Question 1

Calculate the following matrices and matrix-vector multiplication.[1]

(a) $\begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} 2 & 1 & 0 \\ 1 & 5 & 2 \end{pmatrix}$

(b) $\begin{pmatrix} 0 & 0 & 1 \\ 3 & 1 & 1 \\ -2 & 3 & 0 \end{pmatrix} \begin{pmatrix} 2 & 0 & 5 \\ 1 & 3 & 1 \\ 4 & 1 & 2 \end{pmatrix}$

(c) $\begin{pmatrix} 5 & 3 & 2 \end{pmatrix} \begin{pmatrix} 2 & 7 & 1 \\ 0 & 4 & 5 \\ 2 & 3 & 4 \end{pmatrix}$

(d) $\begin{pmatrix} 2 & 3 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

Question 2

Being formal and explicit in the rules of matrix algebra (e.g. when things are commutative, distributive, etc.) solve the following equations for $x \in \mathbb{R}^N$, with vector $b \in \mathbb{R}^N$, matrices A, D, Q, R all $\mathbb{R}^{N \times N}$, and scalar $m \in \mathbb{R}$,

(a) $Ax + \left(x^T D\right)^T = b$

(b) $Q^{-1}\left(A\,x\,m + mb\right) = Rx$

Question 3

Transform the following linear equations into a linear system with matrices/vectors

(a) $\begin{cases} 2x + 3y = 2 \\ x - 2y = -1 \end{cases}$ where $\{x, y\}$ are variables

(b) $\begin{cases} 2x - 3y = 1 \\ 3x + my = -2 \end{cases}$ where $\{x, y\}$ are variables

(c) $\begin{cases} 2a + b = 1 \\ 3b + 4c = 2 \\ -2a + c = 0 \end{cases}$ where $\{a, b, c\}$ are variables

(d) $\begin{cases} a + b = -3 \\ c - 2 - 4b = 0 \end{cases}$ where $\{a, b, c\}$ are variables (this will not be of full rank)

Question 4

(a) Find a linear transformation $G \in \mathbb{R}^2$ such that $G \cdot \begin{bmatrix} a & b \end{bmatrix}^T$ always returns the second element, b.

[1] No need to hand this question in. This is just for your own practice.

Question 5

Use undetermined coefficients example to solve the following functional and difference equations.[2]

(a) Take a simple linear ODE: $\boldsymbol{\partial} f(z) = f(z)$. Guess that $f(z) = C_1 e^z + C_2$ and use undetermined coefficients to solve for C_1 and C_2.

(b) Take the functional equation $[f(z)]^2 = z^2 + 2z + 1$. Guess that the solution is of the form $f(z) = C_1 z + C_2$. Use undetermined coefficients to find C_1 and C_2.

(c) Take the difference equation $z_{t+1} = g z_t$. Guess $z_t = C_1 C_2^t + C_3$. Show that C_1 is indeterminate and find C_2 and C_3. What if we add subject to $z_0 = A$? Show how this pins down C_1?.

Question 6

Let X and Y be random variables such that $X \in \{0, 1\}$ and $Y \in \{1, 2\}$. These are correlated such that[3]

$$\mathbb{P}(X = 0 \text{ and } Y = 1) = .1 \tag{1}$$
$$\mathbb{P}(X = 0 \text{ and } Y = 2) = .3 \tag{2}$$
$$\mathbb{P}(X = 1 \text{ and } Y = 1) = .4 \tag{3}$$
$$\mathbb{P}(X = 1 \text{ and } Y = 2) = .2 \tag{4}$$

(a) Calculate the (unconditional) $\mathbb{E}[X]$, $\mathbb{E}[Y]$, and $\mathbb{E}[XY]$?

(b) Calculate the (conditional) $\mathbb{E}[X \mid Y = 1], \mathbb{E}[X \mid Y = 2]$, and $\mathbb{E}[XY \mid Y = 1]$?

Question 7

Solve the following optimization problems. Please be <u>explicit</u> in your transformation to our canonical form of constrained optimization, and be <u>formal</u> with Lagrange multipliers, first order necessary conditions, inequalities, etc.

(a)

$$\max_x \left\{-x^2 + 2x + 3\right\} \tag{5}$$
$$\text{s.t. } x \geq 0 \tag{6}$$

(b)

$$\min_x \{2x + 3\} \tag{7}$$
$$\text{s.t. } x \leq 1 \tag{8}$$

[2]Remember the notation $\boldsymbol{\partial} f(z) \equiv \frac{df(z)}{dz}$. You don't need to know anything about differential equations to do this problem.

[3]Make sure to show the correct setup with numbers in the equations, but I don't need to see intermediate steps in the calculation after that.