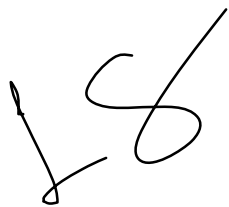


Discrete Maths

A handwritten signature or set of initials in black ink, located below the title. It appears to be a stylized 'V' followed by a cursive 'S'.

Example

We have 7 rooms

assign 3 identical
terminals

← ~~4 rooms~~

← ~~4 programmers~~

3 rooms

How many ways possible?

$7 \times 6 \times 5 \times 4$

1st programmer

can be kept into
any 7 rooms.

Hence, 7 ways

2nd programmer
to remaining but
any 6 rooms

Hence 6 ways

3rd programmer

5 ways

4th programmer

4 ways

Because, all terminals
are identical
ordering is not possible
and hence,
all 3 terminals
assigned into separate
room the very single
way.

fix. 1

Total number
of ways

$$7 \times 6 \times 5 \times 4 \times 1 = 7 \times 6 \times 5 \times 4$$

What if terminals are not identical

3 terminal

3 rooms

$$z = n = 3$$

No repetition

Order is of importance

Number of ways

$$3! = 3 \times 2 \times 1$$

So, total number of ways, we want to assign all of them
Exp prog Exp terminal not either

~~rule of multiplication~~

$$\frac{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{\text{EA}} \times \frac{6!}{\text{EB}} \approx 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 \approx 7! \approx (4+3)!$$

Example

Given 2^x number of binary sequences that is
(0,1)
(repetition) length is x .

How many of them are having even number
of 1's ?

— without loss of generality

let's pair off binary sequence

2^{th} digit is ϕ

\Rightarrow digit is 1

Rest are all
Same.

— ~~One of this will be having even number of 1's~~

Assume this has

even number of
1's

\Rightarrow This one will have
odd number of
1's

bcz of extra 1 at
mismatch.

Thus let us observe that
exactly half will be having even number of 1's
And rest half will be odd number of 1's

$$\frac{1}{2} 2^x$$

\approx

$$2^{x-1}$$

11

$$2 \quad 2^2 \rightarrow \begin{Bmatrix} 00 \\ 01 \\ 10 \\ 11 \end{Bmatrix} \leftarrow \quad 2^1 \Rightarrow 2$$

3

$2^3 \rightarrow$

{

000 ←

001

010

011 ←

100

101 ←

110 ←

111

4 $2^4 \rightarrow \left\{ \begin{array}{l} 0000 \\ 1111 \end{array} \right.$ 999

Example

schedule 3 exams within 5-day period
with restriction ~~one exam~~ per day only.

e_1, e_2, e_3

d_1, d_2, d_3, d_4, d_5

Note that
there will be no repetitions of exams. That is
once e_1 is placed on a given day, no more same e_1
any remaining days.

e_1 can be kept 5 ways (on any day from 5)

~~e_2 can be kept 4 ways~~ (one of the 5 days
taken by e_1)

e_3

3 ways

and we
have restriction
one exam max
per day.

$$\text{Hence, } 5 \times 4 \times 3 \Rightarrow \frac{5 \times 4 \times 3 \times 2 \times 1}{2 \times 1} = \frac{5!}{2!} = \frac{5!}{(5-3)!} = \frac{n!}{(n-r)!}$$

What is your understanding of n & r ?

Have you observed
all exams (e_1, e_2, e_3) must be selected.
~~That if~~ none of them can be skipped

But only once, that is
there will be few days
when there will be no exams
out of 5 days.

free/empty days can be anywhere from
1st to 5th in various answers.

Example 3 exams \rightarrow 3 days How many ways?

$$3 \times 2 \times 1 \Rightarrow \frac{3!}{1} \Rightarrow \frac{3!}{0!} \Rightarrow \frac{3!}{(3-3)!} \Rightarrow \frac{n!}{(n-r)!}$$

So, same formula but when $\boxed{n=r}$ $\Rightarrow n!$ or $r!$
no repetitions one and the same.

Example

Looks like exams are identical.

Schedule 3 exams

with no-restriction on number of exams
per day.

