Discrete Maths

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Combinatorics

Combination (C. Selection)

~~Group~~

Order is of no importance

Permutation P. Arrangement)

Order is of importance.

~~consuming the thing~~ vs ~~Renewable~~

~~(repetition is allowed or not.)~~

Ordinary generating function OGF

Exponential ~~generating~~ functions EGF without repetition

$$^nP_r = {}^nC_r \times \underline{r!} = \frac{n!}{(n-r)!}$$

$$^nC_r \Rightarrow \frac{n!}{(n-r)! \cdot r!}$$

Number of ways of arranging n items in r slots without repetition ^nP_r

two steps

1 → selecting r out of n items ^nC_r

2 → Arranging the 'r' selected items in the r slots.

Permutation

Ⓐ with repetition

n items to be arranged in r places $\boxed{n^r}$

"Same item can go to multiple places"

bcoz, it is renewing, 1 slot

(repetition is allowed)

r slots can be filled with n items

$n \times n \times n \times \cdots$ r times $\boxed{n^r}$

Ⓑ without repetition

$n * (n-1) * (n-2) * \cdots (n-(r-1))$

$n * (n-1) * \cdots (n - r - 1)$

$r <= n$

$$\Rightarrow \boxed{\frac{n!}{(n-r)!}}$$

Ex

find OGF for sequence

$(c_0 , c_1 , c_2 , \ldots\ldots , c_n)$

where $c_k \binom{n}{k}$

clothes line

$a_0 x^0 + a_1 x^1 + \cdots\cdots + a_n x^n$

OGF $\sum_{r=0}^{\infty} a_r x^r$

$= \binom{n}{0} x^0 + \binom{n}{1} x^1 + \binom{n}{2} x^2 + \cdots\cdots + \binom{n}{n} x^n$

$$(a+b)^n = \binom{n}{0} a^0 b^n + \binom{n}{1} a^1 b^{n-1} + \dots + \binom{n}{n} a^n b^0$$

$1 \quad x$

$x \quad 1$

[∵ Binomial expansion]

$$OGF = (1+x)^n$$

Numerical ~~function~~

Ex

Find OGF for $a_r = \frac{1}{r!}$

Numerical Function

$$\text{OGF} = \frac{1}{0!}x^0 + \frac{1}{1!}x^1 + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \ldots$$

$$= 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \ldots$$

$$= e^x$$ [∵ Exponential series]

Info

$$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots \approx \log(1+x)$$

Relation between ~~ordinary~~ binomial co-efficient and extended ~~binary~~ binomial --

$$\binom{-n}{r} = (-1)^r \binom{n+r-1}{r}$$

Ex

Find 5th term in expansion of

$$\left(\underset{a}{2x^2} + \underset{b}{\frac{3}{2x}} \right)^{\overset{n}{8}}$$

P.S.:
Binomial
Expansion
starts from zero.

$(a+b)^n$

${}^nC_0$

rth term $\binom{n}{r} a^r b^{n-r}$

r is 4. n is 8.

$${}^8C_4 \, (2x^2)^4 \left(\frac{3}{2x} \right)^{8-4}$$

$$= {}^8C_4 \, 3^4 \; x^4$$

coefficient

Find the middle term/s in the expansion of

$$\left(2x - \frac{1}{4x}\right)^9$$

n is odd

the expansion will have

$(n+1) \Rightarrow (9+1) = 10$ terms

$$\underbrace{t_1 \quad t_2 \quad \ldots \quad t_4} \quad \boxed{\underline{t_5 \quad t_6}} \quad \underbrace{7 \quad 8 \quad 9 \quad t_{10}}$$

5th term $^{n}C_4 \quad (2x)^4 \quad \left(-\frac{1}{4x}\right)^5$

6th term $= \quad ^{n}C_5 \quad (2x)^5 \quad \left(-\frac{1}{4x}\right)^5$

Extended ~~bin exp~~

$$(a + \overline{b})^n$$

$$|\underline{a}| < |b|$$

?