## 3.4 Denotational Semantic

We will now expore the semantics of the formal language. To do so, we first define a new context.

**Definition 3.1: Variable Context**

Let $\Gamma$ be a type context.

—

$\Delta : \mathcal{V} \nrightarrow \bigcup_{T \in \mathcal{T}} \mathrm{value}_\Gamma(T)$ is called a *variable context.*

The semantics of the type signature was already defined in the last section, as the semantic of a type signature is its type. We therefore define the same concept but now in a denotational style.

**Definition 3.2: Type Signature Semantic**

Let $T, T' \in \mathcal{T}$, $c, a_0, a \in \mathcal{V}$. Let $t_0, t_1, t_2 \in \texttt{<type>}$, $ltf \in \texttt{<list-type-fields>}$ and $lt \in \texttt{<list-type>}$. Let $\Gamma$ be a type context.

—

Let $s \in (\mathcal{V} \times \mathcal{T})^*$ for the following function.

$$\begin{aligned}
[\![.]\!]_\Gamma :& \texttt{<list-type-fields>} \to (\mathcal{V} \times \mathcal{T})^* \\
[\![\texttt{""}]\!]_\Gamma = s :\Leftrightarrow & s = (\ ) \\
[\![a_0 \quad \texttt{":"} \quad t_0 \quad \texttt{","} \quad ltf]\!]_\Gamma = s :\Leftrightarrow & T_0 = [\![t_0]\!]_\Gamma \\
& \wedge s = ((a_0, T_0), \ldots, (a_n, T_n)) \\
& \wedge [\![ltf]\!]_\Gamma = ((a_1, T_1), \ldots, (a_n, T_n)) \\
& \text{where } n \in \mathbb{N} \text{ and } T_i \in \mathcal{T}, a_i \in \mathcal{V} \text{ for all } i \in \mathbb{N}_0^n
\end{aligned}$$

Let $s \in \mathcal{T}^*$ for the following function.

$$\begin{aligned}
[\![.]\!]_\Gamma :& \texttt{<list-type>} \to \mathcal{T}^* \\
[\![\texttt{""}]\!]_\Gamma = s :\Leftrightarrow & s = (\ ) \\
[\![t_0 \ \ lt]\!]_\Gamma = s :\Leftrightarrow & T_0 = [\![t_0]\!]_\Gamma \\
& \wedge [\![lt]\!]_\Gamma = (T_1, \ldots, T_n) \\
& \wedge s = (T_0, \ldots, T_n) \\
& \text{where } n \in \mathbb{N} \text{ and } T_i \in \mathcal{T} \text{ for all } i \in \mathbb{N}_0^n
\end{aligned}$$

Let $s \in \mathcal{T}$ for the following function.

$$
\begin{aligned}
[\![.]\!]_\Gamma :& \texttt{<type>} \rightarrow \mathcal{T} \\
[\![\texttt{"Bool"}]\!]_\Gamma = s :\Leftrightarrow & s = \mathit{Bool} \\
[\![\texttt{"Int"}]\!]_\Gamma = s :\Leftrightarrow & s = \mathit{Int} \\
[\![\texttt{"List"} \quad t]\!]_\Gamma = s :\Leftrightarrow & T = [\![\mathrm{t}]\!]_\Gamma \wedge s = \mathit{List}\ T \\
& \text{where } T \in \mathcal{T} \\
[\![\texttt{"("} \quad t_1 \quad \texttt{","} \quad t_2 \quad \texttt{")"}]\!]_\Gamma = s :\Leftrightarrow & T_1 = [\![t_1]\!]_\Gamma \wedge T_2 = [\![t_2]\!]_\Gamma \wedge s = (T_1, T_2) \\
& \text{where } T_1, T_2 \in \mathcal{T} \\
[\![\texttt{"\{"} \quad \mathit{ltf} \quad \texttt{"\}"}]\!]_\Gamma = s :\Leftrightarrow & [\![\mathit{ltf}]\!]_\Gamma = ((a_1, T_1), \dots, (a_n, T_n)) \\
& \wedge s = \{a_1 : T_1, \dots, a_n : T_n\} \\
& \text{where } n \in \mathbb{N} \text{ and } T_i \in \mathcal{T}, a_i \in \mathcal{V} \text{ for all } i \in \mathbb{N}_0^n \\
[\![t_1 \quad \texttt{"->"} \quad t_2]\!]_\Gamma = s :\Leftrightarrow & [\![t_1]\!]_\Gamma = T_1 \wedge [\![t_2]\!]_\Gamma = T_2 \wedge s = T_1 \rightarrow T_2 \\
[\![c \ \ \mathit{lt}]\!]_\Gamma = s :\Leftrightarrow & (c, T) \in \Gamma \\
& \wedge (T_1, \dots, T_n) = [\![\mathit{lt}]\!]_\Gamma \\
& \wedge T' = \overline{T}\ T_1 \dots T_n \\
& \wedge s = T' \\
& \text{where } n \in \mathbb{N}, T, T' \in \mathcal{T} \text{ and } T_i \in \mathcal{T} \text{ for all } i \in \mathbb{N}_1^n \\
[\![a]\!]_\Gamma = s :\Leftrightarrow & s = \forall b.b
\end{aligned}
$$

