Beijing-Dublin International College

SEMESTER II FINAL EXAMINATION – 2020/2021

Beijing-Dublin International College
BDIC1042J Maths 3 (Advanced Mathematics; Finance)

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MODULE LECTURER: Xin Liu

Time Allowed: 90 minutes

Instructions for Candidates

Answer ALL questions. The marks that each question carry is written as shown.
Write **ALL** your answers on the **EXAMINATION BOOK** provided.

BJUT Student ID: ________________ **UCD Student ID:** ________________

I have read and clearly understand the Examination Rules of both Beijing University of Technology and University College Dublin. I am aware of the Punishment for Violating the Rules of Beijing University of Technology and/or University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I accept the punishment thereof.

Honesty Pledge： ____ **(Signature)**

Instructions for Invigilators

Non-programmable calculators are permitted. NO dictionaries are permitted.
No rough-work paper is to be provided for candidates.

SECTION A — TRUE-OR-FALSE QUESTIONS

In each question, judge the statement is **True** or **False**.
This section is worth a total of **9** marks, with each question worth **3** marks.

1. Let S be a surface in three dimensions. Then at every point P on S, there exists a unique direction $\boldsymbol{n}$ normal to S, and thus a unique tangent plane of S at P.

True **False**

2. Let S be a surface in three dimensions, and P a point on S. If the normal direction $\boldsymbol{n}$ of S at P is unique, then the tangent plane of S at P is unique.

True **False**

3. Let Σ be a plane in three dimensions, and P a point on Σ. Then we can find only one surface S that has Σ as its tangent plane at P.

True **False**

SECTION B — MATCHING-ITEM QUESTIONS

In each question, for every item of **Group A**, find a matching item in **Group B**.
This section is worth a total of **8** marks, with the marks of each question as shown.

4. (**3 marks**) Let $z = f(x, y)$ be a function of two variables.

	Group A		**Group B**
1.	The direction of fastest increasing for z is given by	a.	$\hat{\nabla} f = \mathbf{0}$
2.	The direction of fastest decreasing for z is given by	b.	$\hat{\nabla} f$
3.	The direction of zero change for z is given by	c.	$-\hat{\nabla} f$

5. (**5 marks**) Let $z = f(x, y)$ be a function of two variables, and (x_0, y_0) a local extremum point of $f(x, y)$. Let $D = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix}$ be the discriminant determinant.

Then for (x_0, y_0) we have the following classifications.

	Group A		**Group B**
1.	$D = 0$	a.	It is a local maximum point.
2.	$D < 0$	b.	It contains information useless for the classification.
3.	$D > 0$ and $f_{yy} > 0$	c.	It is a local minimum point.
4.	$D > 0$ and $f_{yy} < 0$	d.	The information is useful but we still cannot draw a conclusion.
5.	$D > 0$ and $f_{xy} > 0$	e.	It is a saddle point.

SECTION C — MULTIPLE CHOICE QUESTIONS

In each question, choose **at most one** option.
This section is worth a total of **33** marks, with each question worth **3** marks.

6. Evaluate the limit:

$$\lim_{(x,y)\to(0,0)} \frac{\sin\left(x^2 + 2y^2\right)}{x^2 + y^2}.$$

(a) 0; (b) 1; (c) 2; (d) no limit.

7. Evaluate the limit:

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2 + 2y}.$$

(a) 0; (b) 1; (c) 2; (d) no limit.

8. Evaluate the limit:

$$\lim_{(x,y)\to(0,0)} \frac{x^2 - y^2}{9\sqrt{x^2 + y^2}}.$$

(a) 0; (b) $\frac{1}{9}$; (c) $\frac{1}{3}$; (d) no limit.

9. Find the distance between the following two planes:

$$\Sigma_1 : \quad 10x + 2y - 2z = 5, \qquad\qquad \Sigma_2 : \quad 5x + y - z = 1.$$

(a) 0; (b) $\frac{\sqrt{3}}{6}$; (c) $\frac{\sqrt{3}}{3}$; (d) $\frac{\sqrt{6}}{2}$.

