Question 1: (Variations on Financing Government Expenditures)

The consumer values consumption, and provides 1 unit of labor inelastically. The period utility be $u(c) = \log(c)$.

Take our standard neoclassical growth model, with the possibility of consumption taxes, τ_{ct}, lump-sum taxes, τ_{ht}, and labor taxes, τ_{nt}. First, assume that taxes and government expenditures are 0 (i.e., $\bar{g} = \bar{\tau}_h, = \bar{\tau}_c$) and that the economy is in a steady state (i.e. $\bar{k}$ as the steady state capital, and $\bar{c} = f(\bar{k}) - \delta\bar{k}$. Let the capital at time 0 be this steady state capital, i.e. $k_0 = \bar{k}$.

There is a <u>sudden</u> announcement that $g_t = \bar{g}$ for all $t \geq 0$, where $\bar{g} = \frac{1}{4}(f(k_0) - \delta k_0)$, and the government expenditures are financed entirely through <u>lump-sum taxes</u>, $\bar{\tau}_h$.

(a) Calculate the new steady state $\bar{c}$ and $\bar{k}$.

(b) What is the transition path of c_t and k_t from the k_0 initial condition?

(c) What is the behavior of c_{t+1}/c_t and $R_{t+1} \equiv f'(k_{t+1}) + 1 - \delta$ along this transition path?

(d) Argue that the timing of the lump-sum taxes is irrelevant (i.e., any τ_{ht} fulfilling the long-run government budget constraint gives the same allocation).

(e) Now, consider the alternative policy that the government finances its expenditures entirely through <u>consumption taxes</u>. First assume that consumption taxes are constant(i.e., $\tau_{ct} = \bar{\tau}_c$ for all $t \geq 0$). Find the new steady state $\bar{c}$ and $\bar{k}$ and the transition path from k_0, .

(f) In this case, would the timing of the consumption tax matter (i.e., does any τ_{ct} fulfilling the long-run government budget constraint deliver the same allocations $\{c_t, k_{t+1}\}$ along the transition dynamics?) If not, why?

(g) Without solving the full model, would financing expenditures entirely through constant <u>labor taxes</u> $\bar{\tau}_n$ have the same steady state as that of lump-sum taxes? What about the transition dynamics?

Question 2: (Variation on the Search Model)

Each period, a previously unemployed worker draws two offers to work forever at wage w from the cumulative distribution function ('cdf')

$$F(w) = \Big(w/B\Big)^{\frac{1}{2}}, \quad 0 \leq w \leq B$$

where $F(w) = \text{Prob}(\tilde{w} \leq w)$ where $\tilde{w}$ is a particular wage offer. Successive draws within a period and across periods are identically and independently distributed. The unemployed worker is free to inspect both offers in a period and, if he or she wants, accept the highest among offers he or she has drawn that period. Offers from past periods cannot be recalled. The offers are to work at the accepted wage forever. There is no option to quit after an offer has been accepted, and there is no prospect of being fired. The worker wants to maximize

$$\mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t y_t\right], \quad 0 < \beta < 1,$$

where

$$y_t = \begin{cases} w & \text{if employed at wage } w, \\ c & \text{if unemployed} \end{cases}$$

where w is the wage, c is unemployment compensation, and $\mathbb{E}[\ldots]$ is an expected value before the offers are drawn.

(a) Verify that $F(w) = \Big(w/B\Big)^{\frac{1}{2}}$ is a legitimate cdf. Find the cdf for the maximum of 2 draws (i.e., find the cdf of $z \equiv \max\{z_1, z_2\} \in \tilde{F}(z)$) and verify it is a cdf.

(b) Find the worker's optimal search strategy and show that it has a 'reservation wage' form.[1] Draw the value function for a worker given a particular w.

(c) Let $\bar{w}$ be the reservation wage. Find a formula for the reservation wage as a function of B, β, c.[2]

(d) Given this $\bar{w}$, find a formula for ψ = probability that an unemployed worker leaves unemployment this period as a function of $\bar{w}, B, \beta, c$ (eliminating $\bar{w}$ if you found a closed form solution in part c).

[1]Hints: (1) setup the model recursively in a way isomorphic to the model from class, (2) For any n and independent draws, $z_1, \ldots, z_n$ from cdf $F(z)$, the cdf of the maximum of these is $z \equiv \max\{z_1, \ldots z_n\} \sim F(z)^n$

[2]Hint: While you could solve for $\bar{w}$ directly, feel free to leave it in an *implicit* form if you are finding the algebra difficult. However, you should be able to eliminate any recursive value functions

Question 3: (Rationalizing Consumption Patterns)

Assume a completely standard neoclassical growth model where the government policy is some $\{g_t, \tau_{kt}\}_{t=0}^{\infty}$. Define

$$R_t \equiv (1-\delta) + (1-\tau_{kt+1})f'(k_{t+1})$$

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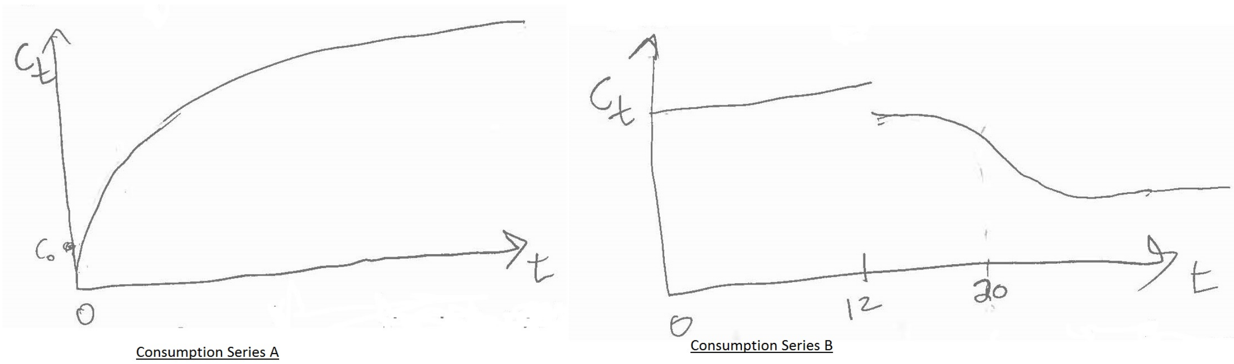

Figure 1: Two Different Consumption Patterns

(a) Make up any story you like on sequences of g_t and τ_{kt} which rationalizes the path of consumption in Figure 1, Consumption Series (A). You are free to come up with any explanation you want that is consistent with the model and these observations.