An Elm program is nothing more than an expression. Semantics of an expression is therefore the heart piece of this section.

**Definition 3.3: Expression Semantic**

Let $\Gamma$ be a type context and let $\Delta, \Theta$ be variable contexts. Let $a, a_0, a_1 \in \mathcal{V}$, $e, e_1, e_2, e_3 \in$ `<exp>`. Let $\mathit{lef} \in$ `<list-exp-field>`, $t \in$ `<type>`, $p \in$ `<pattern>`, $\mathit{lc} \in$ `<list-case>`, $b \in$ `<bool>`, $\mathit{nr} \in \mathbb{N}$, $\mathit{le} \in$ `<list-exp>` and $\mathit{mes} \in$ `<maybe-expression-sign>`.

—

Let $s \in (\mathcal{V} \times \bigcup_{T \in \mathcal{T}} \text{value}_\Gamma(T))^*$ for the following function.

$$
\begin{aligned}
[\![.]\!]_{\Gamma,\Delta} :& \texttt{<list-exp-field>} \rightarrow (\mathcal{V} \times \bigcup_{T \in \mathcal{T}} \text{value}_\Gamma(T))^* \\
[\![a \quad \texttt{"="} \quad e]\!]_{\Gamma,\Delta} = s_1 :\Leftrightarrow & s_2 = [\![e]\!]_{\Gamma,\Delta} \wedge \{a = s_2\} = s_1 \\
& \text{where } s_2 \in \text{value}_\Gamma(T) \text{ for } T \in \mathcal{T} \\
[\![a_1 \quad \texttt{"="} \quad e \quad \texttt{","} \quad \mathit{lef}]\!]_{\Gamma,\Delta} = s_3 :\Leftrightarrow & \{a_1 = s_1\} = [\![a \quad \texttt{"="} \quad e]\!]_{\Gamma,\Delta} \\
& \wedge \{a_2 = s_2, \dots, a_n = s_n\} = [\![\mathit{lef}]\!]_{\Gamma,\Delta} \\
& \wedge \{a_1 = s_1, \dots, a_n = s_n\} = s_3 \\
& \text{where } \ n \in \mathbb{N} \text{ and } a_i \in \mathcal{V}, s_i \in \text{value}_\Gamma(T_i) \\
& \ \text{for } T_i \in \mathcal{T} \text{ for } i \in \mathbb{N}_0^n
\end{aligned}
$$

$$\llbracket . \rrbracket : \texttt{<maybe-exp-sign>} \rightarrow (\ )$$
$$\llbracket \texttt{""} \rrbracket = s :\Leftrightarrow (\ ) = s$$
$$\llbracket a \quad \texttt{":"} \quad t \quad \texttt{";"} \rrbracket = s :\Leftrightarrow (\ ) = s$$

Let $s \in \bigcup_{T \in \mathcal{T}} \text{value}_\Gamma(T))$ for the following function.

