Let $\mathcal{C}$ be abelian, remind $s\mathcal{C}$ – simplicial objects in $\mathcal{C}$, $C_*(\mathcal{C})$ – chain complexes over $\mathcal{C}$.

Definition 1 (s-morphism)**.** *$X_\bullet, Y_\bullet \in s\mathcal{C}$. For a simplicial set $K \in s$Set a map which associates $F(\sigma) : X_n \to Y_n$ to any $\sigma \in K_n$ is called s-morphism (denote $F : K \times X_\bullet \to Y_\bullet$) if for any $\alpha : [m] \to [n]$ in Δ we have $F(K(\alpha)(\sigma)) \circ X(\alpha) = Y(\alpha)F(\sigma)$.*

Observe that when $X_\bullet, Y_\bullet$ are in sSet then s-morphisms are simplicial maps $K_\bullet \times X_\bullet \to Y_\bullet$.

Example 2. If $K = \Delta[0]$, then s-morphism is just a simplicial morphism $X_\bullet \to Y_\bullet$.

Example 3. If $K = \Delta[1]$ then s-morphism is called *a homotopy* between $F(0)$ and $F(1)$.

Remark 4. $T : \mathcal{C} \to \mathcal{C}'$ functor induces $T : s\mathcal{C} \to s\mathcal{C}'$, if $X_\bullet \in s\mathcal{C}$, then $T(X)_n = T(X_n), T(X)(\alpha) = T(X(\alpha))$,

Definition 5. *If F is an s-morphism $F : K_\bullet \times X_\bullet \to Y_\bullet$, then it defines $TF : K_\bullet \times T(X_\bullet) \to T(Y_\bullet)$ defined by $TF(\sigma) = T(F(\sigma))$.*

Remark 6. Any functor $T : \mathcal{C} \to \mathcal{C}'$ sends homotopic maps to homotopic ones.

Definition 7. *Let $\mathcal{C}$ be abelian, then there are functors*

$$s\mathcal{C} \underset{K}{\overset{N}{\rightleftarrows}} C_*(\mathcal{C})$$

defined as follows.

Normalization N is defined, for $X_\bullet \in s\mathcal{C}$, as

$$N(X)_n = \bigcap_{i=1}^{n} \ker(d_i : X_n \to X_{n-1})$$

(e.g. $\ker(X_n \xrightarrow{\prod d_i} \prod X_{n-1})$*), with*

$$d : N(X)_n \to N(X)_{n-1}$$

induced by d_0.

K is defined in such a way. If $\alpha : [n] \to [q]$, then $d(\alpha) = n$ and $r(\alpha) = q$. Notice for any α there is unique $\alpha = \varepsilon \circ \eta$, where ε is an injection and η is a surjection. For $C \in C_*(\mathcal{C})$, take

$$K(C)_n = \bigoplus_{\eta : d(\eta) = n} C_{r(\eta)},$$

Now for $\alpha : [m] \to [n]$ define

$$KC(\alpha) : K(C)_n \to K(C)_m$$

on every $C_{r(\eta)}$ in such a way: $\eta\alpha = \varepsilon'\eta'$, let $KC(\alpha)$ map $C_{r(\eta)}$ into $C_{r(\eta')}$ via the formula

$$K(\eta, \alpha) = \begin{cases} \mathrm{id}_{C_{r(\eta)}} & \text{for } \varepsilon' = \mathrm{id}_{[r(\eta)]} \\ d : C_{r(\eta)} \to C_{r(\eta)-1} = C_{r(\eta')} & \text{for } \varepsilon' = \varepsilon^0 \\ 0 & \text{otherwise} \end{cases}.$$

Remark 8. Observe that if $f : C \to D$ in $C_*(\mathcal{C})$, then the induced map $KC \to KD$ is simplicial.

Theorem 9 (Dold-Kan)**.** *The functors N and K give an equivalence of $s\mathcal{C}$ and $C_*(\mathcal{C})$.*

Remark 10. It was somehow convenient to define, for $X \in s\mathcal{C}$, $\bar{X} \in s\mathcal{C}$ via $\bar{X}_n = \ker(d_{n+1} : X_{n+1} \to X_n)$.

Lemma 11. *Let $f : X_\bullet \to Y_\bullet$ be a simplicial morphism which satisfies $(Nf)_i$ is mono(epi) for $i \leqslant n$. Then $f_n : X_n \to Y_n$ is mono(epi) for $i \leqslant n$.*