Discrete Maths

Arithmetic ~~progression~~

1, 2, 3, ...

$d \Rightarrow 2-1 \Rightarrow 3-2 \Rightarrow 1$

$$\sum n = \frac{n(n+1)}{2}$$

~~Addition~~

Geometric ~~Progression~~ Converge $|ratio| < 1$

multiplication (~~for ratio~~ > 1, ∞ diverge)

$$\frac{term2}{term1} = \frac{term3}{term2} \Rightarrow \text{Common ratio}$$

$$1 \cancel{+x+x^2+x^3} + \ldots \infty \qquad \frac{1}{1-\text{commonratio}} = \frac{1}{1-x}$$

$$\frac{x}{1} = \frac{x^2}{x} = \frac{x^3}{x^2} \Rightarrow x$$

$$\text{firstterm}\left(\frac{cr^n-1}{cr-1}\right)$$

$1 \cdot x^0 + 1 \cdot x^1 + 1 \cdot x^2 + 1 \cdot x^3 + 1 \cdot x^4 + \cdots$

$1 + x + x^2 + x^3 + x^4 + \cdots \infty = \frac{1}{1-x}$

$1, 1, 1, 1, 1 \cdots \infty$

$F(x) = 1 + x + x^2 + x^3 + x^4 + \cdots \infty$ —— (A)

$xF(x) = x + x^2 + x^3 + x^4 + x^5 + \cdots \infty$ —— (B)

From (A) & (B)

$F(x) - xF(x) = 1 + x + x^2 + x^3 + x^4 + \cdots \infty$
$\quad - x - x^2 - x^3 - x^4 - \cdots \infty$

$F(x)(1-x) = 1$

$F(x) = 1/1-x$

Given $G(x)$

Ex : $= \frac{1}{1+x}$, ~~generate series~~

$\frac{1}{1-(-x)}$ Common ratio is $(-x)$

$= a_0(-x)^0 + a_1(-x)^1 + a_2(-x)^2 + a_3(-x)^3 + a_4(-x)^4 + a_5(-x)^5 + \cdots$

$= a_0 1 - a_1 x + a_2 x^2 - a_3 x^3 + a_4 x^4 - a_5 x^5$

$=$ $\because (-1)^2 = 1$

$1, -1, 1, -1, 1, -1, \cdots$

Ex:

$$G(x) = \frac{1}{a - x}$$ give me series $\sum$...

$$G(x) = \sum_{r=0}^{\infty} a_r x^r$$ series terms

$$G(x) = \frac{1/a}{1 - x/a}$$

$$\left(\therefore \frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots \right)$$ G.P

$$= \frac{1}{a}\left(\frac{1}{1 - x/a} \right)$$

$$= \frac{1}{a}\left(1 + \left(\frac{x}{a}\right) + \left(\frac{x}{a}\right)^2 + \left(\frac{x}{a}\right)^3 + \cdots \infty \right)$$

$$= \frac{1}{a}x^0 + \frac{1}{a^2}x + \frac{1}{a^3}x^2 + \frac{1}{a^4}x^3 + \cdots$$

$$a_r = k\,l^r = \frac{1}{a}\left(\frac{1}{a}\right)^r = \left(\frac{1}{a}\right)^{r+1}$$

$$= \left(\frac{1}{a}\right)$$

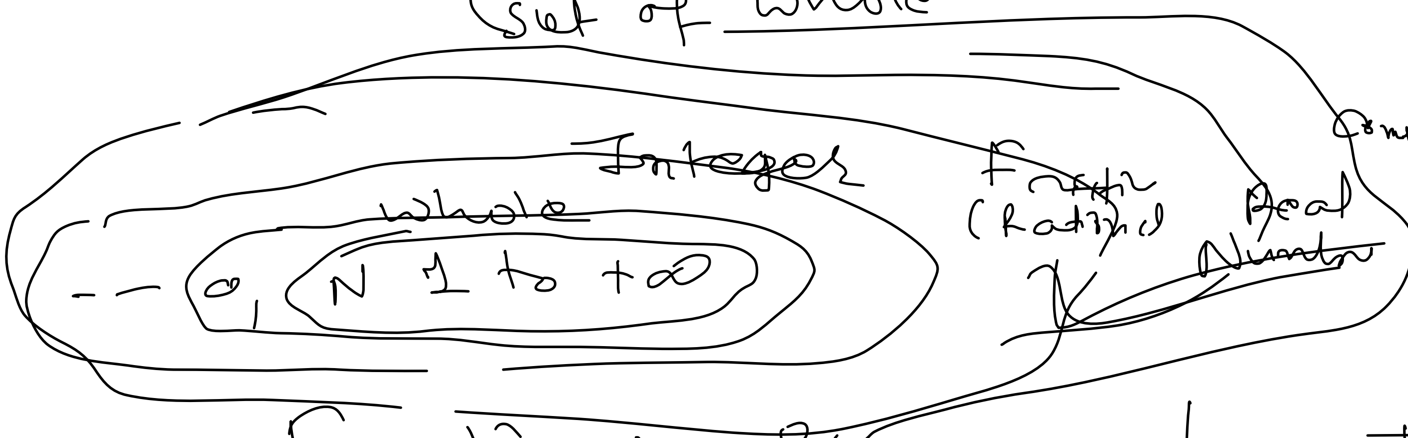
$$\left(\frac{1}{a}\right)^{0+1}, \left(\frac{1}{a}\right)^{1+1}, \left(\frac{1}{a}\right)^{2+1}, \left(\frac{1}{a}\right)^{3+1}, \left(\frac{1}{a}\right)^{4+1} \cdots$$