Let $s \in \text{value}_\varnothing(\mathit{Bool})$ for the following function.

$$\llbracket . \rrbracket : \texttt{<bool>} \rightarrow \text{value}_\varnothing(\mathit{Bool})$$
$$\llbracket b \rrbracket = s :\Leftrightarrow \begin{cases} \mathit{True} & \text{if } b = \texttt{"True"} \\ \mathit{False} & \text{if } b = \texttt{"False"} \end{cases}$$

Let $s \in \text{value}_\varnothing(\mathit{Int})$ for the following function.

$$\llbracket . \rrbracket : \texttt{<int>} \rightarrow \text{value}_\varnothing(\mathit{Int})$$
$$\llbracket \texttt{"0"} \rrbracket = s :\Leftrightarrow 0 = s$$
$$\llbracket \texttt{"-"} \quad nr \rrbracket = s :\Leftrightarrow \mathit{Neg}\ \mathit{Succ}^{nr}\ 0$$
$$\llbracket nr \rrbracket = s :\Leftrightarrow \mathit{Pos}\ \mathit{Succ}^{nr}\ 0$$

Let $s \in \bigcup_{T \in \mathcal{T}} \text{value}_\Gamma(T))^*$ for the following function.

$$\llbracket . \rrbracket_{\Gamma,\Delta} : \texttt{<list-exp>} \rightarrow \bigcup_{T \in \mathcal{T}} \text{value}_\Gamma(T))^*$$
$$\llbracket \texttt{""} \rrbracket_{\Gamma,\Delta} = s :\Leftrightarrow \mathit{Empty} = s$$
$$\llbracket e \quad \texttt{","} \quad le \rrbracket_{\Gamma,\Delta} = s :\Leftrightarrow s_1 = \llbracket e \rrbracket_{\Gamma,\Delta} \wedge s_2 = \llbracket le \rrbracket_{\Gamma,\Delta} \wedge \mathit{Cons}\ s_1\ s_2 = s$$
$$\text{where} \quad n \in \mathbb{N} \quad \text{and} \quad s_i \in \text{value}_\Gamma(T_i), T_i \in \mathcal{T} \text{ for each } i \in \mathbb{N}_0^n$$

Let $s \in \bigcup_{T \in \mathcal{T}} \text{value}_\Gamma(T)$ for the following function.

$$\llbracket . \rrbracket_{\Gamma,\Delta} : \texttt{<exp>} \rightarrow \bigcup_{T \in \mathcal{T}} \text{value}_\Gamma(T)$$
$$\llbracket \texttt{"foldl"} \rrbracket_{\Gamma,\Delta} = s :\Leftrightarrow s = \lambda f.\lambda e_1.\lambda l_1. \begin{cases} e_1 & \text{if } [\ ] = l_1 \\ f(e_2, s(f, e_1, l_2)) & \text{if } \mathit{Cons}\ e_2\ l_2 = l_1 \end{cases}$$
$$\text{where} \quad e_1 \in \text{value}_\Gamma(T_1), e_2 \in \text{value}_\Gamma(T_2)$$
$$\text{and } l_1, l_2 \in \text{value}_\Gamma(\mathit{List}\ T_2) \text{ and}$$
$$f \in \text{value}_\Gamma(T_2 \rightarrow T_1 \rightarrow T_1) \text{ for } T_1, T_2 \in \mathcal{T}$$

$$[\![\texttt{"(::)"}]\!]_{\Gamma,\Delta} = s :\Leftrightarrow s = \lambda e.\lambda l.\mathit{Cons}\ e\ l$$
$$\text{where } e \in \text{value}_\Gamma(T) \text{ and } l \in \text{value}_\Gamma(\mathit{List}\ T) \text{ for } T \in \mathcal{T}$$

$$[\![\texttt{"(+)"}]\!]_{\Gamma,\Delta} = s :\Leftrightarrow s = \lambda n.\lambda m.n + m \quad \text{where } n, m \in \mathbb{Z}$$

$$[\![\texttt{"(-)"}]\!]_{\Gamma,\Delta} = s :\Leftrightarrow s = \lambda n.\lambda m.n - m \quad \text{where } n, m \in \mathbb{Z}$$

$$[\![\texttt{"(*)"}]\!]_{\Gamma,\Delta} = s :\Leftrightarrow s = \lambda n.\lambda m.n * m \quad \text{where } n, m \in \mathbb{Z}$$

$$[\![\texttt{"(//)"}]\!]_{\Gamma,\Delta} = s :\Leftrightarrow \begin{cases} s = \lambda n.\lambda m.\left\lfloor \frac{n}{m} \right\rfloor & \text{if } m \neq 0 \\ 0 & \text{else} \end{cases}$$
$$\text{where } n, m \in \mathbb{Z}$$

