Discrete Maths

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Binomial Expansion

$$(a+b)^2 = a^2 + 2ab + b^2$$

a, b two products

(1)	a a	Observe
(2)	a b	→ Repetation vs run
(3)	b a	
(4)	b b	→ order matters or not

slots

$(a+b)^0 \Rightarrow 1$ 1

$(a+b)^1 \Rightarrow (a+b)$ 1 1

$(a+b)^2 \Rightarrow (a^2+2ab+b^2)$ 1 2 1

$(a+b)^3 \Rightarrow a^3+3a^2b+3ab^2+b^3$ 1 3 3 1

$(a+b)^4 \Rightarrow 1a^4+4a^3b+6a^2b^2+4ab^3+b^4$

$\binom{4}{0}$ $\binom{4}{1}$ 1 4 $\binom{4}{2}$ 6 4 $\binom{4}{3}$ $\binom{4}{4}$

$(a+b)^n =$ Selecting place itself with ~~repeation~~ for placing item a

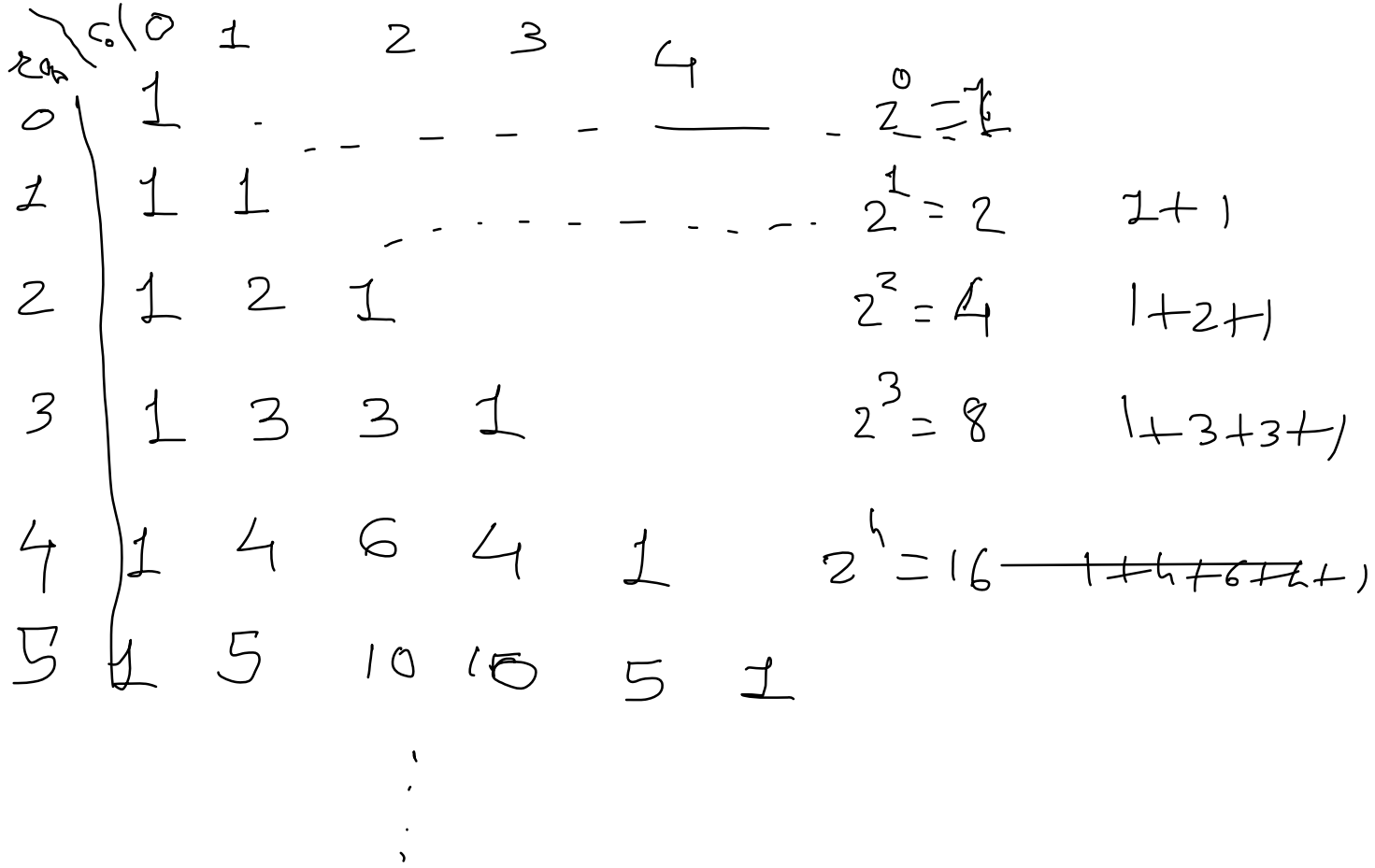
$(a+b)^n$ will have $(n+1)$ terms.

