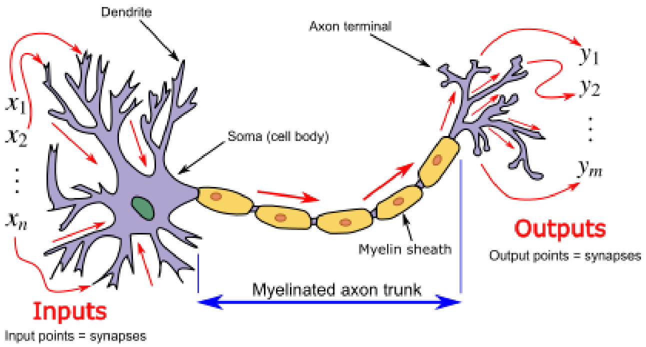
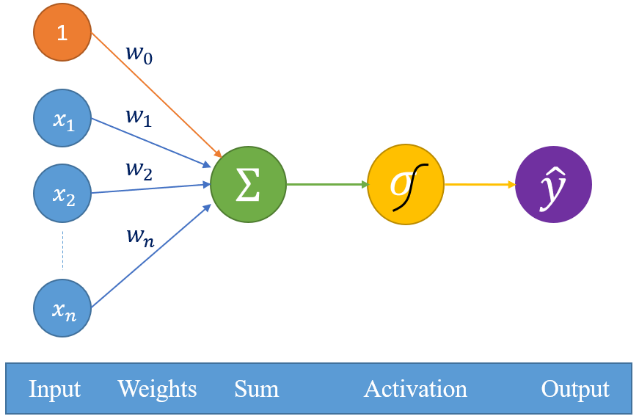
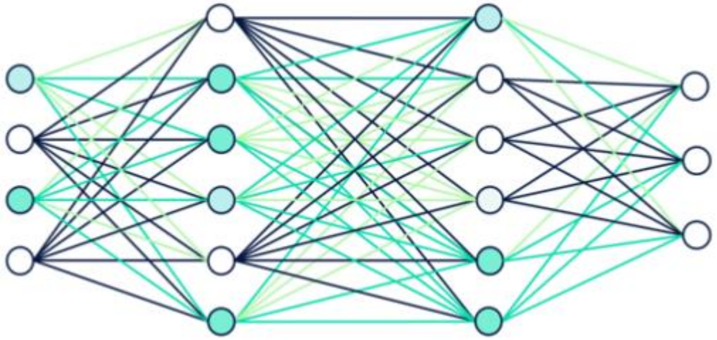
Road map!

- Module 1- Introduction to Deep Forecasting
- Module 2- Setting up Deep Forecasting Environment
- Module 3- Exponential Smoothing
- Module 4- ARIMA models
- Module 5- Machine Learning for Time series Forecasting
- Module 6- Deep Neural Networks
- Module 7- Deep Sequence Modeling (RNN, LSTM)
- Module 8- Prophet and Neural Prophet

Module 4 - Part I
Deep Neural Networks
Basics

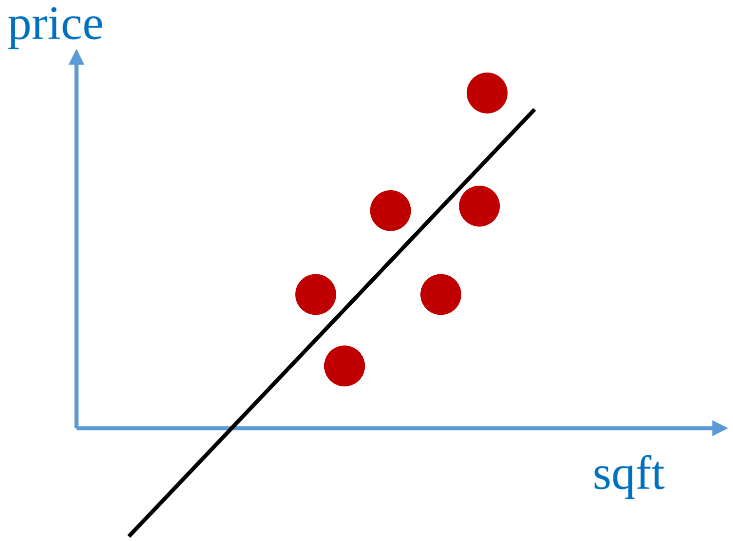


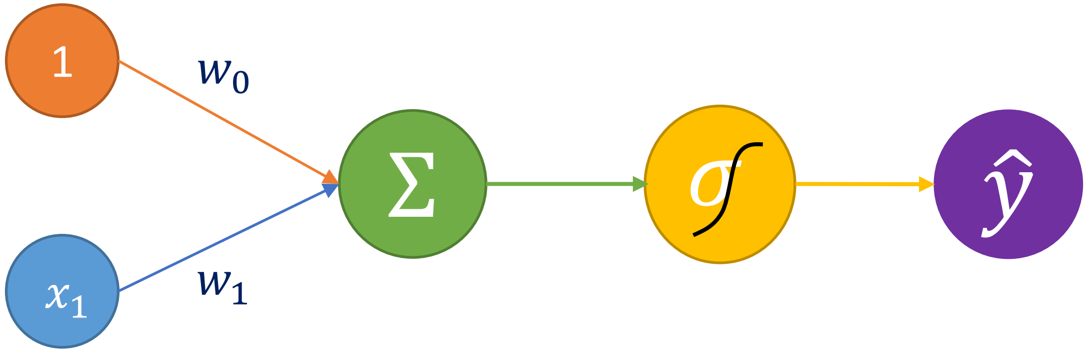




A simple example!

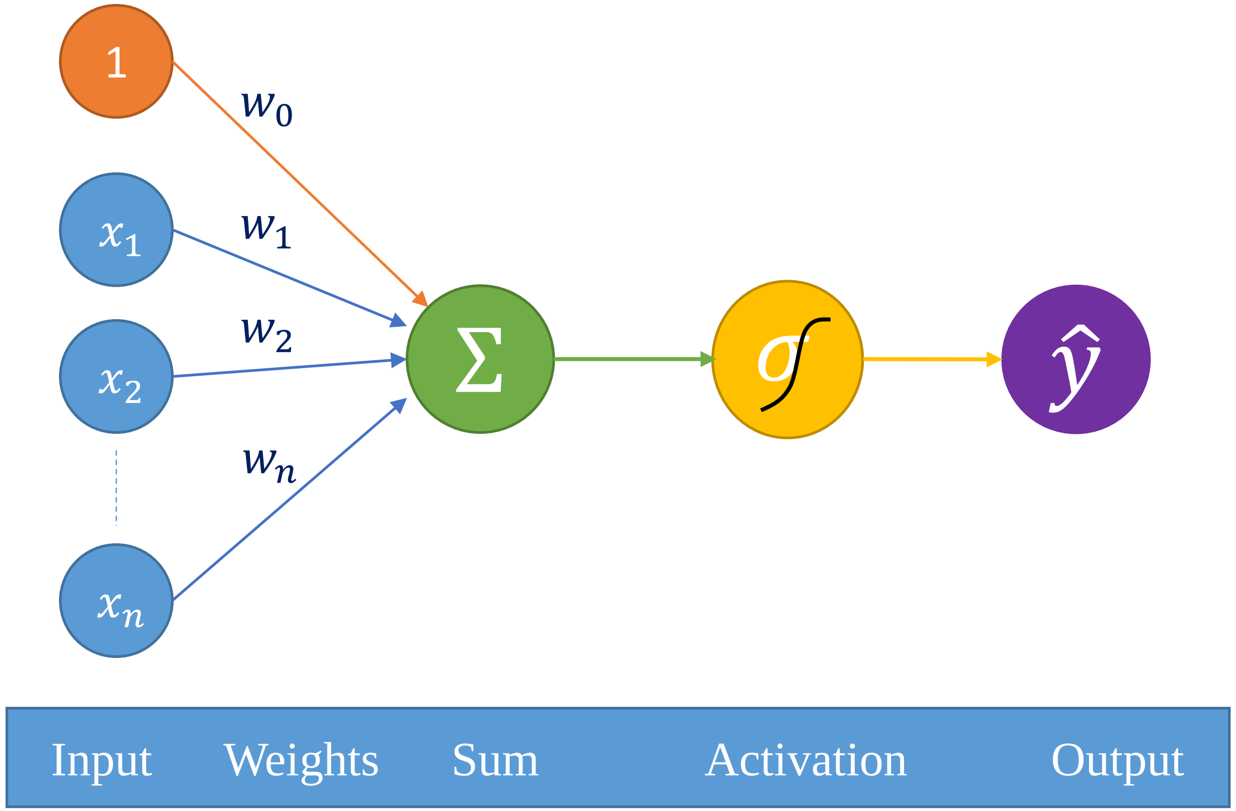
- Consider the housing price prediction! $price = f(sqft)$





Forward Propagation

- Forward propagation is the process of calculating the output of a neural network, given an input.
- w_0 is the bias term which allows shifting Σ to the left or right.



What are Neurons?

- Neurons, are the simplest elements or building blocks in a neural network. They are inspired by biological neurons that are found in the human brain.

- What happens inside the neurons?

