

Applications of stable derived functors

Theorem 1. $T : R\text{-mod} \rightarrow R\text{-mod}$, then

$$L_i^s T(A) = \lim_n \pi_{i+n}(T(\tilde{R}[S^n] \otimes P_*)) \lim_n H_{i+n}(T(\tilde{R}[S^n] \otimes P_*)),$$

where S^n is any simplicial model of n -sphere, $\tilde{R}[\gamma] = R[\gamma]/R[*]$ a simplicial set, P_* is any projective resolution of A .

The limit is taken via suspension

$$\pi_{i+n}(T(\tilde{R}[S^n] \otimes P_*)) \rightarrow \pi_{i+n+1}(S^1 \wedge T(\tilde{R}[S^n] \otimes P_*)) \rightarrow \pi_{i+n+1}(T(\tilde{R}[S^{n+1}] \otimes P_*)).$$

In general for $S^1 \wedge F(X) \rightarrow F(S^1 \wedge X)$ one has to have for any $z \in S^1$, $F(X) \rightarrow F(S^1 \wedge X)$, $X \rightarrow S^1 \wedge X$, $x \rightarrow z \wedge x$.

One takes $R = \mathbb{Z}/p$ or $R = \mathbb{Z}$.

$$L_i^s T(\mathbb{Z}/p) = \lim \pi_{i+n} T(\mathbb{Z}/p[S^n]), \text{ but } \widetilde{\mathbb{Z}/p}[S^n] = K(\mathbb{Z}/p, n), \tilde{\mathbb{Z}}[S^n] = K(\mathbb{Z}, n).$$

Stalk skewed gra..itions on $H^*(\bullet, \mathbb{Z}/p)$ is

$$H_*^s(K(\mathbb{Z}/p), \mathbb{Z}/p) = H_*^s(K(\mathbb{Z}/p, n), \mathbb{Z}/p) = L_*^s \mathbb{Z}_p[.](\mathbb{Z}/p). (?)$$

Theorem 2. Let SP^i be the i -th symmetric power functor, and SP_p^i the p -reduced i -th symmetric power, and $SP_p^* = \bigoplus SP_p^i / \langle x^p - 1 \rangle$.

$$\text{Then } L_*^s SP^*(\mathbb{Z}/p) = H_*^s(K(\mathbb{Z}), \mathbb{Z}/p), L_*^s SP_p^*(\mathbb{Z}/p) = H_*^s(K(\mathbb{Z}/p), \mathbb{Z}/p).$$

Calculations: Let Γ be a category of functors $T : \text{finite pointed sets} \rightarrow \mathbb{Z}/p\text{-vect}$, $T(*) = 0$. $L \in \Gamma$ is defined as $L(X) = \widetilde{\mathbb{Z}/p}[X]$.

Lemma 3. Let $T : \mathbb{Z}/p\text{-vect} \rightarrow \mathbb{Z}/p\text{-vect}$. Then $L_i^s T(\mathbb{Z}/p) = \text{Tor}_i^\Gamma(L^*, T \circ L)$, where $L^*(X) = L(X)^*$, and

OK, I am blown up. Break.