Discrete Maths

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Discrete Probabilities.

(having it's own importance)

[Discrete Vs Continuous

any value within a range: spectrum band]

Probability is a science of predicting the likelyhood of occurrences.

Generally, between 0 and 1.

✓ Event ✓ Number of outcomes

✓ sample space : set of all possible outcomes i.e. U

S

$$P(A) = \frac{n(A)}{n(S)}$$

(Probability)

What background/pre-requisite we ~~need~~ to ~~master before~~ being able to do probability?

To my knowledge,

- permutations and combinations

In other words,
~~One~~ of the applications of permutation and combination can be finding probability.

To be able to compute number of ways/outcomes of an experiment.

Independent Events

In the occurrence or non-occurrence of the event A does not affect the occurrence or non-occurrence of the event B then two events A and B are said to be independent events.

[P. S. Note that the word ~~dependable~~ has a meaning of reliable, can count on)

i.e. 3 students appeared in an exam

passing of student 1 is independent of passing of student 2.

i.e. Drawing of two cards ~~one after~~ another from a pack of cards

[With replacement given]

Complementary event

~~Let~~ A be an event of a given sample space S

~~The event~~ A^c is said to be a complementary event iff A^c consists of all the sample points of S which are not in A.

$$A \cap A^c = \phi$$

$$A \cup A^c = S$$

~~observe, that~~ elementary set-theoretic concepts enable us to introduce new definitions precisely and concisely.

$A \cap B$,

corresponds to event that both A and B occurs.

If A and B have no sample points common

$A \cap B = \phi$, empty set

this leads to

Mutually Exclusive or disjoint events

bcoz, occurrence of A excludes that of B and vice versa.

i.e. toasting of a die turns up

↳ even number that is 1, 3, 5

↳ odd number 2, 4, 6

$A \cup B$, ~~Both~~ A or B or both events occur together. corresponding to samples of $A \cup B$.

$A - B$, The event A occurs but B does ~~not~~ not correspond to set of samples $A - B$.

$A \oplus B$, event that one A or B but not both occurs correspond to set of samples $A \oplus B$

~~Collectively~~ Exhaustive events

Two or more events that are said to be collectively exhaustive if ~~[illegible]~~ at least one of the events must occur.

In other words, their union must cover all the ~~events~~ within entire sample space

Rolling a dice

(A) getting even (B) getting odd

Imp: None is not an option.

Because, $\{1,3,5\} \cup \{2,4,6\} = \{1,2,3,4,5,6\}$

Odds in favor

$$\frac{p}{1-p}$$

odds in against

$$\frac{1-p}{p}$$

Discrete sample space

Sample space S that has a finite numbers or countably infinite numbers of samples. (sample points)

For now, we are not talking completely infinite sample space cases.

Experimental model

- Assumption
  All events to be mutually exclusive and collectively exhaustive.

head, tail [~~Standing~~ on its side not a valid case]

- only one at a time
- can't be 'None' of them.

Event

is a subset of the outcomes

SIMPLE event

contains only one sample point.

Compound event

contains more than one sample points.

A different perspective

- can we think of probability of -ve or larger than 1 value ?

If the probability associated with a sample is a measure of frequency of occurrence of the ~~outcome~~ of an experiment then -ve probability not possible.

- can we think sum of all probabilites other than 1 ?

work it out yourself!

for Timebeing let's
ASSUME
the probabilities of the outcomes of an experiment are given to us by
either
— based on statistical data
(experiments)
i.e. ~~fair~~ coin vs special coin
(regular)
(like, sholey movie Amitabh had)
regular die vs crooked die
(like mama shakuni had in Mahabharata)

– simply ~~on~~ one's intuitive guesstimation

(smart estimation)

In that case,

probability is still ~~counted~~ same way but ~~values considered~~ are different.

i.e., For a CROOKED die

given that

probability of getting 1 → 1/3

remaining 2, 3, 4, 5, 6 → 2/15

~~We know~~ matters

$1/3 + 5(2/15) \Rightarrow 1/3 + 2/3 = 3/3 = 1$ still feels good.

But the probability ~~of~~ getting odds numbers

1, 3, 5

$$\frac{1}{3} + \frac{2}{15} + \frac{2}{15}$$

$$= \frac{5}{15} + \frac{2}{15} + \frac{2}{15}$$

$$= \frac{9}{15}$$

$$= \frac{3}{5} = 0.60$$

Getting even

$$\frac{2}{5} = 0.40$$

in a regular die

this is odd

$$\frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6}$$

$$= \frac{1}{2}$$

0.50

even

2, 4, 6

$$\frac{1}{6} + \frac{1}{6} + \frac{1}{6} \quad 0.50$$

Example

Rolling 2 dice

What is the probability of getting sum as 9?

