UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS 126: Probability and Random Processes

Discussion 1
Spring 2021

1. Miscellaneous Review

(a) Show that the probability that exactly one of the events A and B occurs is $\Pr(A) + \Pr(B) - 2\Pr(A \cap B)$.

(b) If A is independent of itself, show that $\Pr(A) = 0$ or 1.

2. Balls & Bins

Let $n \in \mathbb{Z}_{>1}$ (i.e. n is an integer greater than 1). You throw n balls, one after the other, into n bins, so that each ball lands in one of the bins uniformly at random.

(a) What is an appropriate sample space to model this scenario?

(b) What is the probability that "ball i falls in bin i, for each $i = 1, \ldots, n$".

3. Colored Sphere

Consider a sphere that has $\frac{1}{10}$ of its surface colored blue, and the rest is colored red. Show that, no matter how the colors are distributed, it is possible to inscribe a cube in the sphere with all of its vertices red.

Hint: Carefully define some relevant events.

4. [Extra] The Countable Union Bound

Let $A_1, A_2, \ldots$ be a countable sequence of events. Prove that the union bound holds for countably many events:

$$\Pr\left(\bigcup_{i=1}^{\infty} A_i\right) \leq \sum_{i=1}^{\infty} \Pr(A_i).$$