Discrete Maths

= 23

Applications note

- generating functions can help find co-efficients of terms ~~smartfully~~

- permutations / combinations ~~number~~ of outcomes ~~required~~ for probability formula.

- probability can be used to ~~fill~~ handle missing values via most probable value during cleaning (pre-processing) of data before actually performing qualitative data science / analytics / ML

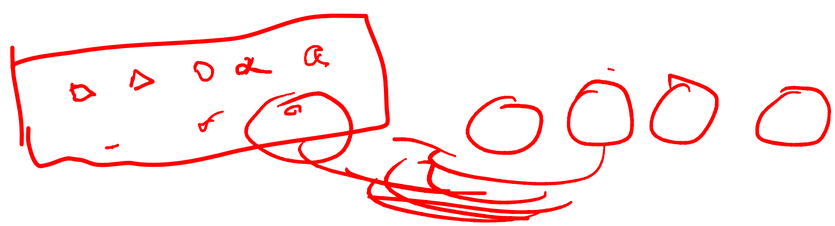
pg. 88 Data mining book

- realtime fraud detection is based on existing patterns vs abnormal activity and probability/chances.

- ~~Investments based on probability~~

- schemes based on maths.

  i.e. - Games in Casino
  - hit/miss balloon in a fair
  - profit/loss of game in fair



- Why would Google have ~~data centers~~
  minimum one per continent?
  speed & safety

~~prediction~~

- Parity, error handling in data transmission network / EC.

- PBE (Probability based Ensemble)

- Bayes Theorem
  Bayes classification methods
  Bayesian classifiers are statistical classifiers.

They can predict class membership probabilities such as the probability that a given tuple belongs to a particular class.

pg. 350
Classification chapter of Data Mining concepts & techniques
By Jiawei Han, Kamber, Pei. book

- shall bank give xyz loan amount to person p? risky Vs safe application

- Market Basket Analysis involving relation amongst Products i.e fruit jam, bread, butter, milk. while ordering stock as well as arranging offers, etc.

- Application of generating permutations and ~~combination~~, algorithms ~~is possibly~~ to generate ~~sample~~ sample space while considering probability.
- Application of generating function in finding p.d.f.

Example:

4 tosses

sum of numbers

$S = \{1111, \ldots, 6666\}$

$X = \{4, 5, 6, \ldots, 24\}$

$|S| = 6^4 = 1296$

p.d.f

$f(x=4) = \frac{1}{1296}$ ← only one way count 4 is possible that is 1111

$f(x=5) = \frac{4}{1296}$ ← 4 ~~ways count~~ 5 is possible

1112 2111

1121

1211

$f(x=6) = \frac{10}{1296}$

10 ways sum 6 is possible

1 1 1 3
1 1 3 1
1 3 1 1
3 1 1 1

4 ways

+

1 1 2 2
1 2 2 1
2 2 1 1
1 2 1 2
2 1 2 1
2 1 1 2

6 ways

$f(x=7) =$ ____

$f(x=8) =$

$f(x=9) =$

$\vdots$

$f(x=24) =$

How will you find there all?

Note that $\sum_{x=4 \text{ to } 24} = 1296/1296 = 1$

Generating function

$$(x^1+x^2+x^3+x^4+x^5+x^6)^4$$

$$(1+x+x^2+x^3+x^4+x^5+x^6-1)^4$$

a 1 $\quad$ n 7

r x

$$=\left(1\cdot\frac{(1-x^7)}{1-x}-1\right)^4$$

$$=\frac{x^4(1-x^6)}{(1-x)^4}=(x^4-x^{10})(1-x)^{-4}$$

$$= (x^4 - x^{10})\Big(1 + \binom{4}{1}x + \binom{5}{2}x^2 + \binom{6}{3}x^3$$

$$+ \binom{7}{4}x^4 + \binom{8}{5}x^5 + \binom{9}{6}x^6$$

$$+ \binom{10}{7}\cancel{x^7} + \binom{11}{8}x^8 + \binom{12}{9}x^9$$

$$+ \binom{13}{10}x^{10} + \binom{14}{11}x^{11} + \binom{15}{12}x^{12}$$

$$+ \binom{16}{13}x^{13} + \binom{17}{14}x^{14} + \dots\Big)$$

If we do smart work, finding co-efficient of terms having power 4, 5, 6 ..., 24 ..

i.e $(1-x)^{-3}$

$= 1 \times 3x \times 6x^2 \times 10x^3 \times 15x^4 \times \dots$

0	1									
1	1	1								
2	1	2	1							
3	1	3	3	1						
4	1	4	6	4	1					
5	1	5	10	10	5	1				
6	1	6	15	20	15	6	1			
7	1	7	21	35	35	21	7	1		
8	1	8	28	56	70	56	28	8	1	
9	1	9	36	84	126	126	84	36	9	1
	0	1	2	3	4	5	6	7	8	9

Note that binomial of -ve integer index has result as diagonal values.

