Question 1
(Basic Unemployment Calculations)
There are two states: U for unemployment and E for employment.

- With probability $\lambda \in (0,1)$, a person unemployed today becomes employed tomorrow.
- With probability $\alpha \in (0,1)$, a person employed today becomes unemployed tomorrow.

(a) Let $N \geq 1$ be the number of periods until a currently unemployed person becomes employed. Calculate $\mathbb{E}[N]$.

(b) Let $M \geq 1$ be the number of periods until a currently employed person becomes unemployed. Calculate $\mathbb{E}[M]$.

(c) Please compute the fraction of time an infinitely lived person can expect to be unemployed and the fraction of time they can expect to be employed.

Question 2
(Multiple Employment States)
An economy has 3 states for workers:

- U: unemployment.
- V: if they have found a potential employer and are being verified to see if they are a good fit.
- E: if a worker has been verified and is employed.

The probabilities that they jump between these states each period is:

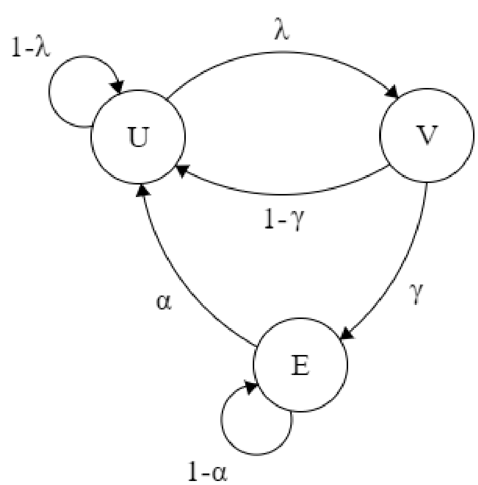

Figure 1: Markov Chain

i.e. probability γ they are a good fit, and the verification takes 1 period.

(a) Write a Markov transition matrix for this process, P.

(b) Write an expression for the stationary distribution across states in the economy, $\pi \in \mathbb{R}^3$ (You can leave in terms of P).

(c) If a worker is U today, write an expression for the probability they will be employed exactly j periods in the future (considering any possible transitions which end in employment at j periods).[1].

(d) Assume that $\alpha = 0, \lambda = 0$. Is the stationary distribution unique? If not, describe the sorts of distributions that could exist.

Question 3

(Asset Price Forecasts)

Let $y_t \in \mathbb{R}$ be a sequence of dividends such that

$$y_{t+1} = y_t + w_{t+1}$$

where $w_{t+1} \sim N(0, \sigma^2)$ for some $\sigma > 0$. i.e. $\mathbb{E}_t [w_{t+1}] = 0$ and $\mathbb{E}_t \left[w_{t+1}^2\right] = \sigma^2$. An agent prices an asset as the expected PDV of dividends,

$$p_t = \mathbb{E}_t \left[\sum_{j=0}^{\infty} \beta^j y_{t+j}\right]$$

Define the forecast error as the deviation in the expected price tomorrow calculated at time t from the actual price tomorrow. i.e.,

$$FE_{t+1|t} \equiv p_{t+1} - \mathbb{E}_t [p_{t+1}]$$

(a) Setup in our canonical Linear Gaussian State Space model.

(b) Solve for p_t in terms of y_t and model intrinsics.

(c) Find the <u>expected forecast error</u>: $\mathbb{E}_t \left[FE_{t+1|t}\right] = \mathbb{E}_t [p_{t+1} - \mathbb{E}_t [p_{t+1}]]$

(d) Find the <u>variance of forecast errors</u>:

$$\begin{aligned}\mathbb{V}_t \left(FE_{t+1|t}\right) &\equiv \mathbb{E}_t \left[FE_{t+1|t}^2\right] - \left(\mathbb{E}_t \left[FE_{t+1|t}\right]\right)^2 \\ &= \mathbb{E}_t \left[(p_{t+1} - \mathbb{E}_t [p_{t+1}])^2\right] - (p_{t+1} - \mathbb{E}_t [p_{t+1}])^2\end{aligned}$$

(e) Setup the problem recursively as p_t define in terms of p_{t+1}. Test, using your solution from early in the question, if the stochastic process p_t a martingale (i.e., just a random walk in this case)? Recall that a martingale is a process such that $\mathbb{E}_t [p_{t+1}] = p_t$. Give any intuition you can on this result.

Question 4

(Ricardian Equivalence)

Consider a variation of the example in class where the government makes a surprise announces that it will borrow money, give the money to the consumers as a "stimulus",

[1] Note: This is only looking at j periods into the future. i.e. this is **not** the probability that they become at least employed once during the j periods.

and eventually pay it back through taxing consumers (i.e., no chance of a government default).

The agent's income is $y_t = \delta^t$ for $\delta > 1$, and assume that: $\beta R = 1$, $u'(c) > 0$, and $u''(c) < 0$. Furthermore, assume that the consumer's and government face the same interest rate $R > 0$.

The government makes there announcement between time 0 and time 1 (i.e., after the consumer has already chosen c_0 and F_1 thinking that their income will follow y_t). The precise announcement is that at time 1, the consumer's are given $\alpha > 0$ as *extra* income as a stimulus (thereby increasing their y_1 from what they had previously anticipated). That is income is now,

$$y_1 = \delta + \alpha$$

Instead of paying back deterministically, the government will pay back the loan at period k (which will be stochastic). To pay the loan, the government taxes the total value of the loan + interest. For example, if they paid it off in period k, then the labor income of a consumer at period k would be

$$y_k = \delta^k - \alpha R^{k-1}$$

Otherwise, the consumer's income follows the same y_t process. While the consumer doesn't know exactly when the loan will be repaid, they know the correct distribution of payment dates upon the announcement 1:

$$\mathbb{P}\,(\text{pay at k}) = p(k) \geq 0$$

where $\sum_{k=2}^{\infty} p(k) = 1$.

(a) First, assuming the standard permanent income model, calculate the optimal sequence $\{c_t\}_{t=0}^{\infty}$ at $t = 0$, before the government announces the policy. Note that at this point, the consumer believes they have a deterministic income stream and that the government is not going to borrow or tax.

(b) After the surprise announcement, what is the new optimal path of consumption chosen?[2] What is $c_1 - c_0$? Interpret the effect of the stimulus on consumption.

(c) Does it matter if the consumer knows the true distribution of payment dates, or the timing of the taxes to pay for the loan?

[2]Hint: at time 1 the consumer's income is now stochastic, but it is very linear and simple.