$$[\![\texttt{"(<)"}]\!]_{\Gamma,\Delta} = s :\Leftrightarrow s = \lambda n.\lambda m.n < m \quad \text{where } n, m \in \mathbb{Z}$$

$$[\![\texttt{"(==)"}]\!]_{\Gamma,\Delta} = s :\Leftrightarrow s = \lambda n.\lambda m.(n = m) \quad \text{where } n, m \in \mathbb{Z}$$

$$[\![\texttt{"not"}]\!]_{\Gamma,\Delta} = s :\Leftrightarrow s = \lambda b.\neg b \quad \text{where } b \in \text{value}_\Gamma(\mathit{Bool})$$

$$[\![\texttt{"(\&\&)"}]\!]_{\Gamma,\Delta} = s :\Leftrightarrow s = \lambda b_1.\lambda b_2.b_1 \wedge b_2 \quad \text{where } b_1, n_2 \in \text{value}_\Gamma(\mathit{Bool})$$

$$[\![\texttt{"(||)"}]\!]_{\Gamma,\Delta} = s :\Leftrightarrow s = \lambda b_1.\lambda b_2.b_1 \vee b_2 \quad \text{where } b_1, n_2 \in \text{value}_\Gamma(\mathit{Bool})$$

$$[\![e_1 \quad \texttt{"|>"} \quad e_2]\!]_{\Gamma,\Delta} = s :\Leftrightarrow s' = [\![e_1]\!]_{\Gamma,\Delta} \wedge f = [\![e_2]\!]_{\Gamma,\Delta} \wedge f(s_1) = s$$
$$\text{where } f \in \text{value}(T_1 \to T_2), s' \in \text{value}(T_1) \text{ for } T_1, T_2 \in \mathcal{T}$$

$$[\![e_1 \quad \texttt{">>"} \quad e_2]\!]_{\Gamma,\Delta} = s :\Leftrightarrow g = [\![e_1]\!]_{\Gamma,\Delta} \wedge f = [\![e_2]\!]_{\Gamma,\Delta} \wedge f \circ g = s$$
$$\text{where } g \in \text{value}(T_1 \to T_2), f \in \text{value}(T_2 \to T_3) \text{ for } T_1, T_2, T_2 \in \mathcal{T}$$

$$\left[\!\!\left[\begin{matrix}\texttt{"if"}\ e_1\ \texttt{"then"} \\ e_2\ \texttt{"else"}\ e_3\end{matrix}\right]\!\!\right]_{\Gamma,\Delta} = s :\Leftrightarrow b = [\![e_1]\!]_{\Gamma,\Delta} \wedge s = \begin{cases} [\![e_2]\!]_{\Gamma,\Delta} & \text{if } b \\ [\![e_3]\!]_{\Gamma,\Delta} & \text{if } \neg b \end{cases}$$
$$\text{where } b \in \text{value}(\mathit{Bool})$$

$$[\![\texttt{"\{"} \quad \mathit{lef} \quad \texttt{"\}"}]\!]_{\Gamma,\Delta} = s :\Leftrightarrow [\![\mathit{lef}]\!]_{\Gamma,\Delta} = s$$

$$[\![\texttt{"\{ \}"}]\!]_{\Gamma,\Delta} = s :\Leftrightarrow \{\} = s$$

$$[\![\texttt{"\{"} \quad a \quad \texttt{"|"} \quad \mathit{lef} \quad \texttt{"\}"}]\!]_{\Gamma,\Delta} = s :\Leftrightarrow \{a_1 = s_1, \ldots, a_n = s_n\} = [\![\mathit{lef}]\!]_{\Gamma,\Delta}$$

$$\wedge\ (a, \begin{Bmatrix} a_1 = \_, \ldots, a_n = \_, \\ a_{n+1} = s_{n+1}, \ldots, a_m = s_m \end{Bmatrix}) \in \Delta$$

$$\wedge\ \{a_1 = s_1, \ldots, a_m = s_m\} = s$$

where $n, m \in \mathbb{N}$ such that $n \leq m$ and $a_i \in \mathcal{V}$, $s_i \in \mathrm{value}(T_i), T_i \in \mathcal{T}$ for $i \in \mathbb{N}_0^m$

$$[\![a_0 \quad \texttt{"."} \quad a_1]\!]_{\Gamma,\Delta} = s :\Leftrightarrow \Delta(a_0) = \{a_1 : s', \ldots\} \wedge s' = s$$

