We began with some motivation-discussion. Ain't got time for writing that nicely (in a motivating way).

Theorem 1 (definition of derived category)**.** *Let $\mathcal{C}$ be an abelian category and $\mathrm{Kom}(\mathcal{C})$ denote the category of cochain complexes over $\mathcal{C}$. Then there is a category $\mathrm{D}(\mathcal{C})$ (derived category of $\mathcal{C}$) and a functor $\mathcal{Q} : \mathrm{Kom}(\mathcal{C}) \to \mathrm{D}(\mathcal{C})$ such that*

1. *For every quasi-isomorphism $f \in \mathrm{Mor}(\mathrm{Kom}(\mathcal{C}))$, $\mathcal{Q}(f)$ is an isomorphism.*
2. *$\mathcal{Q}$ is universal with respect to 1, i.e. for every $\mathcal{A}$ and $F : \mathrm{Kom}(\mathcal{C}) \to \mathcal{A}$, such that for every quasi-isomorphism f the map $F(f)$ is invertible, there exists $\mathcal{Q}F$ making the diagram commutative:*

$$\begin{tikzcd} \mathrm{Kom}(\mathcal{C}) \arrow[r, "\mathcal{Q}"] \arrow[dr, "F"'] & \mathrm{D}(\mathcal{C}) \arrow[d, "\mathcal{Q}F"] \\ & \mathcal{A} \end{tikzcd}$$

$\mathrm{D}(\mathcal{C})$ *is called the* derived category *of $\mathcal{C}$.*

Definition 2 (localisation of a category)**.** *$\mathcal{B}$ is a category, S a class of morphisms in $\mathcal{B}$. We can find a new category $\mathcal{B}[S^{-1}]$ and a functor $L : \mathcal{B} \to \mathcal{B}[S^{-1}]$ such that for any functor $F : \mathcal{B} \to \mathcal{B}'$ which takes any $s \in S$ to an isomorphism there exists a functor $LF : \mathcal{B}[S^{-1}] \to \mathcal{B}'$ such that*

$$\begin{tikzcd} \mathcal{B} \arrow[r, "L"] \arrow[dr, "F"'] & \mathcal{B}[S^{-1}] \arrow[d, "LF"] \\ & \mathcal{B}' \end{tikzcd}$$

Fact 3. $\mathrm{D}(\mathcal{C}) = \mathrm{Kom}(\mathcal{C})[(\mathrm{q}-\mathrm{iso})^{-1}]$

Definition 4. *The class $S \subset \mathrm{Mor}(\mathcal{B})$ is localising if it satisfies*

- $\forall_{X \in \mathrm{Ob}(\mathcal{B})}\, \mathrm{id}_X \in S$,
- $s, t \in S \implies s \circ t \in S$,
- $\forall_{s \in S, f \text{ any}} \exists_{t \in S, g \text{ any}}$

$$\begin{tikzcd} W \arrow[r, "g"] \arrow[d, "t"] & Z \arrow[d, "s"] \\ X \arrow[r, "f"] & Y \end{tikzcd}$$

- $\forall_{t \in S, g \text{ any}} \exists_{s \in S, f \text{ any}}$ *as above,*
- $f, g : X \to Y$, *then* $\exists_{s \in S}\, sf = sg \iff \exists_{t \in S}\, ft = gt$.

Lemma 5. *If S is localizing in $\mathcal{B}$, then we can present any morphism in $\mathcal{B}[S^{-1}]$ as a triangle $X \xleftarrow{s} Z \xrightarrow{f} Y$ with equivalence $(s, f) \sim (t, g) \iff \exists r \in S, h$*

$$\begin{tikzcd}
 & Z'' \arrow[dl, dotted, "r"'] \arrow[dr, dotted, "h"] & \\
Z \arrow[d, "s"'] \arrow[drr, "f"' near start] & & Z' \arrow[d, "g"] \arrow[dll, "t" near start] \\
X & & Y
\end{tikzcd}$$

Also, an equivalent statement with left fractions is true.

Lemma 6 (composition)**.** *Like that.*

$$\begin{tikzcd}
 & & X'' \arrow[dl, dotted, "t'"] \arrow[dr, dotted, "f'"] & & \\
 & X' \arrow[dl, "s"] \arrow[dr, "f"] & & Y' \arrow[dl, "t"] \arrow[dr, "g"] & \\
X & & Y & & Z
\end{tikzcd}$$

Remark 7. Class of quasi-isomorphisms is not localising in $\mathrm{Kom}(\mathcal{C})$.