

Remark 1. $C_*(\mathcal{C})$ does not have enough projective objects.

Theorem 2. The sequence of functors $\{H_i\}_{i=0}^\infty$ gives us a universal δ -functor (takes short exact sequences to long exact sequences), i.e. if we have another sequence T_i such that $T_0 = H_0$, then $\forall_i H_i^* = T_i^*$.

Lemma 3. For a given $C_* \in C_*(\mathcal{C})$ there exists $P_* \rightarrow C_*$ such that $H_i(P_*) = 0$ for $i > 0$.

Remark 4. If $p + q = n$, let $f_{pq} : X_{nn} \rightarrow X_{pq}$ be defined as $d_{p+1}^h \circ \dots \circ d_n^h \circ d_0^v \circ \dots \circ d_0^v$, and then the Alexander-Whitney map $\sum_{p+q=n} f_{pq} : X_{nn} \rightarrow \bigoplus_{p+q=n} X_{pq}$ gives a chain homotopy equivalence of $k(X_{pp})$ and $\text{tot}(kX_{pq})$.

Remark 5. We may take a projective simplicial resolution P_* of A of degree $n > i$, then $L_i^s T(A) = H_{n+i}(T(P_*))$.

Theorem 6. $\deg L_i T(\bullet, n) \leq \lfloor \frac{i}{n} \rfloor$.

Remark 7. Or theorem? Or proof? It is written that $T((A, n) \oplus (B, n)) = T(A, n) \oplus T(B, n) \oplus V$ where V is trivial below $2n$.

Proposition 8. $\forall_i L_i^s T$ is an additive functor.

Proposition 9. Let $0 \rightarrow A \rightarrow B \rightarrow C \rightarrow 0$ be exact in \mathcal{C} . Then we have a long exact sequence $\dots \rightarrow L_{q+1}^s T(C) \rightarrow L_q^s T(A) \rightarrow L_q^s T(B) \rightarrow L_q^s T(C) \rightarrow \dots$

Proposition 10. If $0 \rightarrow T' \rightarrow T \rightarrow T'' \rightarrow 0$ is an exact sequence of functors, then we have a long exact sequence of functors $\dots \rightarrow L_{i+1}^s T'' \rightarrow L_i^s T' \rightarrow L_i^s T \rightarrow L_i^s T'' \rightarrow \dots$

Proposition 11. Let U be an additive functor, then for any functor T we have $\text{Hom}_{sth}(T, U) \simeq \text{Hom}_{sth2}(L_0^s T, U)$.