where $s' \in \mathrm{value}(T)$for $T \in \mathcal{T}$

$$\left[\!\!\left[\begin{matrix}\texttt{"let"}\ \mathit{mes}\ a\ \texttt{"="}\ e_1 \\ \texttt{"in"}\ e_2\end{matrix}\right]\!\!\right]_{\Gamma,\Delta} = s :\Leftrightarrow s' = [\![e_1]\!]_{\Gamma,\Delta} \ \wedge [\![e_2]\!]_{\Gamma,\Delta\cup\{(a,s')\}} = s$$

where $s' \in \mathrm{value}(T) for T \in \mathcal{T}$

$$[\![e_1 \ \ e_2]\!]_{\Gamma,\Delta} = s :\Leftrightarrow s_1 = [\![e_1]\!]_{\Gamma,\Delta} \wedge s_2 = [\![e_2]\!]_{\Gamma,\Delta} \wedge s_1(s_2) = s$$

where $s_1 \in \mathrm{value}_\Gamma(T_1 \to T_2)$ and $s_2 \in \mathrm{value}_\Gamma(T_1)$for $T_1, T_2 \in \mathcal{T}$

$$[\![b]\!]_{\Gamma,\Delta} = s :\Leftrightarrow [\![b]\!] = s$$

$$[\![i]\!]_{\Gamma,\Delta} = s :\Leftrightarrow [\![i]\!] = s$$

$$[\![\texttt{"["} \quad \mathit{le} \quad \texttt{"]"}]\!]_{\Gamma,\Delta} = s :\Leftrightarrow (s_1, \ldots, s_n) = [\![\mathit{le}]\!]_{\Gamma,\Delta} \wedge [s_1, \ldots, s_n] = s$$

where $n \in \mathbb{N}$ and $s_i \in \mathrm{value}_\Gamma(T)$ for $T \in \mathcal{T}$

$$[\![\texttt{"("} \quad e_1 \quad \texttt{","} \quad e_2 \quad \texttt{")"}]\!]_{\Gamma,\Delta} = s :\Leftrightarrow s_1 = [\![e_1]\!] \wedge s_2 = [\![e_2]\!] \wedge (s_1, s_2) = s$$

where $s_1 \in \mathrm{value}_\Gamma(T_1)$ and $s_2 \in \mathrm{value}_\Gamma(T_1)$

$$[\![\texttt{"\textbackslash"} \quad a \quad \texttt{"->"} \quad e]\!]_{\Gamma,\Delta} = s :\Leftrightarrow \lambda b.[\![e]\!]_{\Gamma,\Delta\cup\{(a,b)\}} = s$$

where $b \in \mathcal{V}$

$$[\![c]\!]_{\Gamma,\Delta} = s :\Leftrightarrow (c, s) \in \Delta$$

$$[\![a]\!]_{\Gamma,\Delta} = s :\Leftrightarrow (a, s) \in \Delta$$

Statements are, semantically speaking, just functions that either map the type- or variable-context.

**Definition 3.4: Statement Semantic**

Let $\Gamma$ be a type context. Let $a, a_0 \in \mathcal{V}$, $t \in$ `<type>`, $\mathit{lsv} \in$ `<list-statement-var>`, $\mathit{lt} \in$ `<list-type>`, $\mathit{lss} \in$ `<list-statement-sort>`, $\mathit{st} \in$ `<statement>`, $\mathit{ls} \in$ `<list-statement>`, $\mathit{mss} \in$ `<maybe-statement-sign>` and $\mathit{mms} \in$ `<maybe-main-sign>`. Let $\mathcal{S}$ be the class of all finite sets.

