

Fundamentals of Materials Science Homework 10

Name: Xiao, Liyang

Date: 03/19/2017

Student #: 15090215

Homework Problems:

1. Show that the minimum cation-to-anion radius ratio for a coordination number of 6 is 0.414. [Hint: use the NaCl crystal structure, and assume that anions and cations are just touching along cube edges and across the face diagonals.]

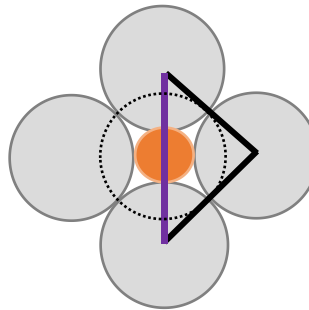
Solution:

The figure shows that:

$$2r_{Na^+} + 2r_{Cl^-} = \sqrt{2}a; a = 2r_{Cl^-}$$

$$\therefore 2r_{Na^+} + 2r_{Cl^-} = 2\sqrt{2}r_{Cl^-}$$

$$\therefore \frac{r_{Na^+}}{r_{Cl^-}} = \frac{2\sqrt{2} - 2}{2} = 0.414$$



2. On the basis of ionic charge and ionic radii given in the Table in your textbook or lecture note, predict crystal structures for the following materials:

(a) CaO, (b) MnS, (c) KBr, and (d) CsBr.

Justify your selections.

Solution:

(a)CaO: $r_{Ca^{2+}} = 0.100nm; r_{O^{2-}} = 0.140nm$

$$\therefore \frac{r_{Ca^{2+}}}{r_{O^{2-}}} = \frac{0.100nm}{0.140nm} = 0.714 ; \text{ based on this ratio, coord\#}=6, \text{ structure=sodium chloride.}$$

(b)MnS: $r_{Mn^{2+}} = 0.067nm; r_{S^{2-}} = 0.184nm$

$$\therefore \frac{r_{Mn^{2+}}}{r_{S^{2-}}} = \frac{0.067nm}{0.184nm} = 0.364 ; \text{ based on this ratio, coord\#}=4, \text{ structure=zinc blende.}$$

(c)KBr: $r_{K^+} = 0.138nm; r_{Br^-} = 0.196nm$

$$\therefore \frac{r_{K^+}}{r_{Br^-}} = \frac{0.138nm}{0.196nm} = 0.704 ; \text{ based on this ratio, coord\#}=6, \text{ structure}=\text{sodium chloride.}$$

(d)CsBr: $r_{Cs^+} = 0.170nm; r_{Br^-} = 0.196nm$

$$\therefore \frac{r_{Cs^+}}{r_{Br^-}} = \frac{0.170nm}{0.196nm} = 0.867 ; \text{ based on this ratio, coord\#}=8, \text{ structure}=\text{cesium chloride.}$$

- 3. The unit cell for Al_2O_3 has hexagonal symmetry with lattice parameters $a = 0.4759 \text{ nm}$ and $c = 1.2989 \text{ nm}$. If the density of this material is 3.99 g/cm^3 , calculate its atomic packing factor.**

Solution:

Atomic weight of Al is 26.98 g/mol ; Ionic radius of Al^{3+} is 0.053 nm

Atomic weight of O is 16.00 g/mol ; Ionic radius of O^{2-} is 0.140 nm

$$\therefore \rho = \frac{nA}{VN_A} \therefore n = \frac{\rho VN_A}{A} = \frac{\rho VN_A}{(2A_{Al} + 3A_O)}$$

$$n = \frac{3.99 \text{ g/cm}^3 \times \frac{\sqrt{3}}{2} \times (0.4759 \text{ nm})^2 \times 1.2989 \text{ nm} \times 6.02 \times 10^{23} \text{ atoms/mol}}{2 \times 26.98 \text{ g/mol} + 3 \times 16.00 \text{ g/mol}} = 18$$

$$APF = \frac{nV_1}{V} = \frac{18 \times \left[\frac{4}{3} \pi \times (0.053 \text{ nm})^3 + \frac{4}{3} \pi \times (0.140 \text{ nm})^3 \right]}{\frac{\sqrt{3}}{2} \times (0.4759 \text{ nm})^2 \times 1.2989 \text{ nm}} = 0.84$$

- 4. Iron oxide (FeO) has the rock salt crystal structure and a density of 5.70 g/cm^3 .**
- (a) Determine the unit cell edge length.**
- (b) How does this result compare with the edge length as determined from the radii in Table 4.4, assuming that the Fe^{2+} and O^{2-} ions just touch each other along the edges?**

Solution:

(a).Atomic weight of Fe is 55.85 g/mol ; Ionic radius of Fe^{2+} is 0.077 nm

Atomic weight of O is 16.00 g/mol ; Ionic radius of O^{2-} is 0.140 nm

$$V = \frac{nA}{\rho N_A} = \frac{4 \times (55.85 \text{ g/mol} + 16.00 \text{ g/mol})}{5.70 \text{ g/cm}^3 \times 6.02 \times 10^{23} \text{ atoms/mol}} = 0.0837 \text{ nm}^3$$

$$a = \sqrt[3]{0.0837 \text{ nm}^3} = 0.437 \text{ nm}$$

$$(b). a = 2r_{Fe^{2+}} + 2r_{O^{2-}} = 2 \times 0.077nm + 2 \times 0.140nm = 0.434nm$$

5. Compute the theoretical density of ZnS, given that the Zn-S distance and bond angle are 0.234 nm and 109.5°, respectively. How does this value compare with the measured density?