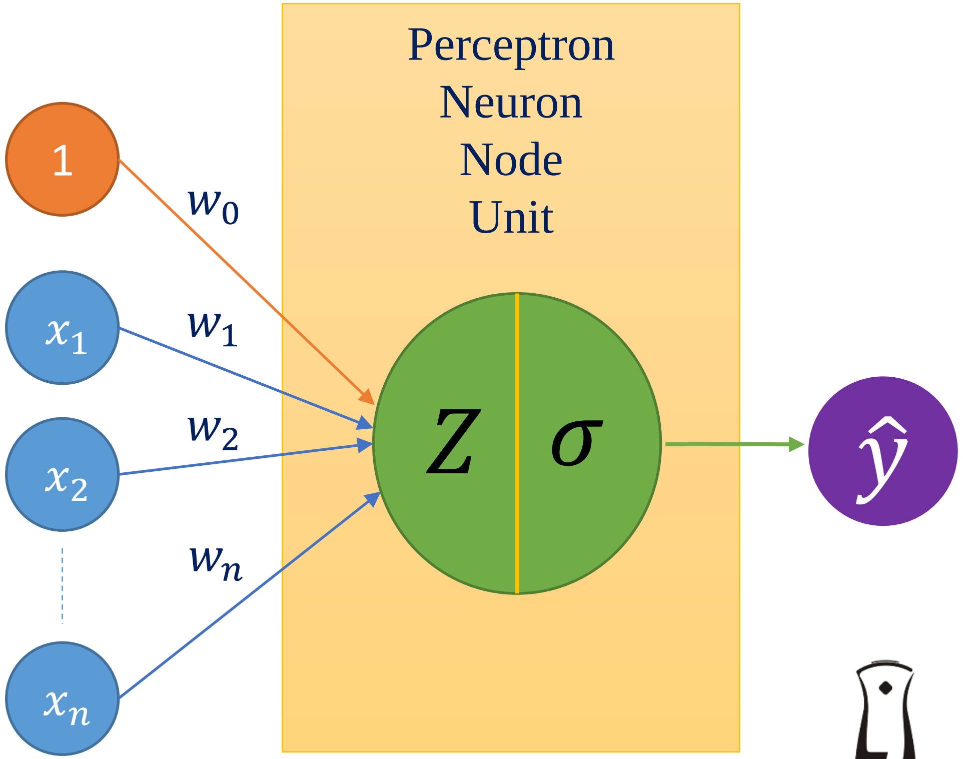
Activation function

Bias

$$\hat{y} = \sigma(w_0 + \sum_{i=1}^{n} w_i x_i)$$

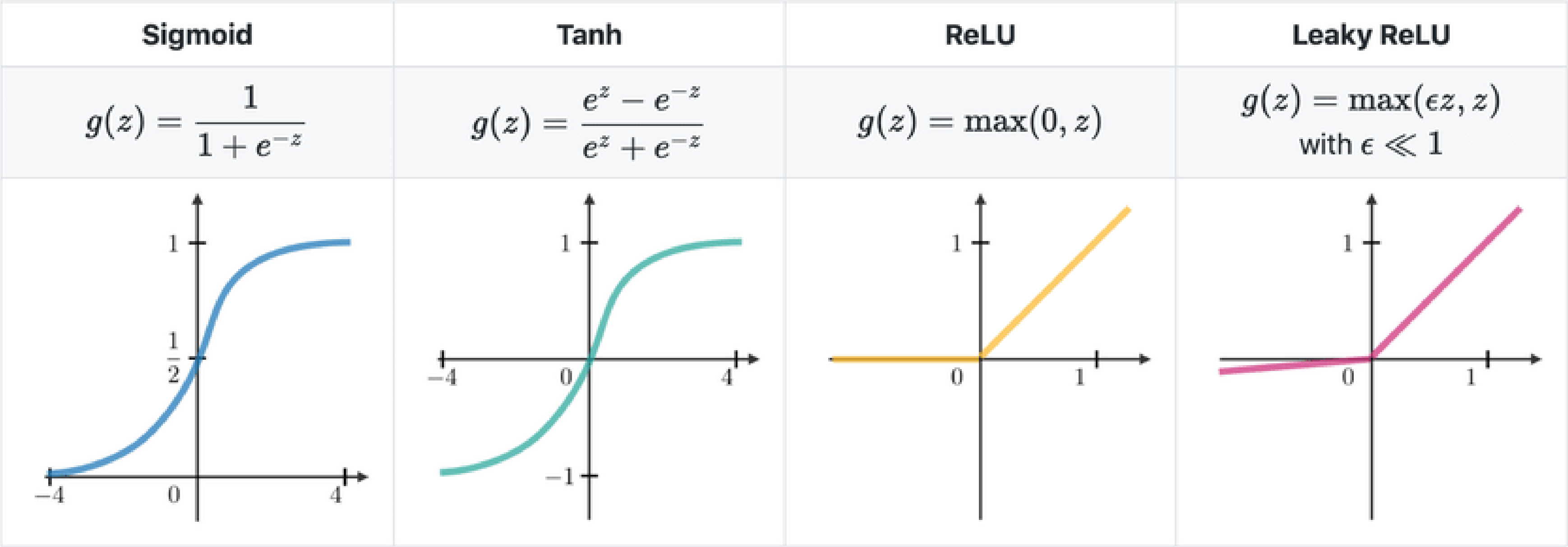
$$\hat{y} = \sigma(w_0 + W^T X)$$

$$\hat{y} = \sigma(Z)$$



Activation function

- An activation function is a mathematical function that is applied to the output of each neuron in a neural network
- Activation functions allow the network to learn non-linearities and complex patterns from the data and make accurate predictions

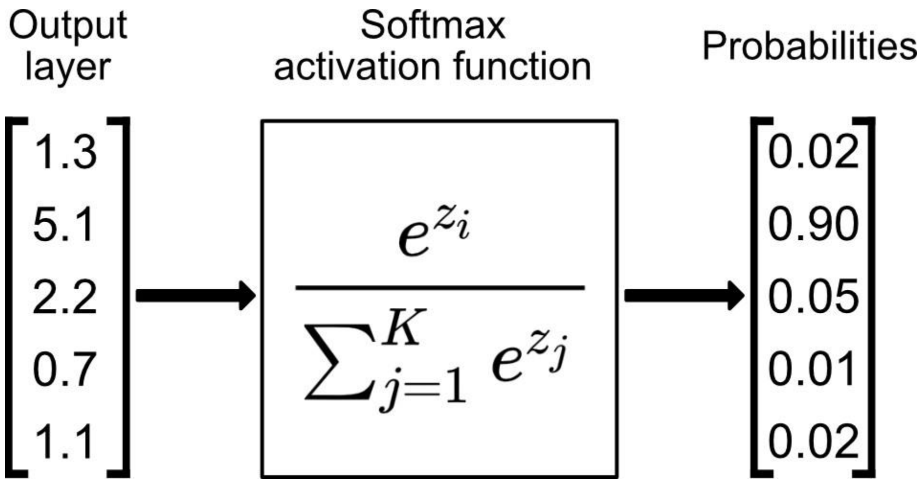


Sigmoid	Tanh	ReLU	Leaky ReLU
$g(z) = \frac{1}{1+e^{-z}}$	$g(z) = \frac{e^z - e^{-z}}{e^z + e^{-z}}$	$g(z) = \max(0, z)$	$g(z) = \max(\epsilon z, z)$ with $\epsilon \ll 1$

1
$\frac{1}{2}$
−4
0
4
1
−4
0
4
−1
1
0
1
1
0
1

Softmax Activation function

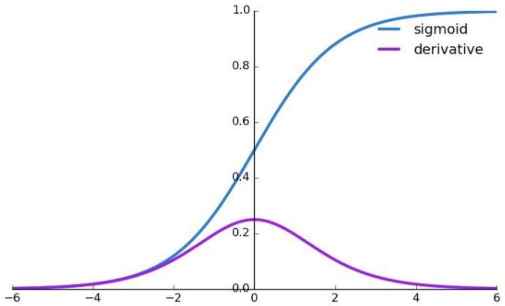
- Softmax activation function is often used in classification tasks, where the goal is to predict which of a fixed set of classes (more than two classes) a particular sample belongs to.
- The Softmax function takes in a vector of real numbers and converts it into a probability distribution, where the sum of all the probabilities is equal to 1.

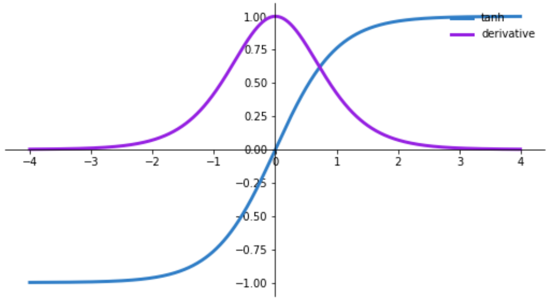

Output layer
Softmax activation function
Probabilities
1.3
5.1
2.2
0.7
1.1
$\frac{e^{z_i}}{\sum_{j=1}^{K} e^{z_j}}$
0.02
0.90
0.05
0.01
0.02

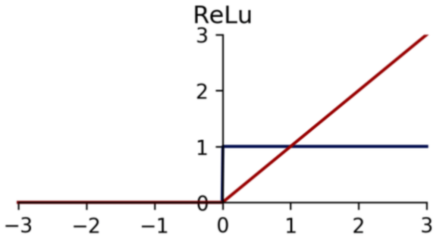
Which activation function?

Again, there is no "right" answer! However:

- For the hidden layers, almost always tanh is better than sigmoid because it center the data for the next layer.
- One downside of both sigmoid or tanh is that the gradient is very small for extreme values.

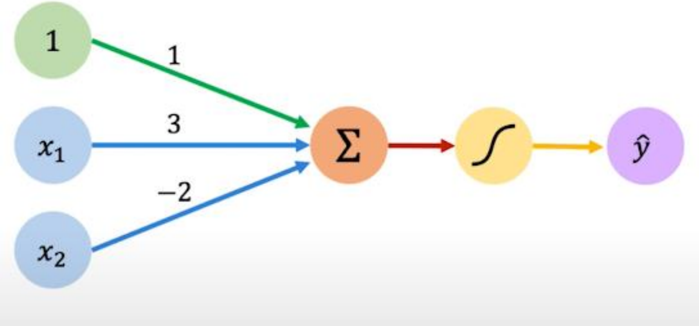






- In practice, RELU works better/faster than sigmoid or tanh for hidden layers.
- Leaky RELU might work better than RELU, but if we have enough number of hidden layers, RELU is just fine.
- Sigmoid is mostly used for output layer!

Sigmoid activation function! example

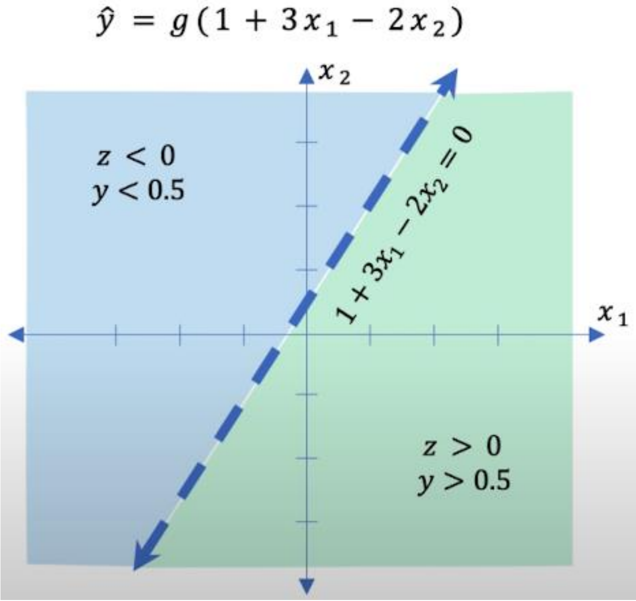


We have: $w_0 = 1$ and $\boldsymbol{W} = \begin{bmatrix}3\\-2\end{bmatrix}$

$$\hat{y} = g(w_0 + \boldsymbol{X}^T\boldsymbol{W})$$

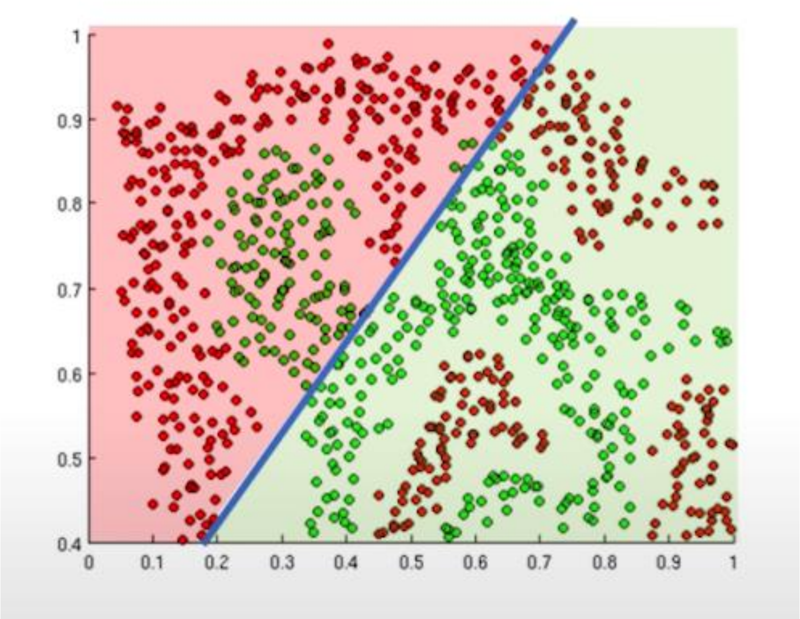
$$= g\left(1 + \begin{bmatrix}x_1\\x_2\end{bmatrix}^T \begin{bmatrix}3\\-2\end{bmatrix}\right)$$

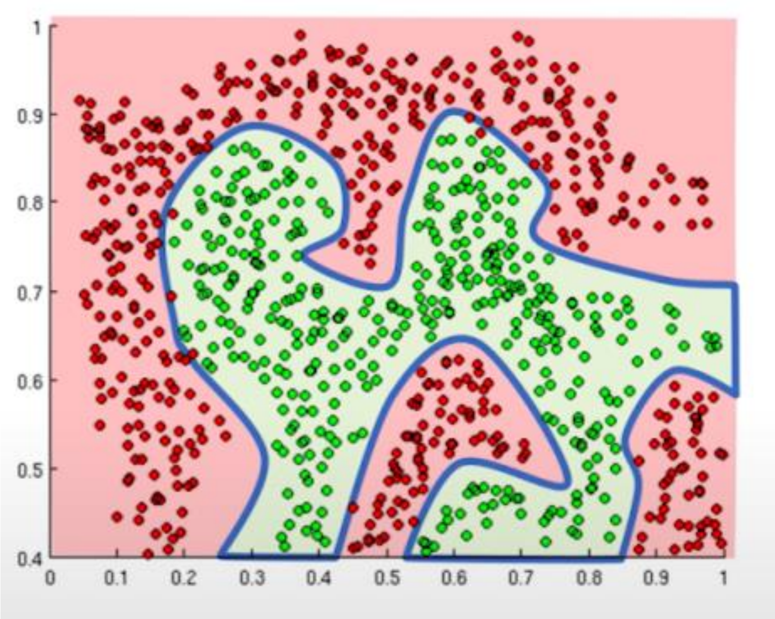
$$\hat{y} = g\underbrace{(1 + 3x_1 - 2x_2)}_{\text{This is just a line in 2D!}}$$



Why do we need non-linear activation functions?

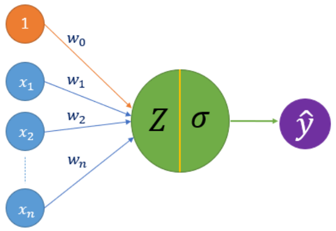
- If we use linear activation function $\sigma(z) = z = w_0 + W^T X$, then the NN will always output a linear function of the inputs regardless of the number of neurons or hidden layers used.
- Linear activation functions produce linear decision boundaries!
- Activation functions allow the network to approximate complex patterns!

