Discrete Maths

Example

We have 7 rooms

assign 3 identical terminals

4 rooms

4 programmers

3 rooms

How many ways possible?

$7 \times 6 \times 5 \times 4$

1st Programmer can be kept into any 7 rooms.
Hence, 7 ways

2nd Programmer to remaining but any 6 rooms
Hence, 6 ways

3rd Prog
5 ways

4th Prgtca
4 ways

Because, all terminals are identical ordering is not possible
And hence,
all 3 terminal assigned into seperate room the very single way.
fix. 1

Total number of ways

$7 \times 6 \times 5 \times 4 \times 1$

$= 7 \times 6 \times 5 \times 4$

Example

Given 2^x number of binary sequences (0,1) (repetition) that is length is x.

How many of them are are having even number of 1's ?

~~Without~~ loss of generality

Let's pair off binary sequence

i^{th} digit is 0 → i^{th} digit is 1

Rest are all same.

~~One of~~ this will be having even ~~number of~~ 1's

Assume this has even number of 1's

⇒ This one will have odd number of 1's

bcoz of extra 1 at mismatch.

Thus let us observe that

exactly half will be having even number of 1's

And rest half will be odd number of 1's

$$\frac{1}{2} \cdot 2^{n}$$

$$\simeq$$

$$2^{n-1}$$

∎

length x | # of seq

1 | 2^1 → { 0 ← No ones. Zero times 1's. ↑ Even ; 1 } | $2^{x-1} \Rightarrow 2^{1-1} \Rightarrow 2^0 \Rightarrow 1$

2 | 2^2 → { 00 ← ; 01 ; 10 ; 11 ← } | $2^1 \Rightarrow 2$

3 | 2^3 → { 000 ← ; 001 ; 010 ; 011 ← ; 100 ; 101 ← ; 110 ← ; 111 } | $2^2 \Rightarrow 4$

4 | 2^4 → { 0000 ; ... ; 1111 } | ???

Example

shedule 3 exams within 5-days period with restriction ~~one exam~~ per day only.

e_1, e_2, e_3 $\quad d_1, d_2, d_3, d_4, d_5$

Note that there will be no repetitions of exams. That is once e_1 is placed on a given day, no more same e_1 any remaining days.

e_1 can be kept 5 ways (on any day from 5)

e_2 ~~can be kept 4 ways~~ (one of the 5 days taken by e_1)

and we have restriction. one exam max per day.

e_3 $\quad$ 3 ways

Hence, $5 \times 4 \times 3 \Rightarrow \frac{5 \times 4 \times 3 \times 2 \times 1}{2 \times 1} = \frac{5!}{2!} = \frac{5!}{(5-3)!} = \frac{n!}{(n-r)!}$

What is your understanding of n & r ?

Have you observed

all exams (e_1, e_2, e_3) must be selected.

~~Treat it~~ none of them can be skipped

But only once, that is

there will be few days

when there will be no exams

out of 5 days.

free/Empty days can be anywhere from

1st to 5th in various answers.

Example 3 exams → 3 days How many ways?

$$3 \times 2 \times 1 \Rightarrow \frac{3!}{1} \Rightarrow \frac{3!}{0!} \Rightarrow \frac{3!}{(3-3)!} \Rightarrow \frac{n!}{(n-r)!}$$

So same formula but when $\boxed{n=r}$ $\Rightarrow n!$ or $r!$

no repetitions one and the same.

Example Looks like exams are identical.

Schedule 3 exams within 5-day period with no restriction ~~on number of~~ exams per day.

→ All exams on day 1 — way 1 — days day $x = y$

or 2 — 1

or 3 — 1

or 4 — 1

or 5 — 1

∴ multiplication rule $5 \times 5 \times 5$

or

2 exams on day x and 1 exam on day y — $x < y$

2 | 1 | 2 — 1 | 2 — 1 | 2 — 1 | 2

1 — 1
2 — 1
3 — 1
4 — 1
5 — 1

n^2

1, 1, 1 exam on day x, y, z — $x < y < z$

1 2 3
1 2 4
1 2 5
1 3 4
1 3 5
2 3 4
2 3 5

$5^3 \Rightarrow 125$

Find your understanding of n and r.

