3.4 Denotational Semantic

We will now define the semantics in a denotational way.

Definition 3.1: Variable Context

Let Γ be a type context.

—

$\Delta : \mathcal{V} \nrightarrow \bigcup_{T \in \mathcal{T}} \text{value}_\Gamma(T)$ is called a *variable context*.

Definition 3.2: Type Signature Semantic

Let $T, T' \in \mathcal{T}$, $c, a_0, a \in \mathcal{V}$. Let $t_0, t_1, t_2 \in$ `<type>`, $ltf \in$ `<list-type-fields>` and $lt \in$ `<list-type>`. Let Γ be a type context.

—

We define the following semantic

$$
\begin{aligned}
[\![.]\!]_\Gamma :&\texttt{<list-type-fields>} \to (\mathcal{V} \times \mathcal{T})^* \\
[\![\texttt{""}]\!]_\Gamma = s :\Leftrightarrow& s = (\,) \\
[\![a_0 \quad \texttt{":"} \quad t_0 \quad \texttt{","} \quad ltf]\!]_\Gamma = s :\Leftrightarrow& T_0 = [\![t_0]\!]_\Gamma \\
&\wedge s = ((a_0, T_0), \ldots, (a_n, T_n)) \\
&\wedge [\![ltf]\!]_\Gamma = ((a_1, T_1), \ldots, (a_n, T_n)) \\
&\text{where } n \in \mathbb{N} \text{ and } T_i \in \mathcal{T}, a_i \in \mathcal{V} \text{ for all } i \in \mathbb{N}_0^n.
\end{aligned}
$$

$$
\begin{aligned}
[\![.]\!]_\Gamma :&\texttt{<list-type>} \to \mathcal{T}^* \\
[\![\texttt{""}]\!]_\Gamma = s :\Leftrightarrow& s = (\,) \\
[\![t_0 \quad lt]\!]_\Gamma = s :\Leftrightarrow& T_0 = [\![t_0]\!]_\Gamma \\
&\wedge [\![lt]\!]_\Gamma = (T_1, \ldots, T_n) \\
&\wedge s = (T_0, \ldots, T_n) \\
&\text{where } n \in \mathbb{N} \text{ and } T_i \in \mathcal{T} \text{ for all } i \in \mathbb{N}_0^n.
\end{aligned}
$$

$$
\begin{aligned}
[\![.]\!]_\Gamma &: \texttt{<type>} \to \mathcal{T} \\
[\![\texttt{"Bool"}]\!]_\Gamma = s &:\Leftrightarrow s = \mathit{Bool} \\
[\![\texttt{"Int"}]\!]_\Gamma = s &:\Leftrightarrow s = \mathit{Int} \\
[\![\texttt{"List"} \quad t]\!]_\Gamma = s &:\Leftrightarrow T = [\![\texttt{t}]\!]_\Gamma \land s = \mathit{List}\ T \\
&\quad\ \text{where } T \in \mathcal{T}. \\
[\![\texttt{"("} \quad t_1 \quad \texttt{","} \quad t_2 \quad \texttt{")"}]\!]_\Gamma = s &:\Leftrightarrow T_1 = [\![t_1]\!]_\Gamma \land T_2 = [\![t_2]\!]_\Gamma \land s = (T_1, T_2) \\
&\quad\ \text{where } T_1, T_2 \in \mathcal{T}. \\
[\![\texttt{"\{"} \quad \mathit{ltf} \quad \texttt{"\}"}]\!]_\Gamma = s &:\Leftrightarrow [\![\mathit{ltf}]\!]_\Gamma = ((a_1, T_1), \ldots, (a_n, T_n)) \\
&\quad\ \land s = \{a_1 : T_1, \ldots, a_n : T_n\} \\
&\quad\ \text{where } n \in \mathbb{N} \text{ and } T_i \in \mathcal{T}, a_i \in \mathcal{V} \text{ for all } i \in \mathbb{N}_0^n. \\
[\![t_1 \quad \texttt{"->"} \quad t_2]\!]_\Gamma = s &:\Leftrightarrow [\![t_1]\!]_\Gamma = T_1 \land [\![t_2]\!]_\Gamma = T_2 \land s = T_1 \to T_2 \\
[\![c \quad \mathit{lt}]\!]_\Gamma = s &:\Leftrightarrow (c, T) \in \Gamma \\
&\quad\ \land (T_1, \ldots, T_n) = [\![\mathit{lt}]\!]_\Gamma \\
&\quad\ \land T' = \overline{T}\ T_1 \ldots T_n \\
&\quad\ \land s = T' \\
&\quad\ \text{where } n \in \mathbb{N}, T, T' \in \mathcal{T} \text{ and } T_i \in \mathcal{T} \text{ for all } i \in \mathbb{N}_1^n. \\
[\![a]\!]_\Gamma = s &:\Leftrightarrow s = \forall b.b
\end{aligned}
$$

