Question 1: (Welfare Cost of Financial Frictions)
Let $\beta = .95$, $R = 1.04$.

Scenario 1 for Consumer The consumer maximizes the following welfare

$$\begin{aligned} U &= \sum_{t=0}^{\infty} \beta^t \log(c_t) && (1) \\ \text{s.t. } F_{t+1} &= R(F_t + y_t - c_t), \quad \forall t \geq 0 && (2) \\ y_t &= y_0 \delta^t, \quad \forall t \geq 0 && (3) \\ F_0 &= 0 && (4) \\ &\text{(transversality condition)} && (5) \end{aligned}$$

Scenario 2 for Consumer The consumer faces the same problem as Scenario 1, except with **no borrowing**: $F_{t+1} \geq 0$ for all $t \geq 0$, and the initial level of y_0 is potentially different (defined as y_0^{NB}. Define the PDV of utility (i.e., the welfare) of this as U^{NB}

(a) Assume $\delta = 1.02, y_0 = 1$, and $y_0^{NB} = 1$. Calculate U and U_{NB}.

(b) Let $y_0 = 1$. Now find a y_0^{NB} such that $U = U^{NB}$. The difference between y_0 and y_0^{NB} is the amount of sacrifice in terms a consumer with a borrowing constraint would pay to be free to borrow. A measure of the welfare loss of the no borrowing constraint.

(c) Maintain $y_0 = 1$. Now, let $\beta = .99, R = 1.04$, and $\delta = 1.01$. What is c_0 and F_1 here under Scenario 1? Repeat part (b) to find y_0^{NB} such that $U = U^{NB}$ with these new parameters. What can you conclude about the welfare cost of no borrowing in this case?

Question 2: (Sequential and Recursive)

Let the consumer have power utility,

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}, \quad \gamma > 1$$

Given $F_0 = 0, B \geq 0, \beta R = 1$, and the deterministic income stream $y_t = \delta^t$, the consumer maximizes

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{6}$$

$$\text{s.t. } F_{t+1} = R(F_t + y_t - c_t), \quad \forall t \geq 0 \tag{7}$$

$$F_{t+1} \geq -B \tag{8}$$

$$F_0 = 0 \tag{9}$$

$$\text{(transversality condition)} \tag{10}$$

(a) Derive the euler equation as an inequality, and the condition for it holding with equality.

(b) Let $\delta > 1$ and $B = \infty$. What is $\{c_t\}_{t=0}^{\infty}$?

(c) Let $\delta > 1$ and $B = 0$. What is $\{c_t\}_{t=0}^{\infty}$?

(d) Let $\delta < 1$ and $B = 0$. What is $\{c_t\}_{t=0}^{\infty}$?

(e) Assume that the consumer optimally eats their entire income each period, i.e., $c_t = y_t = \delta^t$ which implies $c_{t+1} = \delta c_t$. Setup, using dynamic programming, an equation to find the value $V(c)$ recursively.

(f) Guess that $V(c) = k_0 + k_1 c^{1-\gamma}$ for some undetermined k_0 and k_1.[1] Solve for k_0 and k_1 and evaluate $V(1)$ (i.e., the value of starting with $c_0 = 1$.

[1] Note that this equation deliberately is avoiding any t subscripts! This makes it a truly recursive expression.

Question 3: (Search with Firing)

Take the search model we did in class with an endogenous choice of accepting a job, but add in the following elements:

- If you are unemployed and reject a wage, there is only a $\theta \in (0, 1)$ probability to get a wage offer. Otherwise, you can recall a wage you previously rejected last period. (hint: they would never choose to recall a wage in equilibrium, but it may help you write down the Bellman equation cleanly.)
- There is an exogenous probability $\alpha \in (0, 1)$ of being fired at the end of any period you are working. You then draw a new wage as an unemployed agent entering the next period with certainty (i.e., bypass the θ probability going into the next period.

To summarize the timing here: As in our example in class, let $v(w)$ be the value of coming a period unemployed with wage offer w and when they are about to choose to accept or reject.

If they reject an offer, they gain unemployment insurance c, and have the probability θ to gain the draw with expected value

$$Q = \int_0^B v(\hat{w}) f(\hat{w}) \mathrm{d}\hat{w}$$

If they accept they gain the wage w that period, and then have the α chance of being fired as they come into the next period—at which point they get the wage offer draw with certainty, as discussed. (Hint: if they are not fired, the value next period is $v(w)$, the same as if they were first offered w.)

(a) Draw a Markov chain with two states E and U. Let the probability of staying unemployed be λ which will end up endogenous. You will also need the α transition probability

(b) Write the value of a worker with wage offer w who chooses to reject the offer (hint: if they reject they don't necessarily gain a new draw, but could have $v(w)$ as their value next period since they can recall the rejected w).

(c) Write the value of a worker accepting the offer of w. (hint: may need to be recursive now, unlike what we did in class)

(d) Combine the values in the previous two parts to form a Bellman equation with $v(w)$ and the max for the choice.

(e) Write the equation for an indifference point $\bar{w}$, where they are at the threshold of accepting (or rejecting) the wage.[2]

(f) Assuming that you could numerically solve the previous equation to find a $\bar{w}$, what is the expression for the stationary proportion of unemployed workers as a function of $\alpha, \theta, f(\cdot)$, and $\bar{w}$. (Hint: derive the λ, then use older notes on unemployment. However, recall that the $U \to E$ transition only occurs if both a wage offer and endogenous acceptance occur).

[2] As in the case of class, the Bellman equation could be reorganized to eliminate the $v(\cdot)$ function to give an implicit equation in $\bar{w}$ and parameters. This is significantly trickier than what we did, so only try to simplify the indifference equation if you wish.