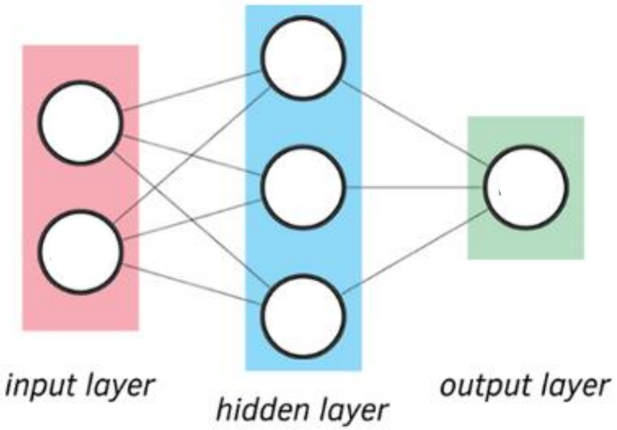




Building a Neural Network

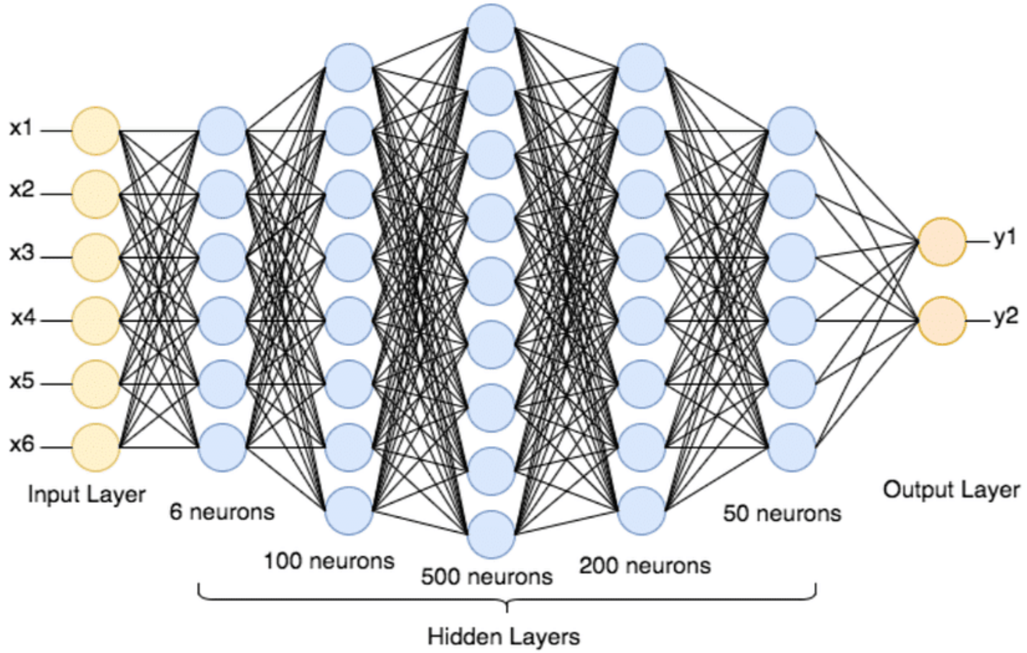
- A neural network is a type of machine learning model that is composed of layers of interconnected "neurons" which process and transmit information.
- Each neuron receives input from other neurons, processes it using a nonlinear activation function, and then transmits the output to other neurons in the next layer. The output of the final layer is the prediction made by the neural network.
- Because all the inputs are densely connected to all outputs, these layers are called Dense layers.
- The output layer can be either single output or multiple output.

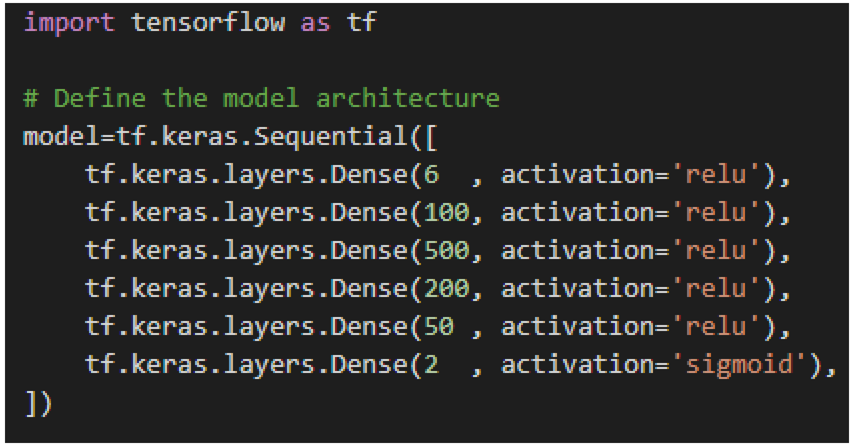




Building a Deep Neural Network

- Deep neural networks are neural networks with a large number of layers, typically consisting of multiple hidden layers.
- Deep neural networks can learn and model very complex patterns in data.
- DNNs have been successful in a wide range of applications, including image and speech recognition, natural language processing, and machine translation.





```
import tensorflow as tf

# Define the model architecture
model=tf.keras.Sequential([
    tf.keras.layers.Dense(6  , activation='relu'),
    tf.keras.layers.Dense(100, activation='relu'),
    tf.keras.layers.Dense(500, activation='relu'),
    tf.keras.layers.Dense(200, activation='relu'),
    tf.keras.layers.Dense(50 , activation='relu'),
    tf.keras.layers.Dense(2  , activation='sigmoid'),
])
```

Training a Neural Network, Backpropagation

- To train a neural network, we present it with many examples and adjust the weights and biases of the connections between neurons so that the network can accurately predict the output for each example and minimize the loss function.
- This process is known as backpropagation, and it is done using an optimization algorithm such as stochastic gradient descent.
- The loss function quantifies the distance between actuals and predictions. It provides feedback to the NN.
- Examples: MSE, Binary Cross Entropy, (Sparse) Categorical Cross Entropy, …
- For the full list, visit https://keras.io/api/losses/#available-losses

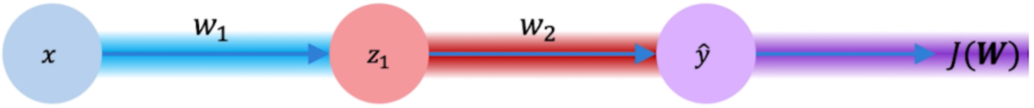
Backpropagation

- Backpropagation is a supervised learning algorithm that adjusts the weights and biases of the connections between neurons in the network to minimize the loss function.

$$\boldsymbol{W}^* = \underset{\boldsymbol{W}}{\operatorname{argmin}} \frac{1}{n}\sum_{i=1}^{n}\mathcal{L}\left(f\left(x^{(i)};\boldsymbol{W}\right), y^{(i)}\right)$$

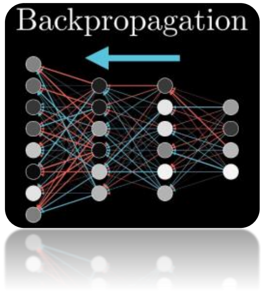
- Steps:

1) Feed the input data through the neural network to compute the output.
2) Calculate the loss or error between the predicted output and the true output.
3) Propagate the error back through the network using the chain rule of calculus to calculate the gradient of the loss function with respect to the weights and biases.
4) Update the weights and biases using the gradient descent algorithm.
5) Repeat the process until the loss reaches a satisfactory level or a predetermined iterations.



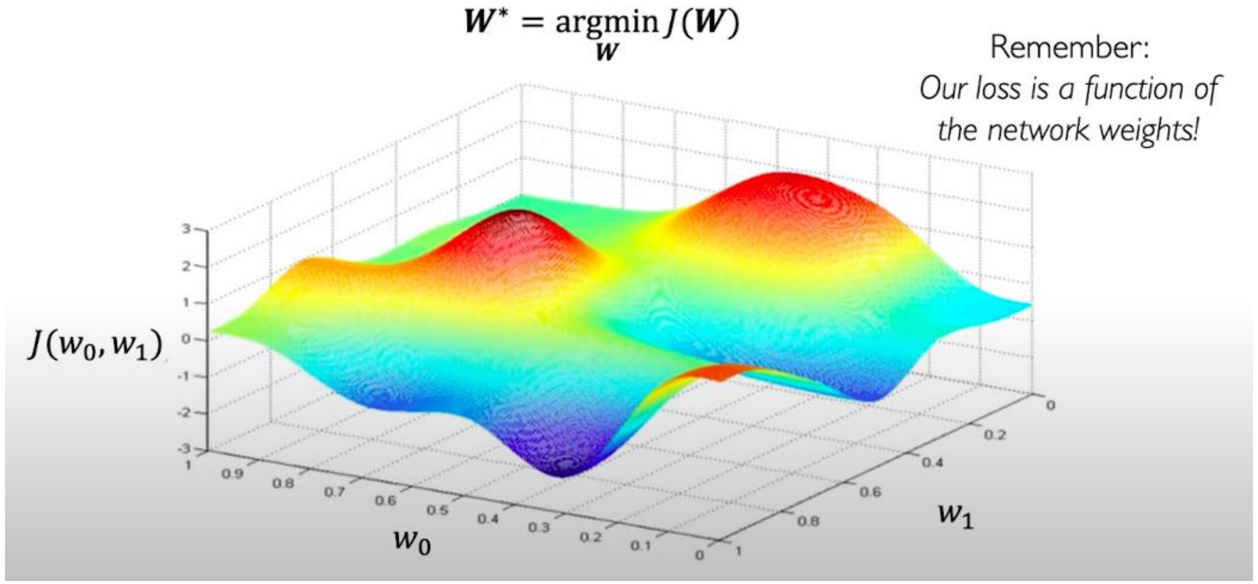
$$\frac{\partial J(\boldsymbol{W})}{\partial w_2} = \frac{\partial J(\boldsymbol{W})}{\partial \hat{y}} * \frac{\partial \hat{y}}{\partial w_2}$$

$$\frac{\partial J(\boldsymbol{W})}{\partial w_1} = \frac{\partial J(\boldsymbol{W})}{\partial \hat{y}} * \frac{\partial \hat{y}}{\partial z_1} * \frac{\partial z_1}{\partial w_1}$$



Loss optimization

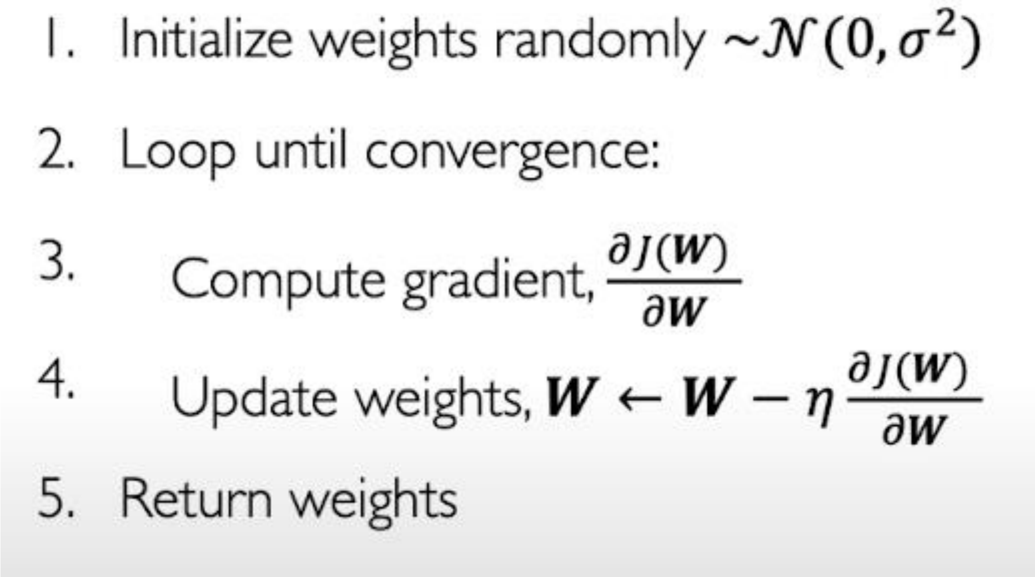
$$\boldsymbol{W}^* = \underset{\boldsymbol{W}}{\operatorname{argmin}} \frac{1}{n} \sum_{i=1}^{n} \mathcal{L}\left(f\left(x^{(i)}; \boldsymbol{W}\right), y^{(i)}\right)$$



$$\boldsymbol{W}^* = \underset{\boldsymbol{W}}{\operatorname{argmin}} J(\boldsymbol{W})$$

Remember:
Our loss is a function of the network weights!

