Refinement Types for Elm

Master Thesis Report

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Background

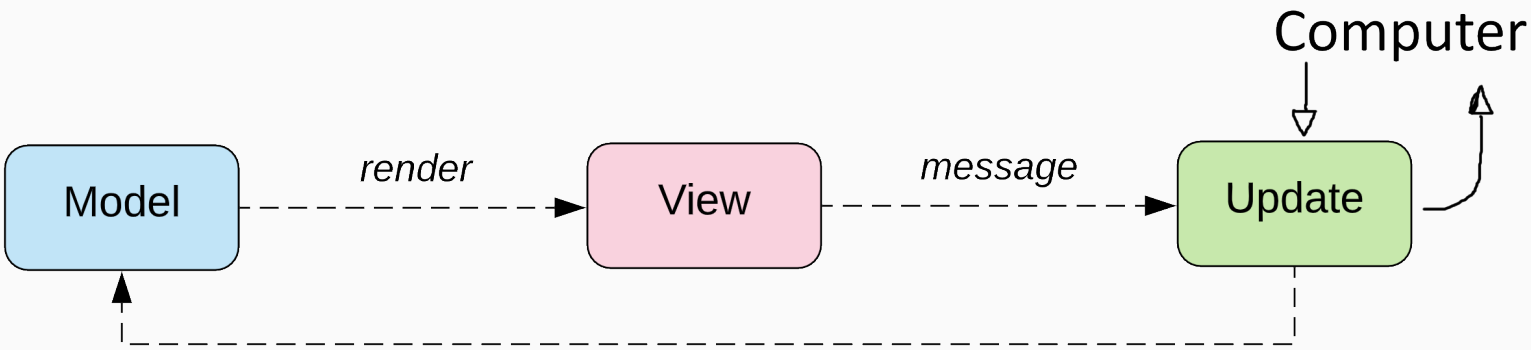
Background: Introduction to Elm

- Invented by Evan Czaplicki as his master-thesis in 2012.
- Website: elm-lang.org

Characteristics

- Pure Functional Language (immutable, no side effect, everything is a function)
- Compiles to JavaScript (in the future also to WebAssembly)
- ML-like Syntax (we say `fun a b c` for $fun(a, b, c)$)
- Simpler than Haskell (no Type classes, no Monads, only one way to do a given thing)
- "No Runtimes errors" (running out of memory, function equality)

Background: Introduction to Elm



Background: Introduction to Refinement Types

Restricts the values of an existing type using a predicate.

Initial paper in 1991 by Tim Freeman and Frank Pfenning

- Initial concept was done in ML.
- Allows predicates with only $\wedge, \vee, =$, constants and basic pattern matching.
- Operates over algebraic types.
- Needed to specify **explicitly** all possible Values.

Example

$$\forall t.\{\nu : \mathit{List}\ t \mid \nu = \mathit{Cons}\ (b : t)\ (c : \mathit{List}\ t) \wedge c = \mathit{Cons}\ (d : t)\ [\,]\}$$

Background: Introduction to Refinement Types

Liquid Types (Logically Quantified Data Types) introduced in 2008

- Invented by Patrick Rondan, Ming Kawaguchi and Ranji Jhala
- Initial concept done in OCaml. Later also C, Haskell and TypeScript.
- Operates over Integers and Booleans, Tuples and Functions.
- Allows predicates with logical operators, comparisons and addition.

Example

$$a : \{\nu : Int \mid 0 \leq \nu\} \rightarrow b : \{\nu : Int \mid 0 \leq \nu\}$$
$$\rightarrow \{\nu : Int \mid 0 \leq \nu \ \wedge \ a - b \leq \nu \ \wedge \ b - a \leq \nu\}$$

Background: Motivation

- Catching Division by zero in compile time
- Catching index-out-of-bounds errors in compile time
- Having natural numbers as a subtype of ingeters

Background: Goals of Thesis

1. Formal language similar to Elm
    - A formal syntax
    - A formal type system
    - A denotational semantic
    - A small step semantic (using K Framework) for rapid prototyping the language
    - Proof that the type system is valid with respect to the semantics.
2. Extension of the formal language with Liquid Types
    - Extending the formal syntax, formal type system and denotational semantic
    - Proof that the extension infers the correct types.
    - A Implementation (of the core algorithm) written in Elm for Elm.