Number of total samples =

which are they?

How will we use number of samples to get ~~answer~~ sum 9?

two dice

each can have

1, 2, 3, 4, 5, 6 $\Rightarrow$ 6 count

Rule of product

The sample space S (U)

$6 \times 6 = 36$

36 outcomes possible.

$n(S) = 36$

1, 2, 3, 4, 5, 6 $\times$ 1, 2, 3, 4, 5, 6

min max min max

Can we use generating function, ~~polynomial~~ binomial, finding co-efficient of a term smartfully any such concept ?

Yes $\Downarrow$

$$(x+x^2+x^3+x^4+x^5+x^6)\times(x+x^2+x^3+x^4+x^5+x^6)$$

$$(x+x^2+x^3+x^4+x^5+x^6)^2$$

$$(\underbrace{1+x+x^2+x^3+x^4+x^5+x^6}-1)^2$$

$a=1$

$r=x$

$n=7$

$$\left(\frac{1\cdot(1-x^7)}{1-x}-1\right)^2$$

$$=\left(\frac{1-x^7-1+x}{1-x}\right)^2=\frac{x^2(1-x^6)^2}{(1-x)^2}$$

∵ Binomial Theorem

$$(1+x)^{-n} = \sum_{r=0}^{\infty} (-1)^r \binom{n+r-1}{r} x^r$$

$$(1+x)^{-2} = \sum_{r=1}^{\infty} r\, x^{r-1}$$

$$= 1 + 2x + 3x^2 + 4x^3 + \cdots$$

Ex conti . . . . $x^2 (1-x^6)^2 (1-x)^{-2}$

$x^2 (1-2x^6+x^{12}) (1+\boxed{2x}+3x^2+4x^3+ \dots$

$= (x^2 - 2x^8 + x^{14}) (\dots 8x^7 + \dots)$

$(2+7)$

$8x^9 - 4x^9$

$= 4$

3, 6
4, 5
5, 4
6, 3

4 pairs having 9 sum.

~ hence,

$$\underset{\text{probability}}{P(A)} = \frac{n(A)}{n(S)}$$

$$= \frac{4}{36}$$

$$= \frac{1}{9}$$

~~Prob~~ability that rolling two dice end up summing to 9 result.
of faces

Ex

There is observation

"Out of 23 people the chance is less than 50-50 that no 2 of them will have the same birthday"

- Assumptions?

→ Sample space

→ What is 23?

let's accept 366 days of year (julian)

366 366 - - - - - -

$\square$ $\square$ $\square$ $\square$ $\square$ $\square$

P1 P2 P3 P4 P5 - - - P23

order is of importance

Sample space 366^{23}

repetition allowed

very similar to
putting decimal (0-9)
into 4 digit space (length) number.

0-9 0-9 0-9 0-9

$\square$ $\square$ $\square$ $\square$

10^4

on other hand
putting binary (0-1)
into 4 digit space (length)

$\square$ $\square$ $\square$ $\square$

2^4

Out of 366^{23} samples,

where we need to find sample count given the sequence, all days from p1 to p23 are different.

That is they have different birthday meaning no two or more have same.

How many ways can we arrange ~~days~~ from 1-366 into 23 without repetition and order is of importance?

Perm(366, 23)

Hence,

Probability is $\frac{\text{Perm}(366, 23)}{366^{23}}$

$= 0.494$

$< .50$ 50 %

The observation is confirmed by maths of probability

~~Example~~

8 students are ~~standing~~ in line
for interview.

Determine probability that there are
exactly 2 freshman [year 1]
2 sophomores [year 2]
2 juniors [year 3]
2 seniors [year 4]
in the line.

- sample space
- Use of perm/combination
variations?

Equiprobable samples!

Sample space

$\cancel{4^8 \text{ samples}}$

4 types of student

8 student count.

Two students from each class

~~number of ways~~

$$\frac{8!}{2!\,2!\,2!\,2!}$$

Probability of event

$$= \frac{\dfrac{8!}{2!\,2!\,2!\,2!}}{48}$$

count of event

sample space

$$= 0.0385$$