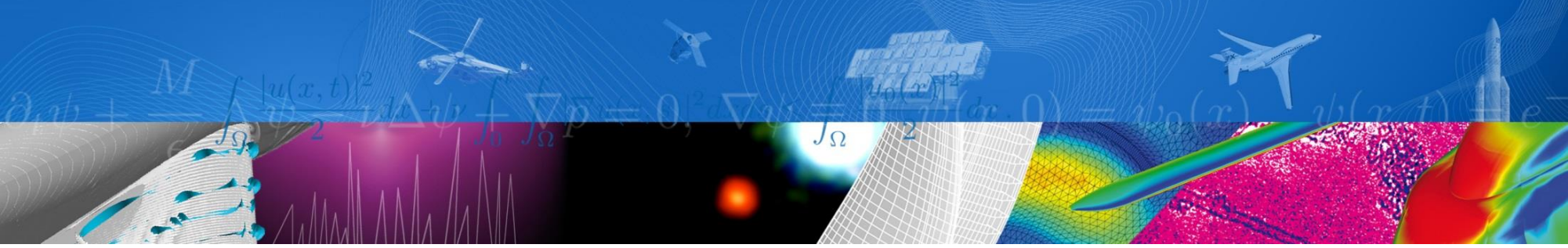

Filtrage Particulaire appliqué à la navigation

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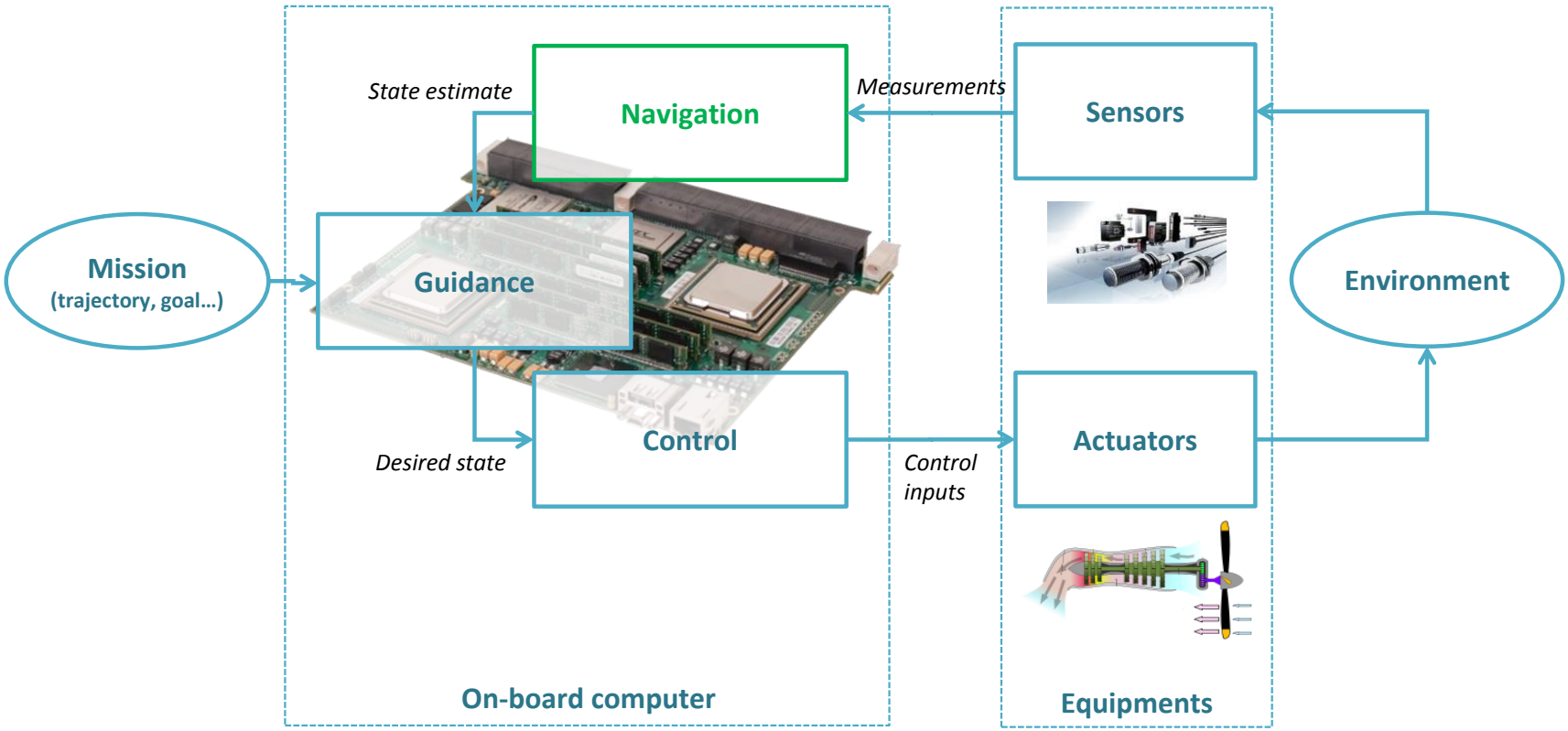
THE FRENCH AEROSPACE LAB

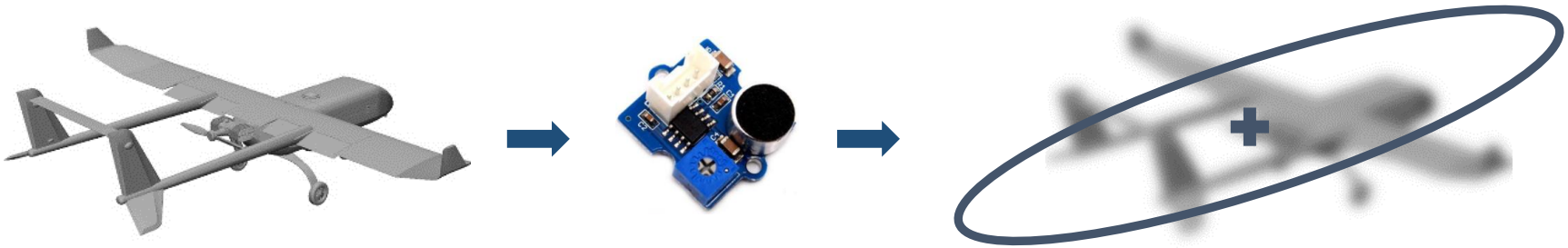
ENSTA
IP PARIS

Autonomous systems need to perform accurate localization

Guidance, Navigation and Control (GNC)



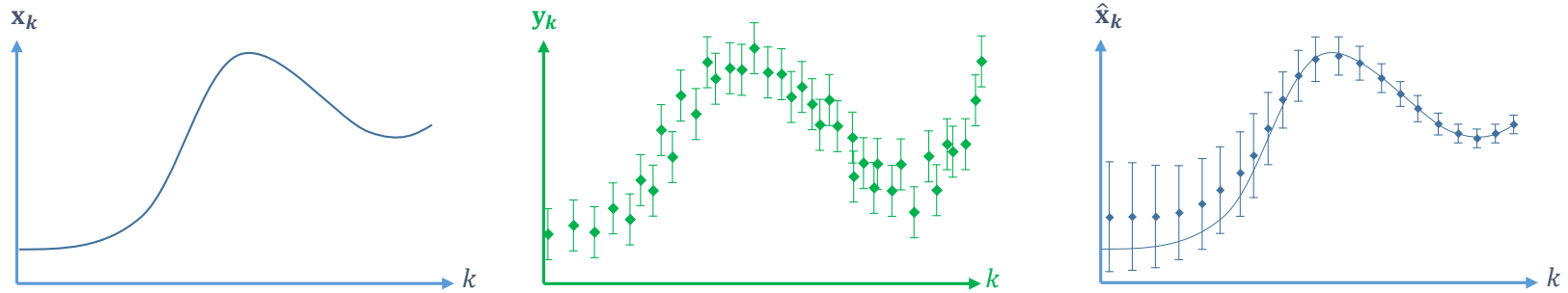
State estimation for navigation



$\mathbf{x_k}$ = position, velocity...

$\mathbf{y_k}$ = measurements

$\hat{\mathbf{x}}_\mathbf{k}$ = estimated position, velocity...



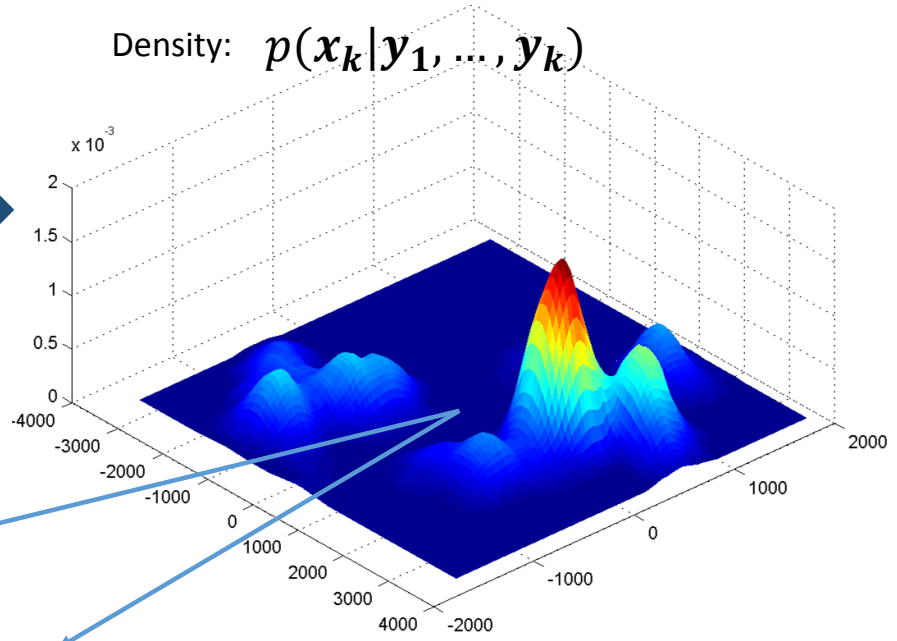
State estimation consists in retrieving the vehicle's state from noisy and incomplete measurements and uncertain evolution model.

