Definition 1. $K^{\leq n}(A), D^{\leq n}(A)$ – full subcategories of objects X for which $H^i(X) = 0$ for i > n.

Analogously one defines $K^{\geqslant n}(\mathcal{A})$, $D^{\geqslant n}(\mathcal{A})$.

We know $\mathcal{A} = D^{\leq 0}(\mathcal{A}) \cap D^{\geq 0}(\mathcal{A}).$

Definition 2. Let C be a triangulated category. Assume that D is a subcategory of C. Then we write $D^{\leq n} = D^{\leq 0}[-n]$, $D^{\geqslant n} = D^{\geqslant 0}[-n]$.

Definition 3 (t-structure). A t-structure on C consists of a pair of full subcategories $(C^{\leq 0}, C^{\geq 0})$ in C which satisfy the following properties:

- $\mathcal{C}^{\leqslant 0} \subset \mathcal{C}^{\leqslant 1}$, $\mathcal{C}^{\geqslant 0} \supset \mathcal{C}^{\geqslant 1}$
- $X \in \mathcal{C}^{\leqslant 0} \land Y \in \mathcal{C}^{\geqslant 1} \implies \operatorname{Hom}(X, Y) = 0$,
- for all $X \in \mathcal{C}$ there is a distinguished triangle $A \to X \to B \to A[1]$ such that $A \in \mathcal{C}^{\leq 0}, B \in \mathcal{C}^{\geqslant 1}$.

Definition 4 (core). $A = C^{\leq 0} \cap C^{\geq 0}$ is called the core of the t-structure on C.

Proposition 5. In D(A), the subcategories $D^{\leq 0}(A)$ and $D^{\geq 0}(A)$ define a t-structure on D(A).

Theorem 6. The core of any t-structure on triangulated C is an additive category.

Lemma 7. There exist functors $\tau_{\leq n}: \mathcal{C} \to \mathcal{C}^{\leq n}$ $(\tau_{\geq n}: \mathcal{C} \to \mathcal{C}^{\geq n})$ which are right (left) adjoint to the inclusion functors.

Moreover, for any $X \in \text{Ob}(\mathcal{C})$ there exists a distinguished triangle

$$\tau_{\leqslant 0}X \to X \to \tau_{\geqslant 1}X \to (\tau_{\leqslant 0}X)[1].$$

Moreover, any distinguished triangle $A \to X \to B \to A[1]$ such that $A \in \mathcal{C}^{\leq 0}, B \in \mathcal{C}^{\geqslant 1}$ is isomorphic to the one above.

Lemma 8. • $\tau_{\leq n}X = 0 \iff X \to \tau_{\geq n+1}X$ is an isomorphism,

• if $m \leq n$, then there are maps

$$\tau_{\leqslant m} X \to \tau_{\leqslant m} \tau_{\leqslant n} X,$$

$$\tau_{\geqslant n}X \to \tau_{\geqslant n}\tau_{\geqslant m}X,$$

which are isomorphisms, if $m \leq n$, then there is a unique isomorphism

$$\tau_{\geqslant m}\tau_{\leqslant n}X \to \tau_{\leqslant n}\tau_{\geqslant m}X (=\tau_{[m,n]}X).$$