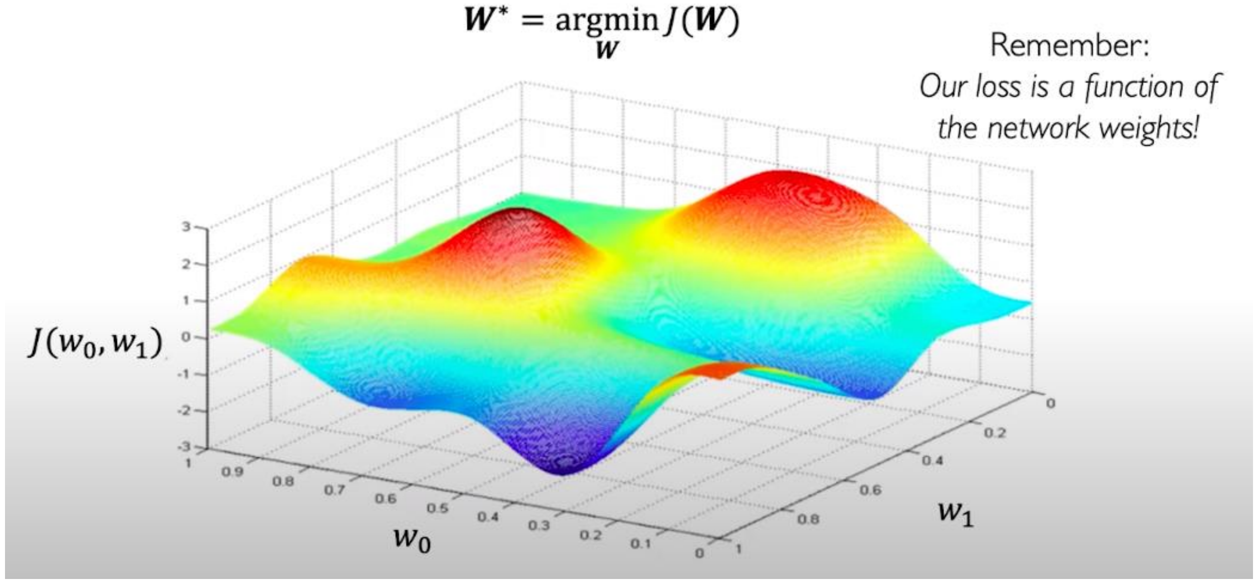
Gradient Descent



1. Initialize weights randomly $\sim\mathcal{N}(0, \sigma^2)$
2. Loop until convergence:
3. Compute gradient, $\frac{\partial J(\boldsymbol{W})}{\partial \boldsymbol{W}}$
4. Update weights, $\boldsymbol{W} \leftarrow \boldsymbol{W} - \eta \frac{\partial J(\boldsymbol{W})}{\partial \boldsymbol{W}}$
5. Return weights

Loss optimization

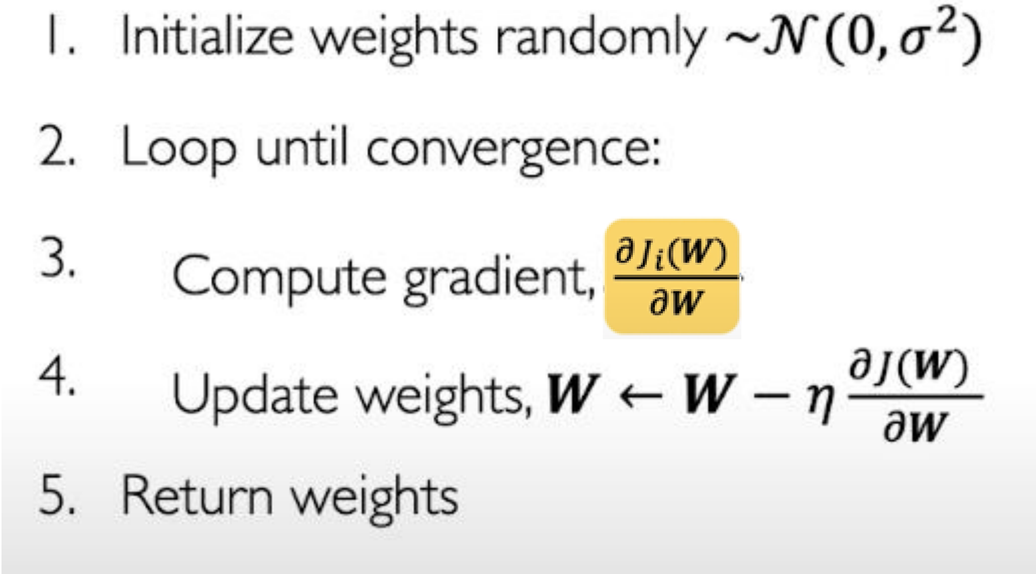
$$W^* = \underset{W}{\operatorname{argmin}} \frac{1}{n}\sum_{i=1}^{n} \mathcal{L}\left(f\left(x^{(i)};W\right), y^{(i)}\right)$$



$W^* = \underset{W}{\operatorname{argmin}}\, J(W)$

Remember:
Our loss is a function of the network weights!

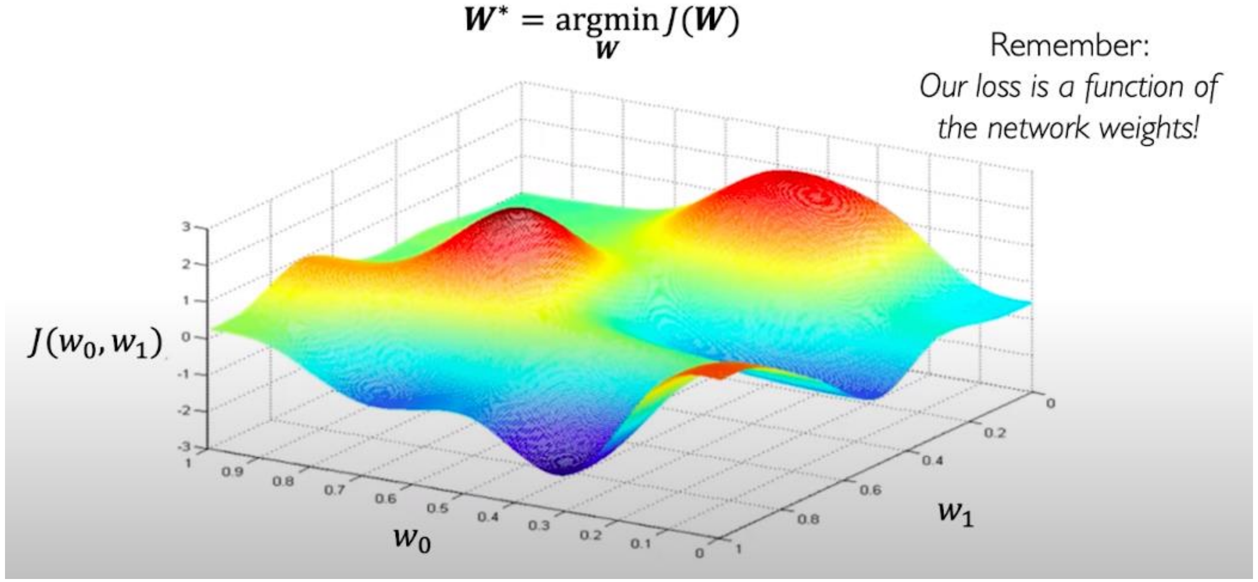
<u>Stochastic GD</u>



1. Initialize weights randomly $\sim\mathcal{N}(0, \sigma^2)$
2. Loop until convergence:
3. Compute gradient, $\frac{\partial J_i(W)}{\partial W}$
4. Update weights, $W \leftarrow W - \eta \frac{\partial J(W)}{\partial W}$
5. Return weights

Loss optimization

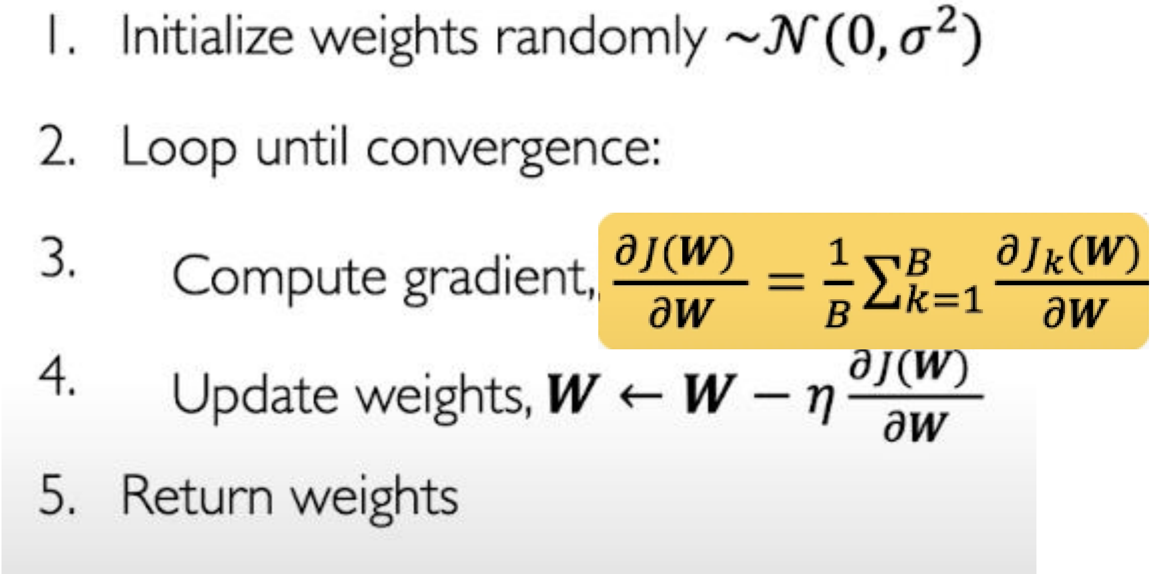
$$W^* = \underset{W}{\operatorname{argmin}} \frac{1}{n}\sum_{i=1}^{n} \mathcal{L}\left(f\left(x^{(i)}; W\right), y^{(i)}\right)$$



$$W^* = \underset{W}{\operatorname{argmin}}\, J(W)$$

Remember:
Our loss is a function of the network weights!

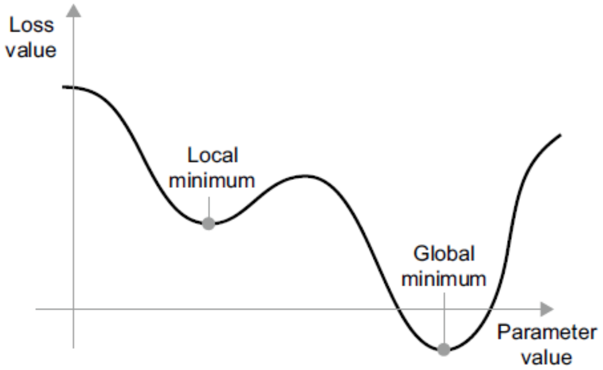
Mini-batch GD

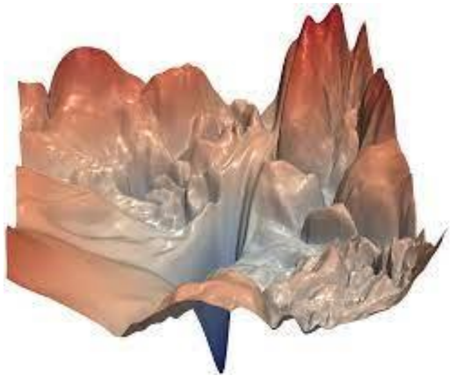


```
1. Initialize weights randomly $\sim\mathcal{N}(0, \sigma^2)$
2. Loop until convergence:
3.     Compute gradient, $\frac{\partial J(W)}{\partial W} = \frac{1}{B}\sum_{k=1}^{B} \frac{\partial J_k(W)}{\partial W}$
4.     Update weights, $W \leftarrow W - \eta \frac{\partial J(W)}{\partial W}$
5. Return weights
```

DNN loss functions can be difficult to optimize!

