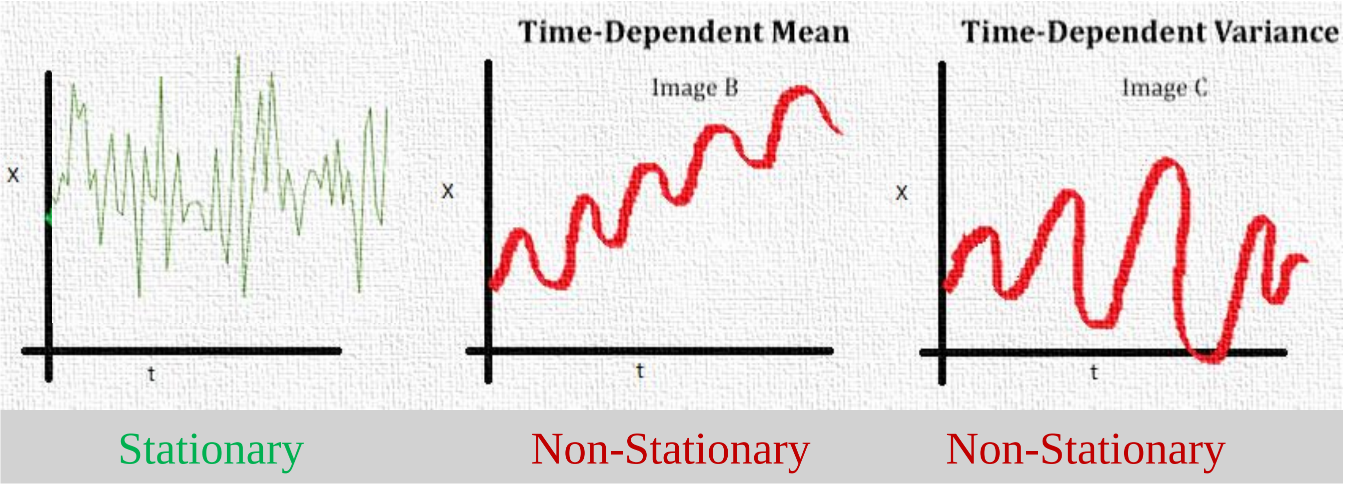
Road map!

- Module 1- Introduction to Deep Forecasting
- Module 2- Setting up Deep Forecasting Environment
- Module 3- Exponential Smoothing
- **Module 4- ARIMA models**
- Module 5- Machine Learning for Time series Forecasting
- Module 6- Deep Neural Networks
- Module 7- Deep Sequence Modeling (RNN, LSTM)
- Module 8- Transformers (Attention is all you need!)
- Module 9- Prophet and Neural Prophet

Module 4 – Part I
ARIMA models' Prerequisites
ACF, PACF, Stationarity, Differencing



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ARIMA models prerequisites

- ARIMA stands for AutoRegressive Integrated Moving Average. It is a class of statistical models for analyzing and forecasting time series data.
- ETS and ARIMA models are two popular models for forecasting time series data. They offer complementary approaches to addressing the challenges of time series forecasting.
- ARIMA models describe autocorrelations in the data, whereas ETS models describe trends and seasonality.
- Let's review some prerequisites before moving forward with the models:

ACF → PACF → Stationarity → Differencing

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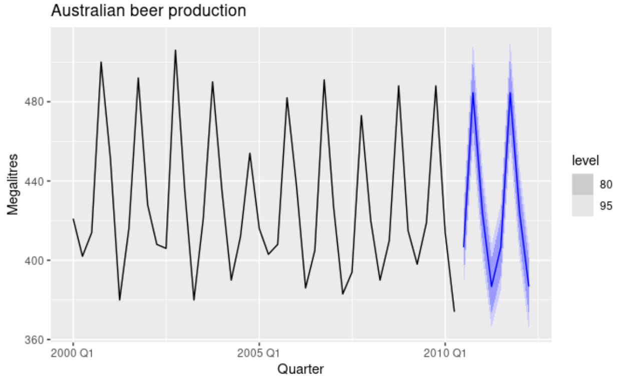
Autocorrelation

- Autocorrelation, also known as serial correlation, is a measure of the correlation between a time series and a lagged version of itself.
- It is used to assess the degree to which the past values of a time series are predictive of its future values.

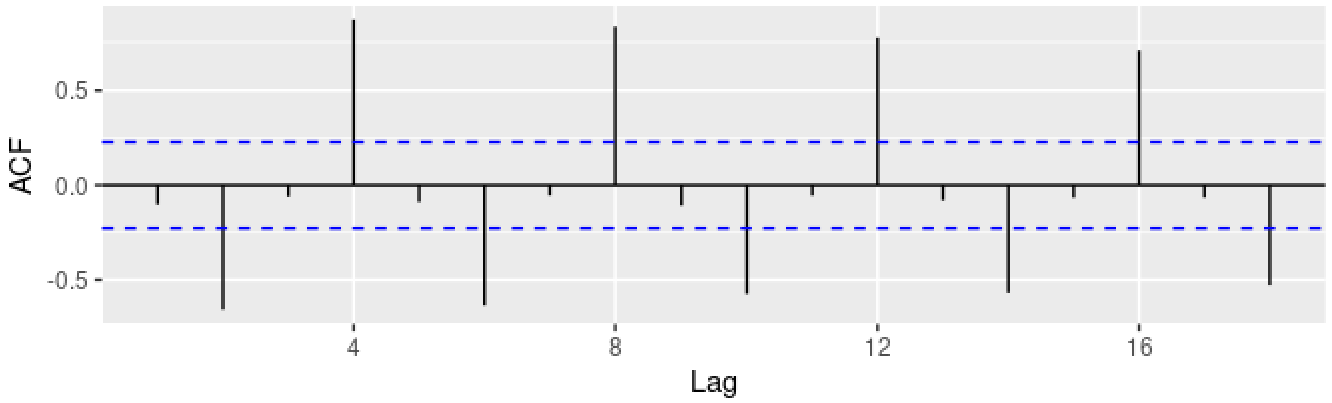
$$r_k = \frac{\sum_{t=k+1}^{T} (y_t - \bar{y})(y_{t-k} - \bar{y})}{\sum_{t=1}^{T} (y_t - \bar{y})^2}$$

ACF: Autocorrelation Function

- The autocorrelation function (ACF) is a statistical tool that can be used to measure the autocorrelation of a time series.
- It calculates the correlation between the time series and lagged versions of itself at different lag periods.



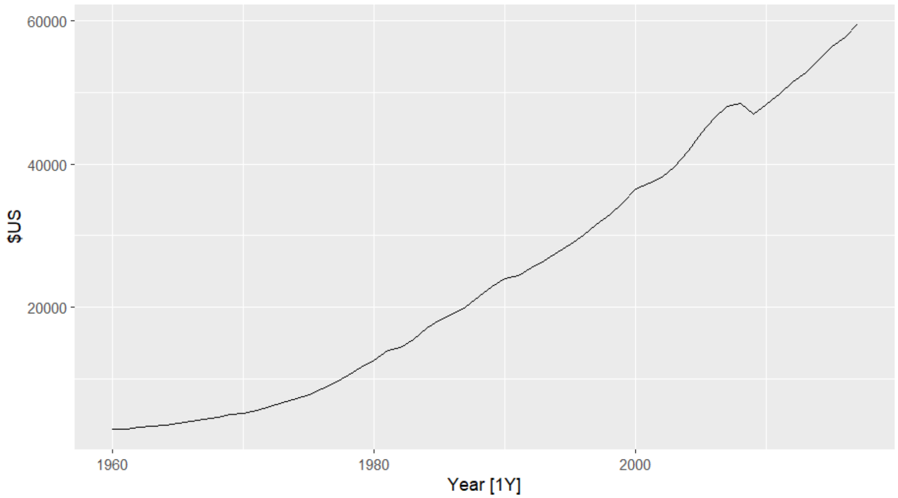
r_1	r_2	r_3	r_4	r_5	r_6	r_7	r_8	r_9
-0.102	-0.657	-0.060	0.869	-0.089	-0.635	-0.054	0.832	-0.108