Sum of powers in every term will be n

$(a+b)^2 = a^2+2ab+b^2$

$(b+a)^2 = b^2+2ab+a^2$

$2ba$



PASCAL TRIANGLE

Ways

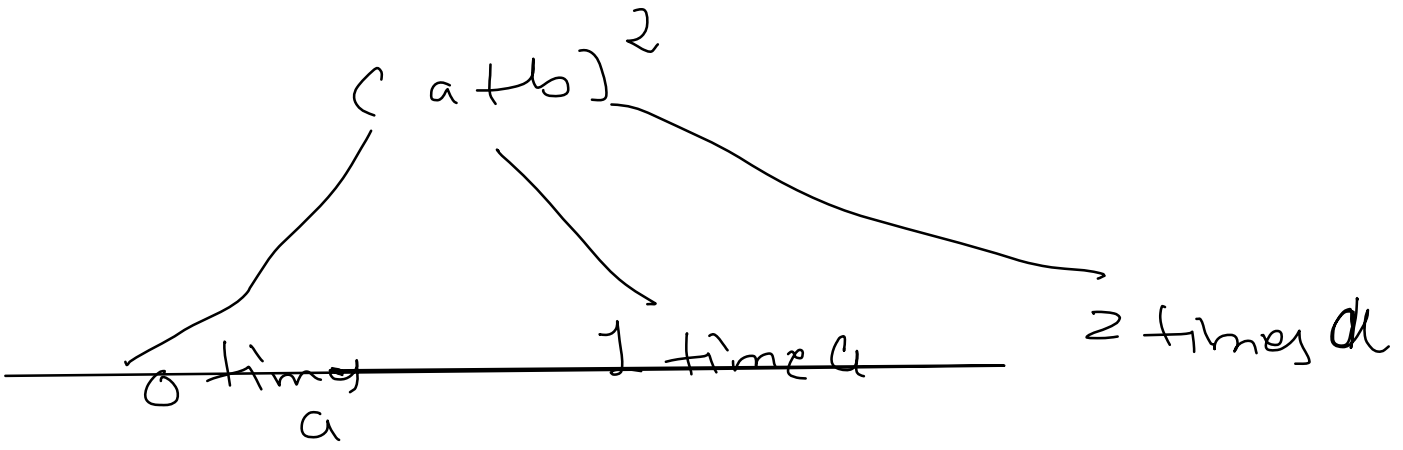
$${}^{n}C_r \Rightarrow \frac{n!}{(n-r)!\,r!}$$