Definition 3.3: Pattern Semantic

Let Γ be a type context and let $\Theta, \Theta_1, \Theta_2, \Theta_3$ be variable contexts. Let $p, p_1, p_2 \in$ `<pattern>`, $\mathit{lpl} \in$ `<list-pattern-list>`, $\mathit{lps} \in$ `<list-pattern-sort>` and $\mathit{lpv} \in$ `<list-pattern-vars>`. Let $b \in$ `<bool>` and $i \in$ `<int>`. Let $c, a \in \mathcal{V}$.

—

$$
\begin{aligned}
\text{match}_\Theta &: (\bigcup_{T \in \mathcal{T}} \text{value}_\Gamma(\mathit{List}\ T)) \times \texttt{<list-pattern-list>} \\
\text{match}_\Theta(s, \texttt{""}) &:\Leftrightarrow [\,] = s \\
\text{match}_{\Theta_3}(s, p\ \texttt{","}\ \mathit{lpl}) &:\Leftrightarrow [a_0, \ldots, a_n] = s \\
&\quad\ \land \text{match}_{\Theta_1}(a_0, p) \land \text{match}_{\Theta_2}(a_1, \ldots, a_n, \mathit{lpl}) \\
&\quad\ \land \varnothing = \Theta_1 \cap \Theta_2 \land \Theta_3 = \Theta_1 \cup \Theta_2 \\
&\quad\ \text{where } n \in \mathbb{N} \text{ and } a_i \in \mathcal{V} \text{ for all } i \in \mathbb{N}_0^n.
\end{aligned}
$$

$$
\begin{aligned}
\text{match}_\Theta : \bigcup_{T \in \mathcal{T}} \text{value}_\Gamma(T)^* \times \texttt{<list-pattern-sort>} \\
\text{match}_\Theta(s, \texttt{""}) &:\Leftrightarrow (\,) = s \\
\text{match}_{\Theta_3}(s, p\ \mathit{lps}) &:\Leftrightarrow (s_1, \ldots, s_n) = s \\
&\quad \wedge \text{match}_{\Theta_1}(s_1, p) \wedge \text{match}_{\Theta_2}(s_2, \ldots, s_n, \mathit{lps}) \\
&\quad \wedge \varnothing = \Theta_1 \cap \Theta_2 \wedge \Theta_3 = \Theta_1 \cup \Theta_2
\end{aligned}
$$

$$
\begin{aligned}
[\![.]\!] :\texttt{<list-pattern-vars>} \to \mathcal{V}^* \\
[\![\texttt{""}]\!] = s &:\Leftrightarrow s = (\,) \\
[\![a_0 \ \ \mathit{lpv}]\!] = s &:\Leftrightarrow (a_1, \ldots, a_n) = [\![\mathit{lpv}]\!] \wedge (a_0, dots, a_n) = s \\
&\quad \text{where } n \in \mathbb{N} \text{ and } a_i \in \mathcal{V} \text{ for all } i \in \mathbb{N}_0^n.
\end{aligned}
$$