Different ways to model the state density

$\boldsymbol{x_k}$ = position, velocity...

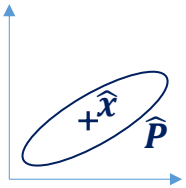
$\boldsymbol{y_k}$ = measurement

Density: $p(\boldsymbol{x_k}|\boldsymbol{y_1}, \dots, \boldsymbol{y_k})$



Kalman Filters

Particle Filters

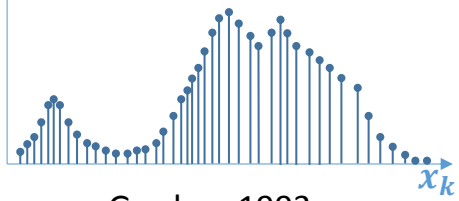


Gaussian approximation

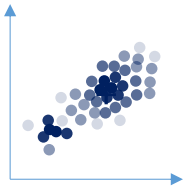
Kalman, 1960

Diracs mixture

$\hat{p}(\boldsymbol{x_k}|\boldsymbol{Y_k}) = \sum_i w_k^i \delta_{x_k^i}$



Gordon, 1993



What we know

Theoretical state evolution (dynamical model):

$$\begin{aligned} \dot{\mathbf{x}} &= F(\mathbf{x}, \mathbf{u}) \\ \mathbf{x}_k &= f(\mathbf{x}_{k-1}, \mathbf{u}_k) \end{aligned} \tag{1}$$

Theoretical observation equation (sensor model):

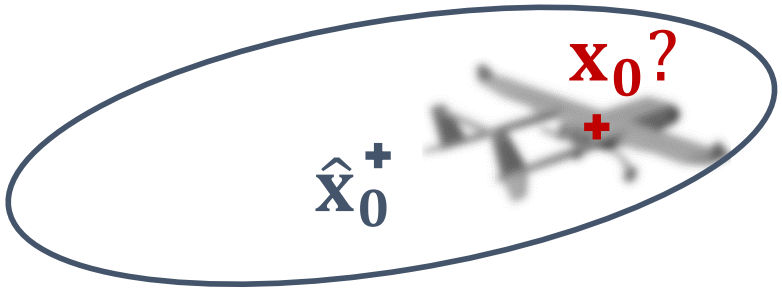
$$\mathbf{y}_k = h(\mathbf{x}_k) \tag{2}$$

However, these equations are not totally representative of the actual system, e.g.:

- unexpected wind, friction, unmodeled dynamics...
- sensor noise, unmodeled disturbances...

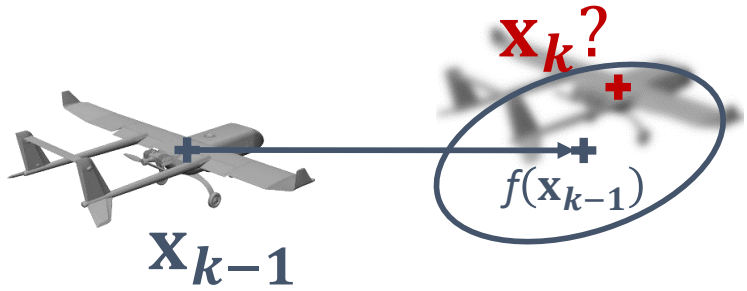
What we **don't** know (uncertainties)

- Initial state uncertainty



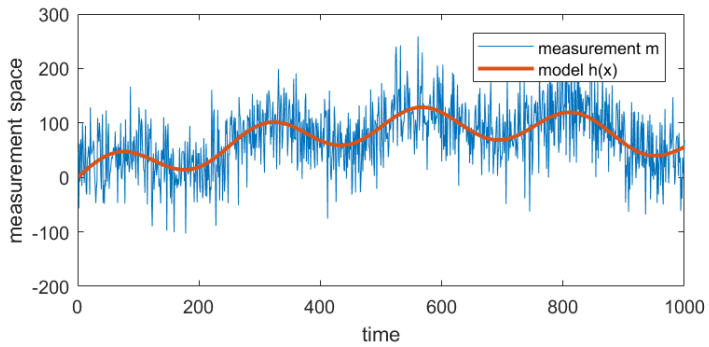
- Process noise (dynamics)

$$\mathbf{x}_k = f(\mathbf{x}_{k-1}, \mathbf{u}_k) + \mathbf{w}_k$$



- Measurement noise (and potentially some bias)

$$\mathbf{y}_k = h(\mathbf{x}_k) + \mathbf{v}_k$$



A way to model uncertainties: probability distribution functions

- State distribution:

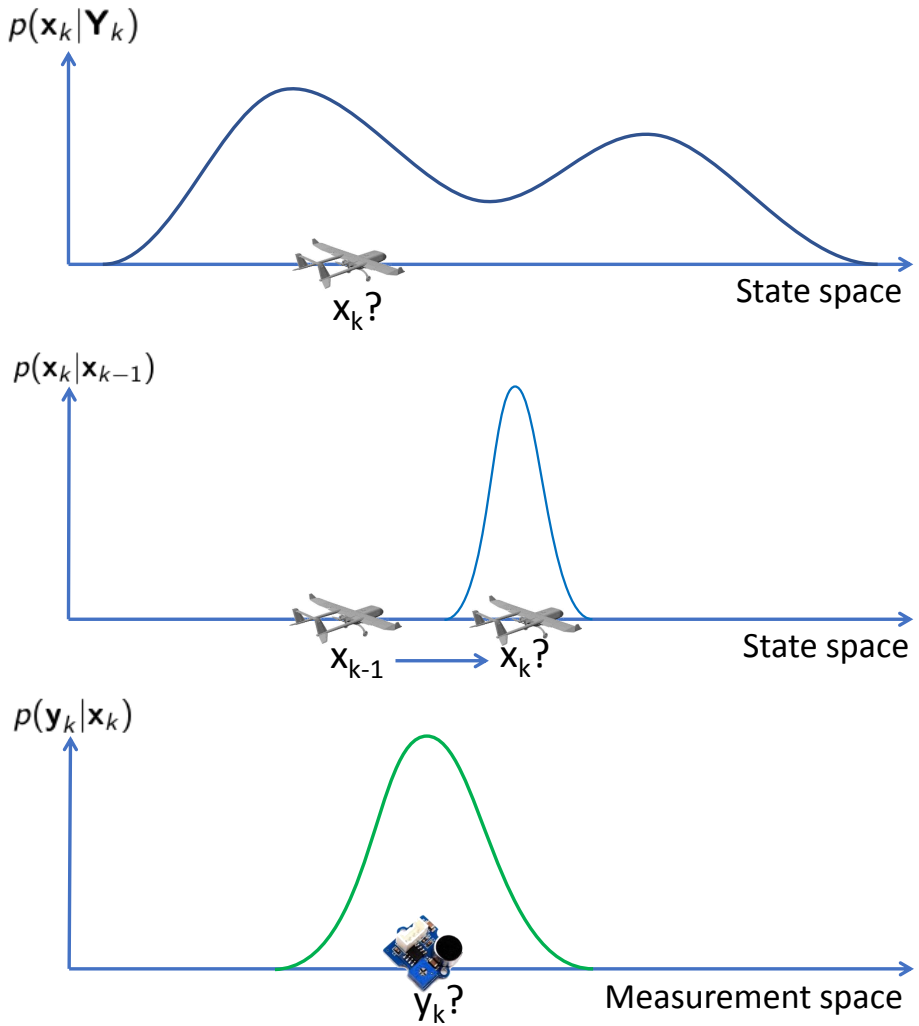
$$p(\mathbf{x}_k|\mathbf{y}_1,\ldots,\mathbf{y}_k) \triangleq p(\mathbf{x}_k|\mathbf{Y}_k)$$

- Process noise distribution

$$p(\mathbf{x}_k|\mathbf{x}_{k-1})$$

- Measurement distribution

$$p(\mathbf{y}_k|\mathbf{x}_k)$$



Optimal filter equations (Bayesian filtering)