Formulizing the Elm Language

Formulizing the Elm Language: Defining the Type System

We will use the Hindley-Milner type system (used in ML, Haskell and Elm)

We say

$$
\begin{aligned}
T \text{ is a } \textit{mono type} :\Leftrightarrow\ & T \text{ is a type variable} \\
\vee\ & T \text{ is a type application} \\
\vee\ & T \text{ is a algebraic type} \\
\vee\ & T \text{ is a product type} \\
\vee\ & T \text{ is a function type} \\
T \text{ is a } \textit{poly type} :\Leftrightarrow\ & T = \forall a.\, T' \\
& \text{where } T' \text{ is a mono type or poly type} \\
& \text{and } a \text{ is a symbol} \\
T \text{ is a } \textit{type} :\Leftrightarrow\ & T \text{ is a mono type } \vee\ T \text{ is a poly type.}
\end{aligned}
$$

Formulizing the Elm Language: Defining the Type System

Example

1. $Nat ::= \mu C.1 \mid Succ\ C$
2. $List = \forall a.\mu C.Empty \mid Cons\ a\ C$
3. splitAt : $\forall a.Nat \to List\ a \to (List\ a, List\ a)$

Formulizing the Elm Language: Defining the Type System

The *values* of a type is the set corresponding to the type:

$$\text{values}(\mathit{Nat}) = \{1, \mathit{Succ}\ 1, \mathit{Succ}\ \mathit{Succ}\ 1, \dots\}$$

$$\text{values}(\mathit{List}\ \mathit{Nat}) = \bigcup_{n \in \mathbb{N}} \text{values}_n(\mathit{List}\ \mathit{Nat})$$

$$\text{values}_0(\mathit{List}\ \mathit{Nat}) = \{[\,]\}$$

$$\text{values}_n(\mathit{List}\ \mathit{Nat}) =$$

$$\{\mathit{Cons}\ a\ b \mid a \in \text{values}(\mathit{Nat}), b \in \text{values}_{n-1}(\mathit{List}\ \mathit{Nat})\}$$

Formulizing the Elm Language: Defining the Type System

Let T_1, T_2 be types.

We define the partial order $\sqsubseteq$ on types as

$$T_1 \sqsubseteq T_2 :\Leftrightarrow T_2 \text{ is an instance of } T_1$$

Example: $\forall a.a \sqsubseteq \forall a.\mathit{List}\ a \sqsubseteq \mathit{List}\ \mathit{Nat}$

Formulizing the Elm Language: Defining the Type System

$$\overline{\Gamma} : \Gamma \to \mathcal{T}$$

$$\overline{\Gamma}(T) := \forall a_1 \ldots \forall a_n . T_0$$

such that $\{a_1, \ldots, a_n\} = \text{free}(T') \setminus \{a \mid (a, \_) \in \Gamma\}$

where $a_i \in \mathcal{V}$ for $i \in \mathbb{N}_0^n$ and T_0 is the mono type of T.

We say $\overline{\Gamma}(T)$ is *the most general type* of T.

Example: $\forall a . \forall b . \text{List}\ (a, b)$ is the most general type of List (a, b).

Formulizing the Elm Language: Defining the Type System

$\Gamma : \mathcal{V} \to \mathcal{T}$ contains type aliases.

$\Delta : \mathcal{V} \to \mathcal{T}$ contains types of occuring variables.

Formulizing the Elm Language: The Max Function

```
max =
  \a -> \b ->
    if
      (<) a b
    then
      b
    else
      a
```

Starting with leaves of the AST: `a` and `b`.

Formulizing the Elm Language: The Max Function

$$\frac{(a, \overline{\Gamma}(T)) \in \Delta}{\Gamma, \Delta \vdash a : T}$$

New rules:

$$\frac{}{\Gamma, \Delta \cup \{(a, \overline{\Gamma}(T))\} \vdash a : T} \quad \frac{}{\Gamma, \Delta \cup \{(b, \overline{\Gamma}(T))\} \vdash b : T}$$

Formulizing the Elm Language: The Max Function

```
max =
  \a -> \b ->
    if
      (<) a b
    then
      b                          --> ?a1
    else
      a                          --> ?a2
```

Next we derive the type for `(<) a b`.

