Discrete Maths

10 -

$$G(x) = \sum_{r=0}^{\infty} a_r x^r$$

<u>Ex</u>. $a_r = (\overset{a_0}{2}, \overset{a_1}{6}, \overset{a_2}{18}, \overset{a_3}{54}, \overset{a_4}{162}, \ldots)$

Find the generating function.

$$G(x) = 2 \cdot x^0 + 6x^1 + 18x^2 + 54x^3 + 162x^4 + \ldots$$

Hint: Geomatric Progression

$$1 + y + y^2 + y^3 + \ldots = \frac{1}{1-y}$$

$$S_n = \underset{\text{first term}}{a} \left(\frac{r^n - 1}{r - 1} \right)$$

Common ratio

n: no. of terms

$$= 2\left[1 + 3x + 9x^2 + 27x^3 + \ldots\ldots\right]$$

$$= \frac{2}{1-3x}$$

$$\therefore \quad G(x) = \frac{2}{1-3x}$$

2. Prove:

$$1+2x+3x^2+4x^3+\dots = \frac{1}{(1-x)^2}$$

$$\frac{d}{dx}\left(\frac{1}{1-x}\right) = 0+1+2x+3x^2+\dots$$

$$\frac{(-1)(-1)}{(1-x)^2} = 1+2x+3x^2+\dots$$

- Fibonacci

$$f(n) = f(n-1) + f(n-2)$$

$$0 \quad 1 \quad \cancel{1} \quad 2 \quad 3 \quad 5 \quad 8 \quad \ldots$$

$$\boxed{G(x) = \frac{1}{1 - x + x^2}}$$

~~Hint:~~ Golden ratio

phi

~~How can I directly find n^{th} term?~~

$x \quad 0.1$

$x \times x \Rightarrow 0.01$

$x \times x \times x \Rightarrow 0.001$

$\vdots$

$x^n \Rightarrow 0.00\text{———}1$

$\vdots$

$x^{\infty} \Rightarrow$ ______

Applications of generating functions

- Find exact formula for the members of seq
- Find a recurrence formula
- Find avg. and other stats
- prove unimodality
- Prove identities
- when any two gfn bear a striking resemblance, you may discover that both problems are related.