10. Try to find the equation of the cone, which has the origin $(0, 0, 0)$ as its apex, a planar curve

$$C : \quad \begin{cases} \frac{x^2}{9} + \frac{z^2}{4} & = & 1 \\ y & = & 6 \end{cases}$$

as the directrix (i.e., the trajectory), and a line l as the generatrix (i.e., the line passing through the apex and a point on the directrix).

(a) $9x^2 + 4z^2 = y^2, \quad x, y, z > 0$; (b) $4x^2 + 9z^2 = y^2, \quad x, y, z > 0$;

(c) $9x^2 + 4y^2 = z^2, \quad x, y, z > 0$; (d) $\frac{x}{3} = \frac{y}{6\sqrt{2}} = \frac{z}{2}, \quad x, y, z > 0$.

11. Consider a curve C in space given by the intersection of two surfaces:

$$\Sigma_1 : \quad x + y + z = 1, \qquad\qquad \Sigma_2 : \quad x^2 + 2y^2 - z = 0.$$

Which of the following statement is INCORRECT, regarding C's projection onto a coordinate plane?

(a) The projection of C onto the xy-plane is $\begin{cases} x^2 + 2y^2 + x + y & = & 1, \\ z & = & 0. \end{cases}$

(b) The projection of C onto the xz-plane is $\begin{cases} x^2 + 2(1 - x - z)^2 - z & = & 0, \\ y & = & 0. \end{cases}$

(c) The projection of C onto the xz-plane is $\begin{cases} x + \sqrt{\frac{z - x^2}{2}} + z & = & 1, \\ y & = & 0. \end{cases}$

(d) The projection of C onto the yz-plane is $\begin{cases} (1 - y - z)^2 + 2y^2 - z & = & 0, \\ x & = & 0. \end{cases}$

12. Consider a function

$$z = \ln \frac{1}{1 - x^2 - y^2}, \qquad x, y \in \mathbb{R}.$$

Which of the following statements is CORRECT for its domain and range?

(a) The domain is the whole xy-plane except the origin $(0, 0)$.

(b) The domain is the unit circular disk on the xy-plane, centered at the origin $(0, 0)$, including the boundary $x^2 + y^2 = 1$ (the unit circle).

(c) The range is $[0, +\infty)$.

(d) The range is $(-\infty, +\infty)$.

13. For a function

$$z = xye^y,$$

which of the following computations is INCORRECT?

(a) $\frac{\partial z}{\partial x} = ye^y$.

(b) $\frac{\partial z}{\partial x} = 1$, when $(x, y) = (1, 0)$.

(c) $\frac{\partial z}{\partial y} = x\,(e^y + ye^y)$.

(d) $\frac{\partial z}{\partial y} = 1$, when $(x, y) = (1, 0)$.

14. Find the total differential of the function

$$z = xye^y$$

at the point $(x, y) = (-1, 1)$.

(a) $dz = e(dx - 2dy)$.

(b) $dz = e(dx + 2dy)$.

(c) $\Delta z = e\Delta x + 2e\Delta y$.

(d) $dz = -e$.

15. The surface described by the equation

$$9x^2 + 36y^2 + 4z^2 - 18x + 72y + 9 = 0$$

is a(n) __________.

(a) ellipsoid; (b) paraboloid; (c) hyperboloid; (d) cone (with the apex at the origin).

16. Consider the intersection curve l between a plane Σ and a surface S:

$$\Sigma: \quad x + y + z = 1, \qquad S: \quad x^2 - 2y^2 - z^3 = 0.$$

Try to find the tangent line of l at the point $(1, 1, -1)$.

(a) $x + 5y + 6z - 10 = 0$.

(b) $x + 5y - 6z - 12 = 0$.

(c) $x - 1 = \frac{y-1}{5}$ and $z = -1$.

(d) $x - 1 = \frac{y-1}{5} = -\frac{z+1}{6}$.

SECTION D — GAP-FILLING QUESTIONS

Write your answers on the **Examination Book** provided.
This section is worth a total of **20** marks, with each question worth **4** marks.

