Discrete Maths

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5 Describe the sample space of 4 tosses

$\square \quad \square \quad \square \quad \square$

$H/T \quad H/T \quad H/T \quad H/T$

$2 \times 2 \times 2 \times 2 = 2^4 = 16$

$|S| = 16$

$\{HHHH, \ldots, TTTT\}$

Ex: Describe the sample space of rolling (a dice 4 times) // (4 dice).

$\square$	$\square$	$\square$	$\square$
1	1	1	1
2	2	2	2
$\vdots$	$\vdots$	$\vdots$	$\vdots$
6	6	6	6

$$6 \times 6 \times 6 \times 6 = 6^4 = 1296$$

$$|S| = 1296$$

$$S = \{1111, \ldots, 6666\}$$

Ex: Fill in two spaces by any cards from pack of cards (52).

$\square \quad \square$

$52 \quad 51$

← Without replacement

That is, once taken first card only 51 are left to choose from for second space.

52×51 ways.

permutation

$$^nP_r \Rightarrow {}^{52}P_2 \Rightarrow \frac{52!}{(52-2)!} = 52 \times 51$$

Order is of importance.

If order is not importance
we are taking combination
that is

$$^{52}C_2 \Rightarrow \frac{52!}{(52-2)!\,2!}$$

$$= \frac{52 \times 51}{2}$$

$$= 26 \times 51$$

no replacement. That is
once taken and not kept back.

Ex.

Sample space english alphabets

$$S = \{a, b, \dots, z\}$$

$$|S| = 26$$

Sample space of vowels

$$S = \{a, e, i, o, u\}$$

$$|S'| = 5$$

Sample space of consonants

$$|S'| = 21$$

deck of cards 52

four suits

spade
club

heart
diamond

two colors

~~black~~ black
26

red
26

types

Ace A
2 10
Jack
Queen
King

13 x 4

= 52

Sample space of
deck of cards

$|S| = 52$

Discrete Random Variable

A random variable defined over a finite sample space or countably infinite sample space consisting of discontinuous elements is known as discrete random variable.

Example

Random variable corresponding to the number of heads obtained in 4 tosses.

$|S| = 2^4 = 16 \quad S = \{HHHH, \ldots, TTTT\}$

Let X = the random variable // either no heads (TTTT), or one, two, three or 4 (HHHH).

$\therefore \quad X = \{0, 1, 2, 3, 4\}$

Example

Find the random variable corresponding to the sum of numbers in rolling of four dice once.

$$S = \{(1,1,1,1), \ldots, (6,6,6,6)\} \qquad |S| = 6^4 = 1296$$

Let X = The random variable

$$= \{4, 5, 6, 7, \ldots, 24\}$$

The purpose of defining discrete random variable is to define

P.d.f (probability distribution function) $\Rightarrow$

Probability Distribution Function (p.d.f)

If X is a discrete random variable with sample space S then a function denoted by

$f(x)$ or $P(X=x)$

and defined as

$f(x) = P(X=x) =$ Probability for the random variable $X=x$, is called

Probability Distribution Function (p.d.f) or simply, probability function.

Example

Find p.d.f corresponding to the number of ~~heads obtained~~ in 4 tosses.

$S = \{\cancel{HHHH, \ldots}, TTTT\}$ $|S| = 2^4 \Rightarrow 16$

The random variable $X = \{0, 1, 2, 3, 4\}$

$\therefore f(0) = P(X=0) =$ Probability of getting zero heads out of 4 tosses.

