✓ Homogeneous Recurr—

$$a_0 t_n + a_1 t_{n-1} + \dots + a_k t_{n-k} = 0$$

✓ Inhomogeneous Recurrences

$$a_0 t_n + a_1 t_{n-1} + \dots + a_k t_{n-k} = b^n p(n)$$

b is a constant and

$p(n)$ is a polynomial in n of degree

i.e $t_n - 2t_{n-1} = 3^n$

Fibonacci

$$f_n = \begin{cases} n & \text{if } n=0,\ n=1 \\ f_{n-1} + f_{n-2} & \text{otherwise } n \geq 2 \end{cases}$$

f_0	f_1	f_2	f_3	f_4	f_5	$f_6 \Rightarrow f_5 + f_4$
0	1	1	2	3	5	$8 \Rightarrow 5+3$

$\Rightarrow 8$

$f_7 = ?$

$f_6 + f_5 \Rightarrow 8+5 \Rightarrow 13$

Rewrite the formula

$$f_n - f_{n-1} - f_{n-2} = 0$$

$\Delta = b^2 - 4ac$

$r_1 =$

r_2

roots $x^2 - x - 1$

$$r_1 = \frac{1+\sqrt{5}}{2} \quad r_2 = \frac{1-\sqrt{5}}{2}$$

General solution

$$f_n = c_1 r_1^n + c_2 r_2^n$$

$$0 = f_0 = c_1 + c_2 \quad \therefore n = 0$$

$\_\_\_\_ \; 1$

$n = 1$

$$f_1 = c_1 r_1 + c_2 r_2 = 1 \quad \text{—— } 2$$

Solving ① & ②

$$c_1 = \frac{1}{\sqrt{5}} \quad \text{and} \quad c_2 = -\frac{1}{\sqrt{5}}$$

$$f_n = \frac{1}{\sqrt{5}}\left[\underbrace{\left(\frac{1+\sqrt{5}}{2}\right)}_{\phi}^{n} - \underbrace{\left(\frac{1-\sqrt{5}}{2}\right)}_{\hat{\phi}}^{n}\right]$$

Golden ratio | De Moivre's formula

$\phi \quad \frac{1+\sqrt{5}}{2} \Rightarrow 1.61803$

ith term of fibonacci $f_i = \frac{\phi^i - \hat{\phi}^i}{\sqrt{5}}$ is conjugate ϕ

$\left\lfloor \frac{\phi^i}{\sqrt{5}} + \frac{1}{2} \right\rfloor$ OR

$\Rightarrow 13$ for $i=7$,

Multiplication as an operation

fn grows exponentially in a number close to ϕ.

If done recursively.

Source: Fundamentals of Algo
By Gilles Brassard & Paul Bratley Pg. 120

characteristic equation for solving recurr...

Four stages

1) Calculate the first few values of the recurrence
2) look for the regularity
3) Guess a suitable general form
4) Finally prove by math/constructive induction

Ex. $T(n) = \begin{cases} 0 & , n=0 \\ 3T(n/2)+n & \text{otherwise} \end{cases}$

n	1	2	4	8	16	32
$T(n)$	1	5	19	65	211	665

$T(4) = 3 \times T(2) + 4$

1	1
2	$3 \times 1 + 2$
2^2	$3^2 \times 1 + 3 \times 2 + 2^2$
2^3	$3^3 \times 1 + 3^2 \times 2 + 3 \times 2^2 + 2^3$
2^4	$\vdots$
2^5	

$$t(2^k) = 3^k 2^0 + 3^{k-1} 2^1 + 3^{k-2} 2^2 + \dots + 3^0 2^k$$

$$= \sum_{i=0}^{k} 3^{k-i} 2^i$$

$$= 3^k \sum_{i=0}^{k} (2/3)^i$$

$$= 3^k \times \left(1 - (2/3)^{k+1}\right) / \left(1 - \frac{2}{3}\right) = 3^{k+1} - 2^{k+1}$$

$$T(n) = T\left(2^{\lg n}\right)$$

$$= 3^{1+\lg n} - 2^{1+\lg n}$$

$$= 3^1 \cdot 3^{\lg n} - 2^1 \cdot 2^{\lg n}$$

$$= 3 \cdot n^{\lg 3} - 2 \cdot n^1 \quad \because \lg_2 2 \Rightarrow 1$$

$$T(n) \in \Theta\left(n^{\lg 3}\right)$$