Refinement Types for Elm

Master Thesis Report

Lucas Payr

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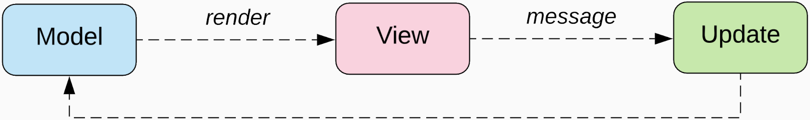
Background: Elm Programming Language

- Invented by Evan Czaplicki as his master-thesis in 2012.
- Goal: Bring Function Programming to Web-Development
- Side-Goal: Learning-friendly design decisions
- Website: elm-lang.org

Characteristics

- Pure Functional Language (immutable, no side effect, everything is a function)
- Compiles to JavaScript (in the future also to WebAssembly)
- ML-like Syntax (we say `fun a b c` for $fun(a, b, c)$)
- Simpler than Haskell (no Type classes, no Monads, only one way to do a given thing)
- "No Runtimes errors" (running out of memory, function equality and non-terminating functions still give runtime errors.)

Background: The Elm Architecture



Example

- Online Editor: ellie-app.com

Background: Refinement Types

Restricts the values of an existing type using a predicate.

Initial paper in 1991 by Tim Freeman and Frank Pfenning

- Initial concept was done in ML.
- Allows predicates with only $\wedge, \vee, =$, constants and basic pattern matching.
- Operates over algebraic types.
- Needed to specify **explicitly** all possible Values.

Example

$$\{a : (Bool, Bool) |\ a = (True, False) \vee (a = (False, True)\}$$

$$\forall t.\{a : List\ t | a = Cons\ (b : t)\ (c : List\ t) \wedge c = Cons\ (d : t)\ [\,]\}$$

Background: Liquid Types

Liquid Types (Logically Quantified Data Types) introduced in 2008

- Invented by Patrick Rondan, Ming Kawaguchi and Ranji Jhala
- Initial concept done in OCaml. Later also C, Haskell and TypeScript.
- Operates over Integers and Booleans. Later also Tuples and Functions.
- Allows predicates with logical operators, comparisons and addition.

Example

$$\{(a : Bool, b : Bool) | (a \vee b) \wedge \neg(a \wedge b)\}$$

$$\{(a : Int, b : Int) | a \leq b\}$$

Goals of Thesis

1. Formal language similar to Elm
   1.1 A formal syntax
   1.2 A formal type system
   1.3 A denotational semantic
   1.4 A small step semantic (using K Framework) for rapid prototyping the language
   1.5 Proof that the type system is valid with respect of the semantics.
2. Extension of the formal language with Liquid Types
   2.1 A formal syntax
   2.2 A formal type system
   2.3 A denotational semantic
   2.4 A small step semantic (using K Framework) for rapid prototyping the type checker
   2.5 Proof that the extension infers the correct types.
3. A type checker implementation written in Elm for Elm.

Problems Addressed by the Type System

- Division by zero errors
- Off by one errors
- Proving the correctness of very simple programs
- Clearer interfaces

Theory: Formalization of the Elm Type System

We will use the Hindley-Milner type system (used in ML, Haskell and Elm)

We say

$$
\begin{aligned}
T \text{ is a } \textit{mono type} :\Leftrightarrow\ & T \text{ is a type variable} \\
\vee\ & T \text{ is a type application} \\
\vee\ & T \text{ is a algebraic type} \\
\vee\ & T \text{ is a product type} \\
\vee\ & T \text{ is a function type} \\
T \text{ is a } \textit{poly type} :\Leftrightarrow\ & T = \forall a. T' \\
& \text{where } T' \text{ is a mono type or poly type} \\
& \text{and } a \text{ is a symbol} \\
T \text{ is a } \textit{type} :\Leftrightarrow\ & T \text{ is a mono type } \vee\ T \text{ is a poly type.}
\end{aligned}
$$

Theory: Formalization of the Elm Type System

Example

1. $Nat ::= \mu C.1 \mid Succ\ C$
2. $List = \forall a.\mu C.Empty \mid Cons\ a\ C$
3. $\text{splitAt} : \forall a.Nat \to List\ a \to (List\ a, List\ a)$

Theory: Formalization of the Elm Type System

The *values* of a type is the set corresponding to the type:

$$\text{values}(\mathit{Nat}) = \{1, \mathit{Succ}\ 1, \mathit{Succ}\ \mathit{Succ}\ 1, \dots\}$$

$$\text{values}(\mathit{List}\ \mathit{Nat}) = \bigcup_{n \in \mathbb{N}} \text{values}_n(\mathit{List}\ \mathit{Nat})$$

$$\text{values}_0(\mathit{List}\ \mathit{Nat}) = \{[\,]\}$$

$$\text{values}_n(\mathit{List}\ \mathit{Nat}) = \{[\,]\} \cup \{\mathit{Cons}\ a\ b | a \in \mathit{Nat}, b \in \mathit{values}_{n-1}(\mathit{List}\ \mathit{Nat})\}$$

Theory: Definition of Liquid Types

Definition (Sketch)
Let T be a Type Application of *Int*, tuples and functions. Let q be a logical formula consisting of

- Logical operations $\neg, \wedge, \vee$
- Logical constants *True*, *False*
- Comparisons $<, \leq, =, \neq$
- Integer operations $+, \cdot c$ where c is a constant
- Integer constants $0, 3, 42, \ldots$
- Bound variables $a, b, c, \ldots$

Then we call the syntactic phrase $\{a : T | q(a)\}$ a *Liquid Type*.

Theory: Definition of Liquid Types

Example
Let $Nat = \{a : Int | a > 0\}$ in

$$\{\ \{(a : Nat, b : Nat)\ | a + b < 42\} \rightarrow \{(c : Nat, d : Nat) | c \leq d\}$$

$$|\ (a = c \wedge b = d) \vee (b = c \wedge a = d)$$

$$\}$$

Theory: Revisiting the Problems

- Division by zero errors

$$(/) : \mathit{Int} \to \{a : \mathit{Int} | a \neq 0\} \to \mathit{Int}$$

- Off by one errors

$$\text{Let } \mathit{Pos} = \{a : \mathit{Int} | 0 \leq a \wedge a < 8\} \text{ in}$$
$$\mathit{get} : (\mathit{Pos}, \mathit{Pos}) \to \mathit{Chessboard} \to \mathit{Maybe\ Figure}$$

- Proving the correctness of very simple programs

$$\text{swap} : \{(a : \mathit{Int}, b : \mathit{Int}) \to (c : \mathit{Int}, d : \mathit{Int}) | b = c \wedge a = d\}$$

- Clearer interfaces

$$\text{length} : \mathit{List}\ a \to \{a : \mathit{Int} | a \geq 0\}$$

Current State

1. Formal language similar to Elm
   1.1 A formal syntax **(DONE)**
   1.2 A formal type system **(DONE)**
   1.3 A denotational semantic **(WORK IN PROGRESS)**
   1.4 A small step semantic (using K Framework) for rapid prototyping the language
   1.5 Proof that the type system is valid with respect of the semantics.
2. Extension of the formal language with Liquid Types
3. A type checker implementation written in Elm for Elm.

Started thesis in July 2019

Expected finish at the end of 2020