Formulizing the Elm Language: The Max Function

$$\frac{}{\Gamma, \Delta \vdash \text{"}(<)\text{"} : \mathit{Int} \to \mathit{Int} \to \mathit{Bool}}$$

$$\frac{\Gamma, \Delta \vdash e_1 : T_1 \to T_2 \quad \Gamma, \Delta \vdash e_2 : T_1}{\Gamma, \Delta \vdash e_1 \ e_2 : T_2}$$

New rule:

$$\frac{\Gamma, \Delta \vdash e_1 : \mathit{Int} \quad \Gamma, \Delta \vdash e_2 : \mathit{Int}}{\Gamma, \Delta \vdash \text{"}(<)\text{"} \ e_1 \ e_2 : \mathit{Bool}}$$

Formulizing the Elm Language: The Max Function

$$\frac{}{\Gamma, \Delta \cup \{(\mathtt{a}, \overline{\Gamma}(T))\} \vdash \mathtt{a} : T} \quad \frac{}{\Gamma, \Delta \cup \{(\mathtt{b}, \overline{\Gamma}(T))\} \vdash \mathtt{b} : T}$$

$$\frac{\Gamma, \Delta \vdash e_1 : \mathit{Int} \quad \Gamma, \Delta \vdash e_2 : \mathit{Int}}{\Gamma, \Delta \vdash \text{"}(<)\text{"}\ e_1\ e_2 : \mathit{Bool}}$$

The most general type of *Int* is *Int*

New rule:

$$\frac{}{\Gamma, \Delta \cup \{(\mathtt{a}, \mathit{Int}), (\mathtt{b}, \mathit{Int})\} \vdash \text{"}(<)\ \mathtt{a}\ \mathtt{b}\text{"} : \mathit{Bool}}$$

Formulizing the Elm Language: The Max Function

```
max =
  \a -> \b ->
    if
      (<) a b              --> Bool
    then
      b                    --> Int
    else
      a                    --> Int
```

Next we apply the rule for if-expressions.

Formulizing the Elm Language: The Max Function

$$\frac{}{\Gamma, \Delta \cup \{(a, \mathit{Int}), (b, \mathit{Int})\} \vdash \text{"}(<)\text{"}\ e_1\ e_2 : \mathit{Bool}}$$

$$\frac{\Gamma, \Delta \vdash e_1 : \mathit{Bool} \quad \Gamma, \Delta \vdash e_2 : T \quad \Gamma, \Delta \vdash e_3 : T}{\Gamma, \Delta \vdash \text{"}\texttt{if}\text{"}\ e_1\ \text{"}\texttt{then}\text{"}\ e_2\ \text{"}\texttt{else}\text{"}\ e_3 : T}$$

New rule:

$$\frac{}{\Gamma, \Delta \cup \{(a, \mathit{Int}), (b, \mathit{Int})\} \vdash \text{"}\texttt{if}(<)\ \texttt{a}\ \texttt{b}\ \texttt{then}\ \texttt{b}\ \texttt{else}\ \texttt{a}\text{"} : \mathit{Int}}$$

Formulizing the Elm Language: The Max Function

```
max =
  \a -> \b ->
    if                          --> Int
      (<) a b
    then
      b                         --> Int
    else
      a                         --> Int
```

Formulizing the Elm Language: The Max Function

$$\frac{\Gamma, \Delta \cup \{(a, \overline{\Gamma}(T_1))\} \vdash e : T_2}{\Gamma, \Delta \vdash \text{"}\backslash\text{"}\ a\ \text{"}->\text{"}\ e : T_1 \to T_2}$$