Partial Autocorrelation

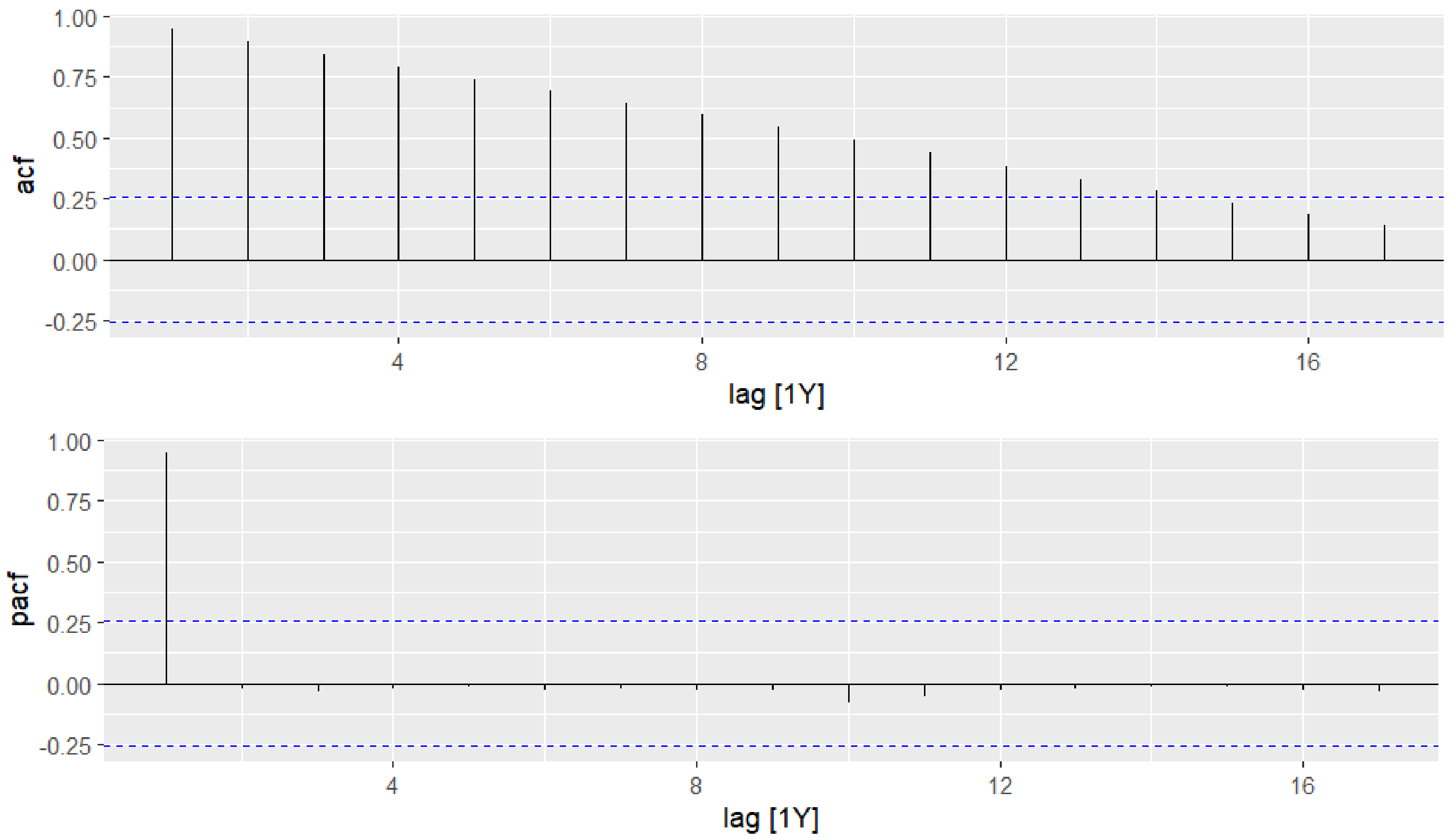
- Partial autocorrelation, also known as partial serial correlation, is a measure of the correlation between a time series and a lagged version of itself, controlling for the effects of intermediate lag periods.
- y_t and y_{t-2} might be correlated, simply because they are both connected to y_{t-1}, rather than because of any new information contained in y_{t-2}. Partial autocorrelation overcomes this problem.

US Annual GDP per capita (1960-2017)



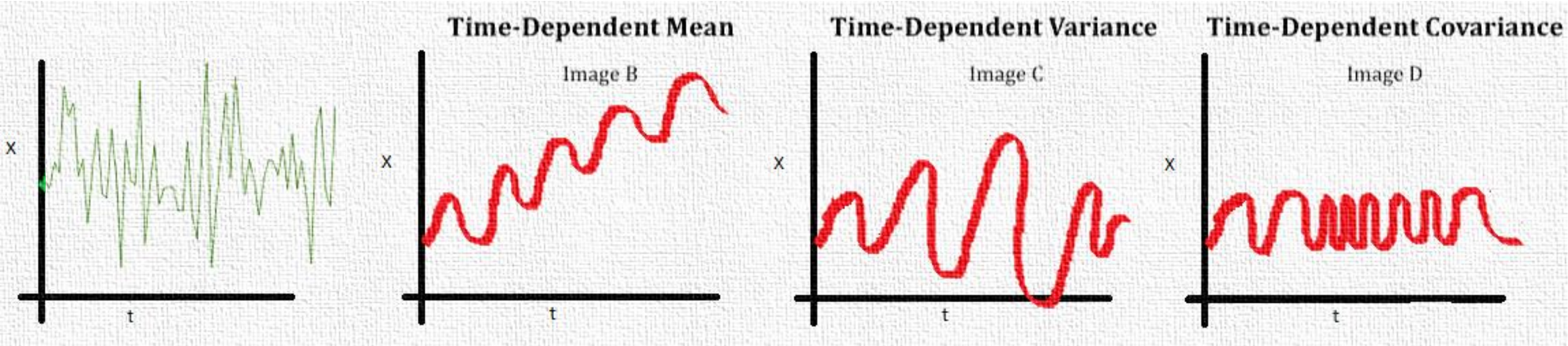
PACF: Partial Autocorrelation Function

- PACF is a statistical tool that can be used to measure the partial autocorrelation of a time series.



Stationarity

- Stationary vs Non-Stationary Data. What makes a data set Stationary?
- In a stationary timeseries, the statistical properties do not depend on the time



Stationary | Non-Stationary | Non-Stationary | Non-Stationary

- Data with trend and seasonality are NOT stationary!

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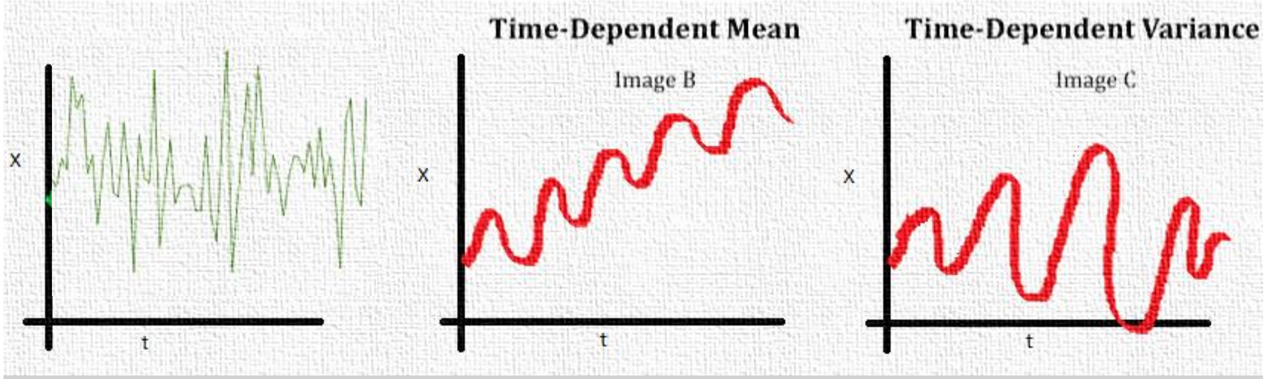
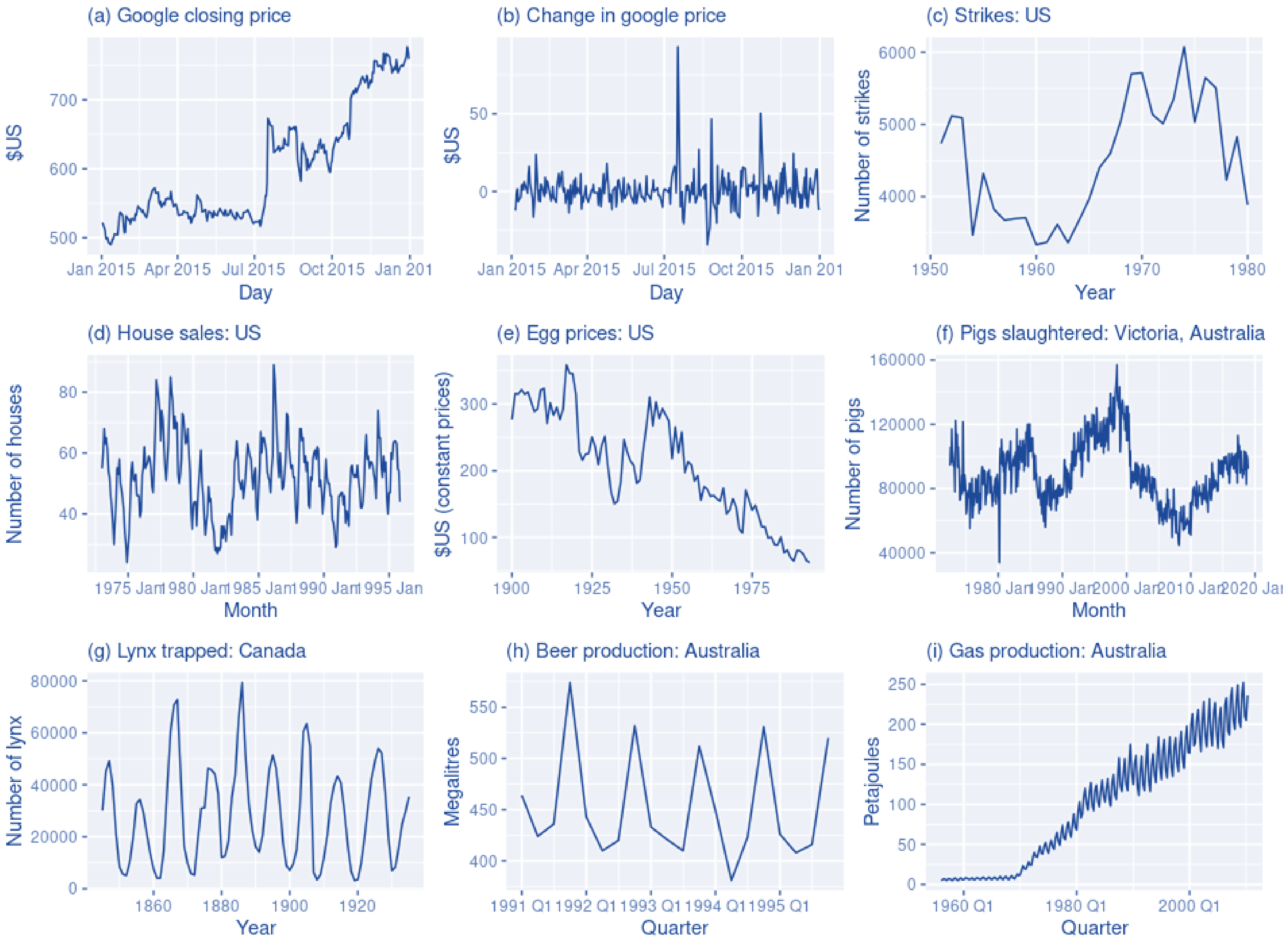
Weak vs Strong Stationarity

- Strong stationarity: mean, variance and autocovariance are constant over time
- Weak stationarity: mean and variance are constant overtime
- ARIMA models require weak stationarity if the autocovariance is not changing too rapidly over time.