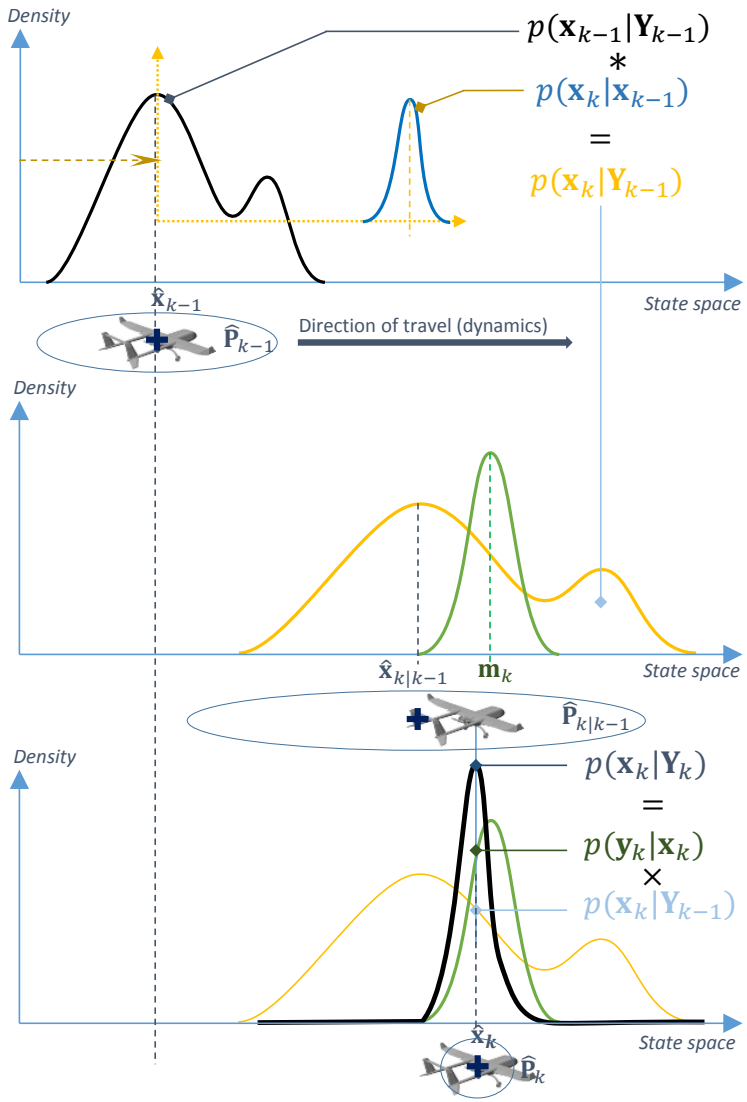
State density propagation (dynamics, Chapman-Kolmogorov equation):

$$p(\mathbf{x}_k|\mathbf{Y}_{k-1}) = \int p(\mathbf{x}_k|\mathbf{x}_{k-1})p(\mathbf{x}_{k-1}|\mathbf{Y}_{k-1})\mathrm{d}\mathbf{x}_{k-1} \tag{3}$$

State density correction/update (measurements, Bayes rule):

$$p(\mathbf{x}_k|\mathbf{Y}_k) \propto p(\mathbf{y}_k|\mathbf{x}_k)p(\mathbf{x}_k|\mathbf{Y}_{k-1}) \tag{4}$$

Optimal filter equations (Bayesian filtering)



(a) Prediction

Convolution of the **prior conditional density** with the state transition density. The transition density accounts for the deterministic dynamics f_k and its uncertainty (process noise $\mathbf{w}_k$).

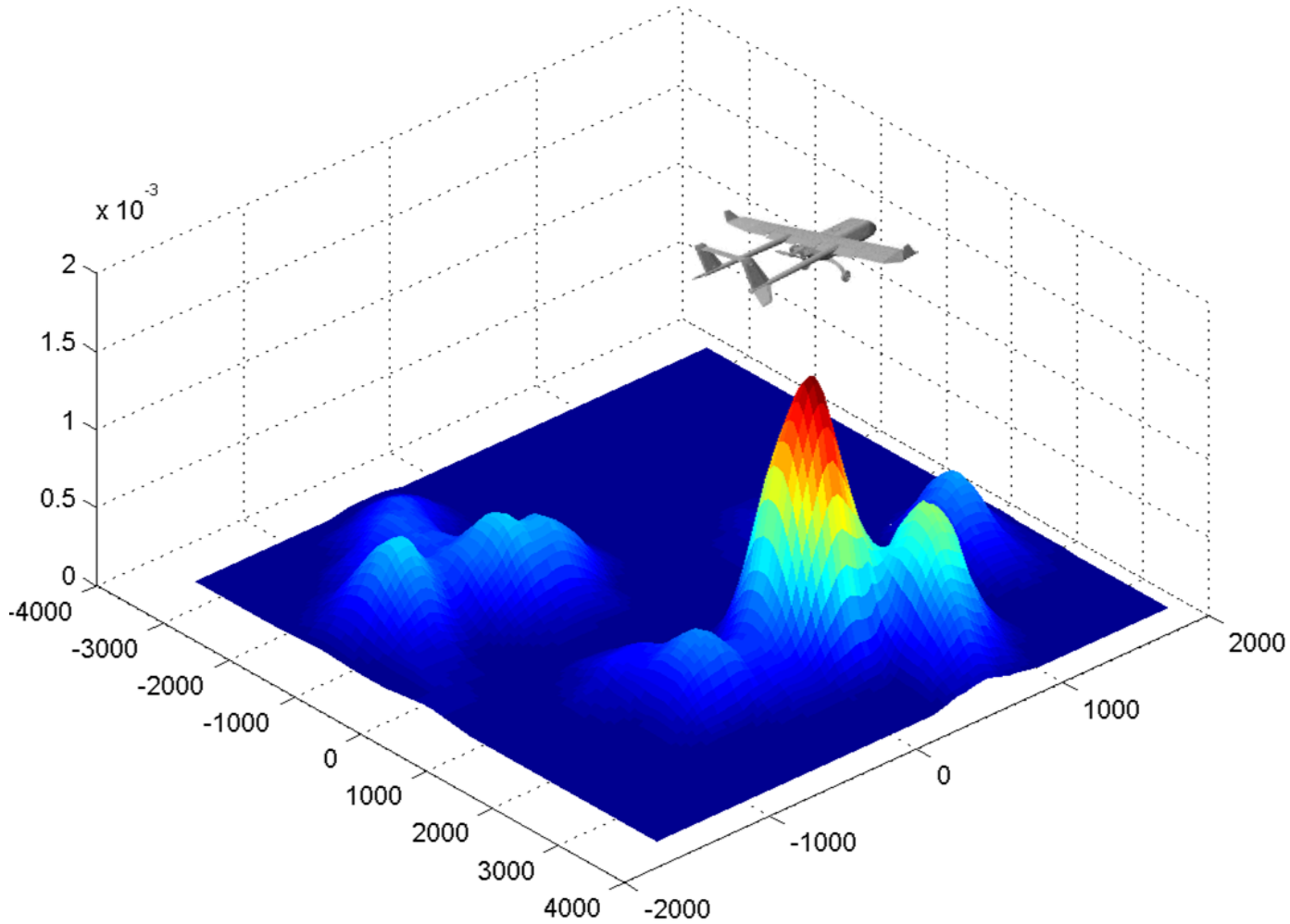
(b) Predicted density, new measurement

Step (a) results in the **predicted conditional density**, whose support is usually larger than the prior density.
A measurement $\mathbf{y}_k$ is now available. It will introduce information in the estimation.

(c) Correction

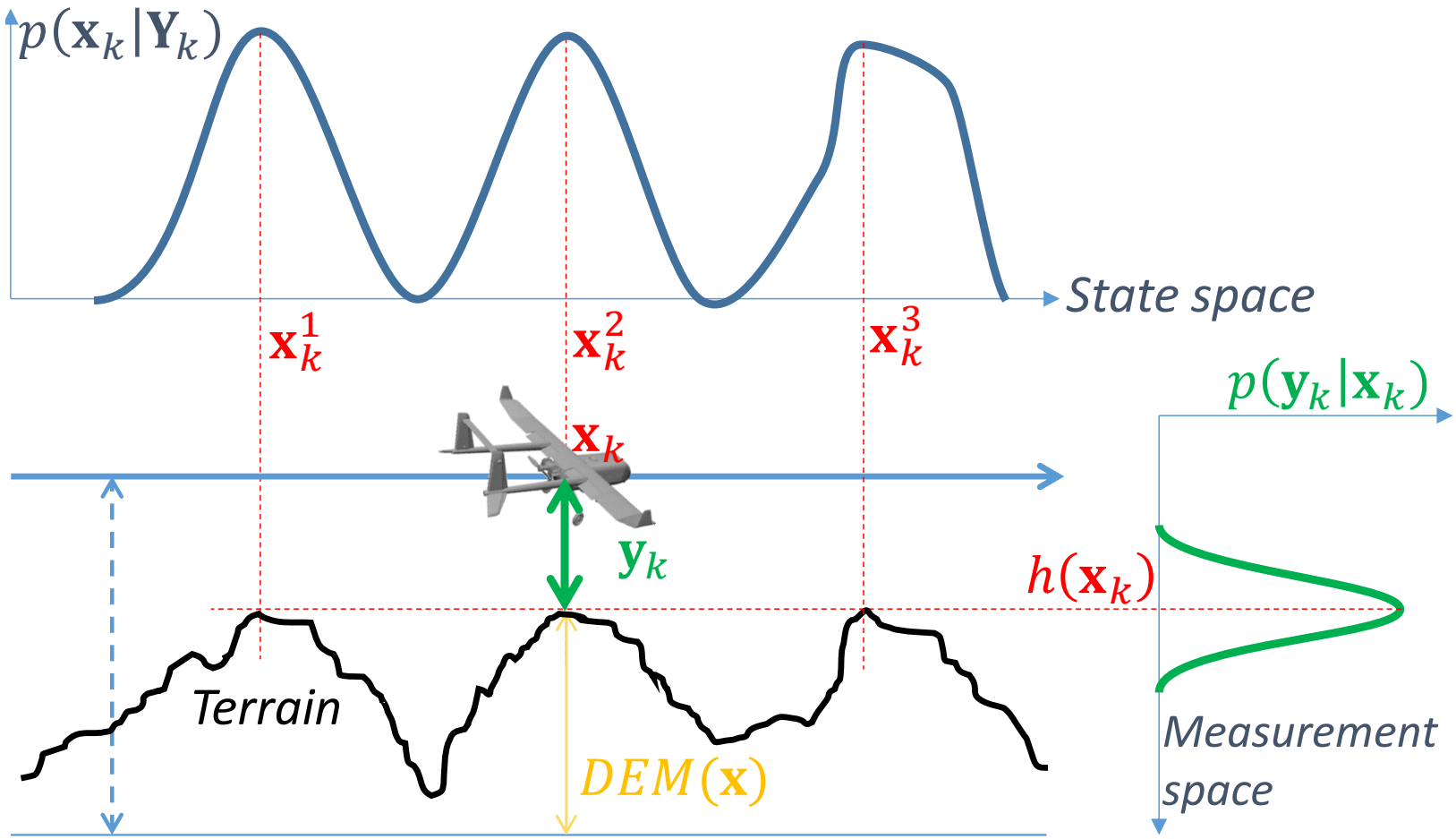
The predicted density is multiplied with the measurement density, leading to the **posterior conditional density**.
Its support is usually smaller than the predicted density. It yields a refined estimate $\hat{\mathbf{x}}_k$ and covariance $\hat{\mathbf{P}}_k$.
Then, one can iterate back to step (a) for further time-steps.

