

Order of Growth

- Abstraction to ease analysis

Best case of insertion sort

- Data is already sorted

$1 + 1 + 1 + 1 + \dots + 1$ ($\sim n$ times)

$\Rightarrow n$ order of growth
 n

Worst the case insertion sort

- Data is reverse sorted

7	5	3	2	1	1	$1+2+3+4+\dots$
	<u>5</u>				\uparrow	$+n$
5	7	<u>3</u>	2	1	2	\approx
		<u>3</u>			\uparrow	$\sum_{j=2}^n j = \sum h$
3	5	7	<u>2</u>	1	3	
			<u>2</u>		\uparrow	
2	3	5	7	<u>1</u>	4	
1	2	3	5	7	5	

$\frac{n(n+1)}{2}$

Look only at the leading term
from mathematical exp

- Drop lower order terms

- Ignore the constant
coefficient in the leading
term

Worst Case

$$\sim \frac{n(n+1)}{2} \Rightarrow \frac{n^2+n}{2} \Rightarrow \frac{n^2}{2} + \frac{n}{2}$$

$$\Rightarrow \underline{a}n^2 + \underline{b}n + \underline{c}$$

Quadratic
Equation

$$\rightarrow n^2$$

Order of growth

Average Case

why
not
1)

9 7 5 3 1 7 9 11 13 19



- Normally because of nature of data to determine which is average case theoretically/mathematically will not be always possible.
- 2) mostly avg case is closer to worst case only. And we really want upper bound.

Learn later on
→ Probabilistic Analysis

Expected running time

⇒ randomized algorithm analysis
pick data various location
design your analysis

Example:

$$\underline{n} + \underbrace{n \log_2 n}_{\geq 6n}$$

1) Drop lower order term - n

$$\Rightarrow \underline{n \log n}$$

2) constant / Coeff
No constant

\Rightarrow Order of growth $n \log n$

Insertion sort was actually
solved with
incremental approach.

Divide and Conquer

Merge Sort

Divide: Splitting into two subarrays

$A[p, q]$ & $A[q+1, r]$

q is halfway point

Conquer: Recursively sorting the two subarrays

Combine: Merging two sorted subarrays to produce a single sorted $A[p, r]$

Merge-sort(A, p, q)

if $p \leq q$ then

checking for
Base

$q \leftarrow \lfloor (p+q)/2 \rfloor$

Divide

Merge-sort(A, p, q)

Conquer

Merge-sort(A, q+1, q)

Conquer

Merge(A, p, q, q)

(Combining
iteratively)

Initial Call mergeSort(A, 1, n)

- Solving recurrences
- Master method