Stationary Non-Stationary Non-Stationary

Which ones are stationary?



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Differencing

- Differencing: Computing the difference between consecutive observations.
- Differencing helps to stabilize the mean of a time series by removing changes in the level and therefore reducing the trend and seasonality.
- Recall: Transformations help to stabilize the variance of a time series.

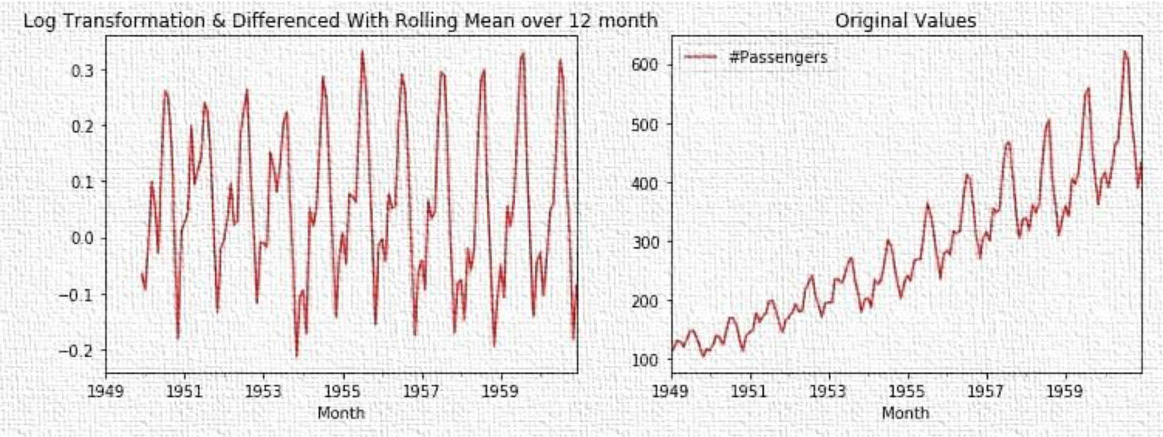
Original data

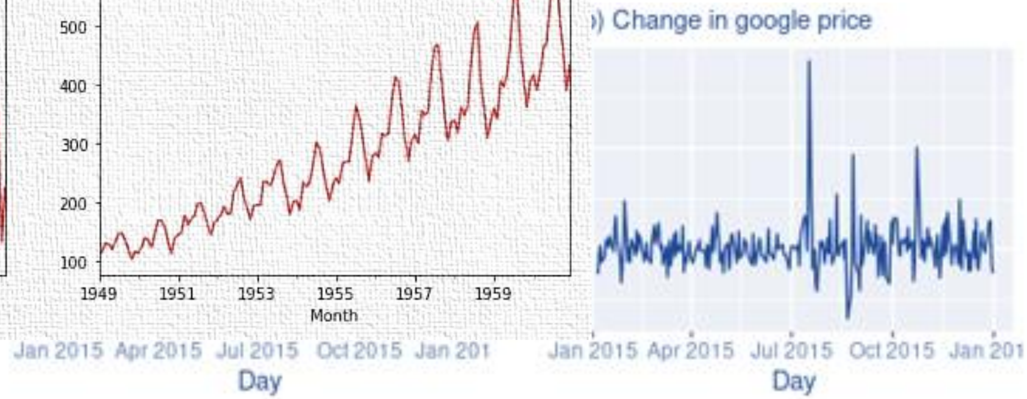
Time1	10
Time2	12
Time3	8
Time4	14
Time5	7

1st diffe

Time1	
Time2	
Time3	
Time4	
Time5	-7

Time5	-13





2^{nd} Differencing

- Occasionally the differenced data will not appear to be stationary, and it may be necessary to difference the data a second time to obtain a stationary series.
- Second differencing is change in change.
- In practice, it is almost never necessary to go beyond second-order differences.

$$
\begin{aligned}
y''_t &= y'_t - y'_{t-1} \\
&= (y_t - y_{t-1}) - (y_{t-1} - y_{t-2}) \\
&= y_t - 2y_{t-1} + y_{t-2}.
\end{aligned}
$$

Seasonal Differencing

- A seasonal difference is the difference between an observation and the previous observation from the same season.

$$y'_t = y_t - y_{t-m}$$

- m is the number of seasons. This is also called lag-m difference.
- If seasonal differenced is white noise, then

$$y_t = y_{t-m} + \varepsilon_t$$

- Recall:
  - Seasonal Naïve forecast: each forecast set to be equal to the last observed value from the same season

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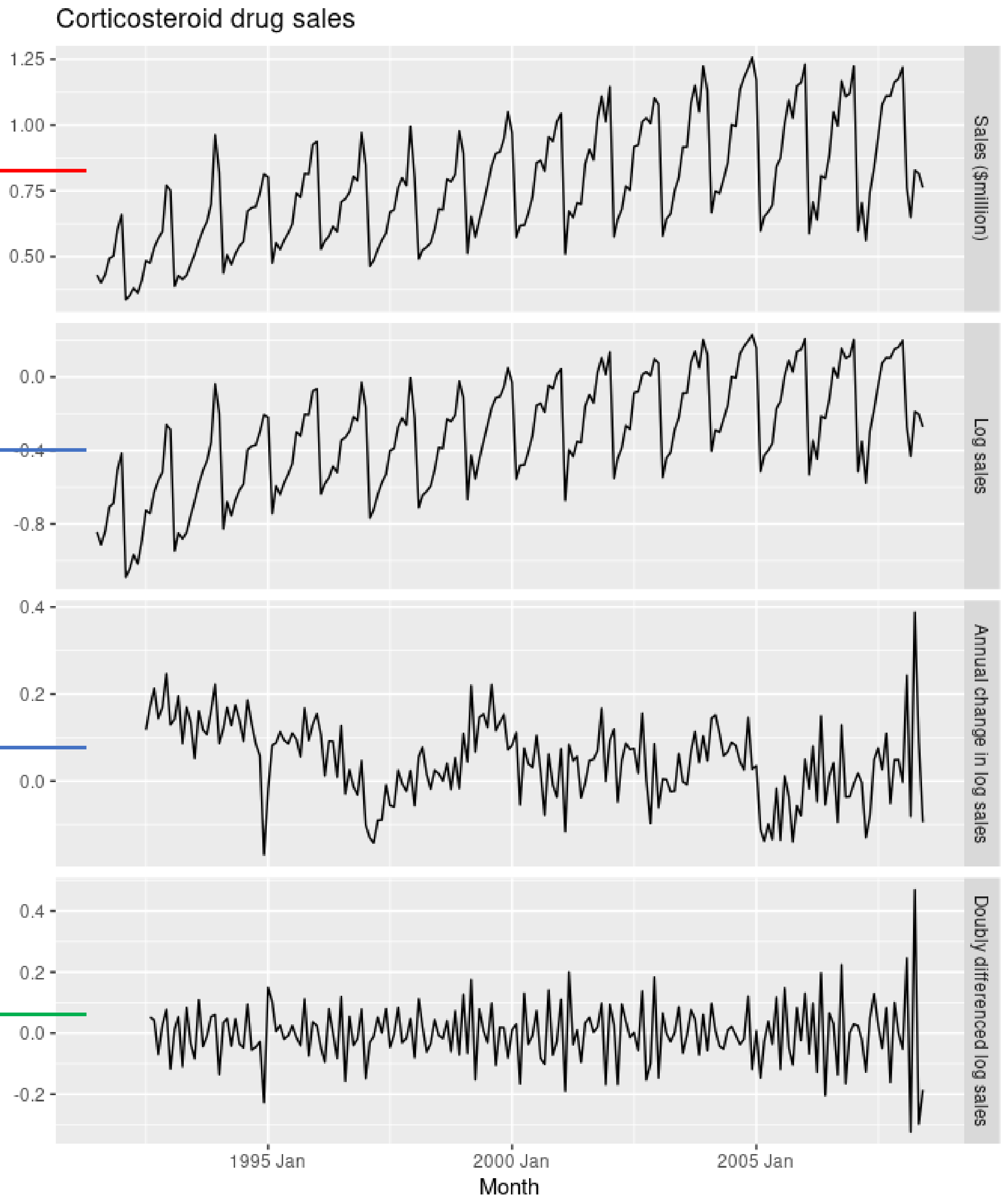
Put it together!