Multimodal state density (example: Terrain Aided Navigation)



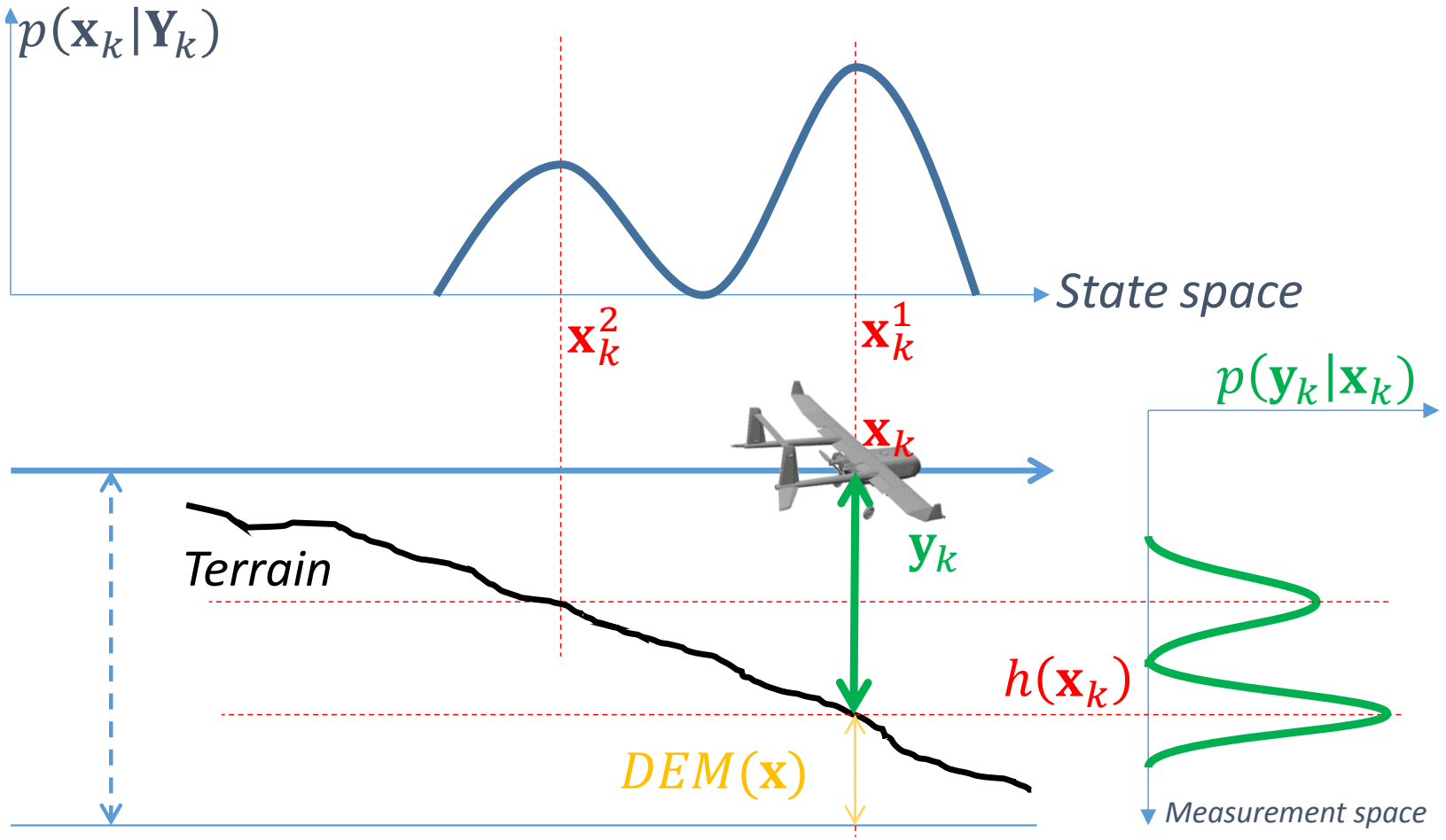
State density $\boldsymbol{p}(\mathbf{x}_k|\mathbf{Y}_k)$

Multimodal state density (example: Terrain Aided Navigation)



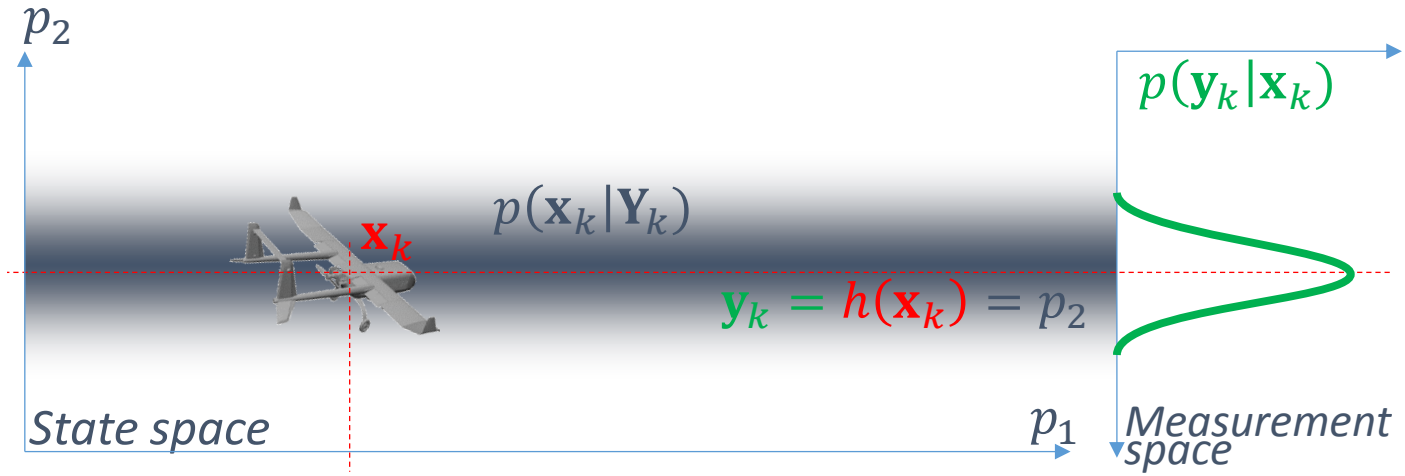
Non injective observation equation (e.g. terrain elevation) may yield several admissible states (peaks in posterior state density).

Multimodal likelihood



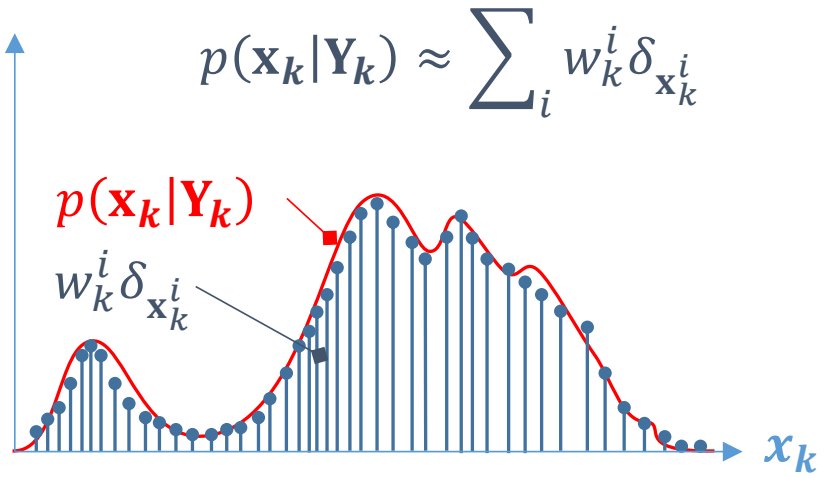
Multimodal measurement noise (likelihood density) yields several admissible states.

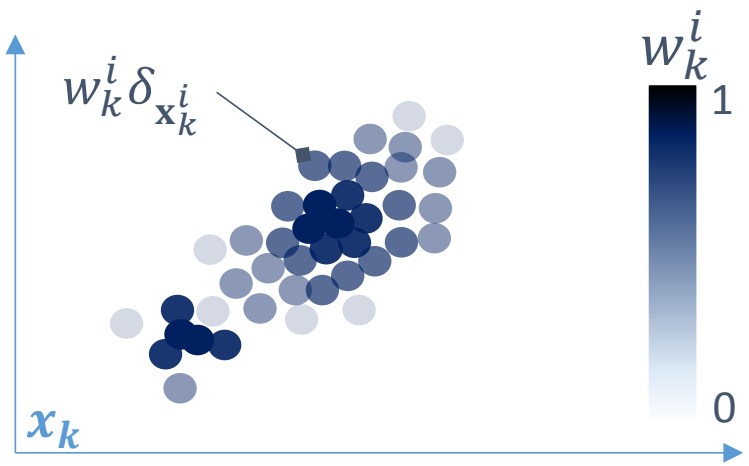
Incomplete measurements



Incomplete measurements (e.g. state variable p_2 not observable) leads to an infinity of admissible states.