$$
\begin{aligned}
\text{match}_\Theta : \bigcup_{T \in \mathcal{T}} \text{value}_\Gamma(T) \times \texttt{<pattern>} \\
\text{match}_\Theta(s, b) &:\Leftrightarrow \wedge\, b \in \texttt{<bool>} \wedge s = [\![\texttt{b}]\!]_{\Gamma,\varnothing} \\
\text{match}_\Theta(s, i) &:\Leftrightarrow \wedge\, i \in \texttt{<int>} \wedge s = [\![\texttt{b}]\!]_{\Gamma,\varnothing} \\
\text{match}_\Theta(s, \texttt{"["}\ \mathit{lpl}\ \texttt{"]"}) &:\Leftrightarrow \text{match}_\Theta(s, \mathit{lpl}) \\
\text{match}_{\Theta_3}(s, \texttt{"("}\ p_1\ \texttt{","}\ p_2\ \texttt{")"}) &:\Leftrightarrow (s_1, s_2) = s \\
&\quad \wedge \text{match}_{\Theta_1}(s_1, p_1) \wedge \text{match}_{\Theta_2}(s_2, p_2) \\
&\quad \wedge \varnothing = \Theta_1 \cap \Theta_2 \wedge \Theta_3 = \Theta_1 \cup \Theta_2 \\
\text{match}_\Theta(s, c\ \mathit{lps}) &:\Leftrightarrow c\ s_1\ dots\ s_n = s \wedge \text{match}_\Theta((s_1, \ldots, s_n), \mathit{lps}) \\
\text{match}_\Theta(s, a) &:\Leftrightarrow s \in \mathcal{V} \wedge \Theta = \{(a, s)\} \\
\text{match}_{\Theta_2}(s, p\ \texttt{"as"}\ a) &:\Leftrightarrow \text{match}_{\Theta_1}(s, p) \\
&\quad \wedge \varnothing = \Theta_1 \cap \{(a, s)\} \wedge \Theta_2 = \Theta_1 \cup \{(a, s)\} \\
\text{match}_\Theta(s, \texttt{"\{"}\ \mathit{lpv}\ \texttt{"\}"}) &:\Leftrightarrow (a_1, \ldots, a_n) = [\![\mathit{lpv}]\!] \\
&\quad \wedge \{a_1 = e_1, \ldots, a_n = e_n\} = s \\
&\quad \wedge \Theta = \{(a_1, e_1), \ldots, (a_n, e_n)\} \\
&\quad \text{where } n \in \mathbb{N} \text{ and } a_i \in \mathcal{V} \text{ for all } i \in \mathbb{N}_0^n. \\
\text{match}_{\Theta_3}(s, p_1\ ::\ p_2) &:\Leftrightarrow (s_1, \ldots, s_n) = s \wedge \text{match}_{\Theta_1}(s_1, p_1) \\
&\quad \wedge \text{match}_{\Theta_2}((s_2, \ldots, s_n), p_2) \\
&\quad \wedge \varnothing = \Theta_1 \cap \Theta_2 \wedge \Theta_3 = \Theta_1 \cup \Theta_2 \\
\text{match}_\Theta(\texttt{"\_"}) &:\Leftrightarrow \varnothing = \Theta
\end{aligned}
$$

Definition 3.4: Expression Semantic

$$
\begin{aligned}
\llbracket . \rrbracket_{\Gamma,\Delta} &: \texttt{<list-exp-field>} \to (\mathcal{V} \times \bigcup_{T \in \mathcal{T}} \mathrm{value}_{\Gamma}(T))^{*} \\
\llbracket a \quad \texttt{"="} \quad e \rrbracket_{\Gamma,\Delta} = s_1 &:\Leftrightarrow s_2 = \llbracket e \rrbracket_{\Gamma,\Delta} \wedge ((a, s_2)) = s_1 \\
\llbracket a_1 \quad \texttt{"="} \quad e \quad \texttt{","} \quad lef \rrbracket_{\Gamma,\Delta} = s_3 &:\Leftrightarrow ((a_1, s_1)) = \llbracket a \quad \texttt{"="} \quad e \rrbracket_{\Gamma,\Delta} \\
&\quad \wedge ((a_2, s_2), \ldots, (a_n, s_n)) = \llbracket lef \rrbracket_{\Gamma,\Delta} \\
&\quad \wedge ((a_1, s_1), \ldots, (a_n, s_n)) = s_3
\end{aligned}
$$