17. Consider a surface S described by the equation $x^2 - 2y^2 - z^3 = 0$. The equation of the plane tangent to S at the point $(1, 1, -1)$ is given by __________.

18. (*Error estimation*) Consider a surface in three dimensions described by the function

$$z = f(x, y) = xye^y$$

and a point P on the surface, $P(x, y, z) = (-1, 1, -e)$. If the errors in measuring x and y are ± 0.01 and ± 0.02, respectively, then at P the value of z with error estimation is given by __________.

19. For a function

$$u = xy^2 + z^3 - xyz,$$

its directional derivative at a point $P(1, 1, 2)$ along a particular direction $\hat{\boldsymbol{l}}$ is __________, where the directional angles of $\hat{\boldsymbol{l}}$ with respect to the x-, y- and z-axis are $\alpha = \frac{\pi}{3}$, $\beta = \frac{\pi}{4}$ and $\gamma = \frac{\pi}{3}$, respectively.

*[Note]: *Directional angles* means $\cos\alpha$, $\cos\beta$ and $\cos\gamma$ are $\hat{\boldsymbol{l}}$'s directional cosines with respect to the x-, y- and z-axis, respectively.

20. The temperature T of a two-dimensional heated circular plate is given by $T = \frac{1}{x^2+y^2}$. At the point $P(1, 1)$, the maximum rate of change of T is __________, occurring in the unit direction $\hat{\boldsymbol{l}} =$ __________.

21. For an implicit function $F(x, y, z) = e^z - xyz = 0$, find $\frac{\partial z}{\partial x} =$ __________ and $\frac{\partial z}{\partial y} =$ __________.

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SECTION E — EXTENDED ANSWER QUESTIONS

Write your answers on the **Examination Book** provided.
This section is worth a total of **30** marks. The marks of each question are as shown.

22. **(8 marks)** Consider an implicit function

$$f(ax+by, cy-az, bz+cx)=0.$$

Try to compute

$$b\frac{\partial z}{\partial x}-a\frac{\partial z}{\partial y}.$$

23. **(7 marks)** Consider two implicit equations

$$\begin{cases} x^2 & +y & -u & +v^2 & =0, \\ x^3 & -2y & +u^2 & -v & =0. \end{cases}$$

Supposing u and v are functions of x and y, i.e., $u=u(x,y)$ and $v=v(x,y)$, try to find $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$.

24. **(7 marks)** Considering a function

$$f(x,y,z)=x^2+xy+y^2+y.$$

find its critical point(s), and classify each one to be a local maximum, minimum or a saddle point.

25. **(8 marks)** Consider a function

$$f(x,y,z)=x^2+y^2+z$$

and two constraints

$$x-y=1, \qquad y^2-z=1.$$

Use the method of Lagrangian multiplier to find the extremum of f, and locate the position (x,y,z) where the extremum occurs.

Glossary

Apex	顶点
Boundary	边界
Circular disk	圆盘
Classification	分类
Cone	锥面
Constraint	约束
Correct	正确
Critical point	临界点
Decreasing	减少
Directional angle	方位角
Directional cosine	方向余弦
Directional derivative	方向导数
Directrix	准线
Discriminant	判别式
Domain	定义域
Ellipsoid	椭球面
Error estimation	误差估计
Extremum (*pl*. extrema)	极值
Generatrix	母线
Heated	受热
Hyperboloid	双曲面
Implicit function	隐函数

Incorrect	不正确
Increasing	增加
Intersection curve	交线
Lagrangian multiplier	拉格朗日乘子
Matching-item	配对，连线
Maximum	极大
Minimum	极小
Normal	垂直于
Occur	发生
Paraboloid	抛物面
Partial derivative	偏导数
Planar curve	平面曲线
Projection	投影
Range	值域
Rate of change	变化率
Saddle	鞍点
Surface	曲面
Tangent plane	切平面
Temperature	温度
Total differential	全微分
Trajectory	轨迹
True-or-false	判断正误
Unique	唯一