6

70

62

07

o 1

way

1

1

1

—

-

$$\lim_{x \rightarrow y} \frac{\sin x - \sin y}{x - y}$$

multiplication
rule
 $5^- \times 5^- \times 5^-$

2 exams on dayx and

1 exam on day 2

$$x < y$$
$$\begin{array}{ccccccc} 2 & 1 & & & & & 1 \\ 1 & | & 2 & & & & 2 \\ & & 1 & | & 2 & & 3 \\ & & & 1 & | & 2 & 4 \\ & & & & 1 & | & 5 \end{array}$$

1

7

カ

7

1

← 1, 1, 1 exam on day x, y, z $x < y < z$

1 2 3

1 2 4

125

2 3 4

235

$$5^3 \Rightarrow 125$$

Find your understanding
of \mathbb{N} and \mathbb{Z} .

Maths

$$n \times (n-1) \times (n-2) \times \dots \times (n-r+1)$$

$$= \frac{n \times (n-1) \times (n-2) \times \dots \times (n-r+1) \times (n-r) \times (n-r-1) \times \dots \times 3 \times 2 \times 1}{(n-r) \times (n-r-1) \times \dots \times 3 \times 2 \times 1}$$

$$= \frac{n!}{(n-r)!}$$

Repetition not allowed.
Consumable items.

place can hold one item only.

$$n > r \quad \text{or} \quad n = r$$

$$\frac{n!}{(n-n)!} \\ \Rightarrow \frac{n!}{0!} = n!$$

Example:

sequence of 4-digit decimal numbers (0-9)
that contain no-repeated digits.

How many ways?

201 202 202 203
 \boxed{x} \boxed{y} \boxed{z} $\boxed{}$

0-9 0-9 0-9 0-9
 $\{x\}$ $\{y, z\}$ $\{x, y, z\}$
 10 ways 9 ways 8 ways 7 ways

8 ways 7 ways

Answer

$$10 \times 9 \times 8 \times 7$$

$$10 P_4 \Rightarrow \frac{10!}{(10-4)!} = \frac{10!}{6!}$$

$$\Rightarrow \frac{10 \times 9 \times 8 \times 7 \times 6!}{6!}$$

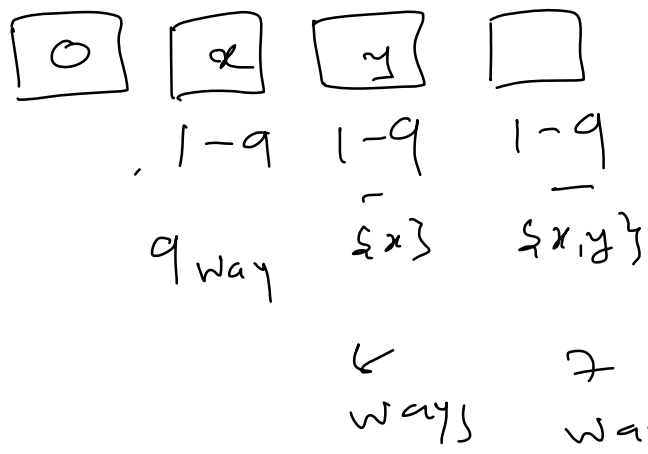
$$\Rightarrow 10 \times 9 \times 8 \times 7$$

$$\Rightarrow 5040$$

Example

4 digit decimal number having fix preceding zero ~~and~~ always.

~~and~~
no repeated digits at all.



$$9 \times 8 \times 7$$

way

$$= 504$$

Example :

4 digit decimal number

Not (never) having preceding zero
and no repeated digits.

$$5040 - 504 = 4536$$

\boxed{x} \boxed{y} \boxed{z} $\boxed{}$

1-9

0-9

0-9

0-9

$\{x\}$

$\{x, y\}$

$\{x, y, z\}$

9
ways

9
ways

8
ways

7
ways

Ex 4 digit decimal sequences with repetition allowed
 0-9 in $\square\square\square\square$ (~~may or may not~~
 include)
 $10^4 \Rightarrow 10000$

Ex 4 digit decimal sequence with repetition not allowed
 $P(10, 4) \Rightarrow \frac{10!}{(10-4)!} = \frac{10 \times 9 \times 8 \times 7 \times 6!}{6!}$

$$= 10 \times 9 \times 8 \times 7$$

$$= 5040$$

Ex 4 digit decimal sequence with repetitions for sure
 one or more
 digits
 repeated.
 $10000 - 5040 \Rightarrow 4960$