Question 1: (Welfare Cost of Financial Frictions)
Let $\beta = .95,\ R = 1.04$.

Scenario 1 for Consumer The consumer maximizes the following welfare

$$U = \sum_{t=0}^{\infty} \beta^t \log(c_t) \tag{1}$$
$$\text{s.t. } F_{t+1} = R(F_t + y_t - c_t), \quad \forall t \geq 0 \tag{2}$$
$$y_t = y_0 \delta^t, \quad \forall t \geq 0 \tag{3}$$
$$F_0 = 0 \tag{4}$$
$$\text{(transversality condition)} \tag{5}$$

Scenario 2 for Consumer The consumer faces the same problem as Scenario 1, except with **no borrowing**: $F_{t+1} \geq 0$ for all $t \geq 0$, and the initial level of y_0 is potentially different (defined as y_0^{NB}. Define the PDV of utility (i.e., the welfare) of this as U^{NB}

(a) Assume $\delta = 1.02, y_0 = 1$, and $y_0^{NB} = 1$. Calculate U and U_{NB}.

(b) Let $y_0 = 1$. Now find a y_0^{NB} such that $U = U^{NB}$. The difference between y_0 and y_0^{NB} is the amount of sacrifice in terms a consumer with a borrowing constraint would pay to be free to borrow. A measure of the welfare loss of the no borrowing constraint.

(c) Maintain $y_0 = 1$. Now, let $\beta = .99, R = 1.04$, and $\delta = 1.01$. What is c_0 and F_1 here under Scenario 1? Repeat part (b) to find y_0^{NB} such that $U = U^{NB}$ with these new parameters. What can you conclude about the welfare cost of no borrowing in this case?

Question 2: (Sequential and Recursive)

Let the consumer have power utility,

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}, \quad \gamma > 1$$

Given $F_0 = 0, B \geq 0, \beta R = 1$, and the deterministic income stream $y_t = \delta^t$, the consumer maximizes

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{6}$$

$$\text{s.t. } F_{t+1} = R(F_t + y_t - c_t), \quad \forall t \geq 0 \tag{7}$$

$$F_{t+1} \geq -B \tag{8}$$

$$F_0 = 0 \tag{9}$$

$$\text{(transversality condition)} \tag{10}$$

(a) Derive the euler equation as an inequality, and the condition for it holding with equality.

(b) Let $\delta > 1$ and $B = \infty$. What is $\{c_t\}_{t=0}^{\infty}$?

(c) Let $\delta > 1$ and $B = 0$. What is $\{c_t\}_{t=0}^{\infty}$?

(d) Let $\delta < 1$ and $B = 0$. What is $\{c_t\}_{t=0}^{\infty}$?

(e) Assume that the consumer optimally eats their entire income each period, i.e., $c_t = y_t = \delta^t$ which implies $c_{t+1} = \delta c_t$. Setup, using dynamic programming, an equation to find the value $V(c)$ recursively.

(f) Guess that $V(c) = k_0 + k_1 c^{1-\gamma}$ for some undetermined k_0 and k_1.[1] Solve for k_0 and k_1 and evaluate $V(1)$ (i.e., the value of starting with $c_0 = 1$.

[1] Note that this equation deliberately is avoiding any t subscripts! This makes it a truly recursive expression.