Discrete Maths

Example

We have 7 rooms

assign 3 identical terminals

4 rooms

4 ~~programmers~~

3 rooms

How many ways possible?

$7 \times 6 \times 5 \times 4$

1st programmer ~~can be kept into~~
any 7 rooms.
Hence, 7 ways

2nd programmer
to remaining but
any 6 rooms
Hence, 6 ways

3rd ~~prog~~
5 ways

4th progra
4 ways

Because, all terminals
are identical
ordering is not possible
and hence,
all 3 terminal
assigned into seperate
room ~~the~~ very single
way.
fix. 1

~~Total number~~
of ways

$7 \times 6 \times 5 \times 4 \times \underline{1}$

$= 7 \times 6 \times 5 \times 4$

What if terminals are not identical

3 terminal 3 rooms

$r = n = 3$

No repetition

order is of importance

Number of ways

$3! = 3 \times 2 \times 1$

So, total number of ways, we want to assign all of them

Exp prog Exper terminal not either

~~rule of~~ multiplication

$\underbrace{7 \times 6 \times 5 \times 4}_{E_A} \times \underbrace{6!}_{E_B} \approx 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 \approx 7!$

$\approx (4+3)!$

Example

Given 2^x number of binary sequences ~~that~~ that is length is x.

(0,1)

(repetition)

How many of them are are having even number of 1's ?

~~Without~~ loss of generality

Let's pair off binary sequence

i^{th} digit is ϕ i digit is 1

Rest are all same.

~~One of this~~ will be having even ~~number of~~ 1's

Assume this has

~~even number of 1's~~

$\Rightarrow$ This one will have odd number of 1's bcoz of extra 1 at mismatch.

Thus let us observe ~~that~~

exactly half will be having even number of 1's

and rest half will be odd ~~number~~ of 1's

$$\frac{1}{2} \, 2^{x}$$

$$\approx$$

$$2^{x-1}$$

length x | # of seq.

1 | $2^1 \rightarrow$ | $\begin{cases} 0 \\ 1 \end{cases}$ | No ones. zero times 1's. ↑ Even | $2^{x-1} \Rightarrow 2^{1-1} \Rightarrow 2^0 \Rightarrow 1$

2 | $2^2 \rightarrow$ | 00, 01, 10, 11 | $2^1 \Rightarrow 2$

3 | $2^3 \rightarrow$ | 000, 001, 010, 011, 100, 101, 110, 111 | $2^2 \Rightarrow 4$

4 | $2^4 \rightarrow$ | 0000 … 1111 | ???

Example

shedule 3 exams within 5 days period with restriction ~~one exam~~ per day only.

e_1, e_2, e_3 d_1, d_2, d_3, d_4, d_5

~~Note~~ that there will be no repetitions of exams. That is once e_1 is placed on a given day, no more same e_1 any remaining days.

e_1 can be ~~kept~~ 5 ways (on any day from 5)

e_2 ~~can be kept 4 ways~~ (~~one of the 5 days taken be e_1~~)

and we have restriction. one exam max per day.

e_3 3 ways

Hence, $5 \times 4 \times 3 \Rightarrow \frac{5 \times 4 \times 3 \times 2 \times 1}{2 \times 1} = \frac{5!}{2!} = \frac{5!}{(5-3)!} = \frac{n!}{(n-r)!}$

What is your understanding of n & r ?

Have you observed
all exams (e_1, e_2, e_3) must be selected.
~~That is~~ none of them can be skipped

But only once, that is
there will be few days
when there will be no exams
out of 5 days.

free/Empty days can be anywhere from
1st to 5th in various answers.

Example 3 exams → 3 days How many ways?

$$3 \times 2 \times 1 \Rightarrow \frac{3!}{1} \Rightarrow \frac{3!}{0!} \Rightarrow \frac{3!}{(3-3)!} \Rightarrow \frac{n!}{(n-r)!}$$

So same formula but when $\boxed{n = r}$ $\Rightarrow$ $n!$ or $r!$
no repetitions one and the same.

Example

Looks like exams are identical.

schedule 3 exams within 5-day period with no-restriction ~~on number of~~ exams per day.

		way	days day $x = y$
→ All exams on	day 1	1	
or	2	1	∴ multiplication rule
or	3	1	$5 \times 5 \times 5$
or	4	1	
or	5	1	

2 exams on day x and 1 exam on day y $\quad x < y$

2 . 1 | 2 1 | 2 1 | 2 1 | 2	1	1
	2	1
	3	1
	4	1
	5	1

n^2

1, 1, 1 exam on day x, y, z $\quad x < y < z$

123   124   125   234   235

$5^3 \Rightarrow 125$

Find your understanding of n and z.

Maths

$$n \times (n-1) \times (n-2) \times \cdots \times (n-r+1)$$

$$= \frac{n \times (n-1) \times (n-2) \times \cdots \times (n-r+1) \times (n-r) \times (n-r-1) \times \cdots \times 3 \times 2 \times 1}{(n-r) \times (n-r-1) \times \cdots \times 3 \times 2 \times 1}$$

$$= \frac{n!}{(n-r)!}$$

Repetition not allowed.
Consumable items.
place can hold one item only.

$n > r$ or $n = r$

$$\frac{n!}{(n-n)!}$$

$$\Rightarrow \frac{n!}{0!} \Rightarrow n!$$

Example:

sequence of 4-digit ~~decimal numbers~~ (0-9) that contain no-repeated digits. How many ~~ways?~~

d_1 d_2 d_2 d_3

$\boxed{x}$ $\boxed{y}$ $\boxed{z}$ $\boxed{\ }$

0-9 0-9 0-9 0-9

$-\{x\}$ $-\{x\}$ $-\{x,y,z\}$

10 ways

9 ways

$-y$

~~8ways~~ 7 ways

Answer

$10 \times 9 \times 8 \times 7$

$${}^{10}P_4 \Rightarrow \frac{10!}{(10-4)!} = \frac{10!}{6!}$$

$$\Rightarrow \frac{10 \times 9 \times 8 \times 7 \times 6!}{6!}$$

$$\Rightarrow 10 \times 9 \times 8 \times 7$$

$$\Rightarrow 5040$$

Example

4 digit decimal number having fix preceding zero ~~as~~ always.
~~and~~
no repeated digits at all.

$\boxed{0}$	$\boxed{x}$	$\boxed{y}$	$\boxed{}$
	1-9	1-9	1-9
		$-\{x\}$	$-\{x, y\}$
	9 way	8 ways	7 ways

$9 \times 8 \times 7$ way

$= 504$

Example :

4 digit decimal number
Not (never) having preceding zero
and no repeated digits.

$5040 - 504 = 4536$

$\boxed{x}$ $\boxed{y}$ $\boxed{z}$ $\boxed{}$

1-9	0-9	0-9	0-9
	$-\{x\}$	$-\{x,y\}$	$-\{x,y,z\}$
9 ways	9 ways	8 ways	7 ways

Ex

4 digit decimal sequences with repetition ~~allowed~~

0–9 in □□□□ (~~may or may not~~ include)

$10^4 \Rightarrow 10000$

Ex 4 digit decimal sequence with repetition not allowed

$$P(10,4) \Rightarrow \frac{10!}{(10-4)!} = \frac{10 \times 9 \times 8 \times 7 \times 6!}{6!}$$

$$= 10 \times 9 \times 8 \times 7$$

$$= 5040$$

Ex 4 digit decimal sequences with repetitions for sure

one or more digits repeated.

$10000 - 5040 \Rightarrow 4960$.