(b) Describe the behavior of k_t, g_t, k_{t+1} and R_t for your story.

(c) Instead, assume that you observe the path of consumption in Figure 1, Consumption Series (B). Make up any story you like on sequences of g_t and τ_{kt} which rationalizes this path of consumption. In doing so, please distinguish between changes in taxes and expenditures that are foreseen vs. unforeseen.

(d) Describe the behavior of k_t, g_t, k_{t+1} and R_t for your story.

Question 4: (Special Permanent Income Model + Asset Pricing)

A consumer faces a time-invariant, risk-free gross interest rate of $R \equiv e^r$ with $r > 0$. The consumer can borrow or lend at this rate up to a "no-Ponzi scheme" condition. The savings (or debt) of the consumer is denoted B_t.

Let the discount factor be β, and define $\beta \equiv e^{-\rho}$ where ρ is the discount rate.[3]

Finally, assume that the consumer's income, Y_t, is a stochastic process following:

$$Y_{t+1} = Y_t \exp(\sigma \epsilon_{t+1})$$

where $\epsilon_{t+1} \sim N(0,1)$. The consumer chooses $\{C_t, B_{t+1}\}_{t=0}^{\infty}$ (which may now be stochastic) to maximize their expected utility, i.e.

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma} \right] \tag{1}$$

$$\text{s.t.}\ C_t + B_{t+1} \leq RB_t + Y_t, \quad \text{for } t \geq 0 \tag{2}$$

With the LOM

$$Y_{t+1} = Y_t e^{\sigma \epsilon_{t+1}} \tag{3}$$

(a) Find the first-order necessary conditions for this problem.[4] Verify that they imply,

$$1 = e^r \mathbb{E}_t [m_{t+1}]$$

where

$$m_{t+1} \equiv \beta \frac{u'(C_{t+1})}{u'(C_t)}$$

(b) Verify that

$$m_{t+1} = \beta \exp(-\gamma(c_{t+1} - c_t))$$

where $c_t \equiv \log C_t$. If it is useful, you could take logs with $y_t \equiv \log Y_t$ in (3) find

$$y_{t+1} - y_t = \sigma \epsilon_{t+1}$$

(c) Assume there is a representative consumer and the consumer is observed to set $C_t = Y_t$ and $B_{t+1} = 0$ for all $t \geq 0$ (thinking of a Lucas-style asset pricing model in general equilibrium). Find the value of constant net interest rate, r, which rationalizes this behavior.[5]

(d) Interpret t the role of aggregate uncertainty, σ, on interest rates, r. Why/when would γ matter? If $\sigma = 0$, why/when would γ matter?

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[3] From a Taylor Series approximation, this is approximately equal to $\beta \equiv \frac{1}{1+\rho}$ for small ρ.

[4] Be careful to keep expectations around when the information set only allows forecasts. As the law of motion in (3) is not a constraint on the choice, you don't put it in as a Lagrange Multiplier. Instead, you should solve the problem with the binding constraint in (2) and then apply (3) as the forecast after you have the Euler Equation.

[5] Hint: If $\epsilon \sim N(0,1)$ then $\mathbb{E}[e^{\sigma\epsilon}] = e^{\sigma^2/2}$. Use the optimality conditions from previous parts and deduce the r necessary to clear the markets.

Question 5: (Transition Dynamics with Risk-Neutral Consumers)

Assume that, given an initial condition $k_0 > 0$, a risk-neutral consumer has the following preferences,[6]

$$\sum_{t=0}^{\infty} \beta^t c_t \tag{4}$$

$$\text{s.t. } c_t + k_{t+1} = f(k_t) + (1-\delta)k_t \tag{5}$$

$$c_t \geq 0 \tag{6}$$

with the standard properties, $f'(\cdot) > 0, f''(\cdot) < 0$, and $f'(0) = \infty$. Define $\beta \equiv \frac{1}{1+\rho}$ with $\rho > 0$.

(a) For a given k_0, formulate the optimal planning problem and find the first-order necessary conditions.[7] Define $\bar{k}$ as the steady state capital

(b) For $k_0 > \bar{k}$, describe the optimal time path of $\{c_t, k_{t+1}\}_{t=0}^{\infty}$

(c) For $k_0 < \bar{k}$, describe the optimal time path of $\{c_t, k_{t+1}\}_{t=0}^{\infty}$

(d) Let the savings rate s_t be defined as

$$k_{t+1} = s_t f(k_t) + (1-\delta)k_t$$

Explain how s_t varies as a function of k_t in this model using the dynamics of parts (a), (b), and (c)

[6] Alternatively, consider the limit of power-utility $u(c) \equiv \lim_{\gamma \to 0} \frac{c^{1-\gamma}}{1-\gamma}$.

[7] Hint: unlike our example in class, you need to keep Lagrange multipliers on some of the inequality constraints. Remember that linear objectives usually mean corner solutions.