Maths

$$n \times (n-1) \times (n-2) \times \cdots \times (n-r+1)$$

$$= \frac{n \times (n-1) \times (n-2) \times \cdots \times (n-r+1) \times (n-r) \times (n-r-1) \times \cdots \times 3 \times 2 \times 1}{(n-r) \times (n-r-1) \times \cdots \times 3 \times 2 \times 1}$$

$$= \frac{n!}{(n-r)!}$$

Repetition not allowed.

Consumable items.

place can hold one item only.

$n > r$ or $n = r$

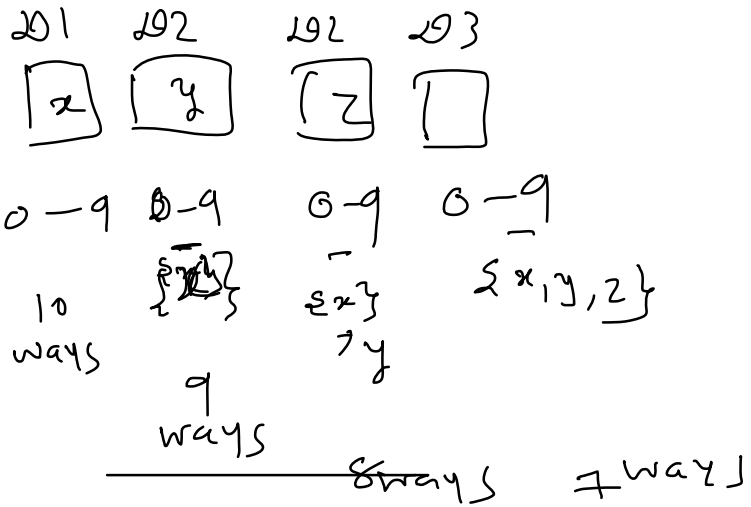
$$\frac{n!}{(n-n)!}$$

$$\Rightarrow \frac{n!}{0!} \Rightarrow n!$$

Example:

sequence of 4-digit decimal numbers (0-9) that contain no-repeated digits.

How many ways?



Answer

$10 \times 9 \times 8 \times 7$

$10P_4 \Rightarrow \frac{10!}{(10-4)!} = \frac{10!}{6!}$

$\Rightarrow \frac{10 \times 9 \times 8 \times 7 \times 6!}{6!}$

$\Rightarrow 10 \times 9 \times 8 \times 7$

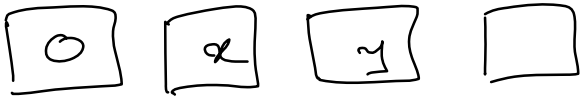
$\Rightarrow 5040$

Example

4 digit decimal number having fix preceding zero always.

and

no repeated digits at all.



1-9 1-9 1-9

9 way {x} {x,y}

↙ 7

ways ways

$9 \times 8 \times 7$

way

$= 504$

Example :

4 digit decimal number
Not (never) having preceding zero
and no repeated digits.

$5040 - 504 = 4536$

x	y	z	□
1-9	0-9	0-9	0-9
	$-\{x\}$	$-\{x,y\}$	$-\{x,y,z\}$
9 ways	9 ways	8 ways	7 ways

Ex 4 digit decimal sequences with repetition allowed

0-9 in □□□□ (~~may or may not~~ include)

$10^4 \Rightarrow 10000$

Ex 4 digit decimal sequence with repetition not allowed

$P(10,4) \Rightarrow \frac{10!}{(10-4)!} = \frac{10 \times 9 \times 8 \times 7 \times 6!}{6!}$

$= 10 \times 9 \times 8 \times 7$

$= 5040$

Ex 4 digit decimal sequences with repetitions for sure

one or more digits repeated.

$10000 - 5040 \Rightarrow 4960$.