$= \dfrac{1}{16}$ ← only TTTT

This way

$f(1), f(2), f(3), f(4)$.

$$f(0) = \frac{{}^4C_0}{16} = \frac{1}{16}$$

$$f(1) = \frac{{}^4C_1}{16} = \frac{4}{16} = 1/4$$

$$f(2) = \frac{{}^4C_2}{16} = \frac{6}{16} = \frac{3}{8}$$

$$f(3) = \frac{{}^4C_3}{16} = \frac{{}^4C_1}{16} = \frac{1}{4}$$

$$f(4) = \frac{{}^4C_4}{16} = \frac{{}^4C_0}{16} = 1/16$$

Two important properties of p.d.f (probability distribution function)

1. The probability distribution function is a non-negative real number, i.e. $f(x) \geq 0$, $\forall x \in X$

2. The sum of the probability distribution function is 1. (one)

$$\sum_{x \in X} f(x) = 1$$

Let's verify these properties from previous example having number of heads in 4 tosses.

$S = \{HHHH, \ldots, TTTT\}$ $\quad |S| = 2^4 = 16$

$X = \{0, 1, 2, 3, 4\}$

$\therefore \quad f(0) = P(X=0) = 1/16 \geq 0$

$f(1) = 1/4 \geq 0$

$f(2) = 3/8 \geq 0$

$f(3) = 1/4 \geq 0$

$f(4) = 1/16 \geq 0$

$f(0) + f(1) + f(2) + f(3) + f(4)$

$\frac{1}{16} + \frac{4}{16} + \frac{6}{16} + \frac{4}{16} + \frac{1}{16}$

$= \frac{16}{16} = 1$ ✓

If the given function does not satiesfy there two properties then it is not a p.d.f. and vice versa

Example

verify that below ~~function~~ is a p.d.f

$$f(x) = \frac{x^2+1}{35}, \quad x = 0, 1, 2, 3, 4$$

Ex

Find the value of c if

$$f(x) = \frac{c}{2^{2x}} \; ; \quad x = 0, 1, 2, \ldots \text{ is a p.d.f given.}$$

solution

As per two important properties of p.d.f

$$\sum_{x=1}^{\infty} f(x) = 1$$

$$\therefore \; f(0) + f(1) + f(2) + \cdots = 1$$

$$\frac{c}{2^0} + \frac{c}{2^2} + \frac{c}{2^4} + \frac{c}{2^8} + \cdots = 1$$

$$c\left(1 + \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^8} + \cdots\right) = 1 \qquad = c\left(\frac{1}{1 - \frac{1}{2^2}}\right)$$

$$= c\left(\frac{4}{3}\right)$$

$$\therefore c = 3/4$$

Distribution Function

if X is a discrete Variable ~~defined~~ over the sample space S with p.d.f $f(x)$ then a function given by

$$F(x) = P(X \leq x)$$

$$= \sum_{x_i \leq x} f(x_i)$$ is called DF of X.

Example

Find DF F when $f(0) = f(1) = 1/3$

when $f(2) = f(3) = 1/6$.

solution

~~From~~ From data values given

Disc random ~~variable~~ $X = \{0, 1, 2, 3\}$

because

$1/3 \geq 0$, $\frac{1}{6} \geq 0$ $\quad f(x) \geq 0, \forall x \in X$

and

$\frac{1}{3} + \frac{1}{3} + \frac{1}{6} + \frac{1}{6} = \frac{6}{6} = 1$ Hence, this

function is satisfying two important properties of p.d.f. and hence it is p.d.f.

now, we can define DF

$$F(0) = \sum_{x \le 0} f(x) = f(0) = 1/3$$

$$F(1) = \sum_{x \le 1} f(x) = f(0) + f(1) = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

$$F(2) = \sum_{x \le 2} f(x) = f(0) + f(1) + f(2) = \frac{1}{3} + \frac{1}{3} + \frac{1}{6} = \frac{5}{6}$$

$$F(3) = \sum_{x \le 3} f(x) = f(0) + f(1) + f(2) + f(3) = 1$$

Hence, the DF is

$$F(x) = \begin{cases} 0, & \text{if } x < 0 \\ 1/3, & \text{if } 0 \le x < 1 \\ 2/3, & \text{if } 1 \le x < 2 \\ \frac{5}{6}, & \text{if } 2 \le x < 3 \\ 1, & \text{if } x \geq 3. \end{cases}$$