

## Applications of stable derived functors

**Theorem 1.**  $T : R\text{-mod} \rightarrow R\text{-mod}$ , then

$$L_i^s T(A) = \lim_n \pi_{i+n}(T(\tilde{R}[S^n] \otimes P_*)) \lim_n H_{i+n}(T(\tilde{R}[S^n] \otimes P_*)),$$

where  $S^n$  is any simplicial model of  $n$ -sphere,  $\tilde{R}[\gamma] = R[\gamma]/R[*]$  a simplicial set,  $P_*$  is any projective resolution of  $A$ .

The limit is taken via suspension

$$\pi_{i+n}(T(\tilde{R}[S^n] \otimes P_*)) \rightarrow \pi_{i+n+1}(S^1 \wedge T(\tilde{R}[S^n] \otimes P_*)) \rightarrow \pi_{i+n+1}(T(\tilde{R}[S^{n+1}] \otimes P_*)).$$

In general for  $S^1 \wedge F(X) \rightarrow F(S^1 \wedge X)$  one has to have for any  $z \in S^1$ ,  $F(X) \rightarrow F(S^1 \wedge X)$ ,  $X \rightarrow S^1 \wedge X$ ,  $x \rightarrow z \wedge x$ .

One takes  $R = \mathbb{Z}/p$  or  $R = \mathbb{Z}$ .

$$L_i^s T(\mathbb{Z}/p) = \lim_n \pi_{i+n} T(\mathbb{Z}/p[S^n]), \text{ but } \widetilde{\mathbb{Z}/p}[S^n] = K(\mathbb{Z}/p, n), \tilde{\mathbb{Z}}[S^n] = K(\mathbb{Z}, n).$$

Stalk skewed gra..itions on  $H^*(\bullet, \mathbb{Z}/p)$  is

$$H_*^s(K(\mathbb{Z}/p), \mathbb{Z}/p) = H_*^s(K(\mathbb{Z}/p, n), \mathbb{Z}/p) = L_*^s \mathbb{Z}_p[.](\mathbb{Z}/p). (?)$$

**Theorem 2.** Let  $SP^i$  be the  $i$ -th symmetric power functor, and  $SP_p^i$  the  $p$ -reduced  $i$ -th symmetric power, and  $SP_p^* = \bigoplus SP_p^i / \langle x^p - 1 \rangle$ .

$$\text{Then } L_*^s SP^*(\mathbb{Z}/p) = H_*^s(K(\mathbb{Z}), \mathbb{Z}/p), L_*^s SP_p^*(\mathbb{Z}/p) = H_*^s(K(\mathbb{Z}/p), \mathbb{Z}/p).$$

Calculations: Let  $\Gamma$  be a category of functors  $T : \text{finite pointed sets} \rightarrow \mathbb{Z}/p\text{-vect}$ ,  $T(*) = 0$ .  $L \in \Gamma$  is defined as  $L(X) = \widetilde{\mathbb{Z}/p}[X]$ .

**Lemma 3.** Let  $T : \mathbb{Z}/p\text{-vect} \rightarrow \mathbb{Z}/p\text{-vect}$ . Then  $L_i^s T(\mathbb{Z}/p) = \text{Tor}_i^\Gamma(L^*, T \circ L)$ , where  $L^*(X) = L(X)^*$ , and

I have found these notes useful in understanding derived functors.

Some detailed constructions are here, and some worked out examples are here in section 2.2.