Original data

Log transformed

Seasonal difference

1st differenced seasonal difference



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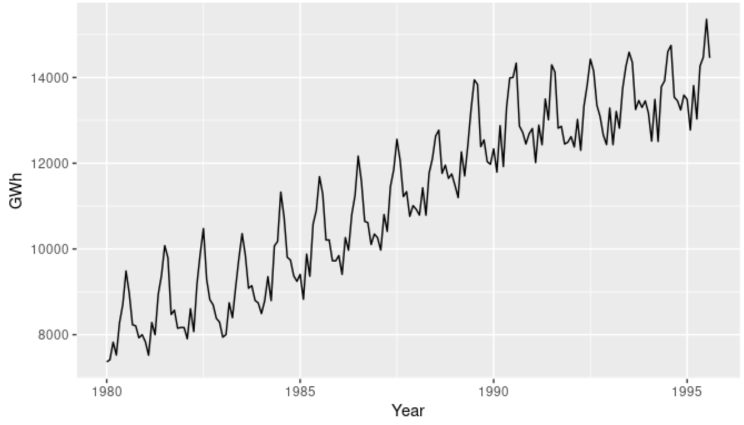
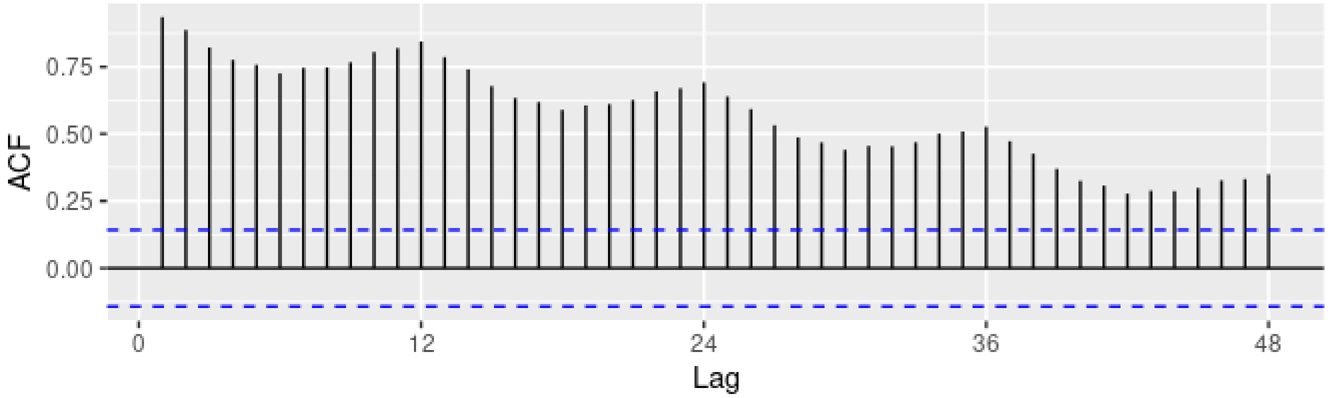
Recall: Trend and seasonality in ACF plots

- Autocorrelation can be useful for identifying patterns and trends in time series data.
- The ACF of trended time series tend to have positive values that slowly decrease as the lags increase.
- Fore seasonal data, the autocorrelations are larger for the seasonal lags than for other lags.

Monthly Electricity Demand





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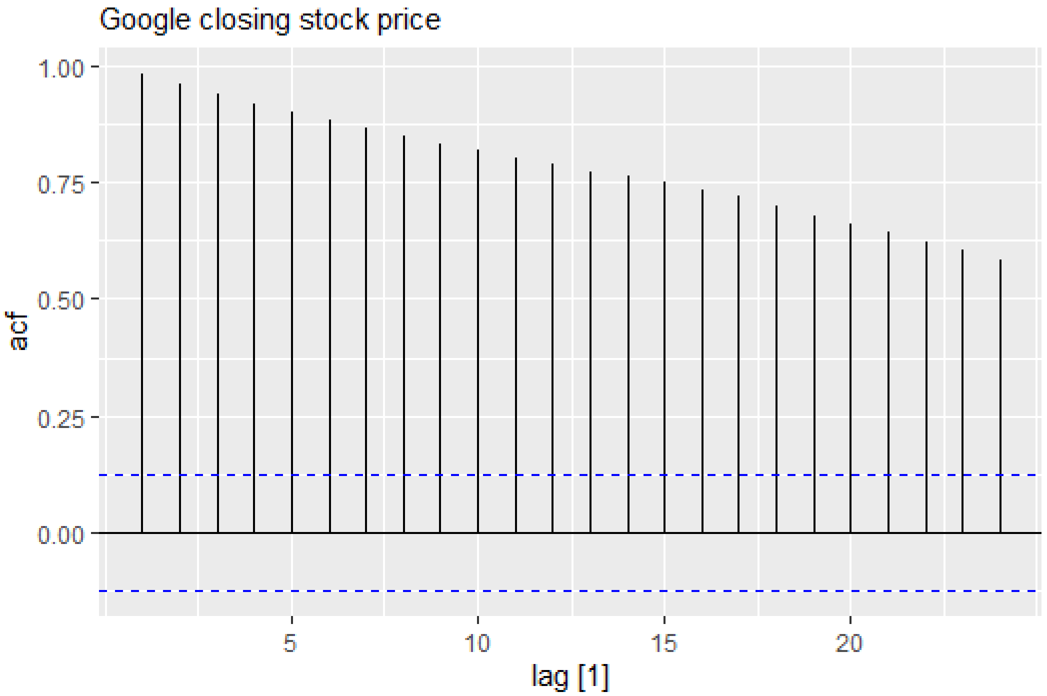
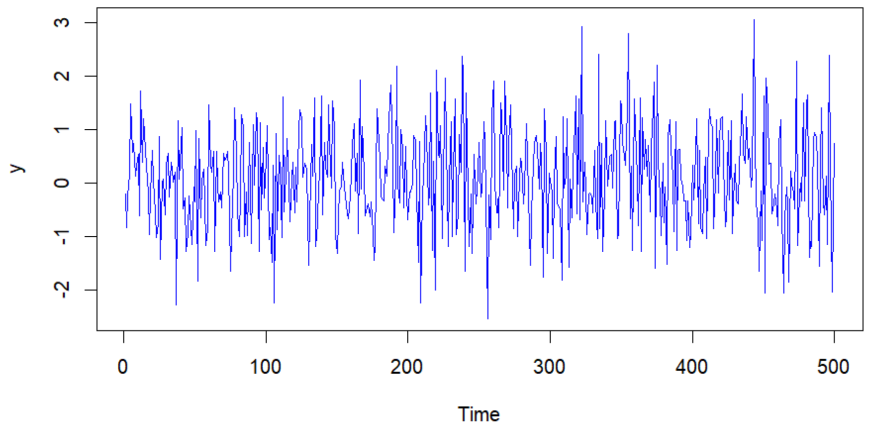
ACF plots and Stationarity

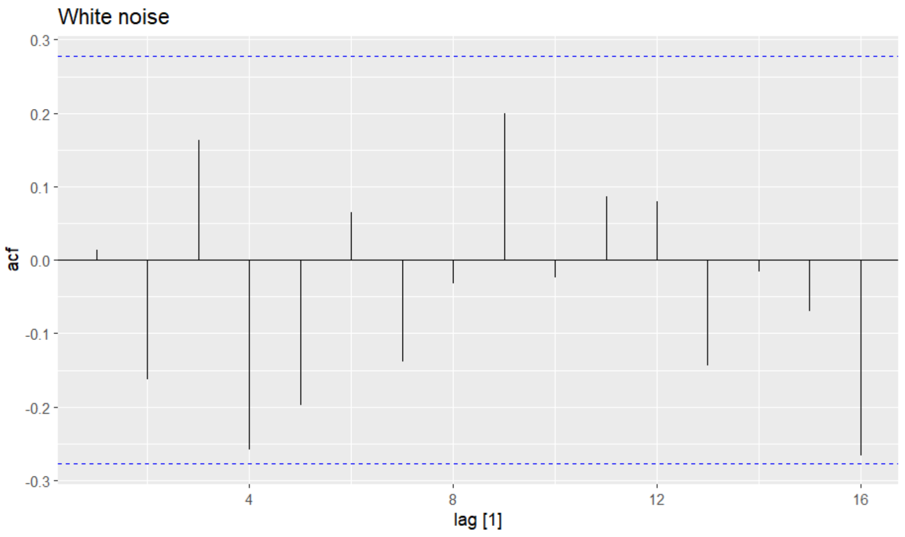
- For stationary data,
  - The ACF plot drops to zero quickly.
  - r_1 is mostly large and positive.



White Noise

- White noise can be thought of as a random sequence of iid values (independent and identically distributed) characterized by a distribution.
- White noise has **zero mean** and **finite variance.** $\epsilon_t \sim D(0, \sigma^2)$
- White noise data show no autocorrelation.





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Random Walk

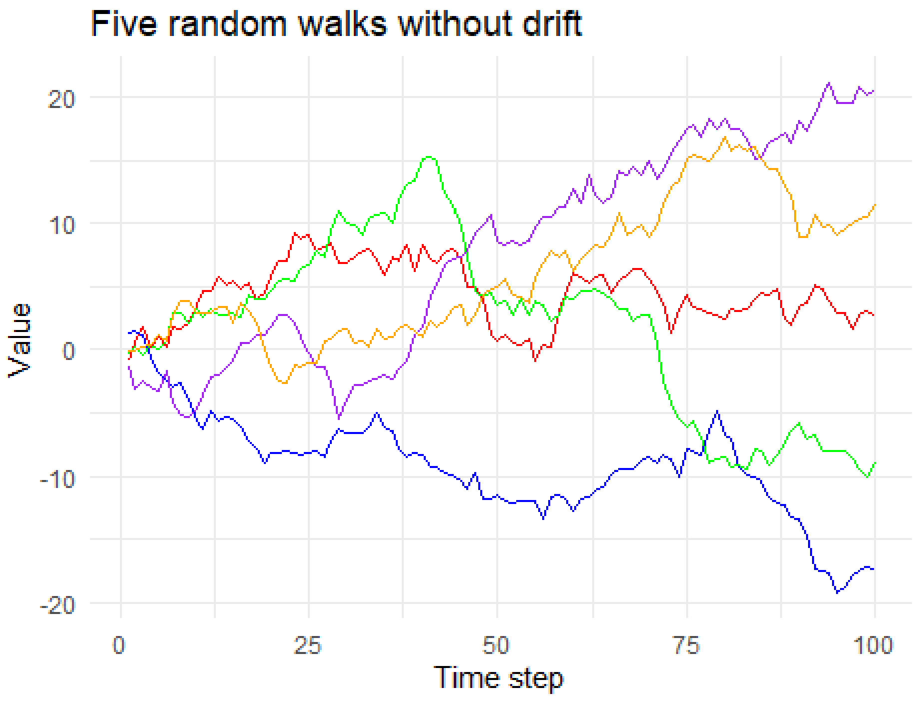
- Random Walk: When the 1st differenced series is white noise

$$y_t - y_{t-1} = \varepsilon_t \longrightarrow \boxed{y_t = y_{t-1} + \varepsilon_t}$$

- Random walk models are widely used for non-stationary data, particularly financial and economic data.

