Beijing-Dublin International College

SEMESTER II FINAL EXAMINATION – 2018/2019

Beijing-Dublin International College

BDIC1042J Maths 3 (Advanced Mathematics; Finance)

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MODULE LECTURER: Xin Liu

Time Allowed: 90 minutes

Instructions for Candidates

Answer ALL questions. The marks that each question carry is written as shown.

BJUT Student ID: ________________ **UCD Student ID:** ________________

I have read and clearly understand the Examination Rules of both Beijing University of Technology and University College Dublin. I am aware of the Punishment for Violating the Rules of Beijing University of Technology and/or University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I accept the punishment thereof.

Honesty Pledge： ____ **(Signature)**

Instructions for Invigilators

Non-programmable calculators are permitted. NO dictionaries are permitted.

No rough-work paper is to be provided for candidates.

SECTION A — MULTIPLE CHOICE QUESTIONS

In each question, choose **at most one** option.

Write your answers on the **Examination Book** provided.

This section is worth a total of **15** marks, with each question worth **3** marks.

1. Consider a periodic function $f(x) = 5 + \cos\left(x - \frac{\pi}{6}\right)$. If its Fourier series reads

$$f(x) = A_0 + \sum_{n=1}^{\infty} A_n \cos nx + \sum_{m=1}^{\infty} B_m \sin mx,$$

we have the coefficients:

(a) $A_0 = 0$; $A_n = 0$ for $n = 1, 2, 3, \cdots$; $B_1 = \frac{1}{2}$, and $B_m = 0$ for $m = 2, 3, 4, \cdots$.

(b) $A_0 = 5$; $A_n = \frac{\sqrt{3}}{2}$ for $n = 1, 2, 3, \cdots$; $B_m = 0$ for $m = 1, 2, 3, \cdots$.

(c) $A_0 = 5$; $A_1 = \frac{\sqrt{3}}{2}$, $B_1 = \frac{1}{2}$; $A_n = B_m = 0$ for $n, m = 2, 3, 4, \cdots$.

(d) $A_0 = 5$; $A_1 = \frac{\sqrt{3}}{2}$, $B_1 = -\frac{1}{2}$; $A_n = B_m = 0$ for $n, m = 2, 3, 4, \cdots$.

2. Regarding the following Fourier transform formulae

$$\begin{cases} f(t) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} G(\omega) e^{i\omega t} d\omega, \\ G(\omega) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt, \end{cases}$$

which statement/understanding is NOT correct?

(a) For an aperiodic function, "aperiodic" means its period $T \to 0$.

(b) Fourier transform can be done for not only an aperiodic function, but also for a periodic function actually.

(c) If regarding $f(t)$ as an original function, then $G(\omega)$ is the corresponding kernel of the Fourier transform. On the contrary, if $G(\omega)$ is regarded as the original function, then $f(t)$ becomes the kernel of the transform.

(d) Fourier transform implies a duality relationship between t and ω, and that between $f(t)$ and $G(\omega)$.

3. Determine the shape of the surface described by the following equation

$$2x^2 + y^2 + 2z^2 + 8x - 2y + 4z + 3 = 0.$$

(a) A sphere; (b) An ellipsoid; (c) A paraboloid; (d) A hyperboloid.

4. Which of the following statements is correct?

(a) In the three-dimensional space, there are only 3 paths to approach a point (x_0, y_0, z_0);

(b) In the two-dimensional xy-plane, there are only 2 paths to approach a point (x_0, y_0);

(c) On the one-dimensional x-axis, there are only 2 paths to approach a point x_0;

(d) On the one-dimensional x-axis, there is only 1 path to approach a point x_0.

5. Find the natural domain of the following function in $\mathbb{R}^2$:

$$z = f(x, y) = \sqrt{\frac{1}{x^2 + y^2 - 4x + 3}}.$$

(a) The domain is the upper z-axis including the origin point.

(b) The domain is the region outside but not including C, where C is the circle centered at $(0, 2)$ with radius 2.

(c) The domain is the region outside and including C, where C is the circle centered at $(2, 0)$ with radius 1.

(d) The domain is the region outside but not including C, where C is the circle centered at $(2, 0)$ with radius 1.