- *Visualizing the loss landscape of neural nets*, Li et all, 2018



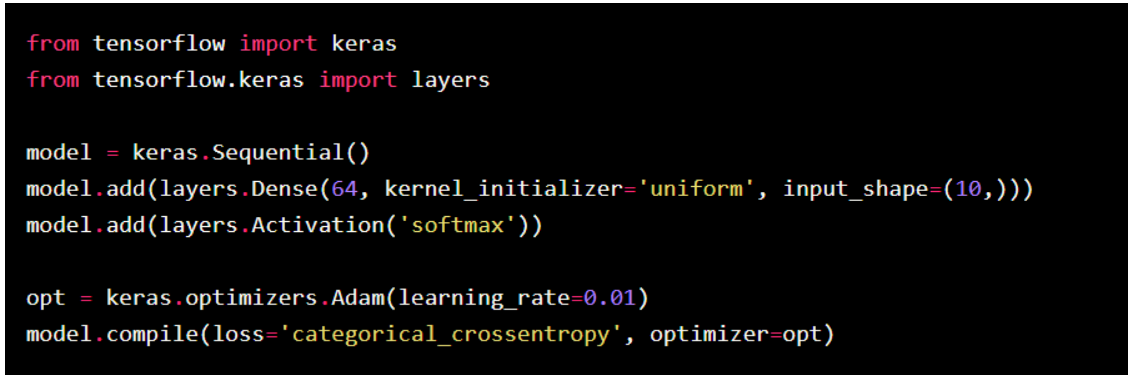


- Solution: Designing an adaptive learning rate that can adapt to the loss landscape. Rather than just looking at the current gradient, take into account the previous weight updates. This is called, momentum!
- Examples: Adam, Adadelta, Adagrad, RMSProp!

Gradient Descent Algorithms

- Available optimizers in Keras:
  - SGD
  - RMSprop
  - Adam
  - AdamW
  - Adadelta
  - Adagrad
  - Adamax
  - Adafactor
  - Nadam
  - Ftrl

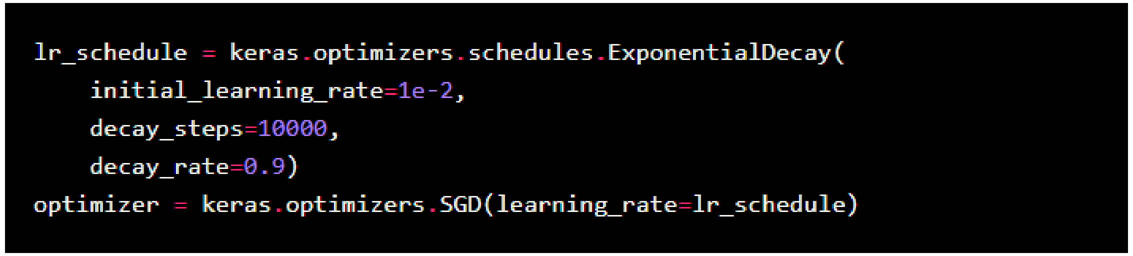
Keras



```python
from tensorflow import keras
from tensorflow.keras import layers

model = keras.Sequential()
model.add(layers.Dense(64, kernel_initializer='uniform', input_shape=(10,)))
model.add(layers.Activation('softmax'))

opt = keras.optimizers.Adam(learning_rate=0.01)
model.compile(loss='categorical_crossentropy', optimizer=opt)
```



```python
lr_schedule = keras.optimizers.schedules.ExponentialDecay(
    initial_learning_rate=1e-2,
    decay_steps=10000,
    decay_rate=0.9)
optimizer = keras.optimizers.SGD(learning_rate=lr_schedule)
```

- Which optimizer? There is no “right” answer.

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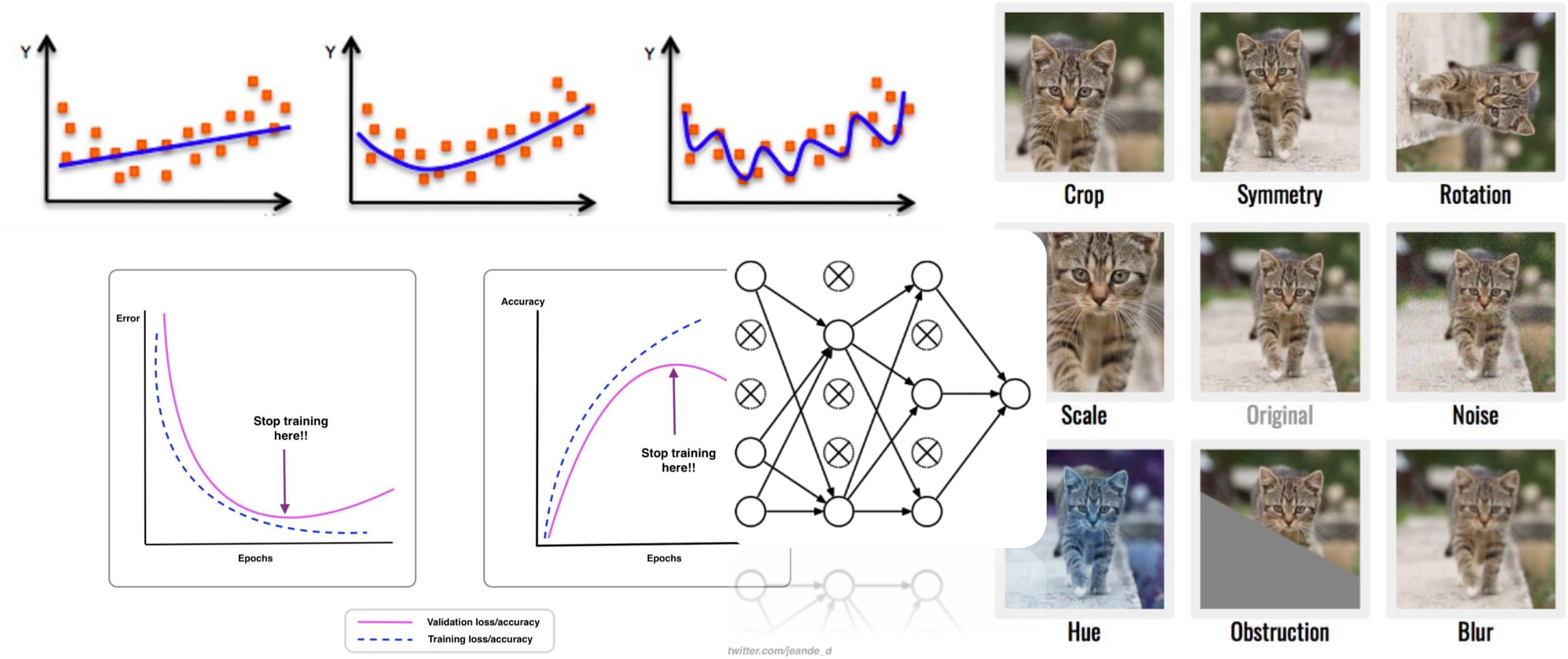
https://keras.io/api/optimizers/

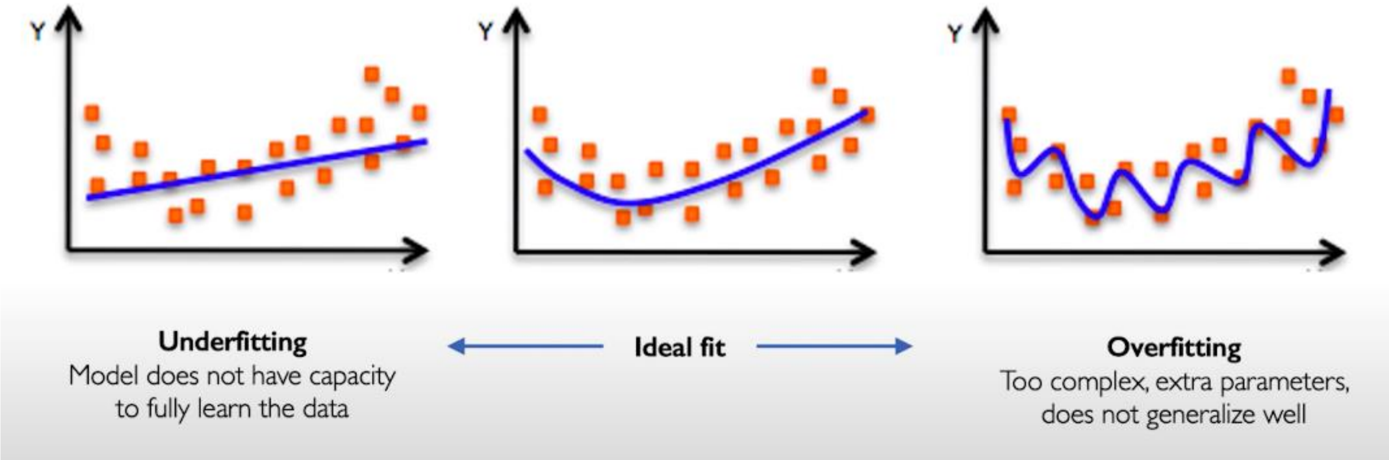
Module 4 - Part II

Deep Neural Networks

Regularization



How to handle overfitting in DNN?


Y
Y
Y
Underfitting
Model does not have capacity to fully learn the data
Ideal fit
Overfitting
Too complex, extra parameters, does not generalize well

- Regularization refers to a set of techniques that are used to prevent overfitting by discouraging complex models.
- Regularization improve generalization of our model on unseen data.
- Methods include, L1, L2, drop out, Early stopping and Data augmentation.

Deep Learning Regularization techniques

L1 regularization: This technique adds a penalty term to the objective function that is proportional to the absolute value of the model weights. This results in a sparse model, with many weights being set to zero.

L2 regularization: This technique adds a penalty term to the objective function that is proportional to the square of the model weights. This results in a model with small, non-zero weights.

Dropout: This technique randomly sets a fraction of the model weights to zero during training, which helps to prevent overfitting

Early Stopping: This technique involves training the model until the performance on a validation set begins to degrade, and then stopping the training at that point. This helps to prevent the model from continuing to fit the training data too closely.

Data Augmentation: This technique involves generating additional training examples by applying random transformations to the existing training data. This can help to prevent overfitting by providing the model with a more diverse training set.

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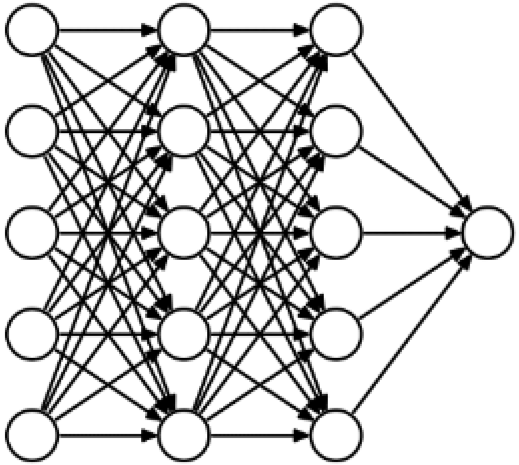
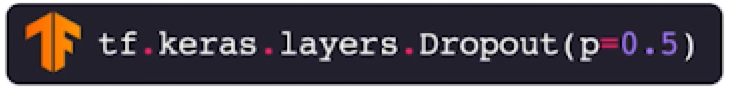
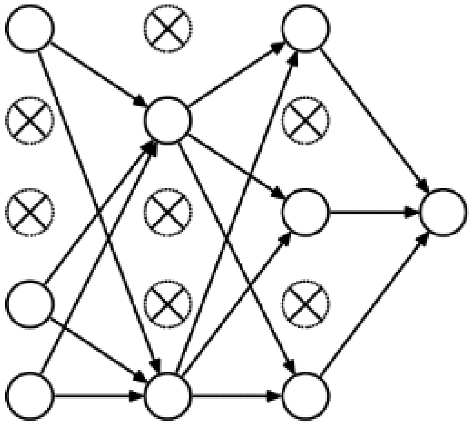
Dropout Regularization

- Dropout is one of the most popular regularization techniques in deep learning.
- At each training iteration, dropout randomly chooses different nodes to ignore
- This prevents the network from relying on any single neuron.