- Random walks typically have long periods of up or down trend + sudden change in direction.

- Random walk with no drift = Naïve forecasting model



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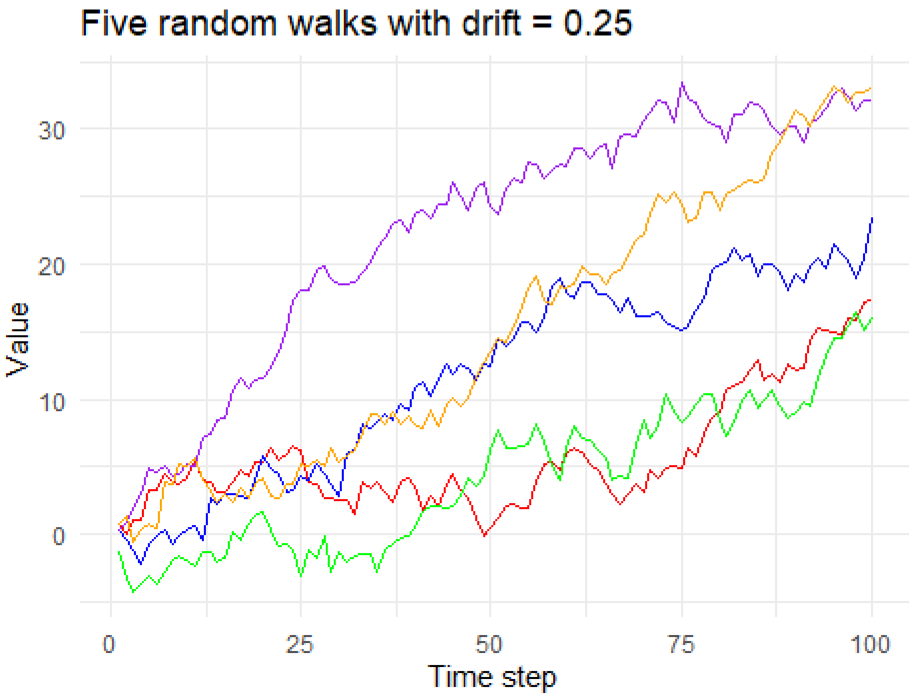
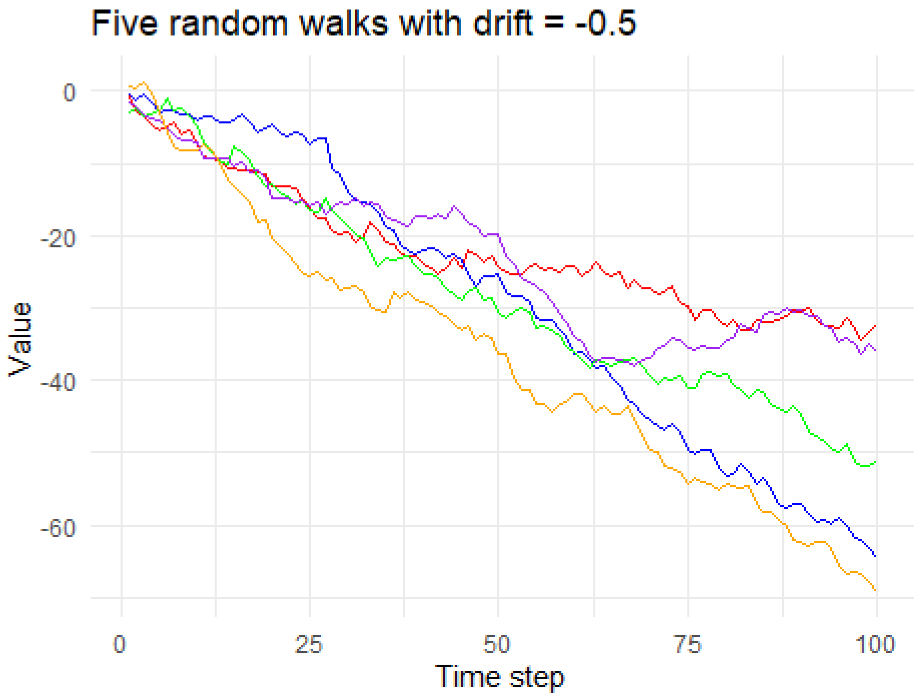
Random Walk with Drift

- Random walk with drift c (the 1st difference does not have zero average):

$$y_t - y_{t-1} = c + \varepsilon_t \quad \text{or} \quad y_t = c + y_{t-1} + \varepsilon_t$$

- C is the average change between consecutive observations.





Testing for Stationarity

- Unite root test is a statistical test used to determine whether a time series has a unit root, which is a characteristic of a non-stationary time series
- There are several different unit root tests including:
  1. Augmented Dickey-Fuller (ADF) test.
  2. Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test.

Hypothesis Test	Null	Alternative	P-value to get stationarity
ADF	Non-Stationary	Stationary	Small
KPSS	Stationary	Non-Stationary	Large

	ADF	KPSS
ADF statistic	-12.533939	0.012944
p-value	0.0	0.1
should we difference?	?	
conclusion		

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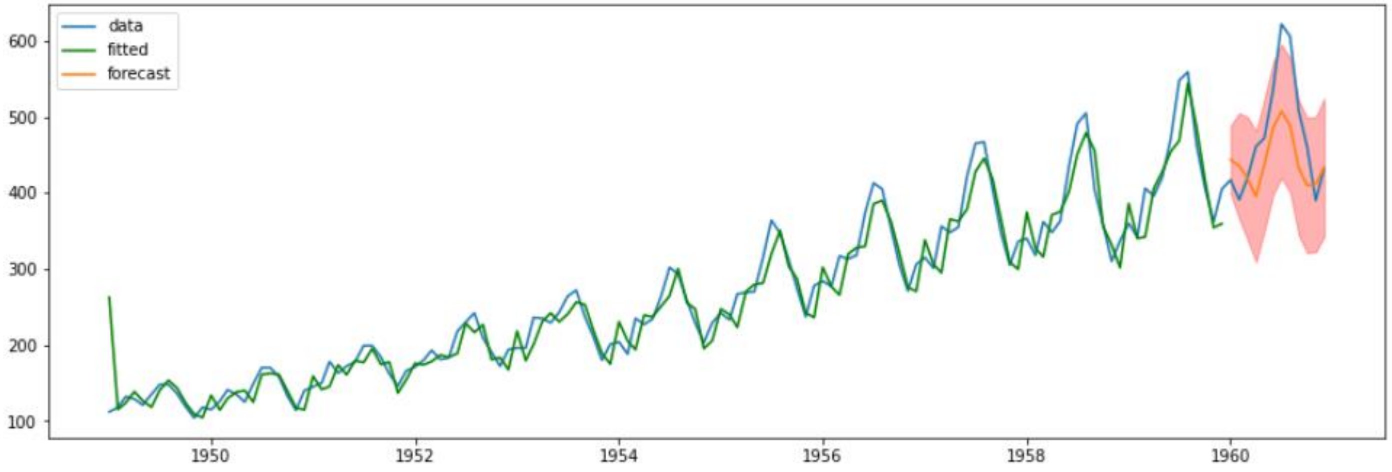
Components of ARIMA model

1. Autoregressive (AR) term - captures the autocorrelation in the data
2. Integrated (I) term - removes the non-stationarity in the data
3. Moving Average (MA) term - captures the error term or noise in the data

How it works?

- The AR term models the current **value** of the time series as a linear combination of its past **values**.
- The I term models the differences between the current **value** and the past **value**.
- The MA term models the current **error** term as a linear combination of the past **error** terms.



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Module 4 – Part II

ARIMA models

Components of ARIMA model

ARIMA

1. Autoregressive (AR) term - captures the autocorrelation in the data
2. Integrated (I) term - removes the non-stationarity in the data
3. Moving Average (MA) term - captures the error term or noise in the data

Autoregressive models

- An autoregressive (AR) model is a statistical model (multiple linear regression model) that uses lagged variable as predictors
- Autoregression = regression of the variable against itself
- AR(p) model, autoregressive model of order p.

$$\boxed{y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + \varepsilon_t}$$

- In AR(1) model:
  - when $\phi_1 = 0$ and $c = 0$, y_t is equivalent to ?
  - when $\phi_1 = 1$ and $c = 0$, y_t is equivalent to ?
  - when $\phi_1 = 1$ and $c \neq 0$, y_t is equivalent to ?
  - when $\phi_1 < 0$, y_t tends to oscillate around the mean.

Autoregressive Models (Example)

$$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + \varepsilon_t$$

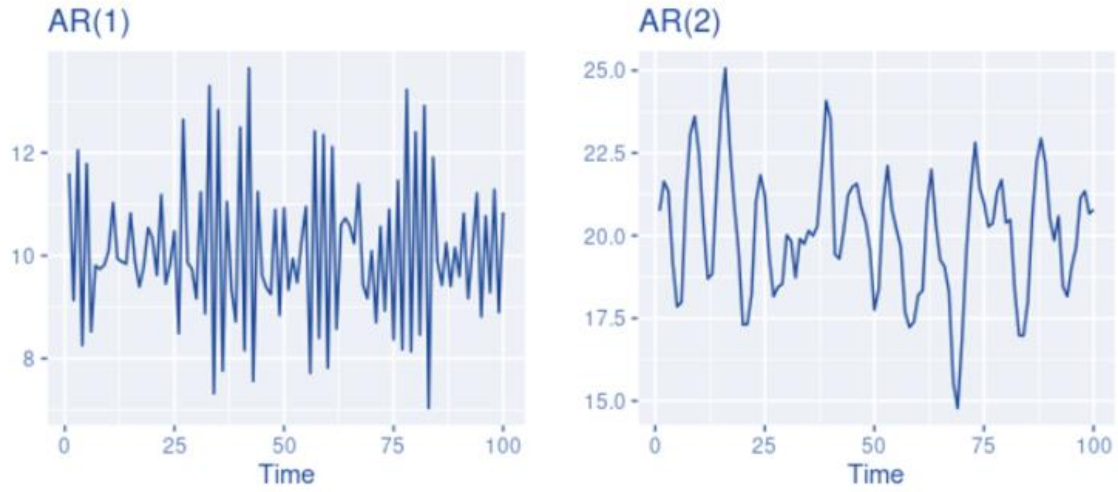

Figure 9.5: Two examples of data from autoregressive models with different parameters. Left: AR(1) with $y_t = 18 - 0.8y_{t-1} + \varepsilon_t$. Right: AR(2) with $y_t = 8 + 1.3y_{t-1} - 0.7y_{t-2} + \varepsilon_t$. In both cases, ε_t is normally distributed white noise with mean zero and variance one.