$$
\begin{aligned}
\llbracket . \rrbracket_{\Gamma,\Delta} &: \texttt{<maybe-exp-sign>} \to (\,) \\
\llbracket \texttt{""} \rrbracket_{\Gamma,\Delta} = s &:\Leftrightarrow (\,) = s \\
\llbracket a \quad \texttt{":"} \ \texttt{t} \quad \texttt{";"} \rrbracket_{\Gamma,\Delta} = s &:\Leftrightarrow (\,) = s
\end{aligned}
$$

$$
\begin{aligned}
\llbracket . \rrbracket_{\Gamma,\Delta} &: \texttt{<exp>} \to \texttt{<list-case>} \to \bigcup_{T \in \mathcal{T}} \mathrm{value}_{\Gamma}(T)) \\
\llbracket e_1, p \quad \texttt{"->"} \quad e_2 \rrbracket_{\Gamma,\Delta} = s &:\Leftrightarrow \mathrm{match}_{\Theta}(e_1, p) \wedge \llbracket e_2 \rrbracket_{\Gamma,\Delta \cup \Theta} = s \\
\llbracket e_1, p \quad \texttt{"->"} \quad e_2 \quad \texttt{";"} \quad lc \rrbracket_{\Gamma,\Delta} = s &:\Leftrightarrow s = \begin{cases} \llbracket e_2 \rrbracket_{\Gamma,\Delta \cup \Theta} & \text{if } \mathrm{match}_{\Theta}(e_1, p) \\ \llbracket e_1, lc \rrbracket_{\Gamma,\Delta} & \text{else} \end{cases}
\end{aligned}
$$

$$
\begin{aligned}
\llbracket . \rrbracket &: \texttt{<Bool>} \to \mathrm{value}_{\varnothing}(\mathit{Bool}) \\
\llbracket b \rrbracket = s &:\Leftrightarrow \begin{cases} \mathit{True} & \text{if } b = \texttt{"True"} \\ \mathit{False} & \text{if } b = \texttt{"False"} \end{cases}
\end{aligned}
$$

$$
\begin{aligned}
\llbracket . \rrbracket &: \texttt{<int>} \to \mathrm{value}_{\varnothing}(\mathit{Int}) \\
\llbracket \texttt{"0"} \rrbracket = s &:\Leftrightarrow 0 = s \\
\llbracket \texttt{"-"} \quad n \rrbracket = s &:\Leftrightarrow \mathit{Neg}\ \mathit{Succ}^{n}\ 1 \\
\llbracket n \rrbracket = s &:\Leftrightarrow \mathit{Pos}\ \mathit{Succ}^{n}\ 1
\end{aligned}
$$

$$
\begin{aligned}
\llbracket . \rrbracket_{\Gamma,\Delta} &: \texttt{<list-exp>} \to \bigcup_{T \in \mathcal{T}} \mathrm{value}_{\Gamma}(T))^{*} \\
\llbracket \texttt{""} \rrbracket_{\Gamma,\Delta} = s &:\Leftrightarrow (\,) = s \\
\llbracket e \quad \texttt{","} \quad le \rrbracket_{\Gamma,\Delta} = s &:\Leftrightarrow s_1 = \llbracket e \rrbracket_{\Gamma,\Delta} \wedge (s_2, \ldots, s_n) = \llbracket le \rrbracket_{\Gamma,\Delta} \wedge (s_1, \ldots, s_n) = s
\end{aligned}
$$

$$[\![.]\!]_{\Gamma,\Delta} : \texttt{<exp>} \rightarrow \bigcup_{T \in \mathcal{T}} \mathrm{value}_{\Gamma}(T)$$