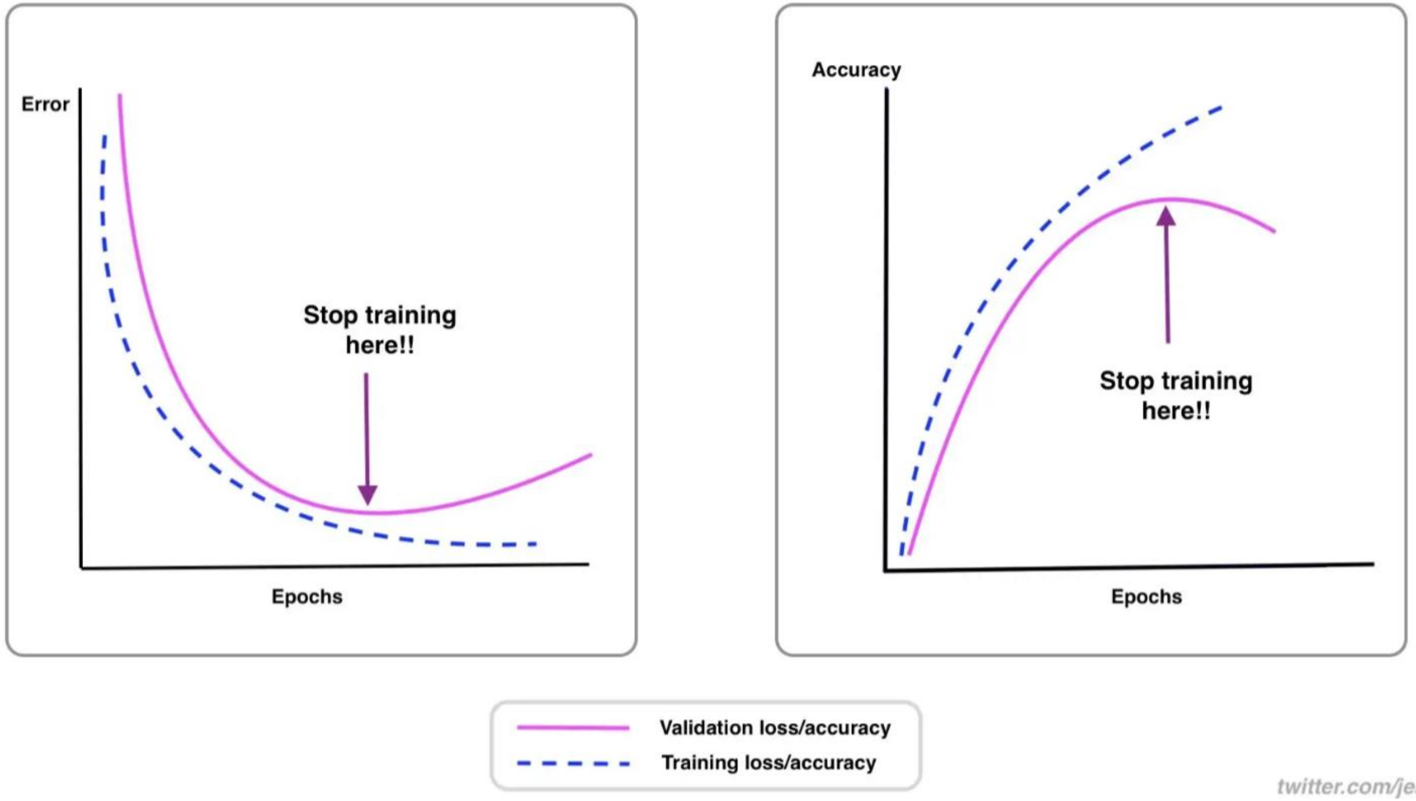


```
tf.keras.layers.Dropout(p=0.5)
```



Early Stopping

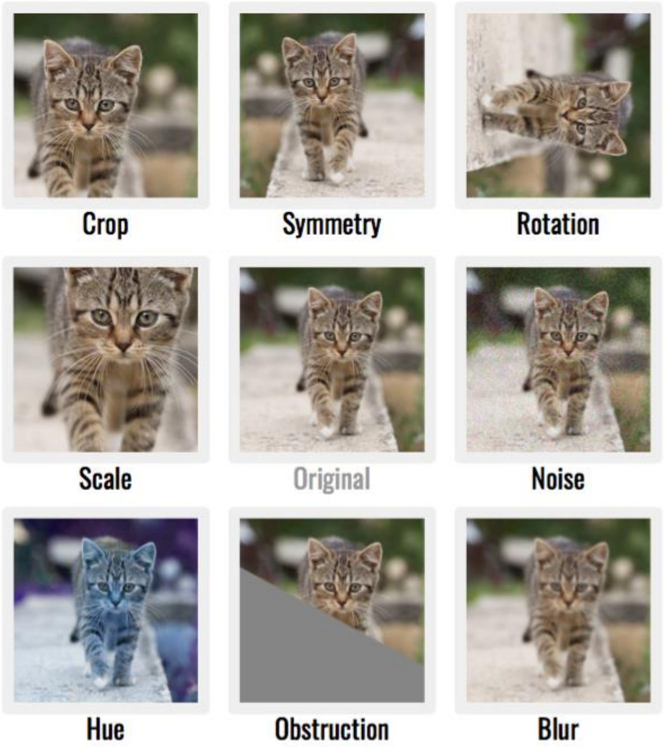
- Stop training when the performance on a validation set begins to degrade!



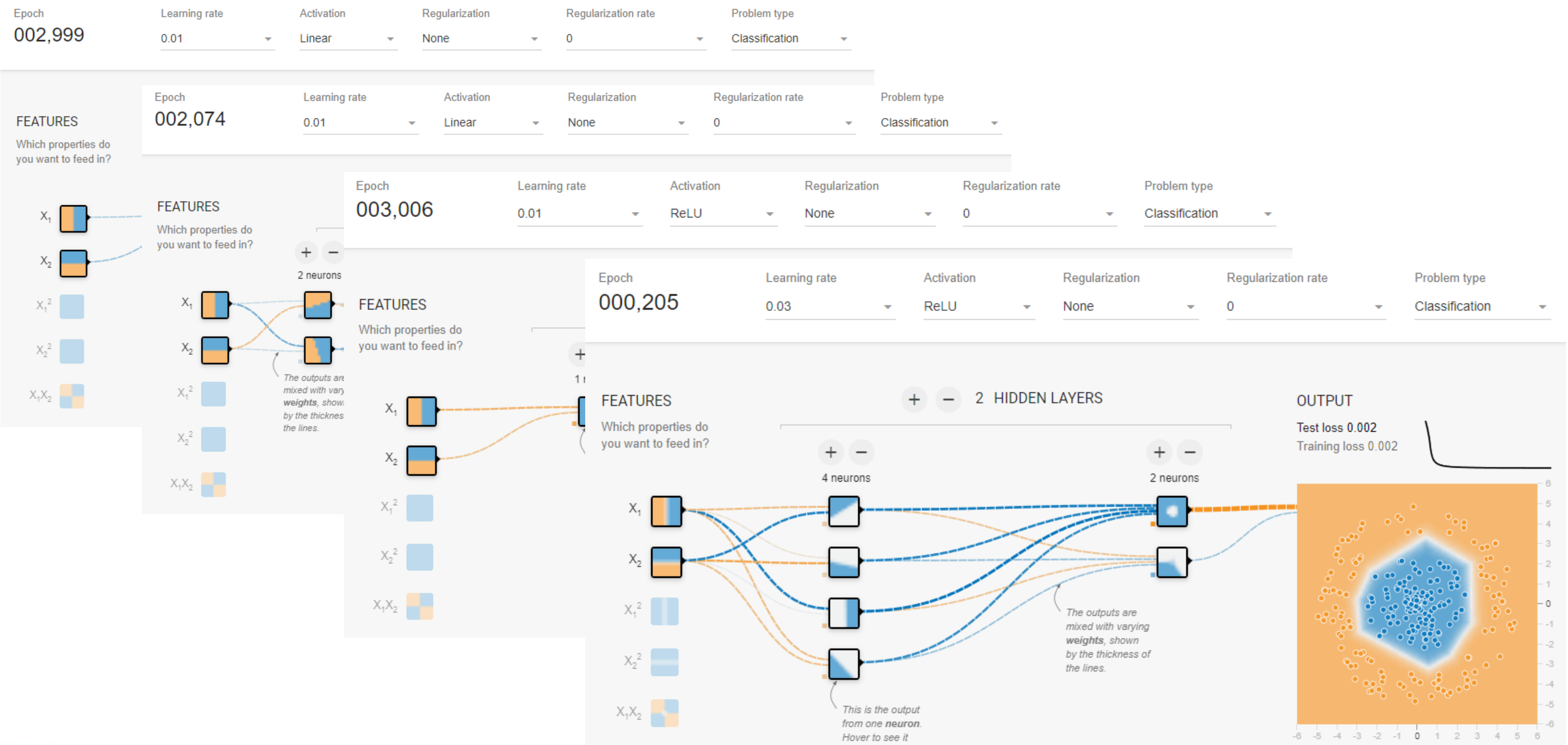
twitter.com/jeande_d

Data Augmentation

- Generating additional training examples by applying random transformations to the existing training data
- The goal of data augmentation is to improve the generalizability and robustness of machine learning models by providing them with more diverse training data.



Neural Network Playground (TensorFlow)



Epoch 002,999 | Learning rate 0.01 | Activation Linear | Regularization None | Regularization rate 0 | Problem type Classification

Epoch 002,074 | Learning rate 0.01 | Activation Linear | Regularization None | Regularization rate 0 | Problem type Classification

Epoch 003,006 | Learning rate 0.01 | Activation ReLU | Regularization None | Regularization rate 0 | Problem type Classification

Epoch 000,205 | Learning rate 0.03 | Activation ReLU | Regularization None | Regularization rate 0 | Problem type Classification

FEATURES

Which properties do you want to feed in?

2 HIDDEN LAYERS

OUTPUT

Test loss 0.002

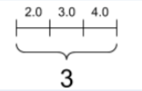
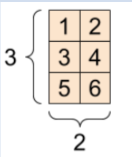
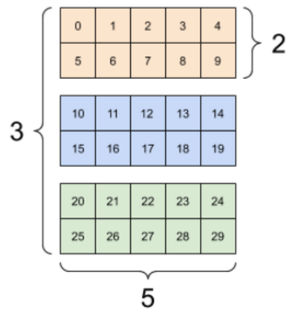
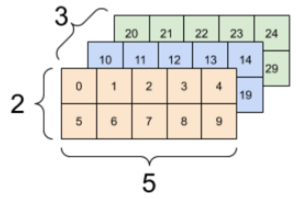
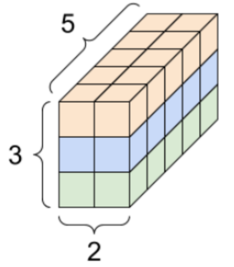
Training loss 0.002

The outputs are mixed with varying **weights**, shown by the thickness of the lines.

This is the output from one **neuron**. Hover to see it

What is a Tensor?

- Tensor: A multi-dimensional array (kind of like np.arrays)
- Rank: Number of tensor axes , Size: The total number of items in the tensor

Tensor	tf code	shape	Example
Scalar (Rank-0 tensor)	tf.constant(4)	Shape = ()	4
Vector (Rank-1 tensor)	tf.constant([2.0, 3.0, 4.0])	Shape = (3 ,)	
Matrix (Rank-2 tensor)	tf.constant([[1, 2],[3, 4],[5, 6]])	Shape = (3 , 2)	
Rank-3 tensor	tf.constant([[[0, 1, 2, 3, 4], [5, 6, 7, 8, 9]], [[10, 11, 12, 13, 14], [15, 16, 17, 18, 19]], [[20, 21, 22, 23, 24], [25, 26, 27, 28, 29]],])	Shape = (3, 2, 5)	 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 2 3 5  3 2 5 20 21 22 23 24 10 11 12 13 14 0 1 2 3 4 5 6 7 8 9 19 29 

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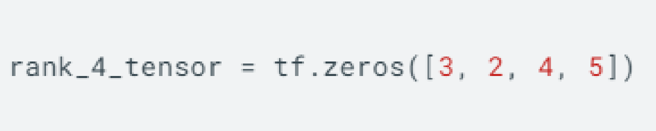
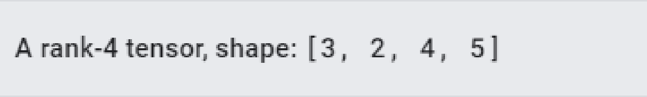
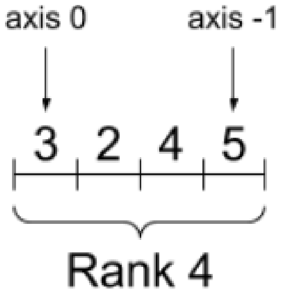
Higher dimension Tensors

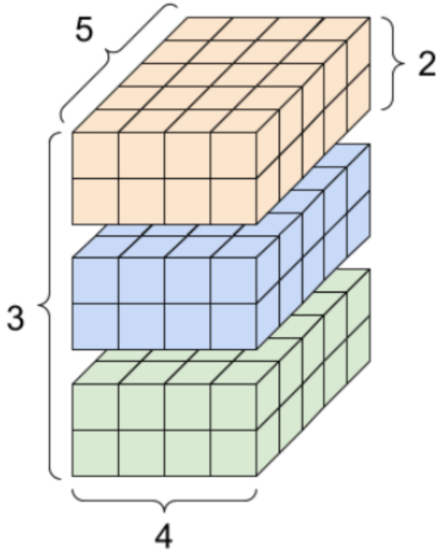


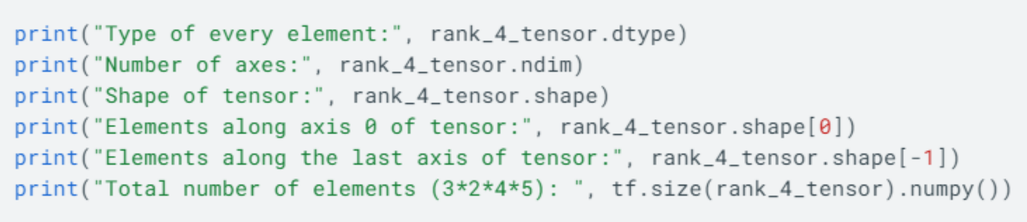
```python
rank_4_tensor = tf.zeros([3, 2, 4, 5])
```