The most general type of *Int* is *Int*

Formulizing the Elm Language: The Max Function

Therefore we conclude

$$\frac{}{\Gamma, \Delta \cup \{(a, \mathit{Int})\} \vdash \text{"}\backslash\texttt{b} - > \texttt{if}\ (<)\ \texttt{a}\ \texttt{b}\ \texttt{then}\ \texttt{b}\ \texttt{else}\ \texttt{a}\text{"} : \mathit{Int} \to \mathit{Int}}$$

$$\frac{}{\Gamma, \Delta \vdash \text{"}\backslash\texttt{a} - > \backslash\texttt{b} - > \texttt{if}\ (<)\ \texttt{a}\ \texttt{b}\ \texttt{then}\ \texttt{b}\ \texttt{else}\ \texttt{a}\text{"} : \mathit{Int} \to \mathit{Int} \to \mathit{Int}}$$

Formulizing the Elm Language: The Max Function

```
max =                          --> Int -> Int -> Int
  \a -> \b ->
    if                         --> Int
      (<) a b
    then
      b                        --> Int
    else
      a                        --> Int
```

Extending the Elm Language

Extending the Elm Language: Defining Liquid Types

$$
\begin{aligned}
\mathit{IntExp} ::= {} & \mathbb{Z} \\
\mid {} & \mathit{IntExp} + \mathit{IntExp} \\
\mid {} & \mathit{IntExp} \cdot \mathbb{Z} \\
\mid {} & \mathcal{V}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{Q} ::= {} & \mathit{True} \\
\mid {} & \mathit{False} \\
\mid {} & \mathit{IntExp} < \mathcal{V} \\
\mid {} & \mathcal{V} < \mathit{IntExp} \\
\mid {} & \mathcal{V} = \mathit{IntExp} \\
\mid {} & \mathcal{Q} \wedge \mathcal{Q} \\
\mid {} & \mathcal{Q} \vee \mathcal{Q} \\
\mid {} & \neg \mathcal{Q}
\end{aligned}
$$

Extending the Elm Language: Defining Liquid Types

T is a *liquid type* $:\Leftrightarrow$ T is of form $\{\nu : \mathit{Int} \mid r\}$

where T_0 is a type, a is a symbol, $r \in \mathcal{Q}$,

$\mathit{Nat} := \mu C.1 \mid \mathit{Succ}\ C$

and $\mathit{Int} := \mu\_.0 \mid \mathit{Pos}\ \mathit{Nat} \mid \mathit{Neg}\ \mathit{Nat}$.

$\vee$ T is of form $a : \{\nu : \mathit{Int} \mid r\} \to T$

where a, b are symbols, $r \in \mathcal{Q}$ and T is a

liquid types.

Extending the Elm Language: Defining Liquid Types

Subtyping Condition

We say c is a *Subtyping Condition* $:\Leftrightarrow c$ is of form $T_1 <:_{\Theta,\Lambda} T_2$ where T_1, T_2 are a liquid types or templates, Θ is a type variable context and $\Lambda \subset \mathcal{Q}$.

Formulizing the Elm Language: Defining the Type System

$\Theta : \mathcal{V} \rightarrow \mathcal{T}$ contains types of liquid type variables (type variable of a dependent function).

$\Lambda : \mathcal{V} \rightarrow \mathcal{T}$ contains guards(conditions of if-expressions).

Extending the Elm Language: Revisiting the Max Function

```
max =
  \a -> \b ->
    if
      (<) a b
    then
      b
    else
      a
```

Again starting at `a` and `b`.

Extending the Elm Language: Revisiting the Max Function

$$\frac{\{\nu : Int|\ \nu = a\} <:_{\Theta,\Lambda} \{\nu : Int|\ r\} \quad (a, \{\nu : Int|\ r\}) \in \Delta \quad (a, \{\nu : Int|\ r\}) \in \Theta}{\Gamma, \Delta, \Theta, \Lambda \vdash a : \{\nu : Int|\ \nu = a\}}$$