$$\begin{aligned}
[\![\texttt{"foldl"}]\!]_{\Gamma,\Delta} = s &:\Leftrightarrow s = \lambda f.\lambda e_1.\lambda l_1. \\
&\qquad \begin{cases} e_1 & \text{if } [\,] = l_1 \\ f(e_2, s(f, e_1, l_2)) & \text{if } \mathit{Cons}\ e_2\ l_2 = l_1 \end{cases} \\
[\![\texttt{"(::)"}]\!]_{\Gamma,\Delta} = s &:\Leftrightarrow s = \lambda e.\lambda l.\mathit{Cons}\ e\ l \\
[\![\texttt{"(+)"}]\!]_{\Gamma,\Delta} = s &:\Leftrightarrow s = \lambda n.\lambda m.n + m \\
[\![\texttt{"(-)"}]\!]_{\Gamma,\Delta} = s &:\Leftrightarrow s = \lambda n.\lambda m.n - m \\
[\![\texttt{"(*)"}]\!]_{\Gamma,\Delta} = s &:\Leftrightarrow s = \lambda n.\lambda m.n * m \\
[\![\texttt{"(//)"}]\!]_{\Gamma,\Delta} = s &:\Leftrightarrow s = \lambda n.\lambda m.\left\lfloor \frac{n}{m} \right\rfloor \\
[\![\texttt{"(<)"}]\!]_{\Gamma,\Delta} = s &:\Leftrightarrow s = \lambda n.\lambda m.n < m \\
[\![\texttt{"(==)"}]\!]_{\Gamma,\Delta} = s &:\Leftrightarrow s = \lambda n.\lambda m.(n = m) \\
[\![\texttt{"not"}]\!]_{\Gamma,\Delta} = s &:\Leftrightarrow s = \lambda b.\neg b \\
[\![\texttt{"(\&\&)"}]\!]_{\Gamma,\Delta} = s &:\Leftrightarrow s = \lambda b_1.\lambda b_2.b_1 \wedge b_2 \\
[\![\texttt{"(||)"}]\!]_{\Gamma,\Delta} = s &:\Leftrightarrow s = \lambda b_1.\lambda b_2.b_1 \vee b_2 \\
[\![e_1 \quad \texttt{"|>"} \quad e_2]\!]_{\Gamma,\Delta} = s_2 &:\Leftrightarrow s_1 = [\![e_1]\!]_{\Gamma,\Delta} \wedge f = [\![e_2]\!]_{\Gamma,\Delta} \wedge f(s_1) = s_2 \\
[\![e_1 \quad \texttt{">>"} \quad e_2]\!]_{\Gamma,\Delta} = s &:\Leftrightarrow g = [\![e_1]\!]_{\Gamma,\Delta} \wedge f = [\![e_2]\!]_{\Gamma,\Delta} \wedge f \circ g = s \\
\left[\!\!\left[\begin{matrix} \texttt{"if"}\ e_1\ \texttt{"then"} \\ e_2\ \texttt{"else"}\ e_3 \end{matrix}\right]\!\!\right]_{\Gamma,\Delta} = s &:\Leftrightarrow b = [\![e_1]\!]_{\Gamma,\Delta} \wedge s = \begin{cases} [\![e_2]\!]_{\Gamma,\Delta} & \text{if } b \\ [\![e_3]\!]_{\Gamma,\Delta} & \text{if } \neg b \end{cases} \\
[\![\texttt{"\{"} \quad \mathit{lef} \quad \texttt{"\}"}]\!]_{\Gamma,\Delta} = s &:\Leftrightarrow ((a_1, s_1), \ldots, (a_n, s_n)) = [\![\mathit{lef}]\!]_{\Gamma,\Delta} \\
