Beijing-Dublin International College

SEMESTER I FINAL EXAMINATION – 2017/2018

School of Mathematics and Statistics
BDIC1026J & BDIC1030J Maths 2 (Advanced Mathematics)

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OTHER EXAMINER: Yuehong FENG

Time Allowed: 90 minutes

Instructions for Candidates

Answer ALL questions. The marks that each question carry is written as shown.

BJUT Student ID: _____ **UCD Student ID: _____**

I have read and clearly understand the Examination Rules of both Beijing University of Technology and University College Dublin. I am aware of the Punishment for Violating the Rules of Beijing University of Technology and/or University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I accept the punishment thereof.

Honesty Pledge：_____ (Signature)

Instructions for Invigilators

Non-programmable calculators are permitted. NO dictionaries are permitted.
No rough-work paper is to be provided for candidates.

NOTE: Answer **ALL** questions.

Time allowed is **90** minutes.

The exam paper has **2** sections on **6** pages, with a full score of 100 marks.

You are required to use only the provided **Examination Book** for answers.

SECTION A — Brief Answer Questions

This section is worth a total of **70** marks, with each question worth **5** marks.

1. Compute

$$\int \frac{1}{1+\sin x} dx = \_\_\_\_\_.$$

2. Compute

$$\int e^{\cos x} \sin x dx = \_\_\_\_\_.$$

3. Compute

$$\int \arctan x dx = \_\_\_\_\_.$$

4. Compute

$$\int \frac{1}{x^2+3x+2} dx = \_\_\_\_\_.$$

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5. Compute

$$\int_0^{\frac{\pi}{2}} \sqrt{1-\sin 2x}dx = \_\_\_\_\_.$$

6. Evaluate the definite integral

$$\int_0^{\frac{\pi}{2}} \frac{\cos^{2017} x}{\cos^{2017} x + \sin^{2017} x}dx = \_\_\_\_\_.$$

7. Evaluate the definite integral

$$\int_0^1 x\sqrt[4]{1-x}dx = \_\_\_\_\_.$$

8. Determine if the following improper integral is convergent or not:

$$\int_e^{+\infty} \frac{1}{x\ln^2 x}dx.$$

If this integral is convergent, find its value: _____.

9. Consider a function $f(x)$ satisfying

$$f(x) = \frac{1}{1+x^2} + x\int_0^1 f(x)dx.$$

Try to solve out this function $f(x)$.

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10. Evaluate the limit

$$\lim_{x\to+\infty} \frac{\int_0^x \arctan t dt}{\sqrt{1+x^2}} = \_\_\_\_\_.$$

11. Try to estimate the following limit by making use of the definition of definite integral:

$$\lim_{n\to\infty} \frac{1}{n+1} \sum_{k=1}^{n} \left\{ \sqrt{\frac{n^2}{n^2+k^2}} \right\}.$$

12. Given the conditions

$$y' = 2xy, \qquad y(0) = 1,$$

try to determine the function $y(x) =$ _____.

13. If $y = C_1 e^{-2x} + C_2 e^{3x}$ is the general solution of a second-order linear homogeneous ODE with constant coefficients, then the ODE can be expressed as _____.

14. Judge the type of the following ODE:

$$(x^2 + xy)dx - (y^2 + 3xy + x^2)dy = 0$$

It is a _____ (first-order/second-order), _____ (linear/non-linear) _____ (homogeneous/non-homogeneous) ordinary differential equation.

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SECTION B — Extended Answer Questions

Write your answers on the **Examination Book** provided.

This section is worth a total of **30** marks. The marks of each question are as shown.

15. (**9 marks**) Find the general solution of the following ODE:

$$y'' - 2y' + 5y = xe^x.$$

16. (**9 marks**) In the two-dimensional plane, consider a region bounded by

the y-axis, the line $y = 1$, and the curve $y = \sqrt{x}$.

(a) Find the area of this region.

[Hint: Write x as a function of y, and integrate it with respect to y.]

(b) Find the volume of the solid obtained by rotating this region about the x-axis.

(c) Express the length of the circumference of the region by means of definite integral.

17. (**6 marks**) Determine whether the following statement is correct or not. If the statement is true, give a proof for it; otherwise, give a counterexample.

If $f(x)$ is a continuous odd function, then its antiderivative is an even function.

18. (**6 marks**) Consider a function satisfying

$$f(x) = \cos 2x + \int_0^x f(t) \sin t dt.$$

Try to solve out $f(x) =$ _____.

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Useful formulae

$$\int \frac{1}{\sqrt{x^2+1}}dx = \ln(x+\sqrt{x^2+1})+c$$

$$\int \frac{1}{\cos^2 x}dx = \tan x + c$$

$$\int \frac{1}{\sin^2 x}dx = -\cot x + c$$

$$\int \tan x dx = -\ln|\cos x| + c$$

$$\int \cot x dx = \ln|\sin x| + c$$

$$\int \frac{1}{1+x^2}dx = \arctan x + c$$

$$\int \frac{1}{x}dx = \ln x + c$$

$$\int \frac{1}{\sqrt{1-x^2}}dx = \arcsin x + c$$

Glossary

Antiderivative	原函数
Boundary	边界
Circumference	周长
Convergent	收敛
Counterexample	反例
Definite integral	定积分
Even function	偶函数
General solution	通解
Homogeneous	齐次的
Improper integral	广义积分
Odd function	奇函数
ODE (Ordinary Differential Equation)	常微分方程