We began with some motivation-discussion. Ain't got time to write that nicely (in a motivating way).

**Theorem 1** (definition of derived category). Let C be an abelian category and Kom(C) denote the category of cochain complexes over C. Then there is a category D(C) (derived category of C) and a functor  $Q : Kom(C) \to D(C)$  such that

- 1. For every quasi-isomorphism  $f \in \text{Mor}(\text{Kom}(\mathcal{C})), \mathcal{Q}(f)$  is an isomorphism.
- 2. Q is universal with respect to 1, i.e. for every A and F:  $Kom(C) \rightarrow A$ , such that for every quasi-isomorphism f the map F(f) is invertible, there exists QF making the diagram commutative:

$$\operatorname{Kom}(\mathcal{C}) \xrightarrow{\mathcal{Q}} \operatorname{D}(\mathcal{C})$$

$$f \searrow \operatorname{\mathcal{Q}F}$$

$$\mathcal{A}$$

D(C) is called the derived category of C.

**Definition 2** (localisation of a category).  $\mathcal{B}$  is a category, S a class of morphisms in  $\mathcal{B}$ . We can find a new category  $\mathcal{B}[S^{-1}]$  and a functor  $L: \mathcal{B} \to \mathcal{B}[S^{-1}]$  such that for any functor  $F: \mathcal{B} \to \mathcal{B}'$  which takes any  $s \in S$  to an isomorphism there exists a functor  $LF: \mathcal{B}[S^{-1}] \to \mathcal{B}'$  such that

$$\mathcal{B} \xrightarrow{L} \mathcal{B}[S^{-1}]$$

$$F \searrow \swarrow : F$$

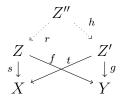
$$\mathcal{B}'$$

Fact 3.  $D(C) = Kom(C)[(q - iso)^{-1}]$ 

**Definition 4.** The class  $S \subset \text{Mor}(\mathcal{B})$  is localising if it satisfies

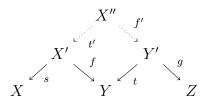
- $\forall_{X \in \mathrm{Ob}(\mathcal{B})} \mathrm{id}_X \in S$
- $s, t \in S \implies s \circ t \in S$
- $\begin{array}{ccc}
  \bullet & \forall_{s \in S, f} \exists_{t \in S, g} \\
  W & \xrightarrow{g} Z \\
  \downarrow t & \downarrow s \\
  X & \xrightarrow{f} Y
  \end{array}$
- $\forall_{t \in S,g} \exists_{s \in S,f} \ as \ above$
- $f, g: X \to Y$ , then  $\exists_{s \in S} sf = sg \iff \exists_{t \in S} ft = gt$ .

**Lemma 5.** If S is localizing in  $\mathcal{B}$ , then we can present any morphism in  $\mathcal{B}[S^{-1}]$  as a triangle  $X \stackrel{s}{\leftarrow} Z \xrightarrow{Y}$  with equivalence  $(s, f) \sim (t, g) \iff \exists r \in S, h$ 



Also, an equivalent statement with left fractions is true.

Lemma 6 (composition). Like that.



**Remark 7.** Class of quasi-isomorphism is not localising in  $Kom(\mathcal{C})$ .