New rule:

$$\frac{\{\nu : Int|\ \nu = \mathtt{a}\} <:_{\Theta,\Lambda} \{\nu : Int|\ r\} \quad (\mathtt{a}, \{\nu : Int|\ r\}) \in \Delta \quad (\mathtt{a}, \{\nu : Int|\ r\}) \in \Theta}{\Gamma, \Delta, \Theta, \Lambda \vdash \mathtt{a} : \{\nu : Int|\ \nu = \mathtt{a}\}}$$

$$\frac{\{\nu : Int|\ \nu = \mathtt{b}\} <:_{\Theta,\Lambda} \{\nu : Int|\ r\} \quad (\mathtt{b}, \{\nu : Int|\ r\}) \in \Delta \quad (\mathtt{b}, \{\nu : Int|\ r\}) \in \Theta}{\Gamma, \Delta, \Theta, \Lambda \vdash \mathtt{b} : \{\nu : Int|\ \nu = b\}}$$

Extending the Elm Language: Revisiting the Max Function

```
max =
  \a -> \b ->
    if
      (<) a b --> Bool
    then
      b       --> {v:Int| v = b }
    else
      a       --> {v:Int| v = a }
```

We skip the rule for `(<) a b`: The inferred type is `Bool`

Liquid Type Inference: Infering the Type of the Max Function

$$\frac{}{\Gamma, \Delta \cup \{(\mathtt{a}, \{\nu : \mathit{Int} |\ r_0\}), (\mathtt{b}, \{\nu : \mathit{Int} |\ r_1\})\}, \Theta, \Lambda \vdash \text{"}(<)\text{"}\ e_1\ e_2 : \mathit{Bool}}$$

$$\frac{\Gamma, \Delta, \Theta, \Lambda \vdash e_1 : \mathit{Bool} \quad e_1 : e_1' \qquad \Gamma, \Delta, \Theta, \Lambda \cup \{e_1'\} \vdash e_2 : T \quad \Gamma, \Delta, \Theta, \Lambda \cup \{\neg e_1'\} \vdash e_3 : T}{\Gamma, \Delta, \Theta, \Lambda \vdash \text{"}\mathtt{if}\text{"}\ e_1\ \text{"}\mathtt{then}\text{"}\ e_2\ \text{"}\mathtt{else}\text{"}\ e_3 : T}$$

New rule:

$$\frac{\{(a, \{\nu : \mathit{Int} | r_0\}), (b, \{\nu : \mathit{Int} \mid r_1\})\} \in \Delta \qquad \Gamma, \Delta, \Theta, \Lambda \cup \{a < b\} \vdash \mathtt{b} : \{\nu : \mathit{Int} \mid r_2\} \qquad \Gamma, \Delta, \Theta, \Lambda \cup \{\neg(a < b)\} \vdash \mathtt{a} : \{\nu : \mathit{Int} \mid r_2\}}{\Gamma, \Delta, \Theta, \Lambda \vdash \text{"}\mathtt{if}\text{"}\ \mathtt{a} < \mathtt{b}\ \text{"}\mathtt{then}\text{"}\ \mathtt{b}\ \text{"}\mathtt{else}\text{"}\ \mathtt{a} : \{\nu : \mathit{Int} \mid r_2\}}$$

Extending the Elm Language: Revisiting the Max Function

We have yet to provide a judgement for the following rules.

$$\{\nu : T \mid \nu = \mathtt{a}\} <:_{\Theta,\Lambda} \{\nu : T \mid r\}$$

$$\frac{(\mathtt{a}, \{\nu : T \mid r\}) \in \Delta \quad (\mathtt{a}, \{\nu : T \mid r\}) \in \Theta}{\Gamma, \Delta, \Theta, \Lambda \vdash \mathtt{a} : \{\nu : T \mid \nu = \mathtt{a}\}}$$

$$\{\nu : T \mid \nu = \mathtt{b}\} <:_{\Theta,\Lambda} \{\nu : T \mid r\}$$

$$\frac{(\mathtt{b}, \{\nu : T \mid r\}) \in \Delta \quad (\mathtt{b}, \{\nu : T \mid r\}) \in \Theta}{\Gamma, \Delta, \Theta, \Lambda \vdash \mathtt{b} : \{\nu : T \mid \nu = b\}}$$

$$\frac{\begin{array}{c}\{(a, \{\nu : Int \mid r_0\}), (b, \{\nu : Int \mid r_1\})\} \in \Delta \\ \Gamma, \Delta, \Theta, \Lambda \cup \{a < b\} \vdash \mathtt{b} : \{\nu : Int \mid r_2\} \\ \Gamma, \Delta, \Theta, \Lambda \cup \{\neg(a < b)\} \vdash \mathtt{a} : \{\nu : Int \mid r_2\}\end{array}}{\Gamma, \Delta, \Theta, \Lambda \vdash \text{"}\mathtt{if}\text{"}\ \mathtt{a} < \mathtt{b}\ \text{"}\mathtt{then}\text{"}\mathtt{b}\ \text{"}\mathtt{else}\text{"}\ \mathtt{a} : \{\nu : Int \mid r_2\}}$$