&\qquad \wedge \{a_1 = s_1, \ldots, a_n = s_n\} = s \\
[\![\texttt{"\{ \}"}]\!]_{\Gamma,\Delta} = s &:\Leftrightarrow \{\} = s \\
[\![\texttt{"\{"} \quad a \quad \texttt{"|"} \quad \mathit{lef} \quad \texttt{"\}"}]\!]_{\Gamma,\Delta} = s &:\Leftrightarrow ((a_1, s_1), \ldots, (a_n, s_n)) = [\![\mathit{lef}]\!]_{\Gamma,\Delta} \\
&\qquad \wedge \left(a, \left\{\begin{matrix} a_1 = _, \ldots, a_n = _, \\ a_{n+1} = s_{n+1}, \ldots, a_m = s_m \end{matrix}\right\}\right) \in \Delta \\
&\qquad \wedge \{a_1 = s_1, \ldots, a_m = s_m\} = s \\
[\![a_0 \quad \texttt{"."} \quad a_1]\!]_{\Gamma,\Delta} = s_2 &:\Leftrightarrow (a_0, \{a_1 : s_1, \ldots\})\Delta \wedge s_1 = s_2 \\
\left[\!\!\left[\begin{matrix} \texttt{"let"}\ \mathit{mes}\ a\ \texttt{"="}\ e_1 \\ \texttt{"in"}\ e_2 \end{matrix}\right]\!\!\right]_{\Gamma,\Delta} = s_2 &:\Leftrightarrow s_1 = [\![e_1]\!]_{\Gamma,\Delta} \ \wedge [\![e_2]\!]_{\Gamma,\Delta \cup \{(a,s_1)\}} = s_2 \\
\left[\!\!\left[\begin{matrix} \texttt{"case"}\ e\ \texttt{"of"} \\ \texttt{"["}\ \mathit{lc}\ \texttt{"]"} \end{matrix}\right]\!\!\right]_{\Gamma,\Delta} = s &:\Leftrightarrow [\![e, \mathit{lc}]\!]_{\Gamma,\Delta} = s \\
[\![e_1 \ e_2]\!]_{\Gamma,\Delta} = e_3 &:\Leftrightarrow s_1 = [\![e_1]\!]_{\Gamma,\Delta} \wedge s_2 = [\![e_2]\!]_{\Gamma,\Delta} \wedge s_1(s_2) \\
[\![b]\!]_{\Gamma,\Delta} = s &:\Leftrightarrow [\![b]\!] = s \\
[\![i]\!]_{\Gamma,\Delta} = s &:\Leftrightarrow [\![i]\!] = s \\
[\![\texttt{"["} \quad \mathit{le} \quad \texttt{"]"}]\!]_{\Gamma,\Delta} = s &:\Leftrightarrow (s_1, \ldots, s_n) = [\![\mathit{le}]\!]_{\Gamma,\Delta} \wedge [s_1, \ldots, s_n] = s \\
[\![\texttt{"("} \quad e_1 \quad \texttt{","} \quad e_2 \quad \texttt{")"}]\!]_{\Gamma,\Delta} = s_3 &:\Leftrightarrow s_1 = [\![e_1]\!] \wedge s_2 = [\![e_2]\!] \wedge (s_1, s_2) = s_3 \\
[\![\texttt{"\textbackslash"} \quad p \quad \texttt{"->"} \quad e]\!]_{\Gamma,\Delta} = s &:\Leftrightarrow \mathrm{match}_{\Theta}(s_1, p) \wedge s_2 = [\![e]\!] \wedge \lambda s_1.s_2 = s \\
[\![c]\!]_{\Gamma,\Delta} = s &:\Leftrightarrow (c, s) \in \Delta \\
[\![a]\!]_{\Gamma,\Delta} = s &:\Leftrightarrow (a, s) \in \Delta
\end{aligned}$$