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Moving Average Models

- A moving average model uses past forecast errors in a regression-like model
- MA(q) model, a moving average model of order q.

$$\boxed{y_t = c + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \cdots + \theta_q \varepsilon_{t-q}}$$

- y_t can be thought of as a weighted moving average of the past few forecast errors
- We require $|\phi| < 1$, the most recent observations carry a greater weight than those from the distant past.

Moving Average Models

$$\boxed{y_t = c + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \cdots + \theta_q \varepsilon_{t-q}}$$

- Do NOT confuse this model with simple moving average method or exponentially weighted moving average method.
- Moving average models is used for forecasting future values!
- Moving average smoothing (SMA, EWMA, …) is used for estimating the trend-cycle of past values.

Moving Average Models (Example)

$$\boxed{y_t = c + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \cdots + \theta_q \varepsilon_{t-q}}$$

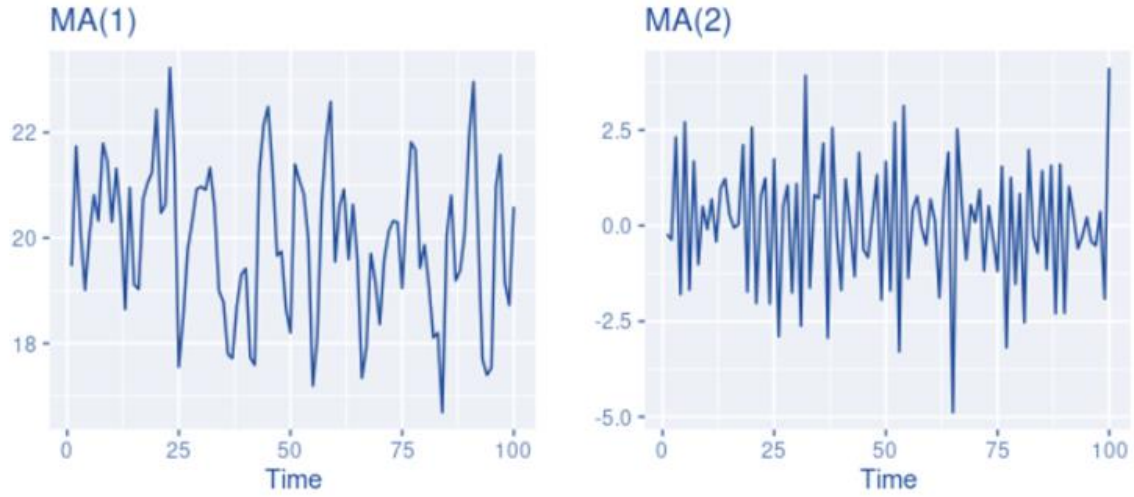

Figure 9.6: Two examples of data from moving average models with different parameters. Left: MA(1) with $y_t = 20 + \varepsilon_t + 0.8\varepsilon_{t-1}$. Right: MA(2) with $y_t = \varepsilon_t - \varepsilon_{t-1} + 0.8\varepsilon_{t-2}$. In both cases, ε_t is normally distributed white noise with mean zero and variance one.

ARIMA (AutoRegressive Integrated Moving Average)

- ARIMA model combines three models, autoregressive (AR) model, an integrated (I) model, and a moving average (MA) model.
- ARIMA(p, d, q) model.

$$\boxed{y'_t = c + \phi_1 y'_{t-1} + \cdots + \phi_p y'_{t-p} + \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q} + \varepsilon_t}$$

- y'_t is the differenced time series.
- d degree of first difference involved.
- Note: p, d, and q are estimated using MLE.

?	ARIMA(0,0,0) with no constant
?	ARIMA(0,1,0) with no constant
?	ARIMA(0,1,0) with a constant
?	ARIMA(p,0,0)
?	ARIMA(0,0,q)

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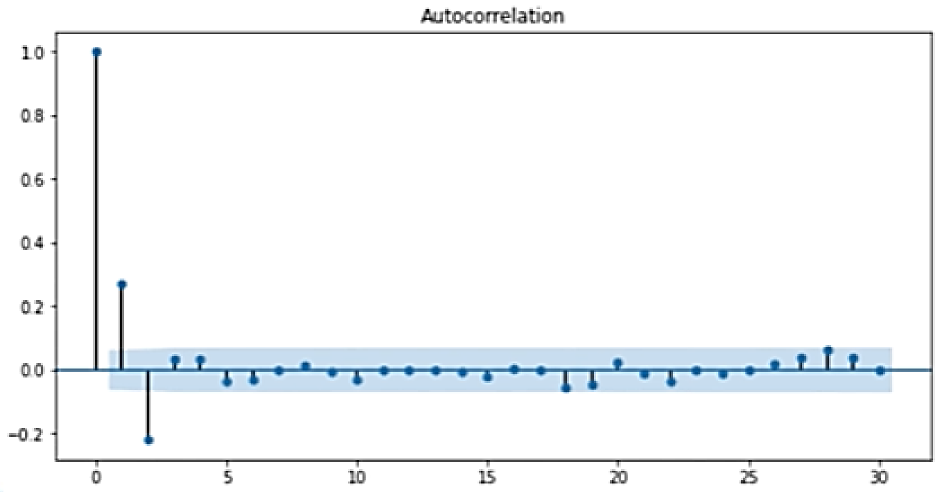
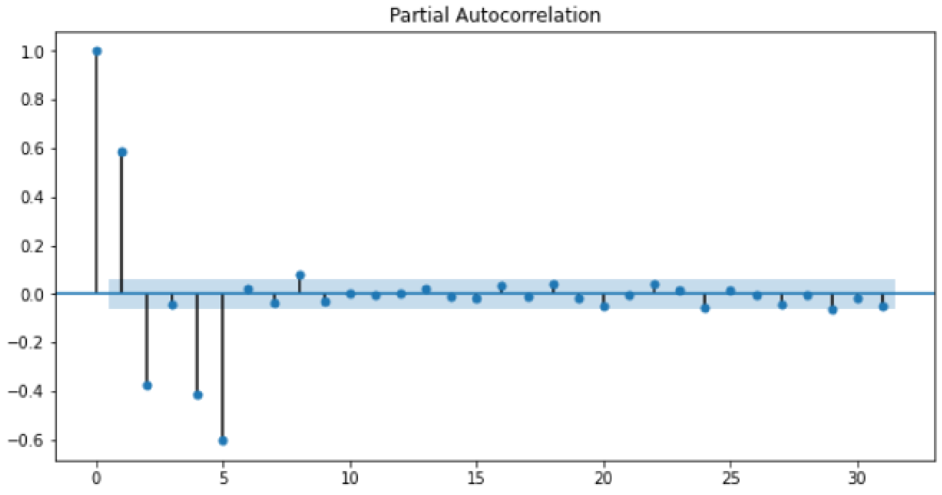
Selecting (p, q) orders using ACF and PAC

- Some rough guidelines:
- Identification of an AR model is often best done with the PACF
  - p set to be the maximum significant non-zero lag in PACF typically followed by a sharp decline.
- Identification of an MA model is often best done with the ACF
  - q set to be the maximum significant non-zero lag in ACF typically followed by a sharp decline.





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Model selection

- Fore model selection we can either use information criteria or any cross validated performance metrics like R^2, MSE, RMSE, MAPE, sMAPE.

Information Criteria	Formula
Akaike's Information Criterion (AIC)	$\text{AIC} = -2\log(L) + 2k$
AIC corrected for small sample bias (AICc)	$\text{AIC}_\text{c} = \text{AIC} + \frac{2k(k+1)}{T-k-1}$
Bayesian Information Criterion (BIC)	$\text{BIC} = \text{AIC} + k[\log(T) - 2]$

- L is the likelihood of the model and K is the total number of parameters (including the variance of residuals)
- The model with the minimum information criteria is often the best model for forecasting

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SARIMA (Seasonal ARIMA) models