State density approximation





$$p(\mathbf{x}_k|\mathbf{Y}_k) \approx \sum_{i=1}^{N} w_k^i \delta(\mathbf{x}_k - \mathbf{x}_k^i) \quad (5)$$

$\mathbf{x}_k$: actual state
$\mathbf{x}_k^i$: particle i state
w_k^i : particle i weight $(\sum_i w_k^i = 1)$

Particle Filter (theory): prediction and correction

State density propagation (dynamics, Chapman-Kolmogorov equation):

$$\begin{array}{ccc} p(\mathbf{x}_k|\mathbf{Y}_{k-1}) & = \int & p(\mathbf{x}_k|\mathbf{x}_{k-1})p(\mathbf{x}_{k-1}|\mathbf{Y}_{k-1})\mathrm{d}\mathbf{x}_{k-1} \\ \sum_{i=1}^N w_{k-1}^i \delta(\mathbf{x}_k - \mathbf{x}_k^i) & \leftarrow & \sum_{i=1}^N w_{k-1}^i \delta(\mathbf{x}_k - (f(\mathbf{x}_{k-1}^i, \mathbf{u}_k) + \mathbf{v}_k^i)) \end{array} \tag{6}$$

State density correction/update (measurements, Bayes rule):

$$\begin{array}{ccc} p(\mathbf{x}_k|\mathbf{Y}_k) & \propto & p(\mathbf{y}_k|\mathbf{x}_k)p(\mathbf{x}_k|\mathbf{Y}_{k-1}) \\ \sum_{i=1}^N w_k^i \delta(\mathbf{x}_k - \mathbf{x}_k^i) & \leftarrow & \sum_{i=1}^N w_{k-1}^i p(\mathbf{y}_k|\mathbf{x}_k^i)\delta(\mathbf{x}_k - \mathbf{x}_k^i) \end{array} \tag{7}$$

Particle Filter (in practice): prediction and correction

State density propagation (dynamics, Chapman-Kolmogorov equation):

$$\mathbf{x}_k^i = f(\mathbf{x}_{k-1}^i, \mathbf{u}_k) + \mathbf{v}_k^i \tag{8}$$

$\mathbf{v}_k^i \quad \forall i \in [1, N]$: random sample of process noise

State density correction/update (measurements, Bayes rule):

$$w_k^i \propto w_{k-1}^i p(\mathbf{y}_k | \mathbf{x}_k^i) \tag{9}$$

Particle Filter: state estimation

Maximum *a posteriori* (hard to compute in the general case):

$$\widehat{\mathbf{x}}_k = \underset{\mathbf{x}_k}{\operatorname{argmax}}\, p(\mathbf{x}_k|\mathbf{Y}_k) \tag{10}$$

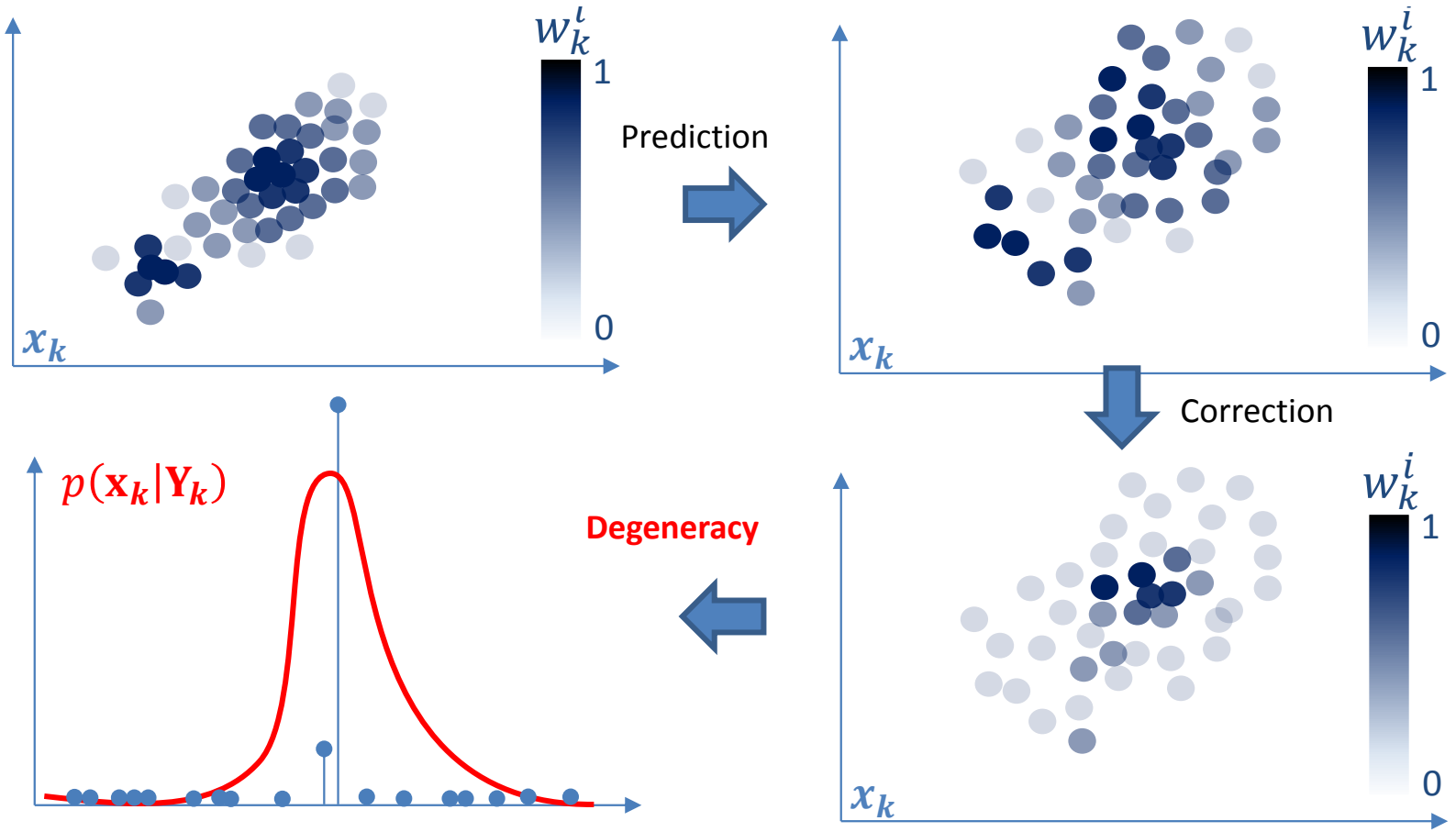
Least square approximation:

$$\widehat{\mathbf{x}}_k \approx \sum_{i=1}^{N} w_k^i \mathbf{x}_k^i \tag{11}$$

Empirical covariance:

$$\widehat{\mathbf{P}}_k = \sum_{i=1}^{N} w_k^i (\mathbf{x}_k^i - \widehat{\mathbf{x}}_k)(\mathbf{x}_k^i - \widehat{\mathbf{x}}_k)^T \tag{12}$$

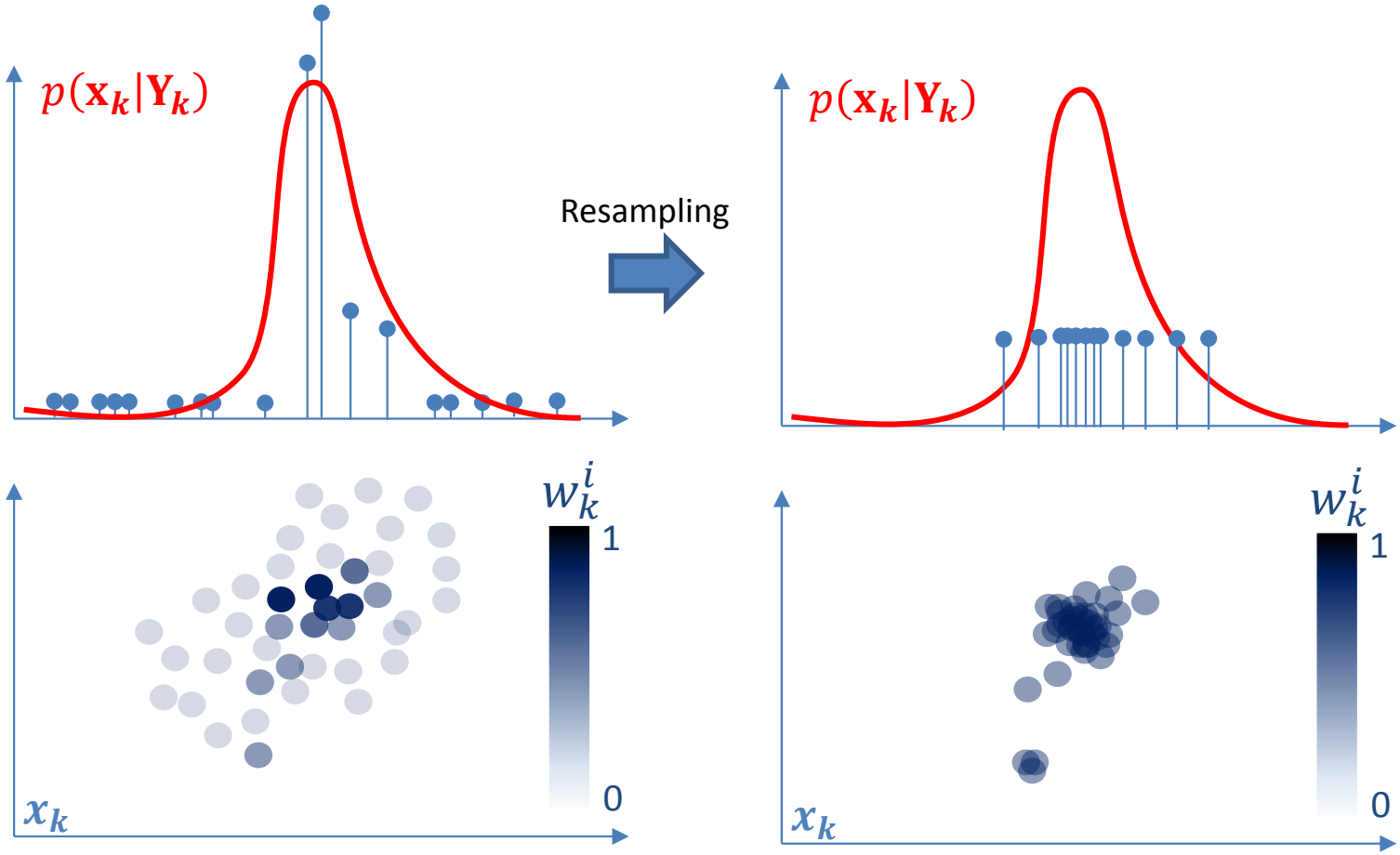
Prediction, correction, and degeneracy phenomenon



