Applications of stable derived functors

Theorem 1. $T : \mathrm{R}-\mathrm{mod} \to \mathrm{R}-\mathrm{mod}$, *then*

$$L_i^s T(A) = \lim_n \pi_{i+n}(T(\tilde{R}[S^n] \otimes P_*)) \lim_n H_{i+n}(T(\tilde{R}[S^n] \otimes P_*)),$$

where S^n is any simplicial model of n-sphere, $\tilde{R}[\gamma] = {}^{R[\gamma]}\big/_{R[*]}$ a simplicial set, P_* is any projective resolution of A.

The limit is taken via suspension
$\pi_{i+n}(T(\tilde{R}[S^n] \otimes P_*)) \to \pi_{i+n+1}(S^1 \wedge T(\tilde{R}[S^n] \otimes P_*)) \to \pi_{i+n+1}(T(\tilde{R}[S^{n+1}] \otimes P_*))$.

In general for $S^1 \wedge F(X) \to F(S^1 \wedge X)$ one has to have for any $z \in S^1$, $F(X) \to F(S^1 \wedge X)$, $X \to S^1 \wedge X$, $x \to z \wedge x$.

One takes $R = \mathbb{Z}/p$ or $R = \mathbb{Z}$.

$L_i^s T(\mathbb{Z}/p) = \lim \pi_{i+n} T(\mathbb{Z}/p[S^n])$, but $\widetilde{\mathbb{Z}/p}[S^n] = K(\mathbb{Z}/p, n)$, $\tilde{\mathbb{Z}}[S^n] = K(\mathbb{Z}, n)$.

Stalk skewed gra..itions on $H^*(\bullet, \mathbb{Z}/p)$ is
$H_*^s(K(\mathbb{Z}/p), \mathbb{Z}/p) = H_*^s(K(\mathbb{Z}/p, n), \mathbb{Z}/p) = L_*^s \mathbb{Z}_p[.](\mathbb{Z}/p)$. (?)

Theorem 2. *Let SP^i be the i-th symmetric power functor, and SP_p^i the p-reduced i-th symmetric power, and $SP_p^* = {}^{\bigoplus SP^i}\big/_{\langle x^p - 1\rangle}$.*

Then $L_*^s SP^*(\mathbb{Z}/p) = H_*^s(K(\mathbb{Z}), \mathbb{Z}/p)$, $L_*^s SP_p^*(\mathbb{Z}/p) = H_*^s(K(\mathbb{Z}/p), \mathbb{Z}/p)$.

Calculations: Let Γ be a category of functors T : finite pointed sets $\to \mathbb{Z}/p-$vect, $T(*) = 0$. $L \in \Gamma$ is defined as $L(X) = \widetilde{\mathbb{Z}/p}[X]$.

Lemma 3. *Let $T : \mathbb{Z}/p-\mathrm{vect} \to \mathbb{Z}/p-\mathrm{vect}$. Then $L_i^s T(\mathbb{Z}/p) = \mathrm{Tor}_i^\Gamma(L^*, T \circ L)$, where $L^*(X) = L(X)^*$, and*

~~OK, I am blown up. Break.~~

I have found these notes useful in understanding derived functors.

Some detailed constructions are here, and some worked out examples are here in section 2.2.