

**Remark 1.**  $C_*(\mathcal{C})$  does not have enough projective objects.

**Theorem 2.** The sequence of functors  $\{H_i\}_{i=0}^\infty$  gives us a universal  $\delta$ -functor (takes short exact sequences to long exact sequences), i.e. if we have another sequence  $T_i$  such that  $T_0 = H_0$ , then  $\forall_i H_i^* = T_i^*$ .

**Lemma 3.** For a given  $C_* \in C_*(\mathcal{C})$  there exists  $P_* \rightarrow C_*$  such that  $H_i(P_*) = 0$  for  $i > 0$ .

**Remark 4.** If  $p + q = n$ , let  $f_{pq} : X_{nn} \rightarrow X_{pq}$  be defined as  $d_{p+1}^h \circ \dots \circ d_n^h \circ d_0^v \circ \dots \circ d_0^v$ , and then the Alexander-Whitney map  $\sum_{p+q=n} f_{pq} : X_{nn} \rightarrow \bigoplus_{p+q=n} X_{pq}$  gives a chain homotopy equivalence of  $k(X_{pp})$  and  $\text{tot}(kX_{pq})$ .

**Remark 5.** We may take a projective simplicial resolution  $P_*$  of  $A$  of degree  $n > i$ , then  $L_i^s T(A) = H_{n+i}(T(P_*))$ .

**Theorem 6.**  $\deg L_i T(\bullet, n) \leq \lfloor \frac{i}{n} \rfloor$ .

**Remark 7.** Or theorem? Or proof? It is written that  $T((A, n) \oplus (B, n)) = T(A, n) \oplus T(B, n) \oplus V$  where  $V$  is trivial below  $2n$ .

**Proposition 8.**  $\forall_i L_i^s T$  is an additive functor.

**Proposition 9.** Let  $0 \rightarrow A \rightarrow B \rightarrow C \rightarrow 0$  be exact in  $\mathcal{C}$ . Then we have a long exact sequence  $\dots \rightarrow L_{q+1}^s T(C) \rightarrow L_q^s T(A) \rightarrow L_q^s T(B) \rightarrow L_q^s T(C) \rightarrow \dots$

**Proposition 10.** If  $0 \rightarrow T' \rightarrow T \rightarrow T'' \rightarrow 0$  is an exact sequence of functors, then we have a long exact sequence of functors  $\dots \rightarrow L_{i+1}^s T'' \rightarrow L_i^s T' \rightarrow L_i^s T \rightarrow L_i^s T'' \rightarrow \dots$

**Proposition 11.** Let  $U$  be an additive functor, then for any functor  $T$  we have  $\text{Hom}_{sth}(T, U) \simeq \text{Hom}_{sth2}(L_0^s T, U)$ .