-4

$x \Rightarrow$ 4, 5, 6, 7, 8, 9, 10, 11, 12, 13

1, 4, 10, 20, 35, 56, ~~83~~ 84, 120, 165, 220, ... c

14, 15, 16, 17, 18, 19, 20, 21, 22, 23

286, 364, 455, 560, 680, 816, 969, 1140, 1330, 1540

24

1771

$(x^4 - x^{10}) \times (1 + 4x + 10x^2 + 20x^3 + 35x^4 \dots + 1540x^{19} + 1771x^{20} + \dots)$

Note that

for $x = 10$

$$(x^4 - x^{10}) * (1 + \dots + 84x^6 + \dots)$$

$$\Rightarrow 84x^{10} - x^{10}$$

$$\Rightarrow 83x^{10}$$

$$\binom{9}{6} x^{10} - \binom{3}{0} x^{10}$$

for $x = 11$

$$(x^4 - x^{10}) * (1 + 4x + \dots + 120x^7 + \dots)$$

$$\Rightarrow x^4 \cdot 120x^7 - 4x \cdot x^{10}$$

$$\Rightarrow 120x^{11} - 4x^{11}$$

$$\Rightarrow 120 - 4 \Rightarrow 116$$

$$\binom{10}{7} x^{11} - \binom{4}{1} x^{11}$$

for $x = 12$

$$(x^4 - x^{10})(\ldots + 10x^2 + \ldots + 165x^8 + \ldots)$$

$$\Rightarrow 165x^{12} - 10x^{12} \qquad \binom{11}{8}x^{12} - \binom{5}{2}x^{12}$$

$$\Rightarrow 155$$

for $x = 13$

$$(x^4 - x^{10})(\ldots + 20x^3 + \ldots + 220x^9 + \ldots)$$

$$\Rightarrow 220x^{13} - 20x^{13} \qquad \binom{12}{9}x^{13} - \binom{6}{3}x^{13}$$

$$\Rightarrow 200x^{13}$$

$$\Rightarrow 200$$

· for $x = 14$

$$\binom{7}{4} \qquad \binom{13}{10}$$

$$(x^4 - x^{10})(\ldots + 35x^4 + \ldots + 286x^{10} + \ldots)$$

$$\Rightarrow 286x^{14} - 35x^{14} \Rightarrow \binom{13}{10}x^{14} - \binom{7}{4}x^{14}$$

$$\Rightarrow 286 - 35$$

$$\Rightarrow 251$$

$x = n$

$$\binom{n-1}{n-1-3}x^n - \binom{n-1-6}{n-1-6-3}x^n$$

for $x = 15$

$$\binom{14}{11}x^{15} - \binom{8}{5}x^{15}$$

$$\simeq \binom{n-1}{n-4}x^n - \binom{n-7}{n-10}x^n$$

$= 364 - 56$

$\Rightarrow 308$

$x = 16 \quad 455 - 84 =$

$x = 17 \quad 560 - 120 =$

$x = 18 \quad 680 - 165 =$

$x = 19 \quad 816 - 220 =$

$x = 20 \quad 969 - 286 =$

$x = 21 \quad 1140 - 364 =$

$x = 22 \quad 1330 - 455 =$

$x = 23 \quad 1540 - 560 =$

$x = 24 \quad 1771 - 680 =$

Unfortunately,

all these

numbers. that is

$4 \ldots\ldots - - - - 24$

$\square + \square + \ldots - - \ldots + \square$

> 1296

Hence, there

is problem in

our maths

yet.

~~Another attempt~~

$1 \le x_1, x_2, x_3, x_4 \le 6$

~~sum of~~ 4 digits to be 4

that is $x_1 + x_2 + x_3 + x_4 = 4$, How many ways possible?

One way that is $1+1+1+1 = 4$

---

sum of 4 digits to be 5

$1 \le x_1, x_2, x_3, x_4 \le 6$

$x_1 + x_2 + x_3 + x_4 = 5$

How many ways

1 1 1 2,

1 1 2 1 / 1 2 1 1, 2 1 1 1

4 ways.

dice outcome

$x_1 + x_2 + x_3 + x_4 = 6$, $1 \le x_1, x_2, x_3, x_4 \le 6$

$\vdots$

You <u>may</u> use principle of inclusion and exclusion.

Give it a try.

The sum rule in probability

$$P(A+B) = P(A) + P(B)$$

Two events, A and B ~~can~~ occur but cannot occur simultaneously

Person be it a student, 0.70

person be it a faculty, 0.05

Person be it a student or faculty

$$0.70 + 0.05 = 0.75$$

If E & F are two mutually exclusive events,
(they can NOT occur at the same time)

then the probability that either event E or event F will occur in a single trial is given by $P(E \text{ or } F)$

$$P(E) + P(F)$$

If the ~~event are~~ not mutually exclusive

then $P(E \text{ or } F) = P(E) + P(F) - P(E \,\&\, F \text{ together})$

Product Rule

$$P(MN) = P(M) \cdot P(N)$$

where M and N are ~~independent~~ events.

that is one event M does not affect outcome ~~of event~~ N.

M getting an ace in cards A $\quad 4/52 = 1/13$ spade, club, diamond, heart

N getting a heart $\quad 13/52 = 1/4$

Probability of getting ace of heart $\frac{1}{4} * \frac{1}{13} = \frac{1}{52}$

(Correct as it's only one).

* the sum rule fails for events occuring at the same time.

* the product rule fails, when events are not independent.