Definition 3.5: Statement Semantic

$$
\begin{aligned}
[\![.]\!] &: \texttt{<list-statement-var>} \to \mathcal{V}^* \\
[\![\texttt{""}]\!] = s &:\Leftrightarrow () = s \\
[\![a_0 \quad lsv]\!] = s &:\Leftrightarrow (a_1, \ldots, a_n) = [\![lsv]\!] \wedge (a_0, \ldots, a_n) = s
\end{aligned}
$$

$$
\begin{aligned}
[\![.]\!]_\Gamma &: \texttt{<list-statement-sort>} \to (\mathcal{V} \times \mathcal{T}^*)^* \\
[\![\texttt{a} \quad lt]\!]_\Gamma = s &:\Leftrightarrow l = [\![lt]\!]_\Gamma \wedge ((a, l)) \\
[\![a_0 \quad lt \quad lss]\!]_\Gamma = s &:\Leftrightarrow ((a_1, l_1), \ldots, (a_n, l_n)) = [\![lss]\!] \\
&\quad \wedge \; l_0 = [\![lt]\!]_\Gamma \\
&\quad \wedge ((a_0, l_0), \ldots, (a_n, l_n)) = s
\end{aligned}
$$

$$
\begin{aligned}
[\![.]\!] &: \texttt{<list-statement>} \to ((\mathcal{V} \nrightarrow \mathcal{T}) \times (\mathcal{V} \nrightarrow \mathcal{S})) \to ((\mathcal{V} \nrightarrow \mathcal{T}) \times (\mathcal{V} \nrightarrow \mathcal{S})) \\
[\![\texttt{""}]\!] = s &:\Leftrightarrow id = s \\
[\![st \quad \texttt{","} \quad ls]\!] = s &:\Leftrightarrow f = [\![st]\!] \wedge g = [\![ls]\!] \wedge g \circ f = s
\end{aligned}
$$

$$
\begin{aligned}
[\![.]\!] &: \texttt{<maybe-statement-sign>} \to () \\
[\![\texttt{""}]\!] = s &:\Leftrightarrow () = s \\
[\![a \quad \texttt{":"} \quad t \; \texttt{";"}]\!] = s &:\Leftrightarrow () = s
\end{aligned}
$$

$$
\begin{aligned}
[\![.]\!] &: \texttt{<statement>} \to ((\mathcal{V} \nrightarrow \mathcal{T}) \times (\mathcal{V} \nrightarrow \mathcal{S})) \to ((\mathcal{V} \nrightarrow \mathcal{T}) \times (\mathcal{V} \nrightarrow \mathcal{S})) \\
[\![mss \quad a \quad \texttt{"="} \quad e]\!](\Gamma, \Delta) = s_2 &:\Leftrightarrow s_1 = [\![e]\!]_{\Gamma,\Delta} \wedge (\Gamma, \Delta \cup \{(a, s_1)\}) = s_2 \\
\left[\!\!\left[\begin{matrix} \texttt{"type alias"} \\ c \; lsv \; \texttt{"="} \; t \end{matrix} \right]\!\!\right](\Gamma, \Delta) = s &:\Leftrightarrow T = [\![\texttt{t}]\!]_\Gamma \wedge (\Gamma \cup \{(c, T)\}, \Delta) = s \\
\left[\!\!\left[\begin{matrix} \texttt{"type"} \; c \\ lsv \; \texttt{"="} \; lss \end{matrix} \right]\!\!\right](\Gamma_1, \Delta_2) = s &:\Leftrightarrow ((c_1, (T_{1,1}, \ldots, T_{1,k_1})), \ldots, (c_n, (T_{n,1}, \ldots, T_{n,k_n}))) \\
&\quad \wedge T_1 = \mu C. c_1 T_{1,1} \ldots T_{1,k_1} | \ldots | c_n T_{n,1} \ldots T_{n,k_n} \\
&\quad \wedge (a_1, \ldots, a_m) = [\![lsv]\!] \\
&\quad \wedge T_2 = \forall a_1 \ldots . \forall a_m T_1 \\
&\quad \wedge \Gamma_2 = \Gamma_1 \cup \{(c, T_2)\} \\
&\quad \wedge \Delta_2 = \left\{ \begin{matrix} (c_1, \lambda t_{1,1} \ldots . , \lambda t_{1,k_1} . c_1 t_{1,1} \ldots t_{1,k_1}) \\ \vdots \\ (c_n, \lambda t_{n,1} \ldots . , \lambda t_{n,k_n} . c_1 t_{n,1} \ldots t_{n,k_n}) \end{matrix} \right\} \\
&\quad \wedge \Delta_3 = \Delta_1 \cup \Delta_2 \\
&\quad \wedge (\Gamma_2, \Delta_3) = s
\end{aligned}
$$

$$
\begin{aligned}
[\![.]\!] :& \texttt{<maybe-main-sign>} \rightarrow (\,) \\
[\![\texttt{""}]\!] = s :\Leftrightarrow& (\,) = s \\
[\![\texttt{"main : "} \quad t \quad \texttt{";"}]\!] = s :\Leftrightarrow& (\,) = s
\end{aligned}
$$

$$
\begin{aligned}
[\![.]\!] :& \texttt{<program>} \rightarrow \bigcup_{T \in \mathcal{T}} \mathrm{value}_{\Gamma}(T) \\
[\![ls \quad mms \quad \texttt{"main = "} \quad e]\!] = s :\Leftrightarrow& (\Gamma, \Delta) = [\![ls]\!](\varnothing, \varnothing) \wedge [\![e]\!]_{\Gamma,\Delta} = s
\end{aligned}
$$