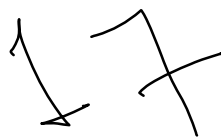


# Discrete Maths



Rule of product

$m \times n$

possible outcomes

Let's

Experiment <sub>A</sub> can take place  
in  $m$  ways

Experiment <sub>B</sub> can take place  
in  $n$  ways

•

~~Both experiments~~  
can take place

$m \times n$  ways.

Rule of Sum

$m + n$

• Exactly one

(either)  
can take place  
 $m + n$  ways.

- ~~selecting representative from Juniors, Seniors.~~

✓ Any two (both) one from juniors and one from seniors

number of ways

$$J \times S$$

total number of juniors  $\times$  total number of seniors

✓ either (exactly one) from either team

$$J + S$$

Ex: Selecting courses from morning, ~~noon~~ schedule

$m \times n$   
one in morning  
and one in noon

$m+n$   
only one  
either morning or noon

Ex

Train station



Assuming no direct train from A to C  
there are  $m \times n$  ways to travel from

A to C via B station.

What if there is a direct train from A to C?  
one or more()



Simplest way to understand this

place 1



0  
or  
1

place 2



0  
or  
1

place 3



0  
or  
1

place 4



0  
or  
1

place 5



0  
or  
1

2 ways

2 ways

2 ways

2 ways

2 ways

We need all five places to be filled  
that is with rule of ~~multiplication~~

2 x 2 x 2 x 2 x 2

5 times

$2^5$

||

Any other way  $\Rightarrow$

Because while choosing from 0 or 1, repetition is allowed

every time you have 2 choices.

either 0 or 1.

And for filling 5 places/slots

we are going to do this 5 times.

Hence, ~~number of~~ ~~ways~~ ways

$$2 \times 2 \times 2 \times 2 \times 2 \Rightarrow 2^5$$

$n$   
 $r$

Your understanding \_\_\_\_\_ What is  $n$  ?  
\_\_\_\_\_ What is  $r$  ?

and relation of  $n$  and  $r$  ?

$$n \times n \times n \times \dots \times n \text{ (} r \text{ times)} = n^r$$

\* choosing an object can be in  
n different ways (each time)

which is  $0, 1 \Rightarrow 2$  ways  $\Rightarrow 2$  items

\* we want to choose 5 times  
that is  $n = 5$  for 5 slots  
that is choosing 5 items

With repetition from 2 (01)

note that surprisingly, for few example

$$\begin{array}{cc|cc} n > r & & n < r & \\ 5 & 2 & 2 & 5 \end{array}$$



## Permutations

- To arrange in all possible ways
- To change the order or arrangement

Note that order is of importance.

meaning ab is differently counted than ba.

## Scenarios to consider

- what to arrange? and availability of this.
- where to arrange? and capacity of this.
- Any constraints? / which limitations?
- How to arrange?
- when to arrange?
- why to arrange?

How knowing the number of ways help practically?

Further generating permutations and utilizing for applications.

Real world English → maths → Result → English world

Example

Factorial number of times performing a task

also conveys number of ways arrangement can be done.

Which scenario?

All must be used.

number of objects to arrange is same

as number of slots available, <sup>And</sup> <sup>Capacity 1 only.</sup>  
(places) # ways

$\left. \begin{array}{l} \text{1st object can be placed anywhere in one of } n \text{ slots} \\ \text{2nd object now can be placed in } n-1 \text{ slots} \\ \text{as one slot is taken before} \\ \text{3rd object now can be placed in } n-2 \text{ slots} \\ \text{as 2 slots taken before} \\ \vdots \\ (n-1)\text{st object} \\ n\text{th object} \end{array} \right\} n \text{ objects}$

$$n \times (n-1) \times (n-2) \times \dots \times 2 \times 1 \Rightarrow n!$$

All  $n$  objects as well as  $n$  slots combined.

$(n - (n-1) - 1)$   
 $= n - n + 1 + 1$   
can be placed in 2 slots  
can be placed last  
 $n - (n-1) = n - n + 1 \Rightarrow 1$   
empty slot.

Example: Set  $\{1, 2, \dots, n\}$  will have

$n!$  number of permutations generated.

$n$	
1	1
2	1 2
(left, right)	2 1
3	1 2 3
(left, between, before)	1 3 2
	3 1 2
	2 1 3
	2 3 1
	3 2 1

$n$   
4  
\_ 1 \_ 2 \_ 3 \_

insert 4, 4 times

in every solution of  $\text{perm}(3)$ .

What is the algorithm?

Is it recursive?

There are two gaps between  
3 objects

$2 + 1 + 1 \Rightarrow 4 \text{ places}$   
↑ before after

P.S For  $x, y \in \mathbb{N}$ , is it  $x^y$  same as  $y^x$ ?

$$2^4 = 16 = 4^2 \quad \text{But} \quad 2^3 = 8 \neq 9 = 3^2$$

Maths, don't rely on any one or two examples for learning maths.

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A cup half full is also half empty.

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A <sup>square or</sup> rectangle shape from universe (adequate distance)  
will look circle (dot) only. !!!!! ???

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If we have a complete list of permutations

for  $\{1, 2, \dots, \underline{n-1}\}$

then we can obtain a complete list of  
permutations for  $\{1, 2, \dots, \underline{n}\}$

How?

$\Rightarrow$  By inserting the number  $n$  in  
' $n$ ' ways to each  
permutation of the list

for  $\{1, 2, \dots, n-1\}$ .

Example:

For term 124653 in the lexicographic order next is

125346

Define: Lexicographic order

Observation:  
Note that  $\begin{matrix} 12 \\ 12 \end{matrix}$  are common going to be because

ordering 1 followed 2.

in ~~2nd~~ place 4 must move to next val <sup>by</sup> 5 because  
~~remaining~~ 653 is last sequence possible for digit 3, 5, 6  
→ 125 —, remaining digits 3, 4, 6 in ascending.