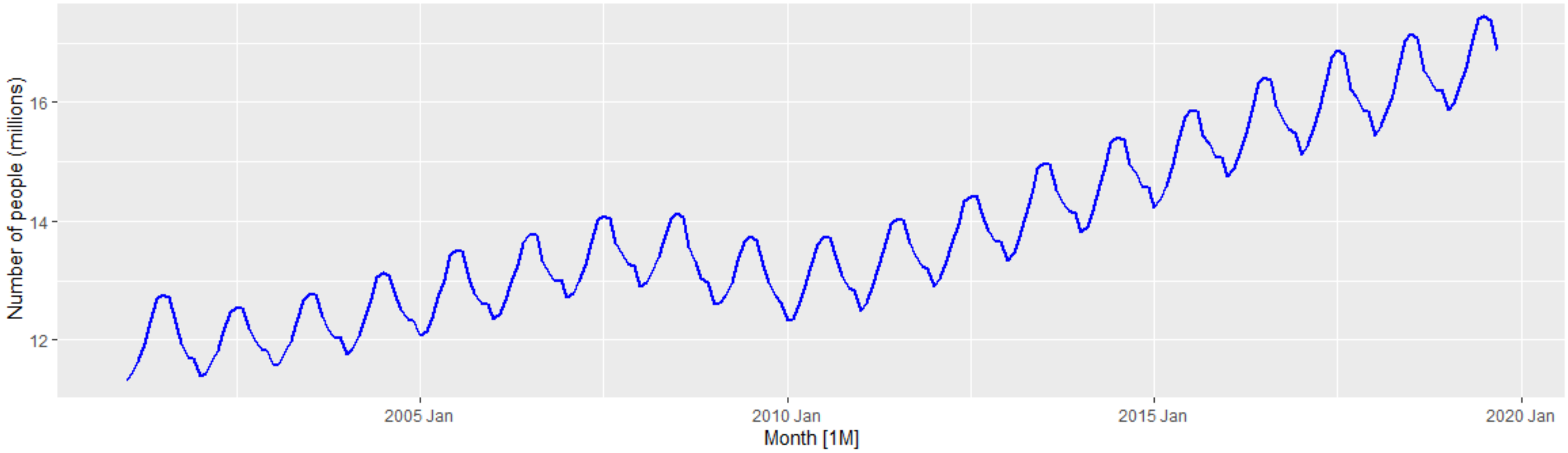
- SARIMA is an extension of an ARIMA model that includes additional seasonal terms.
- It is used to model time series data that exhibits seasonal patterns

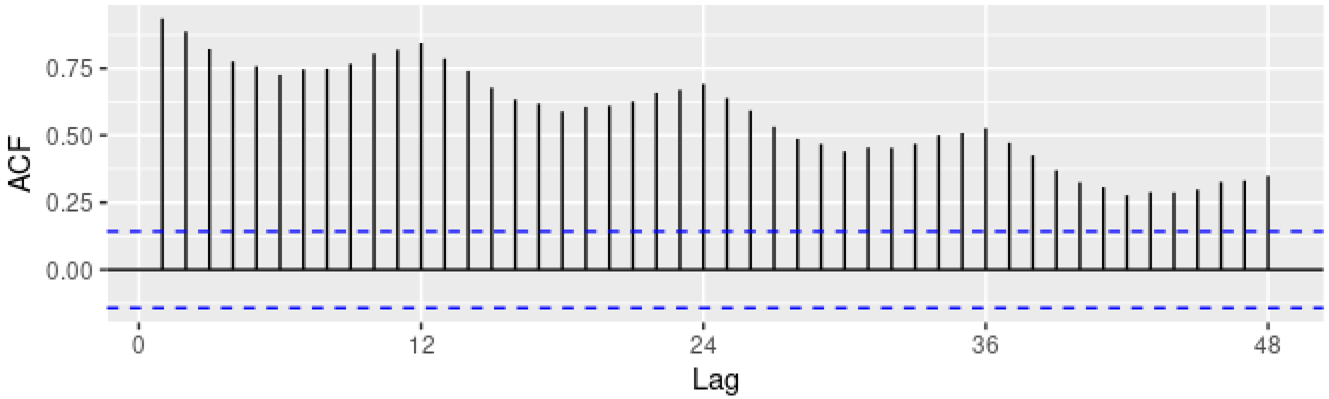
$$\text{ARIMA} \quad \underbrace{(p, d, q)}_{\substack{\uparrow \\ \text{Non-seasonal part} \\ \text{of the model}}} \quad \underbrace{(P, D, Q)_m}_{\substack{\uparrow \\ \text{Seasonal part} \\ \text{of the model}}}$$

- p, d, q are defined as before.
- P is the order of the seasonal autoregressive component
- D is the degree of seasonal differencing
- Q is the order of the seasonal moving average component
- m is the period of the seasonality. m = 4, 12 is for quarterly and monthly seasonality, respectively.

SARIMA example

Monthly US leisure and hospitality employment, 2001-2019.





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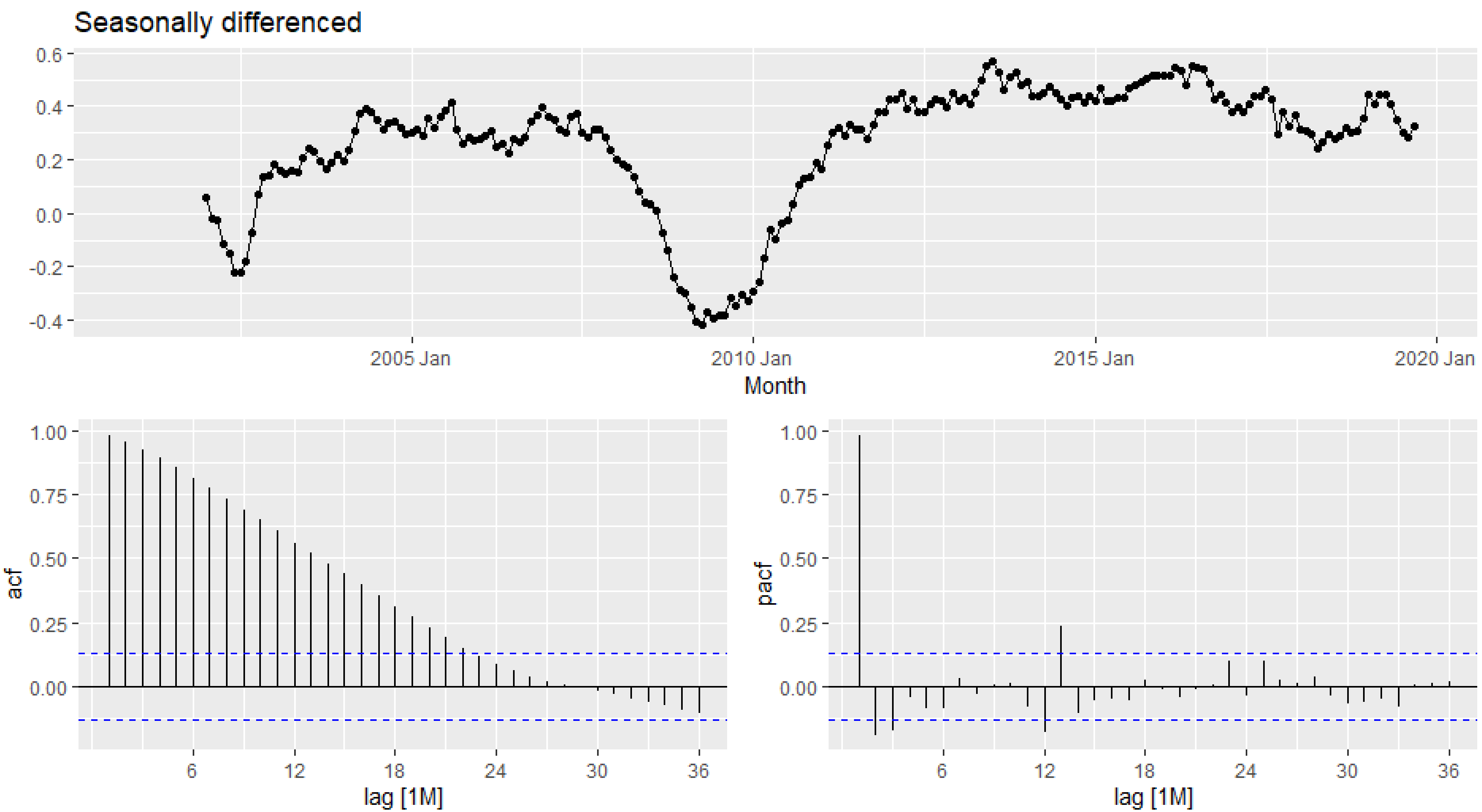
SARIMA example