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SECTION B — GAP-FILLING QUESTIONS

Write your answers on the **Examination Book** provided.

This section is worth a total of **45** marks, with each question worth **3** marks.

6. Consider the plane Σ passing through three points $A\,(0,-1,-1)$, $B\,(1,3,-2)$ and $C\,(1,2,-1)$ in space. The equation of Σ is ________________.

7. Consider two surfaces in three dimensions,

$$\Sigma_1: \quad x^2+(y+2)^2+z^2=1, \qquad\qquad \Sigma_2: \quad x^2+y^2+z^2-2x-2y-2z+2=0.$$

Let P_1 and P_2 be two moving points on Σ_1 and Σ_2, respectively. The shortest distance between P_1 and P_2 (i.e., the shortest distance between Σ_1 and Σ_2) is ________________.

8. The surface of a revolution body Σ is generated by rotating a curve C in the xz-plane about a symmetric axis L in space. If the equation of Σ is given by

$$z=e^{x^2+y^2}-\sin\sqrt{x^2+y^2},$$

the curve C is ________________, with L being the z-axis.

9. Let C be a curve in space

$$C: \left\{ \begin{array}{rcl} z & = & \sin\left(x^2+y^2\right), \\ z & = & x+2y. \end{array} \right.$$

The equation of the projection of C onto the xy-plane is ________________.

10. Let C be a horizontal curve given by $\begin{cases} e^{3(x+y)}\cos(x-y) &= 1, \\ z &= 2. \end{cases}$ Suppose Σ is a cone formed by moving a line L along C — that is, regarding C as a trajectory — while keeping L passing through the origin point $(0,0,0)$.

The equation of the cone Σ is ________________.

11. Evaluate the limit:

$$\lim_{(x,y)\to(0,0)} \frac{\sqrt{3}xy}{x+\sqrt{3}y} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.$$

12. Evaluate the limit:

$$\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{3\sqrt{x^2+y^2}} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.$$

13. Evaluate the limit:

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2-y^2} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.$$

14. Evaluate the limit:

$$\lim_{(x,y)\to(0,-1)} \frac{e^{2xy}\sin(2xy)}{x} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.$$

15. If $f(x,y) = 4x^2y - y^3x + y^2$, then $f_y(1,-1)$ equals _______________.

16. For $f(x,y) = \ln\sin\left(x^3y^2\right)$, find its partial derivative f_y over the natural domain:

$$f_y = \underline{\hspace{4cm}}.$$

17. Let $f(x,y) = ye^{\sin x} + xe^{\sin y}$. The second order partial derivative $f_{yy} =$ _______________.

18. Let $u(x,y) = e^y \sin x$. Compute $u_{xx} + u_{yy} =$ _______________.

19. The total differential of a two-variable function $z = e^{x^2 - \sin y}$ is _______________.

20. The total differential of a three-variable function $u = e^{(x+z)^2 - \sin y}$ is _______________.

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SECTION C — EXTENDED ANSWER QUESTIONS

Write your answers on the **Examination Book** provided.

This section is worth a total of **40** marks. The marks of each question are as shown.

21. (7 marks) Consider a real coefficient polynomial equation in the unknown z:

$$z^5 - 6z^4 + 23z^3 - 34z^2 + 26z = 0, \qquad \text{where} \quad z \in \mathbb{C}.$$

If $z_1 = 2 - 3i$ is a root of this equation, try to find all the other roots.

22. (10 marks) Consider a triangle $\triangle ABC$, with three vertices A, B and C, and three sides a, b and c. The area of $\triangle ABC$ is given by $S = \frac{1}{2}ab\sin C$. When $a = 20$, $b = 30$ and $C = \frac{\pi}{6}$, try to find:

(a) The rate of change of S with respect to a, when b and C are constant. *(3 marks)*

(b) The rate of change of S with respect to C, when a and b are constant. *(3 marks)*

(c) The rate of change of b with respect to a, when S and C are constant. *(4 marks)*

23. (10 marks) (*Physics*) The power consumed in an electrical resistor is given by

$$P = \frac{U^2}{R},$$

where P is the *power* (in the unit of watts), U the *electrical voltage* (in volts), and R the *electrical resistor* (in ohms). The initial values read $U = 200$ volts and $R = 8$ ohms.

