}{1 - \cos x^2} = \underline{\hspace{2cm}}.$$

13. Suppose $f(x)$ satisfies

$$\lim_{x \rightarrow a} \frac{f(x) - f(a)}{(x - a)^4} = -1.$$

Classify $f(a)$ as a _____ local minimum value , local maximum value , or neither of them.

14. Evaluate the definite integral

$$\int_0^{\frac{\pi}{2}} [\sin^8 x + \cos^5 x] dx = \underline{\hspace{2cm}}.$$

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SECTION B — Extended Answer Questions

Write your answers on the **Examination Book** provided.

This section is worth a total of **30** marks.

- 15. (6 marks)** Prove the following inequity :

$$\frac{\ln(1+x)}{\ln x} > \frac{x}{1+x}, \quad \forall x > 1.$$

- 16. (6 marks)** Let D be a planar region bounded by

$$y = \sqrt{x}, \quad \text{a line } x = a \text{ with } a > 0, \quad \text{and the } x\text{-axis.}$$

Let V_x and V_y be the volumes of the solid bodies obtained by rotating D about the x - and y -axis, respectively. Then, if $V_y = 10V_x$, try to determine the value of a .

- 17. (6 marks)** Try to estimate the following limit by making use of the definition of definite integral :

$$\lim_{n \rightarrow \infty} \frac{1}{n+1} \sum_{k=1}^n \frac{n^2}{n^2 + k^2}.$$

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- 18. (6 marks)** Let $f(x)$ be a continuous function over $[0, 1]$, and a derivable function over $(0, 1)$. If

$$f(1) = 3 \int_0^{\frac{1}{3}} f(x) dx,$$

try to show that there exists at least one $\xi \in (0, 1)$ such that $f'(\xi) = 0$.

- 19. (6 marks)** Evaluate the following definite integral

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left[\sqrt{\frac{\pi^2}{4} - x^2} \arctan 3^x + x^3 \cos x \right] dx.$$

**Hint: Try to prove first*

$$\arctan x + \arctan \frac{1}{x} = \frac{\pi}{2}, \quad \forall x > 0.$$

Glossary

Statement	命题
Determine	确定
Even function	偶函数
Odd function	奇函数
antiderivative	原函数
continuous	连续的
Definite integral	定积分
Improper integral	反常积分、广义积分
limit	极限
classify	分类
Local maximum value	极大值
Local minimum value	极小值
prove	证明
Inequality/ inequity	不等式
Planar region	平面区域
Bounded by	由....围成
value	值
Such that	使得
volume	体积
respectively	分别地
suppose	假设
definition	定义
derivable	可微分的/可导的
Show that	证明