$${}^{n}C_0 \Rightarrow 1$$

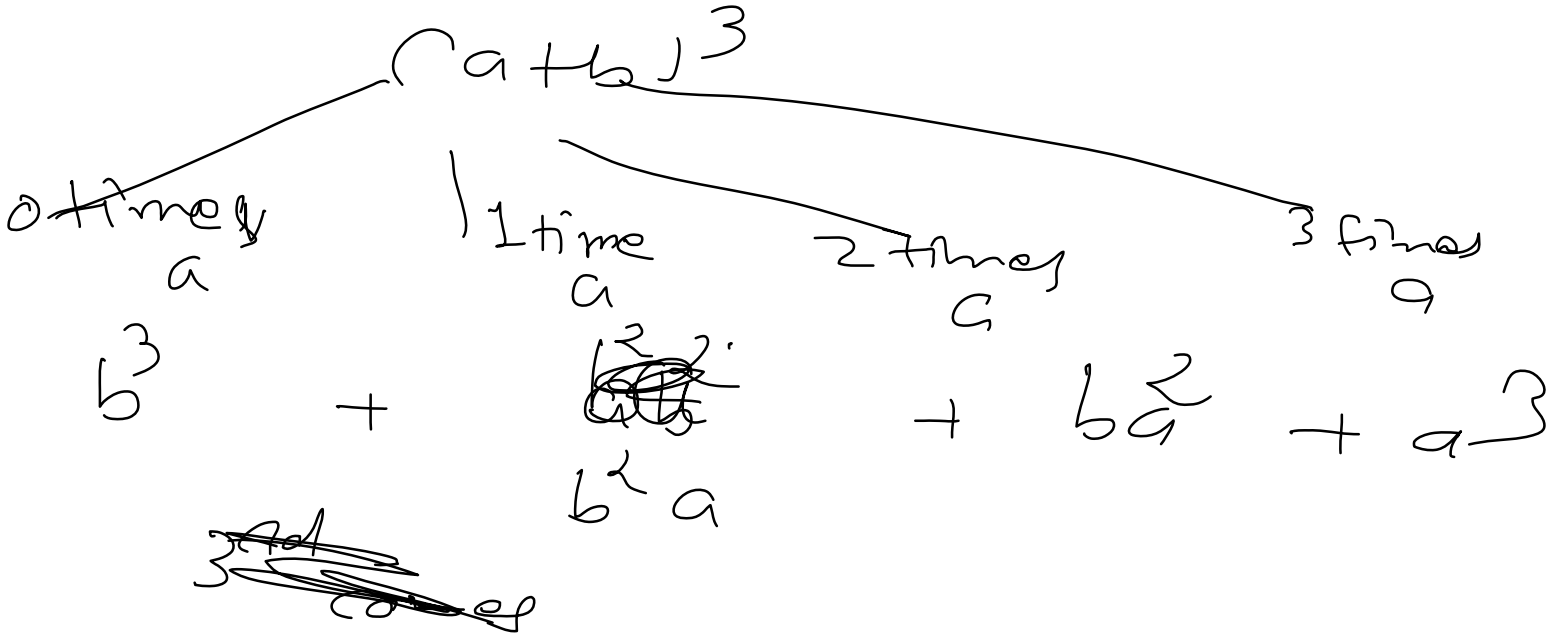
$${}^{n}C_1 \Rightarrow n$$

$${}^{n}C_r \Rightarrow {}^{n-1}C_{r-1} + {}^{n-1}C_r$$

$${}^{n}C_n \Rightarrow {}^{n}C_{n-n} \Rightarrow {}^{n}C_0 \Rightarrow 1$$



$\binom{2}{0}$ $\binom{2}{1}$ $\binom{2}{2}$



$b^3 + ab^2 + ba^2 + a^3$

b^2a

Ordinary Binomial Expansion

$$(a+b)^n = \binom{n}{0} a^n b^0 + \binom{n}{1} a^{n-1} b^1 + \binom{n}{2} a^{n-2} b^2 + \binom{n}{3} a^{n-3} b^3 + \cdots + \binom{n}{n} a^0 b^n$$

$(1+x)^n$

$a \neq b$

$1 + nx + \frac{n(n-1)}{2!}x^2$

$+ \frac{n(n-1)(n-2)}{3!}x^3$

$+ \cdots + \frac{1}{n!}x^n$

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r^{th} term $\Rightarrow \binom{n}{r} a^r b^{n-r}$

Binomial Exp starts from 0

Example: what is the co-efficient of x^{15} in

$$\left(x^2 + x^{-3}\right)^{10}$$

$a \to x^2$, $b \to x^{-3}$, $(a+b)^{10}$

r^{th} term

$$\binom{n}{r} a^r b^{n-r}$$

$$= \binom{10}{r} (x^2)^r (x^{-3})^{10-r}$$

$$= \binom{10}{r} x^{2r-30+3r} = \binom{10}{r} x^{-30+5r}$$

(1)

$-30 + 5r = 15$ $\left(\binom{10}{9} \Rightarrow \binom{10}{1} \Rightarrow 10\right)$

$r = 9.$

(2) x^{12}

$-30+5r=12$

$r=\frac{42}{5}$ $\quad r \notin Z^+$

This term does not exist.

0

Ex what is coeff of $x^6 y^2 z^2$ in $(x+y+z)^{10}$?

theory

$(a+b+c)^n$ n places : arrange a, b, c

$$(a+b+c)^n = \sum \frac{n!}{x!\,y!\,z!}\, a^x b^y c^z$$

$x+y+z = n$

$$(a+b+c+d)^n$$

$$= \sum \frac{n!}{x!\,y!\,z!\,w!}\, a^x b^y c^z d^w$$

$$\therefore x+y+z+w = n$$

$(a+b)^{-n}$

$= \binom{-n}{0}$

n are places.

Extended Binomial ~~Expansion~~

$n \Rightarrow$ replace by $-n$