By how much does the power change, if U is decreased by 5 volts and R is decreased by 0.2 ohm?

24. (13 marks) Consider a function $f = f(t)$, which satisfies an ordinary differential equation

$$\frac{d^2 f}{dt^2} - 5\frac{df}{dt} + 6f = 0. \tag{1}$$

It is known from the theory of differential equations that the general solution to (1) is given by

$$f(t) = C_1 e^{2t} + C_2 e^{3t}, \tag{2}$$

where C_1 and C_1 are two real constants.

Now let us use an alternative method, the *Fourier transform*, to re-solve this differential equation. Try to use the formulae

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} G(\omega)e^{i\omega t}d\omega, \qquad G(\omega) = \int_{-\infty}^{+\infty} f(t)e^{-i\omega t}dt \tag{3}$$

to obtain Solution (2).

[*Hints]:

- Rule of derivative:

$$\frac{d}{dt}\int_{-\infty}^{+\infty} G(\omega)e^{i\omega t}d\omega = \int_{-\infty}^{+\infty} G(\omega)\left(\frac{d}{dt}e^{i\omega t}\right)d\omega. \tag{4}$$

- Rule of the Dirac δ-function:

$$\int_{-\infty}^{+\infty} \delta(\omega - \omega_0)e^{i\omega t}d\omega = e^{i\omega_0 t}. \tag{5}$$

$$0 = \frac{d^2}{dt^2}\left[\frac{1}{2\pi}\int_{-\infty}^{+\infty} G(\omega) e^{i\omega t} d\omega\right] - 5\frac{d}{dt}\left[\frac{1}{2\pi}\int_{-\infty}^{+\infty} G(\omega)e^{i\omega t}d\omega\right] + 6\left[\frac{1}{2\pi}\int_{-\infty}^{+\infty} G(\omega) e^{i\omega t} d\omega\right]$$

$$= \frac{1}{2\pi}\left[\int_{-\infty}^{\infty} G(\omega)\left(\frac{d^2}{dt^2} e^{i\omega t}\right) d\omega - 5\int_{-\infty}^{\infty} G(\omega)\left(\frac{d}{dt} e^{i\omega t}\right)d\omega + 6\int_{-\infty}^{\infty} G(\omega) e^{i\omega t} d\omega\right]$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} G(\omega) e^{i\omega t}\left[(i\omega)^2 - 5i\omega + 6\right] d\omega$$

$$\Rightarrow \frac{1}{2\pi} G(\omega)\left[(i\omega)^2 - 5i\omega + 6\right] = \frac{1}{2\pi}G(\omega)\left[(i\omega)-2\right]\left[(i\omega)-3\right] = 0$$

i.e. $i\omega_1 = 2$ $\quad i\omega_2 = 3$

$$G(\omega) = 2\pi C_1\delta\left(\omega - \frac{2}{i}\right) + 2\pi C_2 \delta\left(\omega - \frac{3}{i}\right)$$

Then $f(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} G(\omega) e^{i\omega t} d\omega$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty}\left[2\pi C_1\delta\left(\omega-\frac{2}{i}\right) + 2\pi C_2\delta\left(\omega-\frac{3}{i}\right)\right]e^{i\omega t}d\omega.$$

$$= C_1 e^{2t} + C_2 e^{3t}$$

Glossary

Aperiodic	非周期的
Approach	逼近
Cartesian	笛卡尔
Coefficient	系数
Complex conjugate	复共轭
Cone	锥面
Cylinder	柱体
Duality	对偶
Ellipsoid	椭球面
Error	误差
Even function	偶函数
Fourier series	傅立叶级数
Fourier transform	傅立叶变换
Horizontal	水平
Hyperboloid	双曲面
Kernel	核（函数）
Modulus	模长
Odd function	奇函数
Ohm	欧姆
Ordinary differential equation	常微分方程
Paraboloid	抛物面
Partial derivative	偏导数
Periodic	周期的
Polynomial	多项式
Power	功率
Projection	投影
Resistor	电阻
Symmetric axis	对称轴
Trajectory	轨迹
Volt	伏特
Voltage	电压
Watt	瓦特