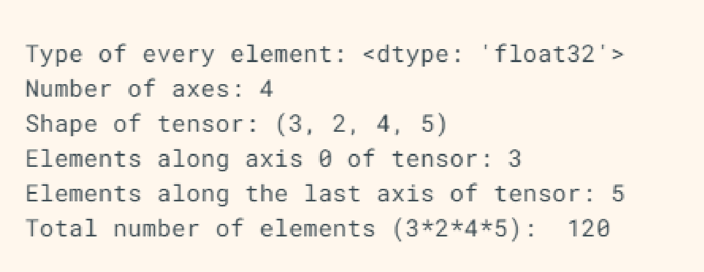
A rank-4 tensor, shape: [3, 2, 4, 5]





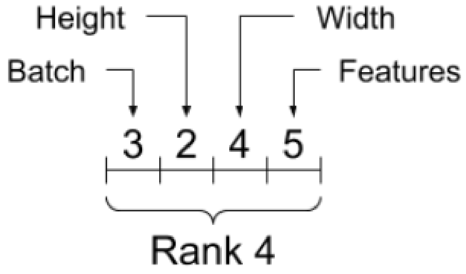


```python
print("Type of every element:", rank_4_tensor.dtype)
print("Number of axes:", rank_4_tensor.ndim)
print("Shape of tensor:", rank_4_tensor.shape)
print("Elements along axis 0 of tensor:", rank_4_tensor.shape[0])
print("Elements along the last axis of tensor:", rank_4_tensor.shape[-1])
print("Total number of elements (3*2*4*5): ", tf.size(rank_4_tensor).numpy())
```



```
Type of every element: <dtype: 'float32'>
Number of axes: 4
Shape of tensor: (3, 2, 4, 5)
Elements along axis 0 of tensor: 3
Elements along the last axis of tensor: 5
Total number of elements (3*2*4*5):  120
```

- Often axes are ordered from global to local.
- The first axes is called the batch axis or batch dimension.



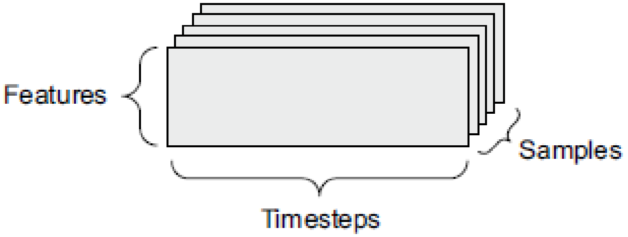
https://www.tensorflow.org/guide/tensor

Real-world examples of data tensors

- The data we’ll manipulate almost always fall into one of the following categories:

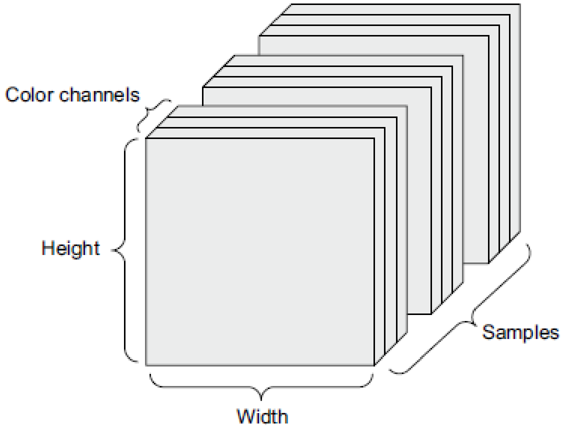
✓ Tabular data: Rank-2 tensors of shape (samples, features), where each sample is a vector of numerical attributes (“features”)

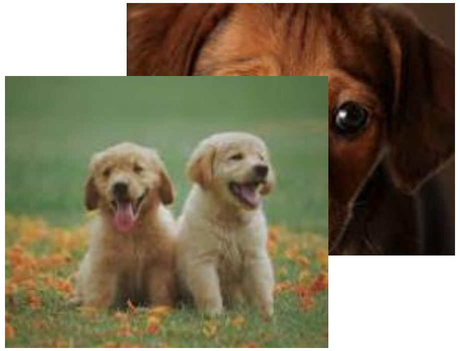
✓ Timeseries data or sequence data: Rank-3 tensors of shape (samples, timesteps, features), where each sample is a sequence (of length timesteps) of feature vectors



Real-world examples of data tensors

- ✓ Images: Rank-4 tensors of shape (samples, height, width, channels), where each sample is a 2D grid of pixels, and each pixel is represented by a vector of values ("channels")



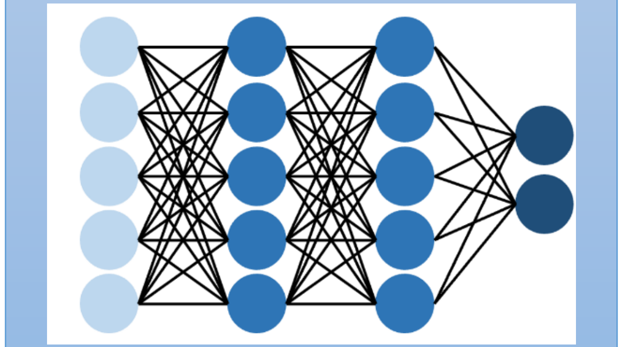


- ✓ Video: Rank-5 tensors of shape (samples, frames, height, width, channels), where each sample is a sequence (of length frames) of images.

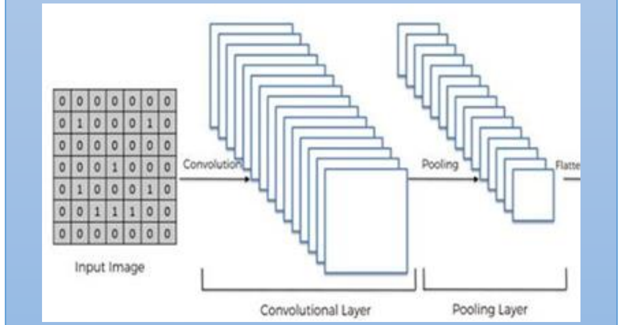
Example: What is the tensor shape for a 60-second, 144*256 video clip sampled at 4 frames per second?

Types of Neural Networks

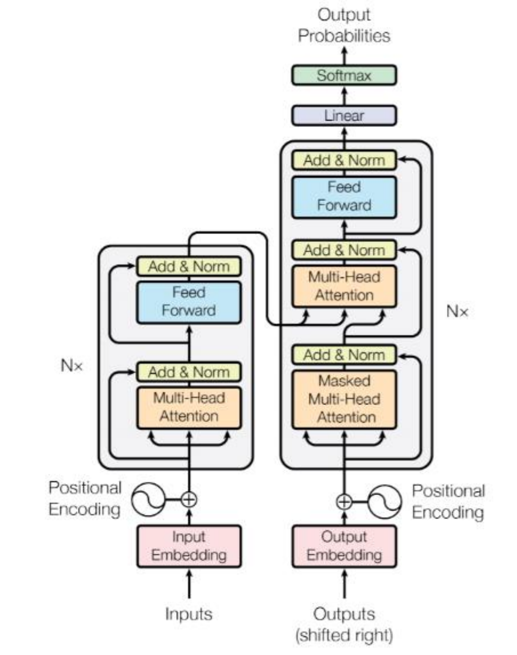
Standard NN



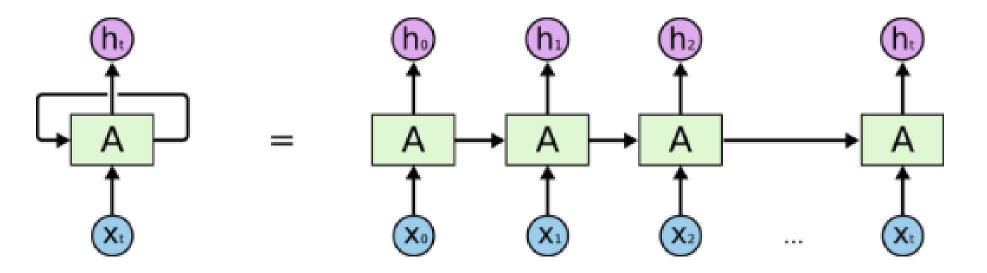
Convolutional NN



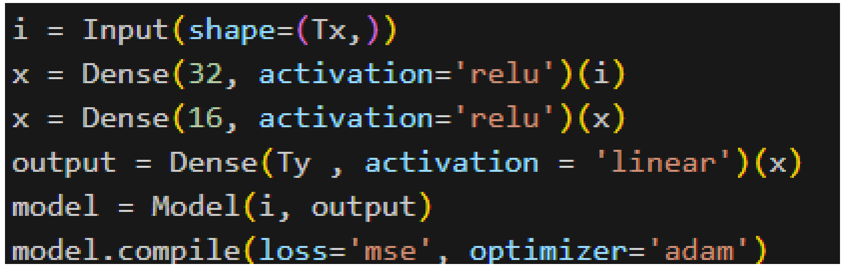
Transformers



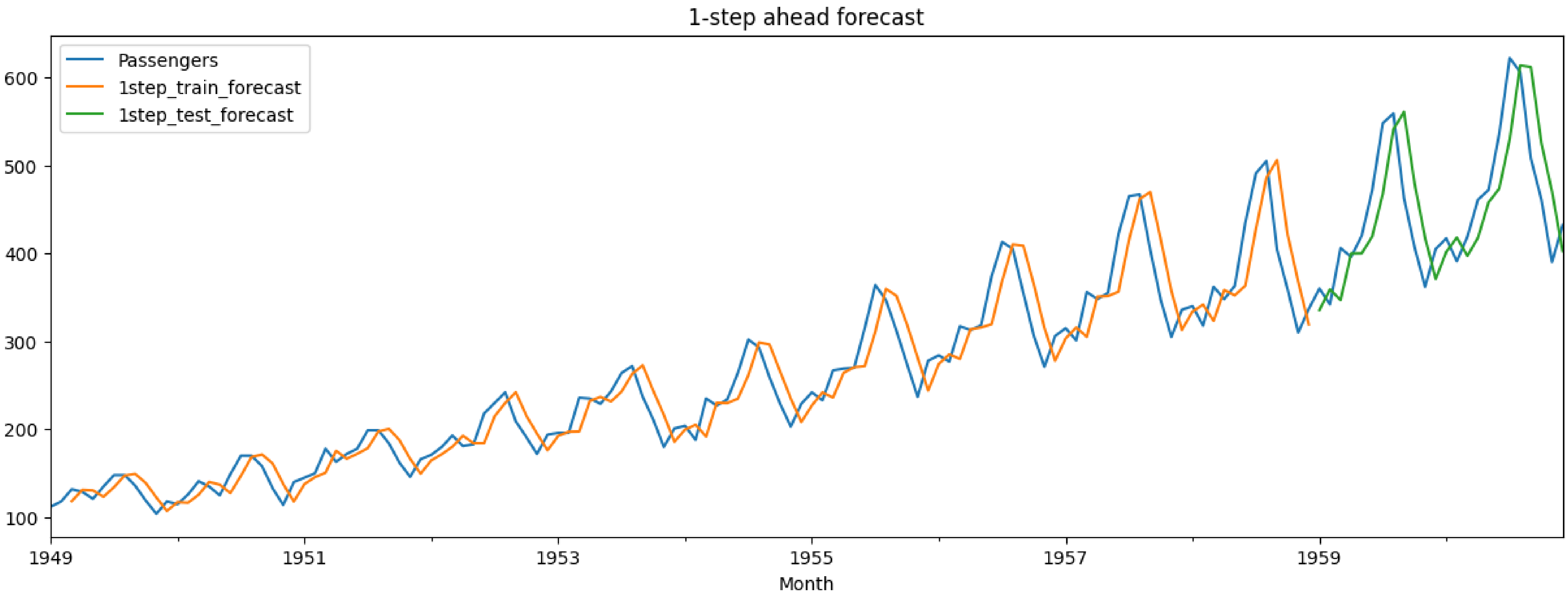
Recurrent NN



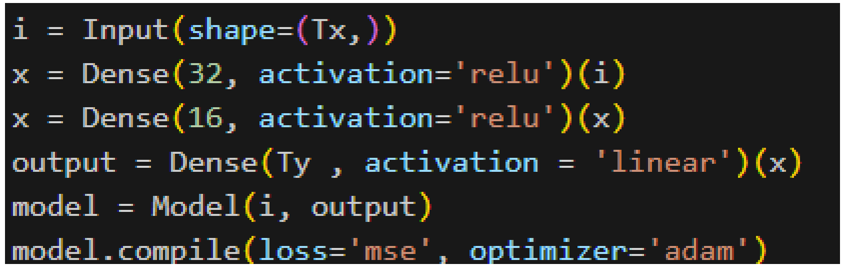
DNN using 2 lags



```
i = Input(shape=(Tx,))
x = Dense(32, activation='relu')(i)
x = Dense(16, activation='relu')(x)
output = Dense(Ty , activation = 'linear')(x)
model = Model(i, output)
model.compile(loss='mse', optimizer='adam')
```