After a number of [prediction-correction] cycles, a majority of particle weights will tend to 0 while a small number (or only one) will tend to 1:

That is the weights degeneracy phenomenon.

Resampling



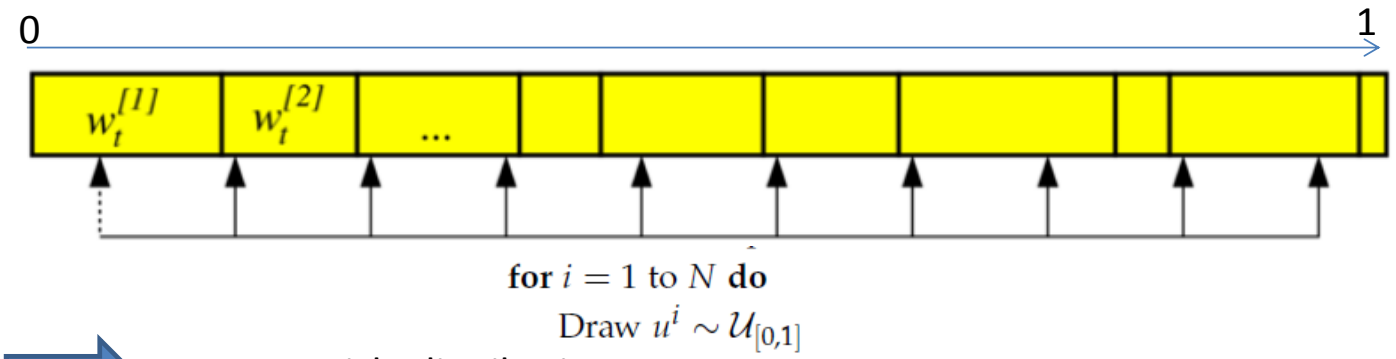
The resampling step aims to duplicate strong-weighted particles to keep an appropriate description of the state density **when degeneracy is about to occur**. Low-weighted particles are destroyed to keep N unchanged.

Multinomial Resampling

The most commonly used resampling technique is Multinomial Resampling:

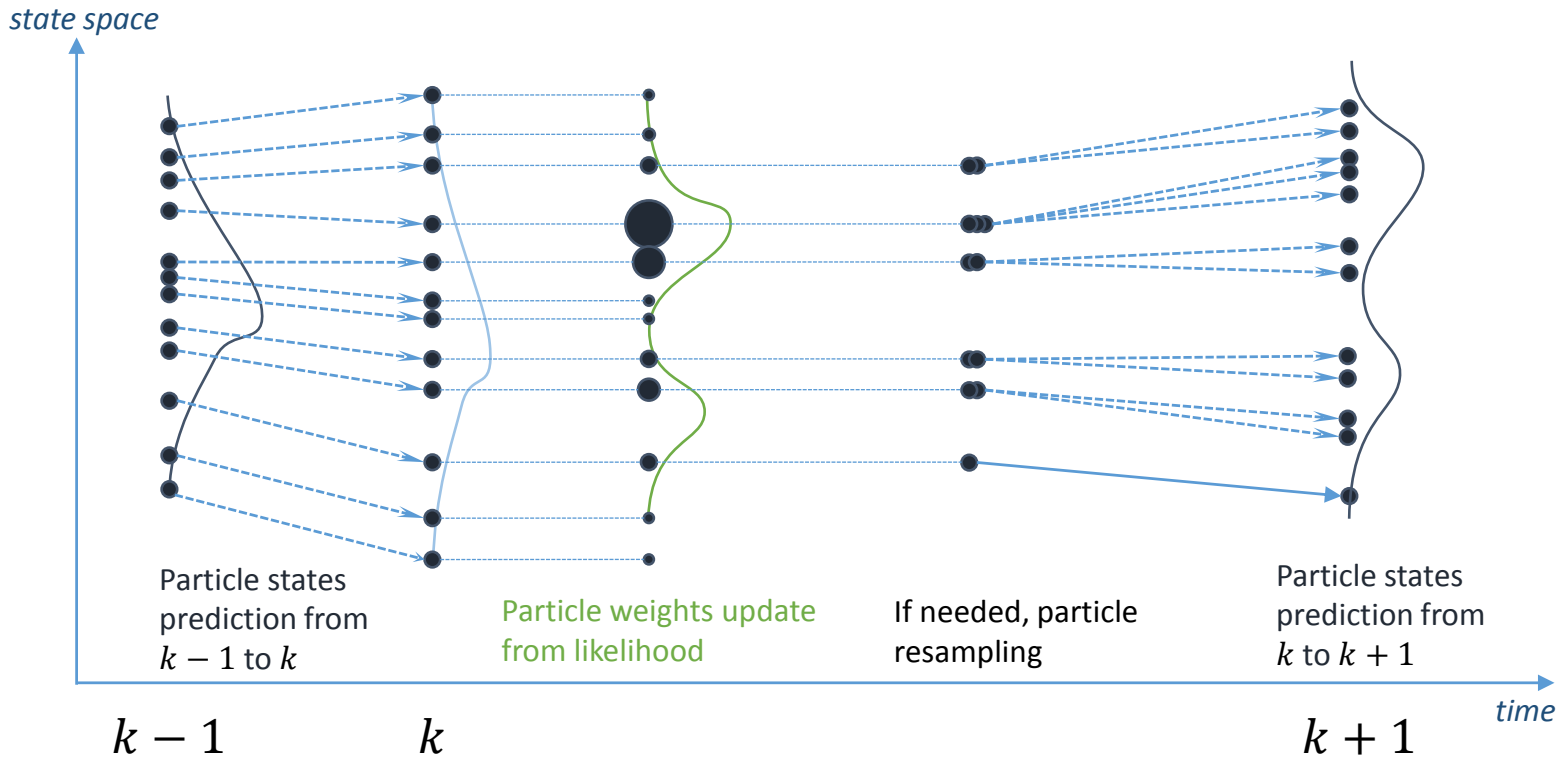
Input: particle weights $\{w^i\}_{i\in[1,N]}$
Output: number of new instances per particles $\{n^i\}_{i\in[1,N]}$

1: Initialise the duplication counters to $n^i = 0\ \forall i \in [1, N]$
2: **for** $i = 1$ to N **do**
3: Draw $u^i \sim \mathcal{U}_{[0,1]}$
4: Find $j \in [1, N]$ such that $u^i \in \left]\sum_{l=1}^{j-1} w^l, \sum_{l=1}^{j} w^l\right]$
5: Count $n^j = n^j + 1$
6: **end for**
7: Return $n^i\ \forall i \in [1, N]$



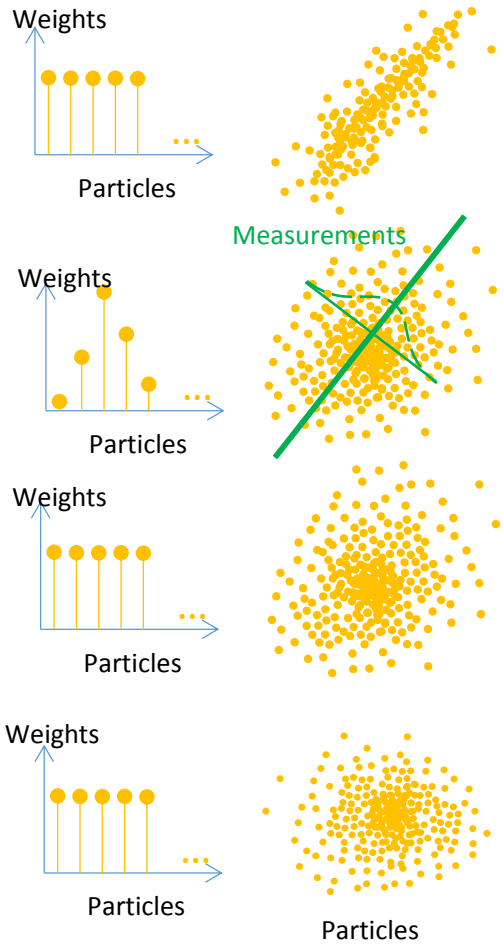
New particle distribution

Prediction, correction, and resampling



SIR-PF scheme

Particle Filter



Low-weighted particles are replaced with copies of high-weighted particles

$$p(\mathbf{x}_k|\mathbf{Y}_k) = \sum_{i=1}^{N} \omega_k^i \delta_{\mathbf{x}_k^i}$$
$$\omega_k^i \propto p\left(\mathbf{y}_k|\mathbf{x}_k^i\right)\omega_{k-1}^i$$

$$p(\mathbf{x}_k|\mathbf{Y}_{k-1}) = \sum_{i=1}^{N} \omega_{k-1}^i \delta_{\mathbf{x}_k^i}$$
$$\mathbf{x}_k^i = f\left(\mathbf{x}_{k-1}^i, \mathbf{u}_k\right) + \mathbf{v}_k^i$$

$$p(\mathbf{x}_0) = \sum_{i=1}^{N} \omega_0^i \delta_{\mathbf{x}_0^i}$$
$$\left\{\mathbf{x}_0^i\right\}_{i=\{1,\dots N\}} \sim p(\mathbf{x}_0)$$