Extending the Elm Language: Revisiting the Max Function

Subtyping Rule

$$\frac{\Gamma, \Delta, \Theta, \Lambda \vdash e : T_1 \quad T_1 <:_{\Theta,\Lambda} T_2 \quad \text{wellFormed}(T_2, \Theta)}{\Gamma, \Delta, \Theta, \Lambda \vdash e : T_2}$$

$$\{a_1 : Int \mid r_1\} <:_{\Theta,\Lambda} \{a_2 : Int \mid r_2\} \;:\Leftrightarrow$$

$$\begin{aligned}
&\text{Let } \{(b_1, T_1), \ldots, (b_n, T_n)\} = \Theta \text{ in} \\
&\forall k_1 \in \text{value}_\Gamma(T_1). \ldots \forall k_n \in \text{value}_\Gamma(T_n). \\
&\forall n \in \mathbb{N}. \forall e \in \Lambda. \\
&\quad [[e]]_{\{(a_1,n),(b_1,k_1),\ldots,(b_n,k_n)\}} \\
&\quad \wedge [[r_1]]_{\{(a_1,n),(b_1,k_1),\ldots,(b_n,k_n)\}} \\
&\Rightarrow [[r_2]]_{\{(a_2,n),(b_1,k_1),\ldots,(b_n,k_n)\}}
\end{aligned}$$

Extending the Elm Language: Revisiting the Max Function

Find $r_2 \in \mathcal{Q}$ such that

$$[[((a < b) \wedge \nu = b) \Rightarrow r_2]]_{\{(a,\{\nu:Int \mid r_0\}),(b,\{\nu:Int \mid r_1\})\}}$$

and

$$[[(\neg(a < b) \wedge \nu = a) \Rightarrow r_2]]_{\{(a,\{\nu:Int \mid r_0\}),(b,\{\nu:Int \mid r_1\})\}}$$

are valid.

Use SMT-Solver to find a solution.

Sharpest solution: $r_2 := ((a < \nu \wedge \nu = b) \vee (\neg(\nu < b) \wedge \nu = a))$
for $r_0, r_1 := True$

Extending the Elm Language: The Inference Algorithm

We say T is a *template* $:\Leftrightarrow$ T is of form $\{\nu : \mathit{Int} \mid k\}$

where $k \in \mathcal{K}$ and $S : \mathcal{V} \nrightarrow \mathcal{Q}$

$\vee$ T is of form $a : \{\nu : \mathit{Int} \mid k\} \to T$

where $k \in \mathcal{K}$, T is a template and $S : \mathcal{V} \nrightarrow \mathit{IntExp}$.

We define $\mathcal{K} := \{\kappa_i \mid i \in \mathbb{N}\}$.

Extending the Elm Language: The Inference Algorithm

1. (Split) Split the subtyping conditions over dependent function into subtyping conditions over simple liquid types.
2. (Init) Compute $Q = \mathit{Init}(V)$ where V is the set of all occurring variables and initiate the mapping A for very key κ_i with the set of resulting predicates with Q.
3. (Solve) Check for very subtyping condition if the current mapping A violates the subtyping condition.
4. (Weaken) If so, weaken the mapping by removing any predicate that violates the subtyping condition and repeat
5. One the algorthm terminates we have obtained the strongest refinements that can be build by conjunction over predicates in $\mathit{Init}(V)$.