DNN using 2 lags



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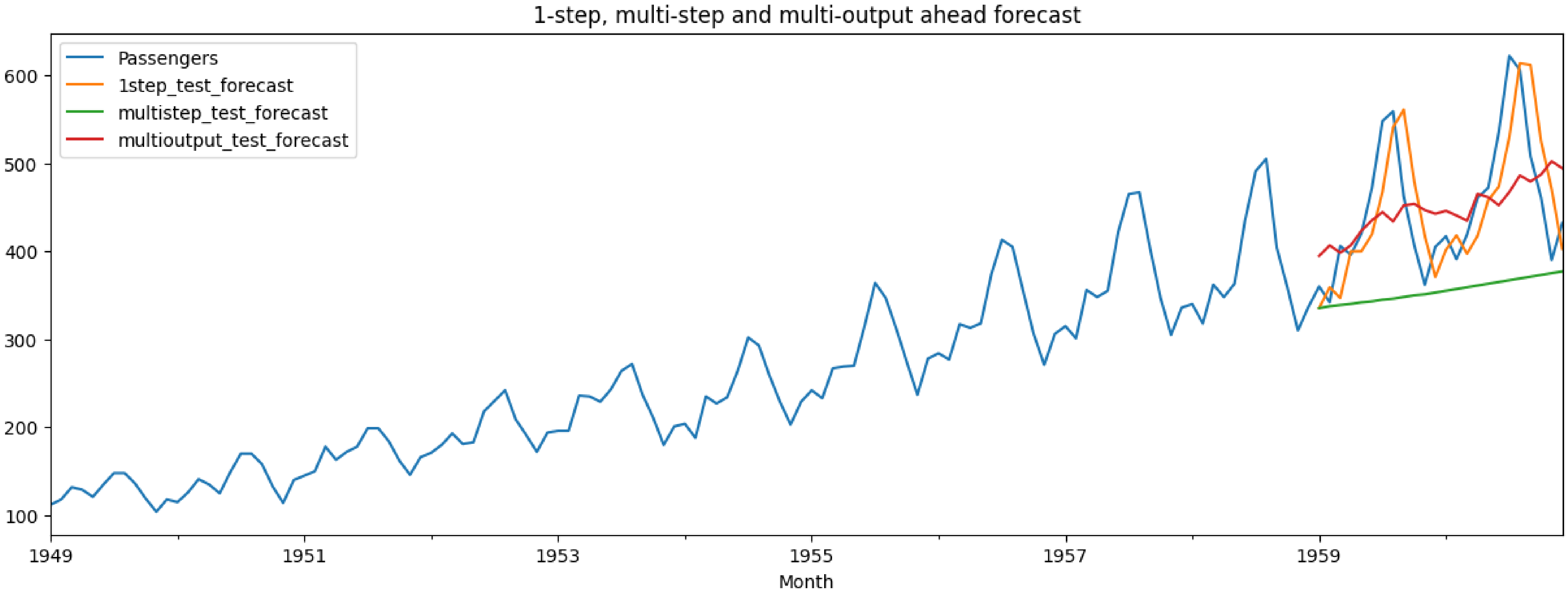
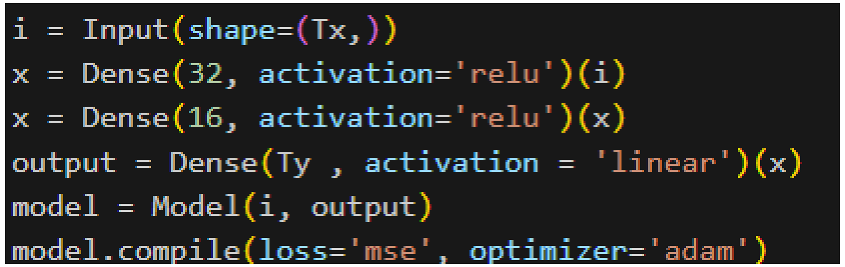


1-step, multi-step and multi-output ahead forecast

DNN using 12 lags



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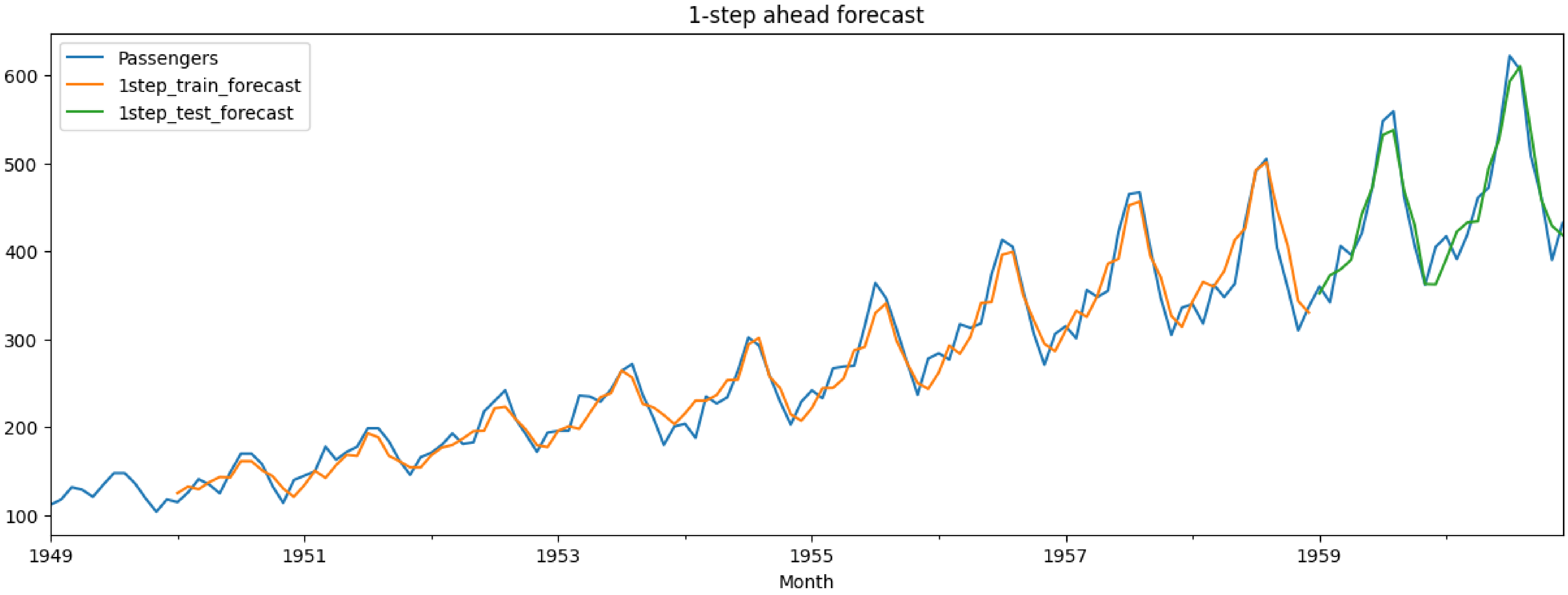
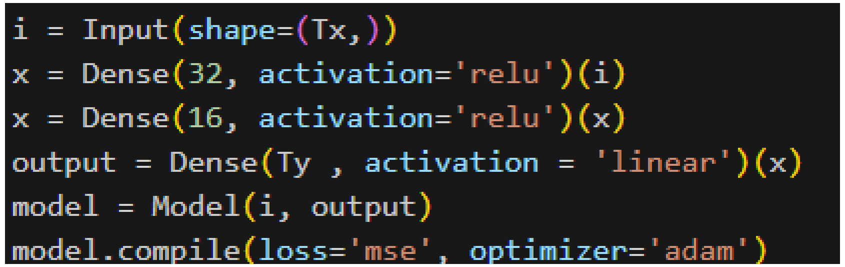
model = Model(i, output)
model.compile(loss='mse', optimizer='adam')
```



DNN using 12 lags



```
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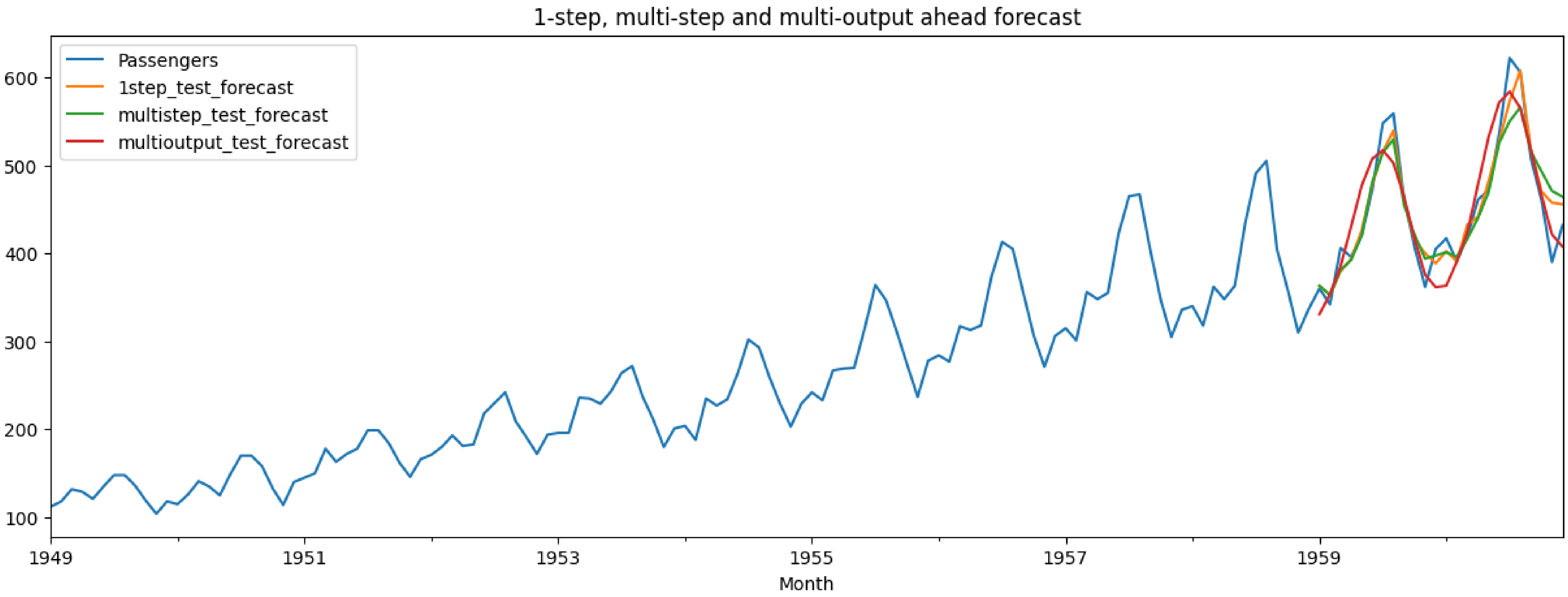
x = Dense(16, activation='relu')(x)
output = Dense(Ty , activation = 'linear')(x)
model = Model(i, output)
model.compile(loss='mse', optimizer='adam')
```



Road map!

- ✓ Module 1- Introduction to Deep Forecasting
- ✓ Module 2- Setting up Deep Forecasting Environment
- ✓ Module 3- Exponential Smoothing
- ✓ Module 4- ARIMA models
- ✓ Module 5- Machine Learning for Time series Forecasting
- ✓ Module 6- Deep Neural Networks
- Module 7- Deep Sequence Modeling (RNN, LSTM)
- Module 8- Prophet and Neural Prophet

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