Steps

Resampling

Correction

Prediction

Initialization

PF formulation example with Gaussian uncertainties

Initialization: draw:

$$\left\{ \mathbf{x}_0^i \sim \mathcal{N}(\widehat{\mathbf{x}_0}, \widehat{\mathbf{P}_0}) \right\}_{i \in [1,N]} \tag{13}$$

Prediction step:

$$\mathbf{x}_k^i = f(\mathbf{x}_{k-1}^i, \mathbf{u}_k) + \mathbf{v}_k^i \tag{14}$$

where $\mathbf{v}_k^i \sim \mathcal{N}(0, \mathbf{Q}_k) \quad \forall i \in [1, N]$

Correction step:

Weights update:

$$\tilde{w}_k^i = w_{k-1}^i \exp\left(-\frac{1}{2} (\mathbf{y}_k - h(\mathbf{x}_k^i))^T \mathbf{R}_k^{-1} ((\mathbf{y}_k - h(\mathbf{x}_k^i)) \right) \tag{15}$$

Weights normalization:

$$w_k^i = \frac{\tilde{w}_k^i}{\sum_i \tilde{w}_k^i} \tag{16}$$

Estimation:

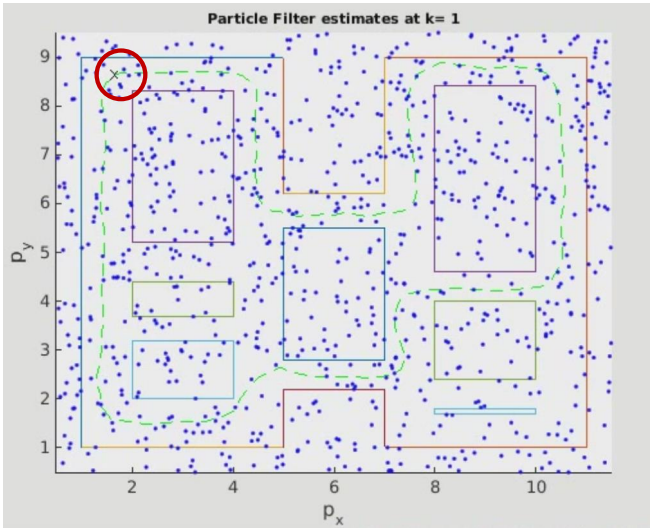
$$\widehat{\mathbf{x}_k} = \sum_{i=1}^{N} w_k^i \mathbf{x}_k^i \qquad \widehat{\mathbf{P}_k} = \sum_{i=1}^{N} w_k^i (\mathbf{x}_k^i - \widehat{\mathbf{x}_k})(\mathbf{x}_k^i - \widehat{\mathbf{x}_k})^T \tag{17}$$

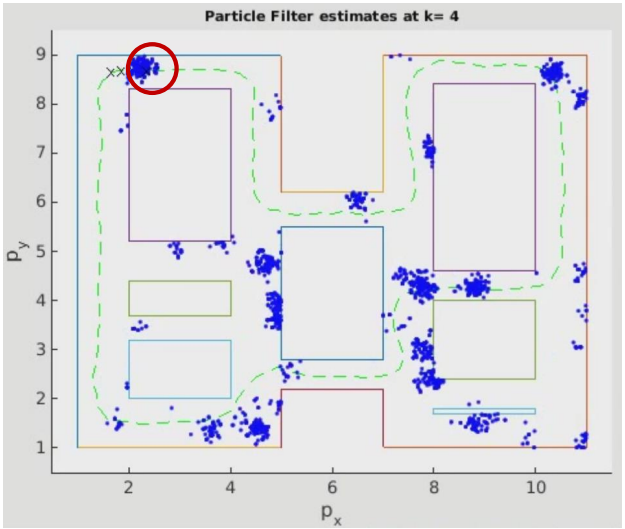
Resampling (cf slide 21) if:

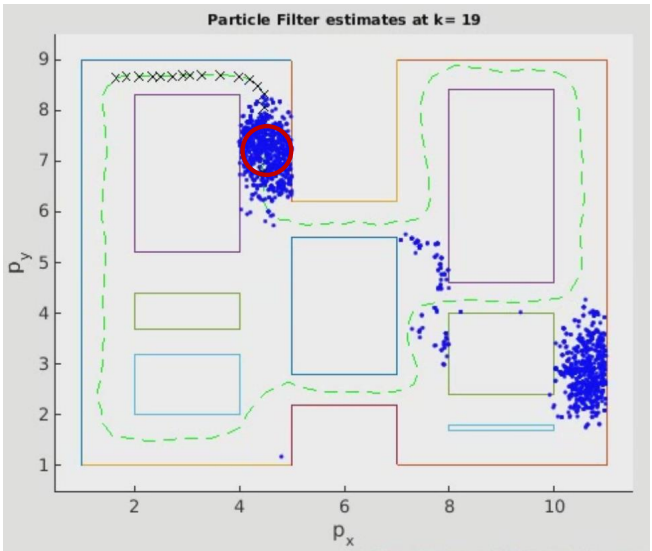
$$N_{eff} = \frac{1}{\sum_i (w_k^i)^2} < \theta_{eff} N \tag{18}$$

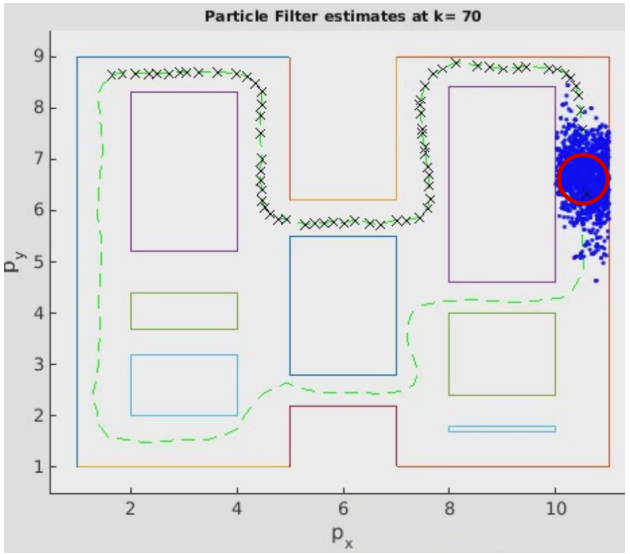
where $\theta_{eff} \in [0, 1]$, (usually 0.5)

