

**Theorem 1.** *The core  $\mathcal{A} = \mathcal{C}^{\geq 0} \cap \mathcal{C}^{\leq 0} \subset \mathcal{C}$  is an abelian category.*

**Definition 2** (cohomology object). *The  $i$ -th cohomology object of  $X \in \mathcal{C}$  is defined as*

$$H^0(X) = \tau_{[0,0]}(X) \in \mathcal{A},$$

$$H^i(X) = H^0(X[i]) \in \mathcal{A}.$$

**Definition 3** (nondegenerate t-structure). *A t-structure on  $\mathcal{C}$  is nondegenerate if  $\bigcap_n \text{Ob } \mathcal{C}^{\geq n} = \bigcap_n \text{Ob } \mathcal{C}^{\leq n} = \{0\}$ .*

**Theorem 4.**  *$H^0$  is a cohomological functor.*

*If additionally the t-structure is nondegenerate, then*

- *$f : X \rightarrow Y$  in  $\mathcal{C}$  is an isomorphism iff  $\forall_i H^i(f)$  is an isomorphism,*
- *$\text{Ob}(\mathcal{C}^{\leq n}) = \{X \in \text{Ob } \mathcal{C} : \forall_{i > n} H^i(X) = 0\}$ ,*
- *$\text{Ob}(\mathcal{C}^{\geq n}) = \{X \in \text{Ob } \mathcal{C} : \forall_{i < n} H^i(X) = 0\}$ .*

**Definition 5** (bounded t-structure). *A t-structure is bounded if it is nondegenerate and for any  $X \in \mathcal{C}$ ,  $H^i(X) \neq 0$  only for a finite number of  $i$ .*

**Definition 6** (Ext).  $\text{Ext}_{\mathcal{C}}^i(X, Y) = \text{Hom}_{\mathcal{C}}(X, Y[i])$

**Definition 7** (multiplication on Ext). *Notice that  $\text{Hom}_{\mathcal{C}}(X, Y[i]) = \text{Hom}_{\mathcal{C}}(X[k], Y[i+k])$  and define multiplication  $\text{Ext}_{\mathcal{C}}^i(X, Y) \times \text{Ext}_{\mathcal{C}}^j(Y, Z) \rightarrow \text{Ext}_{\mathcal{C}}^{i+j}(X, Z)$  in the most obvious way.*

**Theorem 8.** *Let  $\mathcal{A}$  be a core of a bounded t-structure on  $\mathcal{C}$ . Assume  $F : \mathcal{D}^b(\mathcal{A}) \rightarrow \mathcal{C}$  satisfies*

$$F(\mathcal{D}^b(\mathcal{A})^{\geq 0}) \subset \mathcal{C}^{\geq 0},$$

$$F(\mathcal{D}^b(\mathcal{A})^{\leq 0}) \subset \mathcal{C}^{\leq 0},$$

*then  $F$  is an equivalence of categories iff  $\text{Ext}_{\mathcal{C}}$  is generated by  $\text{Ext}_{\mathcal{C}}^1$  under Yoneda multiplication.*

**Remark 9.** In chain complexes  $\text{Ext}_{\mathcal{C}}^i(X, Y) : 0 \rightarrow Y \rightarrow E_i \xrightarrow{d^i} \dots \xrightarrow{d^2} E_1 \xrightarrow{d^1} X \rightarrow 0$ .

**Theorem 10.** *Assume that  $\mathcal{C}$  satisfies additionally:*

- *TR5. Arbitrary coproducts and products exist in  $\mathcal{C}$ .*
- *There is a generating set  $\Lambda$  of objects in  $\mathcal{C}$ , i.e. set  $\Lambda$  such that*
  - *$T(\Lambda) \subset \Lambda$ ,  $T$  – translation functor,*
  - *if  $X \in \mathcal{C}$  and  $\forall_{\lambda \in \Lambda} \text{Hom}(\lambda, X) = 0$ , then  $X \simeq 0$ .*

*Then any homological functor  $H : \mathcal{C} \rightarrow \mathcal{A}$  (where  $\mathcal{A}$  is abelian) which sends coproducts to products is representable, i.e.  $H = \mathcal{C}(\cdot, h)$ .*