—

Let $s \in \mathcal{V}^*$ for the following function.

$$
\begin{aligned}
[\![ . ]\!] :& \texttt{<list-statement-var>} \to \mathcal{V}^* \\
[\![ \texttt{""} ]\!] = s :\Leftrightarrow& (\ ) = s \\
[\![ a_0 \quad lsv ]\!] = s :\Leftrightarrow& (a_1, \ldots, a_n) = [\![ lsv ]\!] \wedge (a_0, \ldots, a_n) = s \\
& \text{where } \ n \in \mathbb{N} \text{ and } a_i \in \mathcal{V} \text{ for } i \in \mathbb{N}_0^n
\end{aligned}
$$

Let $s \in ((\mathcal{V} \nrightarrow \mathcal{T}) \times (\mathcal{V} \nrightarrow \mathcal{S})) \to ((\mathcal{V} \nrightarrow \mathcal{T}) \times (\mathcal{V} \nrightarrow \mathcal{S}))$ for the following function.

$$
\begin{aligned}
[\![ . ]\!] :& \texttt{<list-statement>} \to ((\mathcal{V} \nrightarrow \mathcal{T}) \times (\mathcal{V} \nrightarrow \mathcal{S})) \to ((\mathcal{V} \nrightarrow \mathcal{T}) \times (\mathcal{V} \nrightarrow \mathcal{S})) \\
[\![ \texttt{""} ]\!] = s :\Leftrightarrow& id = s \\
[\![ st \quad \texttt{","} \quad ls ]\!] = s :\Leftrightarrow& f = [\![ st ]\!] \wedge g = [\![ ls ]\!] \wedge g \circ f = s \\
& \text{where } f, g \in ((\mathcal{V} \nrightarrow \mathcal{T}) \times (\mathcal{V} \nrightarrow \mathcal{S})) \to ((\mathcal{V} \nrightarrow \mathcal{T}) \times (\mathcal{V} \nrightarrow \mathcal{S}))
\end{aligned}
$$

$$
\begin{aligned}
[\![ . ]\!] :& \texttt{<maybe-statement-sign>} \to (\ ) \\
[\![ \texttt{""} ]\!] = s :\Leftrightarrow& (\ ) = s \\
[\![ a \quad \texttt{":"} \quad t \ \texttt{";"} ]\!] = s :\Leftrightarrow& (\ ) = s
\end{aligned}
$$

Let $s \in ((\mathcal{V} \nrightarrow \mathcal{T}) \times (\mathcal{V} \nrightarrow \mathcal{S})) \to ((\mathcal{V} \nrightarrow \mathcal{T}) \times (\mathcal{V} \nrightarrow \mathcal{S}))$ for the following function.

$$
\begin{aligned}
[\![ . ]\!] :& \texttt{<statement>} \to ((\mathcal{V} \nrightarrow \mathcal{T}) \times (\mathcal{V} \nrightarrow \mathcal{S})) \to ((\mathcal{V} \nrightarrow \mathcal{T}) \times (\mathcal{V} \nrightarrow \mathcal{S})) \\
[\![ mss \quad a \quad \texttt{"="} \quad e ]\!](\Gamma, \Delta) = s :\Leftrightarrow& s' = [\![ e ]\!]_{\Gamma,\Delta} \wedge (\Gamma, \Delta \cup \{(a, s')\}) = s \\
& \text{where } s' \in \text{value}(T) \text{ for } T \in \mathcal{T} \\
\left[\!\!\left[ \begin{matrix} \texttt{"type alias"} \\ c \ lsv \ \texttt{"="} \ t \end{matrix} \right]\!\!\right](\Gamma, \Delta) = s :\Leftrightarrow& T = [\![ \texttt{t} ]\!]_\Gamma \wedge (\Gamma \cup \{(c, T)\}, \Delta) = s
\end{aligned}
$$

$$
\begin{aligned}
[\![ . ]\!] :& \texttt{<maybe-main-sign>} \to (\ ) \\
[\![ \texttt{""} ]\!] = s :\Leftrightarrow& (\ ) = s \\
[\![ \texttt{"main : "} \quad t \quad \texttt{";"} ]\!] = s :\Leftrightarrow& (\ ) = s
\end{aligned}
$$

Let $s \in \bigcup_{T \in \mathcal{T}} \text{value}_\Gamma(T)$ for the following function.

$$
\begin{aligned}
[\![ . ]\!] :& \texttt{<program>} \to \bigcup_{T \in \mathcal{T}} \text{value}_\varnothing(T) \\
[\![ ls \quad mms \quad \texttt{"main = "} \quad e ]\!] = s :\Leftrightarrow& (\Gamma, \Delta) = [\![ ls ]\!](\varnothing, \varnothing) \wedge [\![ e ]\!]_{\Gamma,\Delta} = s \\
& \text{where } \Gamma \text{ is a type context and } \Delta \text{ is a variable context.}
\end{aligned}
$$