Ex1: Indoor navigation without knowledge of initial position



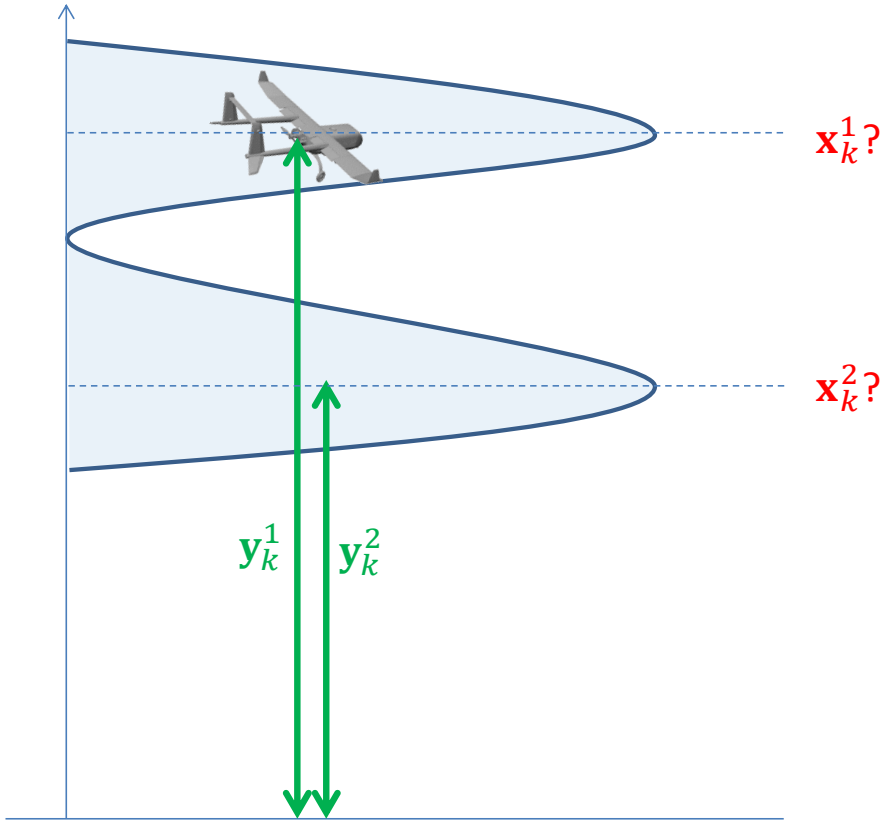




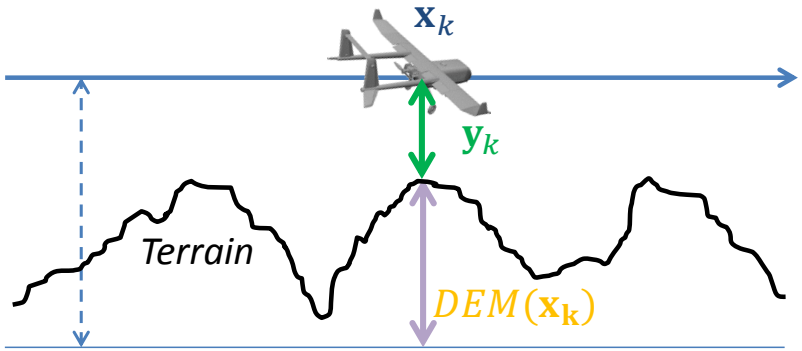


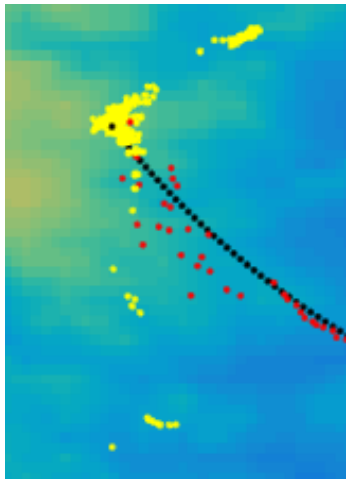
Position estimation using Particle Filter
https://www.youtube.com/watch?v=qsNGoHi7o2U

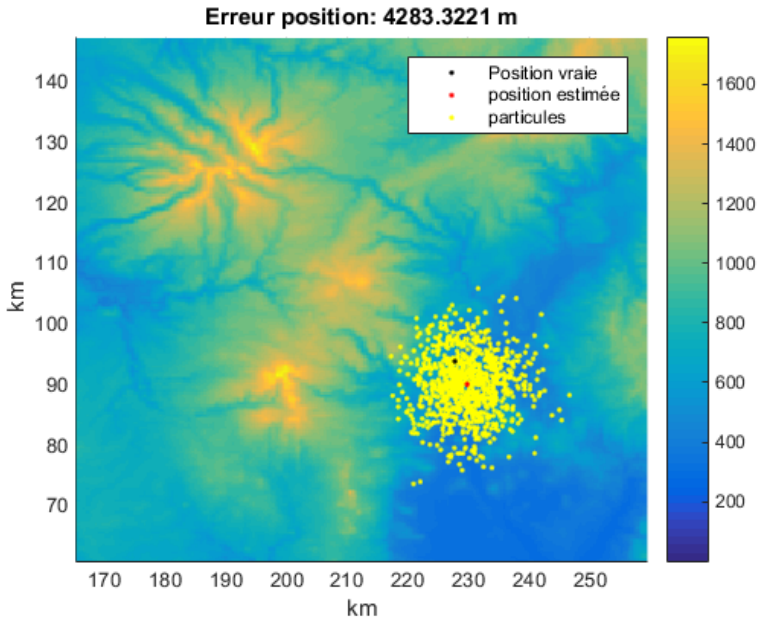
Ex2: Faulty measurement on redundant sensors

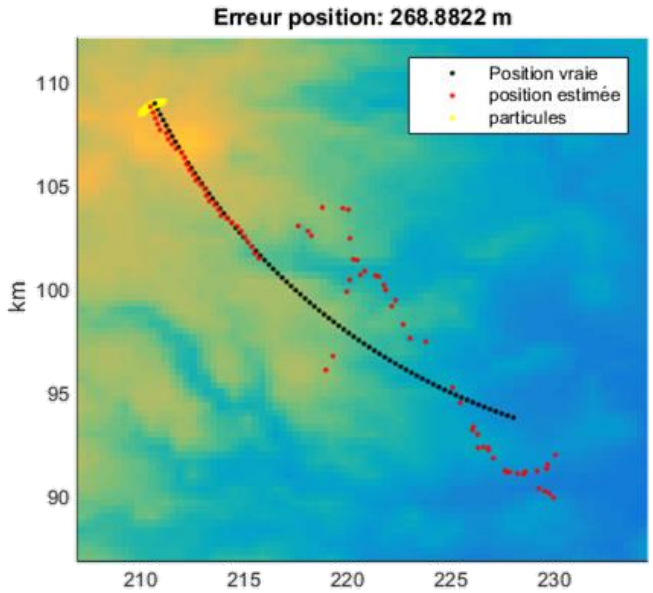


Ex3: Terrain aided navigation





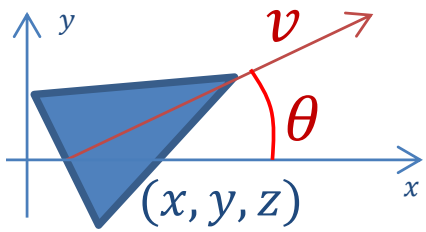


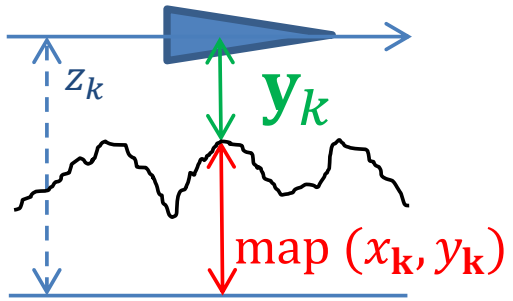


State: $\mathbf{x}_k = [x_k, y_k, z_k, \theta_k]^T$, Control: $\mathbf{u}_k = [v_k, \omega_k]^T$

Dynamics:

$$\mathbf{x}_{k+1} = f(\mathbf{x}_k, \mathbf{u}_k) = \begin{bmatrix} x_k + v_k \Delta t \cos\theta_k \\ y_k + v_k \Delta t \sin\theta_k \\ z_k \\ \theta_k + \Delta t \omega_k \end{bmatrix}$$





Measurements (e.g., radar altimeter or laser telemeter):

$$\mathbf{y}_k = h(\mathbf{x}_k) + \mathbf{v}_k = z_k - \mathrm{lectureCarte}(x_k, y_k) + \mathbf{v}_k$$

where $\mathbf{v}_k \sim \mathcal{N}(0, \mathbf{R})$ and $\mathbf{R} = 20^2$ m^2.

SIR-PF advantages and limitations

Advantages

- Tackles non-linear dynamics and measurement models
- Tackles non-differentiable models (no linearization)
- Tackles non-Gaussian uncertainties and multimodal state densities (ambiguities)
- Can start with huge initial uncertainty
- Relatively easy to implement

Limitations:

- Computationally demanding
- May converge to erroneous local state density maximum
- Can be hard to tune (number of particles, choice of noise modeling, resampling threshold)

More advanced Particle Filters

Original algorithm:

- Sequential Importance Resampling Particle Filter (SIR-PF) [2]

More advanced algorithms:

- Rao-Blackwellized Particle Filter (RBPF) [7]
- Regularized Particle Filter (RPF) [4]
- Kalman Particle Kernel Filter (KPKF) [6]
- (Weighted) Ensemble Kalman Filter (WEnKF/EnKF) [3]
- Adaptive Approximate Bayesian Computational Particle Filter (A2BC-PF) [10]
- Box Particle Filter (BPF) [8]
- Box Regularized Particle Filter (BRPF) [9]
- ...

TP

1. Prendre en main et décrire sous forme de schéma la structure du code *FiltrageParticulaire_terrainNavigation.m* et repérer les différents paramètres (réglage du filtre, simulation...).
2. Compléter le code avec les équations du filtre PF (prédiction, correction, ré-échantillonnage slide 24), le modèle dynamique et le modèle de mesure (slide 28) à l'aide de la fonction *hobs(*$\boldsymbol{x}_k$*, params)* et commenter les résultats;
3. Faire varier le bruit de dynamique du filtre (matrice $\mathbf{Q}_f$) sur les différentes variables d'état et expliquer le comportement des particules.
4. Faire varier le bruit de mesure du filtre (matrice $\mathbf{R}_f$) entre 10^2 et 100^2 et expliquer les résultats.
5. Faire varier le nombre de particules N et expliquer les résultats.
6. Faire varier le seuil de ré-échantillonnage *threshold_resampling*. Pour les valeurs de 0, 0.5 et 1, tracer des histogrammes des poids (*hist(wp)*) et identifier le phénomène de dégénérescence et l'impact du ré-échantillonnage.
7. Simuler un trou de mesures entre $t = 50$ s et $t = 75$ s en utilisant la variable *is_measurementValid* et expliquer les résultats.
8. Modifier la fréquence des mesures (passer à 0.1 Hz) en utilisant la variable *is_measurementValid* et expliquer les résultats.
9. Proposer une autre façon de ré-échantillonner les poids, en remplacement de la fonction select.p (qui s'utilise comme: $indp = select(wp)$ avec wp l'ensemble des poids et $indp$ la liste des indices des nouvelles particules par rapport aux anciennes), la coder puis commenter les résultats. Vous pourrez vous aider de [11].

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