Discrete Maths

17

Rule of Product

$m \times n$

Rule of Sum

$m + n$

possible outcomes

Let's Experiment A can take place m ways

Experiment B can take place n ways

- Both experiments can take place $m \times n$ ways.

- Exactly one (either) can take place $m + n$ ways.

- selecting representative from Juniors, Seniors.

Any two (both) one from juniors and one from seniors

number of ways

$J \times S$

~~total~~ number of juniors $\times$ ~~total number~~ of seniors

either (exactly one) from either team

$J + S$

Ex:

Selecting courses from morning, ~~noon schedules~~

$m \times n$

one in morning

and one in noon

$m+n$

only one

either morning or noon

Ex

Train station

$$A \xrightarrow[m \text{ trains}]{} B \xrightarrow[n \text{ trains}]{} C$$

Assuming no direct train from A to C

there are $m \times n$ ways to travel from A to C via B station.

What if there is a direct train from A to C?

one or more(l)

Can we solve?

Example:

How many ways can you solve the famous "Die Hard 3" water puzzle? How?

Given ① 3-gallon ~~capacity~~ size water jug
② 5-gallon " " "
③ Unlimited supply of water

Get the scientist exactly 4 gallons of water.

~~Example: Given unlimited supply~~ digit of 0's and 1's (any as matter of fact)

How ~~many ways~~ can it be placed into 5 places? Prove.

Make sure to fill all 5 places all the time.
That is multiple times 0 and multiple time 1 can be used (have to be used because 2 < 5)

- $5P_2$ - $5C_2$ - $2P_5$ - $2C_5$ - $2!$ - $5!$ - 5^2 — 2^5

Simplest way to understand this

place 1   place 2   place 3   place 4   place 5

place 1	place 2	place 3	place 4	place 5
0	0	0	0	0
or	or	or	or	or
1	1	1	1	1

2 ways

2 ways

2 ways

2 ways

2 ways

We need all five places to be filled

that is with rule of multiplication

$2 \times 2 \times 2 \times 2 \times 2$ 5 times

2^5

Any other way $\Rightarrow$

Because while choosing from 0 or 1, repetition is allowed

every time you have 2 choices.

either 0 or 1.

And for filling 5 places/slots

we are going to do this 5 times.

Hence, ~~number of~~ ~~comb~~. ways

$$2 \times 2 \times 2 \times 2 \times 2 \Rightarrow 2^5 \qquad n^r$$

Your understanding ~~What~~ What is n? is r?

and relation of n and r?

$$n \times n \times n \times \ldots\ldots (r \text{ times}) = n^r$$

* choosing an object can be in
n ~~different~~ ways (each time)

which is 0, 1 ⇒ 2 ways ⇒ 2 items

* we want to choose 5 times
~~that~~ is $r \Rightarrow 5$ for 5 slots
that is choosing 5 items
with repetition from 2 (0, 1)

note ~~that~~ surprisingly, for this example

r	$>$	n	n	$<$	r
5		2	2		5

Permutations

- To arrang in all possible ways
- To change the order ~~or arrangment~~

Note that order is of importance.
meaning ab is differently counted than ba.

Scenarios ~~to consider~~

- What to arrange? and availability of this.
- where to arrange ~~? and capacity of this.~~
- Any constraints? / which ~~limitations?~~
- How ~~to arrange~~?
- when to arrange?
- why to arrange?

How knowing the number of ways help practically?
Further generating permutations and utilizing for applications: Real world English → maths → maths Result → English Real world

Example

~~Factorial~~ number of times performing a task

also conveys number of ways arrangment can be done.

which scenario?

All must be used.

number of objects to arrange is same as number of slots available, And ~~having~~ capacity 1 only.

(places)

\# ways

1st object can be placed anywhere in one of n slots → = n

2nd object now can be placed in n - 1 slots ~~as one slot is taken~~ before

n slots

n objects

3rd object now can be placed in n - 2 slots as 2 slots taken before

. . .

$(n-(n-1)-1)$
$= n - n + 1 + 1$

$(n-1)^{st}$ objects can be place in 2 slots

n^{th} object can be place is last empty slot.

$n - (n-1) \Rightarrow n - n + 1 \Rightarrow 1$

$n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1 \Rightarrow n!$

All n objects as well as n slots consumed.

Example: set $\{1, 2, \ldots, n\}$ will have $n!$ number of permutations generated.

n	
1	1
2 (left, right)	1 2
	2 1
3 (after, between, before)	1 2 3
	1 3 2
	3 1 2
	2 1 3
	2 3 1
	3 2 1

n

4

_ 1 _ 2 _ 3 _

insert 4, 4 times in every solution of perm(3).

What is the algorithm?

Is it recursive?

There are two gaps between 3 objects

2 + 1 + 1 ~~=> 4 places~~

before after

P.S For $x, y \in \mathbb{N}$, is it x^y same as y^x?

$$2^4 = 16 = 4^2 \quad \text{But} \quad 2^3 = 8 \neq 9 = 3^2$$

Maths, don't rely on any one or two examples for learning maths.

A cup half full is also half empty.

A square or rectangle shape from universe (adequate distance) will look circle (dot) only. !!!! ???

If we have a complete list of permutations

for $\{1, 2, \ldots, n-1\}$

then we can obtaine a complete list of

permutations for $\{1, 2, \ldots, n\}$

How?

$\Rightarrow$ By inserting the numbers n in

'n' ways to each

permutation of the list

for $\{1, 2, \ldots, n-1\}$.

~~Example~~:

For ~~term~~ 124653 in the lexicographic order next is

125346

Define: Lexicographic order

observation:
note that 12 are common going to be because
12

ordering 1 followed by 2.

for 3rd place 4 must move to next val 5 because
~~remaining~~ 653 is last sequence possible for digit 3,5,6
→ 125 ___ ; Remaining digits 3,4,6 in ascending.