Solution:

Atomic weight of Zn is 65.42g/mol

Atomic weight of S is 32.06g/mol

$$\frac{\sqrt{2}}{2}a = 2 \times d \times \sin\left(\frac{180^\circ - \theta}{2}\right) \therefore a = 2\sqrt{2} \times 0.234nm \times \sin 35.25^\circ = 0.540nm$$

$$\therefore V = a^3 = (0.540nm)^3 = 0.157nm^3$$

$$\therefore \rho = \frac{nA}{VN_A} = \frac{4 \times (65.42g/mol + 32.06g/mol)}{0.157nm^3 \times 6.02 \times 10^{23}atmos/mol} = 4.12g/mol$$

6. The zinc blende crystal structure is one that may be generated from close-packed planes of anions.
- Will the stacking sequence for this structure be FCC or HCP? Why?
 - Will cations fill tetrahedral or octahedral positions? Why?
 - What fraction of the positions will be occupied?

Solution:

(a)The stacking sequence of close-packed planes of anions for the zinc blende crystal structure will be the same as FCC (and not HCP) because the anion packing is FCC.

(b)The cations will fill tetrahedral positions since the coordination number for cations is four.

(c)Only one-half of the tetrahedral positions will be occupied because there are two tetrahedral sites per anion, and yet only one cation per anion.

7. Magnesium oxide has the rock salt crystal structure and a density of 3.58 g/cm³.
- Determine the unit cell edge length.
 - How does this result compare with the edge length as determined from the radii in your textbook or lecture note, assuming that the Mg²⁺ and O²⁻ ions just touch each other along the edges?

Solution:

- (a). Atomic weight of Mg is 24.31g/mol ; Ionic radius of Mg^{2+} is 0.072nm
 Atomic weight of O is 16.00g/mol ; Ionic radius of O^{2-} is 0.140nm

$$V = \frac{nA}{\rho N_A} = \frac{4 \times (24.31 \text{ g/mol} + 16.00 \text{ g/mol})}{3.58 \text{ g/cm}^3 \times 6.02 \times 10^{23} \text{ atoms/mol}} = 0.0748 \text{ nm}^3$$

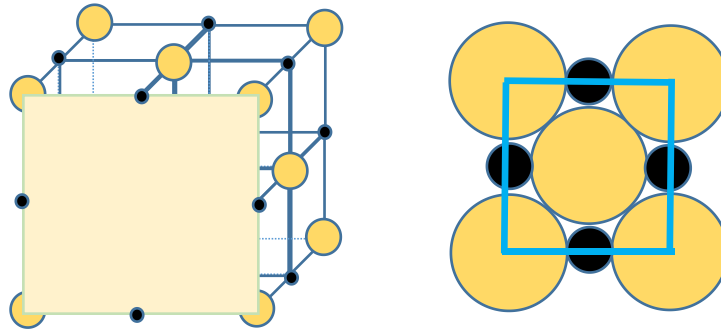
$$a = \sqrt[3]{0.0748 \text{ nm}^3} = 0.421 \text{ nm}$$

(b). $a = 2r_{Mg^{2+}} + 2r_{O^{2-}} = 2 \times 0.072 \text{ nm} + 2 \times 0.140 \text{ nm} = 0.424 \text{ nm}$

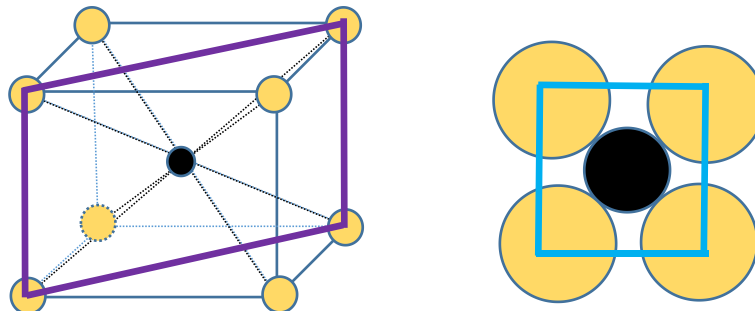
8. For each of the following crystal structures, represent the indicated plane in the manner of Figure 4.20 and 4.21, showing both anions and cations:
- (100) plane for the rock salt crystal structure,
 - (110) plane for the cesium chloride crystal structure.

Solution:

(a)



(b)



9. Nanowires are high aspect-ratio metal or semiconducting wires with diameters on the order of 1 to 100 nanometers and typical lengths of 1 to 100 microns. Nanowires likely will be used in the future to create high-density electronic circuits.
- Nanowires can be fabricated from ZnO. ZnO has the wurtzite structure. The

wurtzite structure is a hexagonal lattice with four atoms per lattice point at Zn (0, 0, 0), Zn (2/3, 1/3, 1/2), O (0, 0, 3/8), and O (2/3, 1/3, 7/8).

- How many atoms are there in the conventional unit cell?
- If the atoms were located instead at Zn (0, 0, 0), Zn (1/3, 2/3, 1/2), O (0, 0, 3/8), and O (1/3, 2/3, 7/8), would the structure be different? Please explain.
- For ZnO, the unit cell parameters are $a=3.24 \text{ \AA}$ and $c=5.19 \text{ \AA}$. (Note: this is not the ideal HCP c/a ratio.) A typical ZnO nanowire is 20 nm in diameter and 5 μm long. Assume that the nanowires are cylindrical. Approximately how many atoms are there in a single ZnO nanowire?

Solution:

(a) Because there are 2 Zn atoms and 2 O atoms per unit cell, there are 4 atoms in the conventional unit cell.

(b) the structure will be the same.

(c) ???