Extending the Elm Language: The Inference Algorithm

$$\mathsf{Split}(a : \{\nu : \mathit{Int} \mid q_1\} \to T_2 <:_{\Theta,\Lambda} a : \{\nu : \mathit{Int} \mid q_3\} \to T_4) =$$
$$\{\{\nu : \mathit{Int} \mid q_3\} <:_{\Theta,\Lambda} \{\nu : \mathit{Int} \mid q_1\}\} \cup \mathsf{Split}(T_2 <:_{\Theta \cup \{(a,q_3)\},\Lambda} T_4\})$$
$$\mathsf{Split}(\{\nu : \mathit{Int} \mid q_1\} <:_{\Theta,\Lambda} \{\nu : \mathit{Int} \mid q_2\}) =$$
$$\{\{\nu : \mathit{Int} \mid q_1\} <:_{\Theta,\Lambda} \{\nu : \mathit{Int} \mid q_2\}\}$$

Extending the Elm Language: The Inference Algorithm

$$
\begin{aligned}
\mathit{Init} : \mathcal{P}(\mathcal{V}) \rightarrow & \mathcal{P}(\mathcal{Q}) \\
\mathit{Init}(V) ::= & \{0 < \nu\} \\
& \cup \{a < \nu \mid a \in V\} \\
& \cup \{\nu < 0\} \\
& \cup \{\nu < a \mid a \in V\} \\
& \cup \{\nu = a \mid a \in V\} \\
& \cup \{\nu = 0\} \\
& \cup \{a < \nu \vee \nu = a \mid a \in V\} \\
& \cup \{\nu < a \vee \nu = a \mid a \in V\} \\
& \cup \{0 < \nu \vee \nu = 0\} \\
& \cup \{\nu < 0 \vee \nu = 0\} \\
& \cup \{\neg(\nu = a) \mid a \in V\} \\
& \cup \{\neg(\nu = 0)\}
\end{aligned}
$$

Extending the Elm Language: The Inference Algorithm

Solve

For a subtyping condition $\{\nu : Int \mid r_1\} <:_{\Theta,\Lambda} \{\nu : Int \mid \kappa_i\}$ and a mapping $A : \mathcal{K} \nrightarrow \mathcal{P}(\mathcal{Q})$, let

- $r_2 := \bigwedge A(\kappa_i)$
- $p := \bigwedge \Lambda$
- $\{(b_1, r_1'), \ldots, (b_n, r_n')\} = \Theta$.

If SMT statement is satifiable, then call **Weaken** on the mapping and repeat

SMT statement:

$$((\bigwedge_{j=0}^{n} [r_j']_{\{(\nu, b_j)\}}) \wedge r_1 \wedge p) \wedge \neg r_2$$

with free variables $\nu \in \mathbb{Z}$ and $b_i \in \mathbb{Z}$ for $i \in \mathbb{N}_1^n$.

Extending the Elm Language: The Inference Algorithm

Weaken

For a subtyping condition $\{\nu : Int \mid r_1\} <:_{\Theta,\Lambda} \{\nu : Int \mid \kappa_i\}$ and a mapping $A : \mathcal{K} \nrightarrow \mathcal{P}(\mathcal{Q})$, let

- $p := \bigwedge \Lambda$
- $\{(b_1, r_1'), \ldots, (b_n, r_n')\} = \Theta$.

Return all predicates $q \in A(\kappa)$ such that SMT statement is satifiable.

SMT statement:

$$\neg((\bigwedge_{j=0}^{n} [r_j']_{\{(\nu, b_j)\}}) \wedge r_1 \wedge p) \vee q$$

with free variables $\nu \in \mathbb{Z}$ and $b_i \in \mathbb{Z}$ for $i \in \mathbb{N}_1^n$.

Conclusion

Positives

- Can catch index-out-of-bounds errors in compile time
- Can catch (some) division by zero errors in compile time
- Can define the natural numbers as a subtype of the integers.

Negatives

- The capabilities of liquid types directly depend on the predicates included in $\mathit{Init}(V)$.
- Increasing the size of $\mathit{Init}(V)$ increases the computation time by a lot. ($\sim$ quadratic time)
- The set of inferrable refinements is always a proper subset of the set of refinements annotatable. Thus, the type system is no longer complete.

Conclusion

I therefore come to the conclusion, that liquid types are not a proper fit for Elm.

Started thesis in July 2019

Expected finish in April 2021