Numeric function

is a function defined from W to R

$f : W \rightarrow R$ ~~a set~~ of real numbers

↑ set of whole numbers



Fractions (rational) p/q, $q \neq 0$, $p, q \in I$

~~rational~~ Irrational

(1.23 $\quad \pi \quad 3.1451$ .... e, $\sqrt{3}$

Rational Irration numbers

Real $\Rightarrow$ $R + IR$

Imaginary $5i$; Complex $a+bi$

Notation of Numeric function : a_r

$\Downarrow$

$f(x) = x \Rightarrow a_r = r$

$f(x) = -x \Rightarrow a_r = -r$

Gen. Fun Closed Form	Numeric Function Form	Series
$\frac{1}{1-x}$	$a_r = 1$	$1, 1, 1, 1, 1, \ldots$
$\frac{1}{1+x}$	$a_r = (-1)^r$	$1, -1, 1, -1, 1, -1, \ldots$
$\frac{k}{1-x}$	$k \cdot (+1)^r$	$k, k, k, k, \ldots$
$\frac{k}{1+x}$	$k \cdot (-1)^r$	$k, -k, k, -k, \ldots$
$\frac{1}{1-lx}$	l^r	$l^0, l^1, l^2, \ldots$
$\frac{1}{1+lx}$	$(-l)^r$	$(-l)^0, (-l)^1, (-l)^2, \ldots$

$$\frac{k}{1+lx}$$

$$a_r = k(-l)^r$$

$$\frac{1}{a-x}$$

$$a_r = \left(\frac{1}{a}\right)^{r+1}$$

$$\frac{1}{1-x^2}$$

$a_r = 1$, if $r = 2i$

$a_r = 0$, if $r \neq 2i$

$i \geq 0$

$1 + x^2 + x^4 + x^6 + x^8 + \ldots$

$1, 0, 1, 0, 1, 0 \ldots$

Ex

$$1 + 2x + 3x^2 + 4x^3 + 5x^4 + \cdots$$

1, 2, 3, 4 ... Arithmetic ~~Progression~~

$d = 1$

$x^0, x^1, x^2, \ldots$ Geometric Progression

AP + GP

$$F(x) = 1 + 2x + 3x^2 + 4x^3 + 5x^4 + \cdots \quad (a)$$

$$xF(x) = x + 2x^2 + 3x^3 + 4x^4 + 5x^5 + \cdots \quad (b)$$

$F(x) - xF(x)$ using (a) & (b)

$$= 1 + 2x + 3x^2 + 4x^3 + 5x^4 + \cdots$$
$$- x - 2x^2 - 3x^3 - 5x^4 - \cdots$$

$F(x) - xF(x) =$

$1 + x + x^2 + x^3 + x^4 + \cdots$

$= \frac{1}{1-x}$ ($\because$ Gen P ($r \cdot x$, ∞))

$F(x)(1-x) = \frac{1}{1-x}$

$F(x) = \frac{1}{(1-x)^2}$ OGF closed form.

$\cancel{a \quad \frac{1}{(1-x)^2}}$

Ex

$0, 1, 2, 3, 4, 5, 6 - - -$

$\cancel{0x^0} + 1x^1 + 2x^2 + - - -$

$x + 2x^2 + 3x^3 + - - -$

$x(1 + 2x + 3x^2 + - - -)$

$x\left(\frac{1}{(1-x)^2}\right)$ ∵ prev. example

$\frac{x}{(1-x)^2}$

$= \frac{x}{(1-x)^2}$

Fibonacci V1

1, 1, 2, 3, 5 ...

$$F(x) = 1x^0 + 1x^1 + 2x^2 + 3x^3 + 5x^4 + \cdots$$

$$= 1 + x + 2x^2 + 3x^3 + 5x^4 + 8x^5 + \cdots$$

$$xF(x) = x + x^2 + 2x^3 + 3x^4 + 5x^5 + 8x^6 + 21x^7 \quad (A)$$

$$x^2F(x) = x^2 + x^3 + 2x^4 + 3x^5 + 5x^6 + 8x^7 \quad (B)$$

$$xF(x) + x^2F(x)$$

$$= x + 2x^2 + 3x^3 + 5x^4 + 8x^5 + 13x^6 + 21x^7$$

$$= (\cdots + 1) - 1$$

$$= F(x) - 1$$

$$x F(x) + x^2 F(x) = F(x) - 1$$

$$F(x)(x + x^2) = F(x) - 1$$

$$1 = F(x) - F(x)(x + x^2)$$

$$\frac{1}{1 - (x + x^2)} = F(x)$$

$$\frac{1}{1 - (x + x^2)}$$

Fibonacci v2

$0, 1, 1, 2, 3, 5, \ldots \Rightarrow \frac{x}{1 - (x + x^2)}$