Beijing-Dublin International College

SEMESTER II FINAL EXAMINATION – 2017/2018

School of Mathematics and Statistics

BDIC1031J Maths 3 (Advanced Mathematics; Engineering)

HEAD OF SCHOOL: Wenying Wu

MODULE LECTURERS: Yanru Ping, Yuehong Feng

Time Allowed: 90 minutes

Instructions for Candidates

Answer ALL questions. The marks that each question carry is written as shown.

BJUT Student ID: ________________ **UCD Student ID:** ________________

I have read and clearly understand the Examination Rules of both Beijing University of Technology and University College Dublin. I am aware of the Punishment for Violating the Rules of Beijing University of Technology and/or University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I accept the punishment thereof.

Honesty Pledge: ____ **(Signature)**

Instructions for Invigilators

Non-programmable calculators are permitted. NO dictionaries are permitted.

No rough-work paper is to be provided for candidates.

NOTE: Answer **ALL** questions.

Time allowed is **90** minutes.

The exam paper has **2** sections on **5** pages, with a full score of 100 marks.

You are required to use the provided **Examination Book** only for answers.

Section A: Fill-in-the-blank Questions

This section is worth a total of **80** marks, with each question worth **5** marks.

1. Determine if the series $\sum_{n=1}^{\infty} \frac{1}{n(1+\ln^2 n)}$ is convergent or divergent.

The answer is __________. Specify why you make this choice.

2. Determine if the series $\sum_{n=1}^{\infty} n \tan \frac{1}{n}$ is convergent or divergent.

The answer is __________. Specify why you make this choice.

3. Determine if the series $\sum_{n=1}^{\infty} \frac{5^n + 4^n}{7^n - 6^n}$ is convergent or divergent.

The answer is __________. Specify why you make this choice.

4. Consider a series $\sum_{n=1}^{\infty} \frac{\cos n\pi}{n^p}$ in the following three cases. Determine the domain of p in the cases, **and then justify your conclusions.**

(a) when $p \in$ __________, the series is absolutely convergent.

(b) when $p \in$ __________, the series is conditionally convergent.

(c) when $p \in$ __________, the series is divergent.

5. Suppose that the series $\sum_{n=1}^{\infty} a_n x^n$ is conditionally convergent when $x = 2$. Then the radius of convergence can be determined as $R =$ __________, with respect to the power series $\sum_{n=1}^{\infty} a_n x^n$.

6. Compute the series $\sum_{n=1}^{\infty} \ln^n x =$ __________, where the domain of the sum function should be $\{x | x \in$ __________$\}$.

7. Express $\sin^2 x$ as a Maclaurin series, i.e., in the form of $\sin^2 x = \sum_{n=0}^{\infty} a_n x^n$.

8. Express $f(x) = \frac{1}{x^2 + 3x + 2}$ as a Maclaurin series: __________. Then calculate the 2018$^{\text{th}}$ order derivative of $f(x)$:

$$f^{(2018)}(0) = \underline{\hspace{3cm}}.$$

9. Consider a function of period 2π:

$$f(x) = \begin{cases} 1 & \text{if} \quad 0 \leq x \leq \pi; \\ 2 & \text{if} \quad \pi < x < 2\pi. \end{cases}$$

Let its corresponding Fourier series expansion be given by

$$S(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

Compute the coefficients

$a_0 =$ ___________, $a_n =$ ___________, $b_0 =$ ___________, where $n = 1, 2, \cdots$.

Then compute $S(\pi) =$ ___________.

10. Consider a complex number $z = 1 - i$. Write down the modulus and principal argument of z:

$|z| =$ ___________, $\arg z =$ ___________.

11. Consider a complex number $z = 1 + i\sqrt{3}$. Compute z^7, expressing the result in the Cartesian form:

$z^7 =$ ___________.

12. Express a complex number $\sqrt{3} + i$ in the polar exponential form: ___________.

13. Let Σ be the surface described by the equation

$$x^2 + y^2 + z^2 + 4x + 6y - 2z = 0.$$

Classify Σ to be a ___________ (a paraboloid, ellipsoid, cylinder, cone or a sphere)

14. Consider a curve

$$C: \quad \begin{cases} y^2 - \dfrac{z^2}{4} &= 1 \\ \qquad\quad x &= 0. \end{cases}$$

Let Σ be the surface obtained by revolving C about the $y-$axis.

Determine the equation of the revolving surface Σ : __________.

15. Consider a horizontal curve C described by

$$\begin{cases} x^2 - y^2 + xy &= 4, \\ \qquad\qquad\quad z &= 5. \end{cases}$$

A cone, denoted as Σ, can be produced by regarding C as the directrix, and a line L as the generating line. That is,

- let P be a moving point on the directrix C, and
- keep L always passing through P and the origin point $(0, 0, 0)$.

Try to determine the equation of the cone Σ : __________.

16. Compute the limit:

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{(-1)^k}{k!} = \underline{\qquad\qquad}.$$

Advanced Mathematics (Module 3)

Section B: Extended Answer Questions

This section is worth a total of **20** marks. The marks of each question are as shown.

17. (**12** marks) Given a power series

$$\sum_{n=1}^{\infty} \frac{nx^n}{3^n}.$$

(a) Determine its interval of convergence.

(b) Find its sum function

$$S(x) = \sum_{n=1}^{\infty} \frac{nx^n}{3^n}.$$

(c) Evaluate the series

$$\sum_{n=1}^{\infty} \frac{n}{3^n}.$$

18. (**8** marks) Consider a real coefficient polynomial equation

$$z^4 - z^3 + z^2 - 11z + 10 = 0.$$

Given that $z = -1 + 2i$ is a root of the equation, try to find all the other three roots.

Glossary

Absolutely convergent	绝对收敛
Cartesian form	坐标形式
Classify	分类，识别
Conditionally convergent	条件收敛的
Cone	锥面
Convergence	收敛
Cylinder	柱面
Directrix	准线
Divergence	发散
Domain	定义域
Ellipsoid	椭球面
Expansion	展开式
Exponential form	指数形式
Fourier series	傅里叶级数
Generating line	母线

Interval of convergence	收敛区间
Maclaurin series	马克老林级数
Modulus	模长
Paraboloid	抛物面
Polar form	极坐标形式
Power series	幂级数
Radius of convergence	收敛半径
Specify	说明
Sphere	球面
Sum function	和函数
Surface of revolution	旋转曲面
The principal argument	辐角主值