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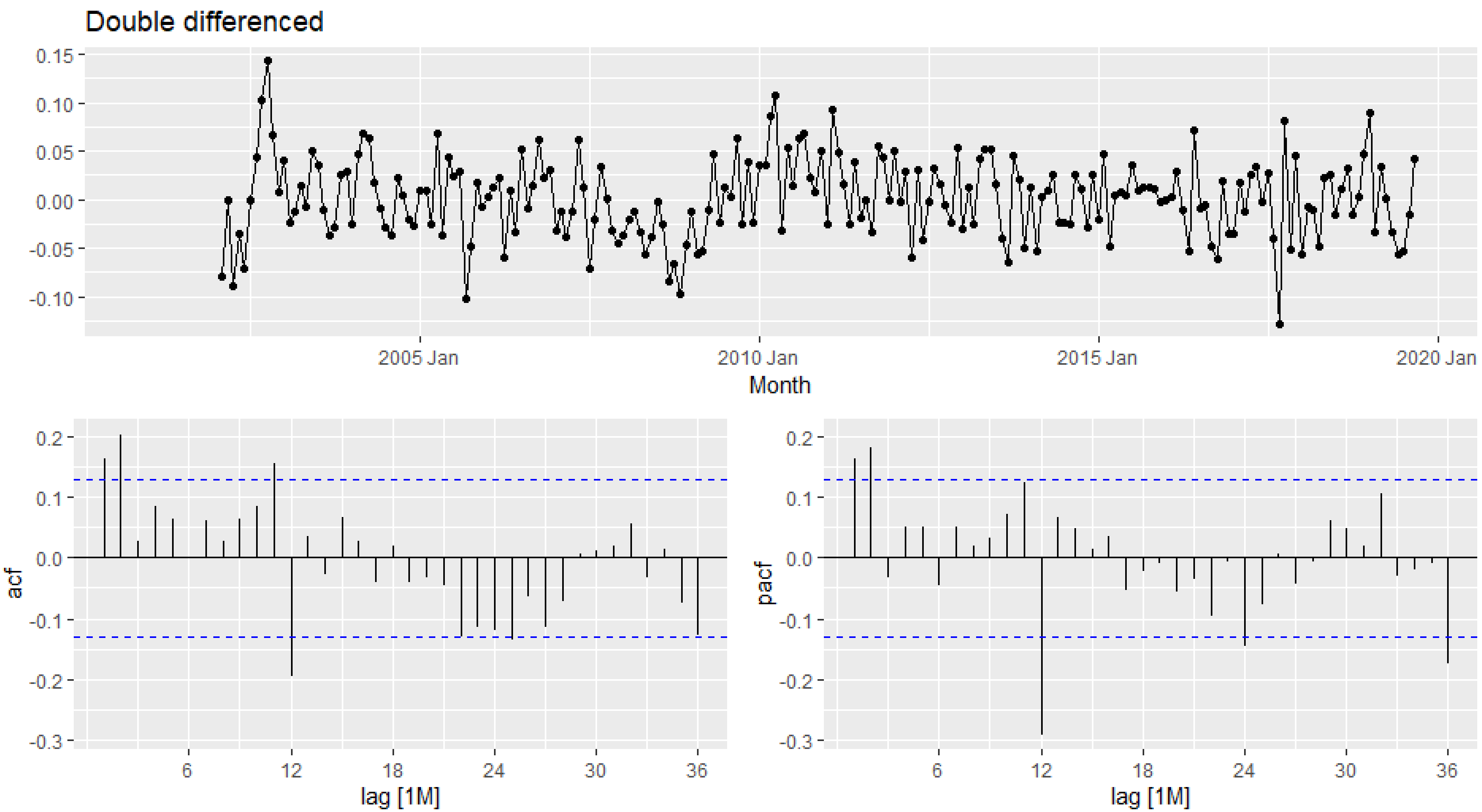
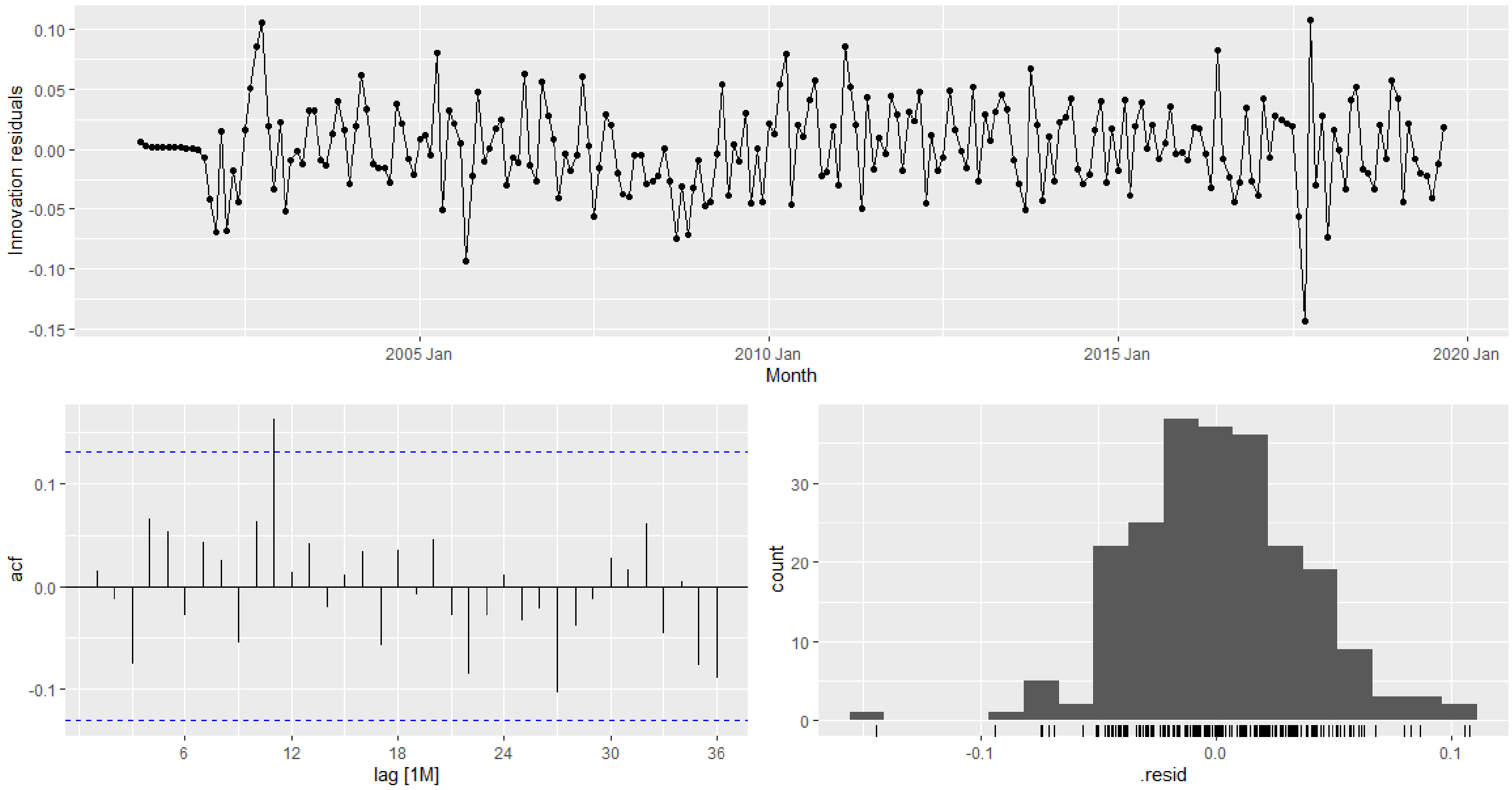
SARIMA example



SARIMA example

- Using Auto-SARIMA, the winning model is $SARIMA(2,1,0)(1,1,1)_{12}$
- Plotting residuals to confirm they are like Gaussian white noise.



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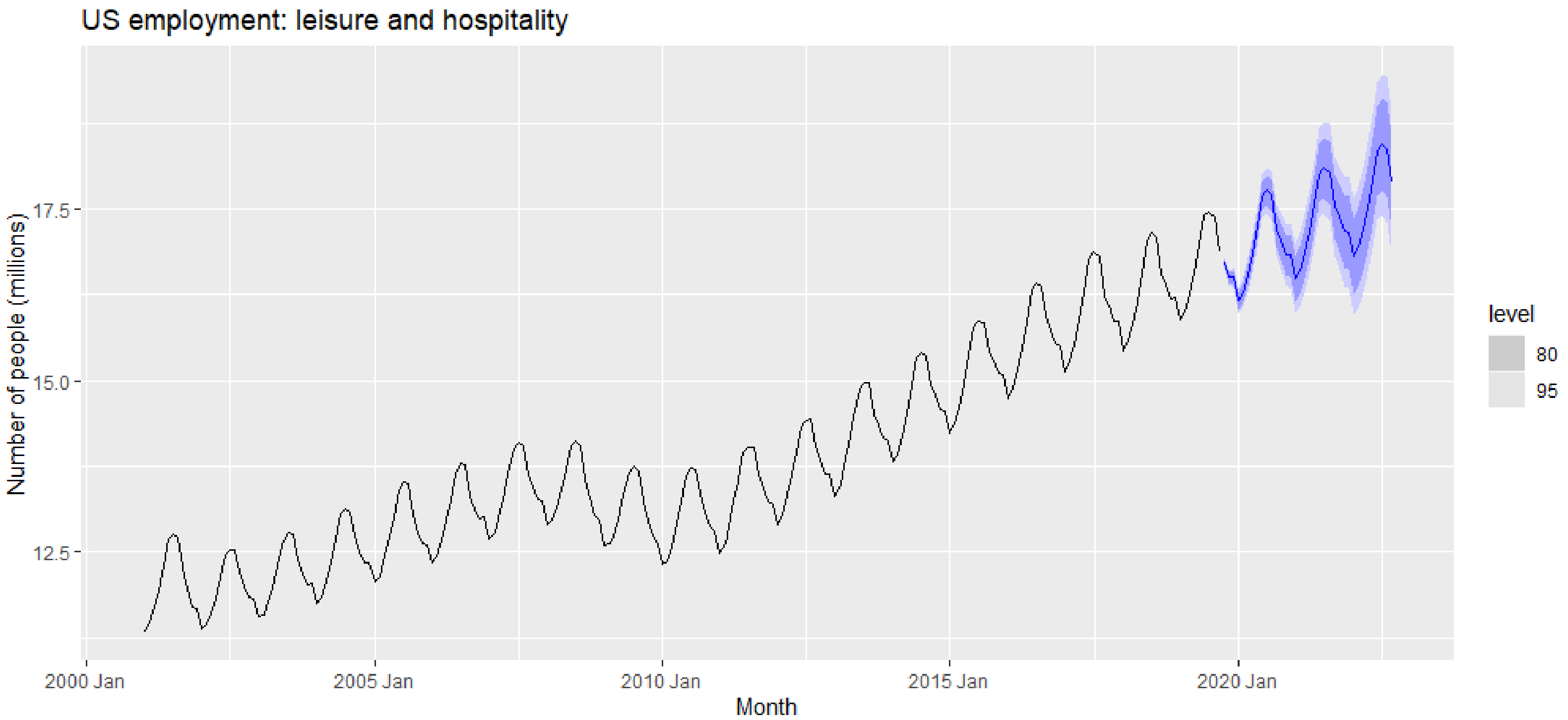
SARIMA example, Forecasting

- We now have a seasonal ARIMA model that passes the required checks and is ready for forecasting.
- The forecasts have captured the seasonal pattern very well, and the increasing trend extends the recent pattern. The trend in the forecasts is induced by the double differencing.



ARIMA: Advantages and Disadvantages

- Flexible and can be used with a wide range of time series data.
- Able to capture both linear and non-linear relationships
- Easy to implement and interpret

- Difficult to tune and get accurate forecasts, especially for time series with complex patterns or multiple seasonality
- It is sensitive to the choice of parameters and can produce unstable forecasts if the parameters are not chosen carefully.

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