

Discrete Maths

22

Conditional Probability

Rolling single die and getting '4'

$$\frac{1}{6} \quad \because |S| = 6 \quad \{1, 2, 3, 4, 5, 6\}$$

How about you were told that
number is even

now what is the probability of
getting '4'

Because,
it's even given already, it can be either
 $S = \{2, 4, 6\}$ ($|S|=3$) $\frac{1}{3}$ (P.S-
 $\frac{1}{3} > \frac{1}{6}$)

A event of getting 4

$$\frac{1}{6} = 0.16667$$

B event of getting even number

$$\frac{3}{6} = \cancel{0.16667}$$

0.500

/

$$P(A|B)$$

Given B, what \rightarrow probability of A

$$= \frac{1}{3}$$

Note that $\frac{1}{3} = 0.333\bar{3} > \frac{1}{6} = 0.16667$

The occurrence of B has effectively changed
the probabilities associated with the
samples in sample space.

that is
for even number like 4
probability has increased.

On the other side
given even number,
probability of 1, 3, 5 has
reduced to 0 from $\frac{1}{6}$ each

$$P(A/B) = \sum_{x_i \in A \cap B} P_B(x_i)$$

$$= \sum_{x_i \in A \cap B} \frac{P(x_i)}{P(B)}$$

$$= \frac{1}{P(B)} \cdot \sum_{x_i \in A \cap B} P(x_i)$$

$$= \frac{P(A \cap B)}{P(B)}$$

$P(A \cap B)$

Both event A and B occurs

$$A \{4\}$$

$$B \{2, 4, 6\}$$

$A \cap B \Rightarrow$ common sample points
 $\{4\}$

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$|S| = 6$$

$$P(A \cap B) = \frac{1}{6}$$

$$\therefore P(A|B) = \frac{\frac{1}{6}}{\frac{1}{2}} = \frac{P(A \cap B)}{P(B)} = \frac{2}{6} = \frac{1}{3}$$

Note that sum of all still remains 1

$$P\left(\frac{\{2\}}{\{2, 4, 6\}}\right) = \frac{1}{3}$$

$$P\left(\frac{\{4\}}{\{2, 4, 6\}}\right) = \frac{1}{3}$$

$$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{3}{3} = 1$$

$$P\left(\frac{\{6\}}{\{2, 4, 6\}}\right) = \frac{1}{3}$$

Example 2.40 +

Out of 1,00,000 people (hundred Thousand · one lakh)

51,500 female \rightarrow 9000 are bald

48,500 male \rightarrow 30,200 are bald

Suppose , talking about a person at random

$P(\text{Female} \& \text{Bald})$

$P(\text{Female} \& \text{with hair})$

$P(\text{male} \& \text{Bald})$

$P(\text{male} \& \text{with hair})$

.... ex. 2.50

Let A denote the event that a
bald person was chosen

B the event that a female was chosen

C the event that a male was chosen.

$P(A|B)$ Given $\frac{B}{A}$ female, what is probability of
being bald (no hair)

$$= \frac{P(A \cap B)}{P(B)} = \underline{\quad}$$

$$P(B) = P(\text{female}) = \frac{51,544}{100,000} = 0.515$$

$$P(A \cap B)$$

treat B ~~com~~

A bald person

$$\begin{array}{r} 9000 \\ + 30,200 \\ \hline 39,200 \end{array} \begin{array}{l} f \\ m \\ \text{total} \end{array}$$

B female 51,500

common points (samples) $\Rightarrow 9000$

$$\frac{9000}{100,000} \Rightarrow 0.09$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

= $\frac{0.09}{0.515}$

= 0.175

Given B
female, being
bald A

$$P(B) = \frac{51,500}{100,000}$$

= 0.515

observe that

$$0.175 < 0.392$$

$$P(A|B) < P(A)$$

$$P(A) = \frac{39,200}{100,000}$$

$$= 0.392$$

On the other hand

$$P(A|C)$$

$$= \frac{P(A \cap C)}{P(C)}$$

$$= \frac{0.302}{0.485}$$

$$= 0.623$$

$$0.623$$

$$P(A|C)$$

$$0.392$$

$$> P(A)$$

Further

$$P(B|A) = \frac{0.090}{0.392} = 0.23$$

$$P(C|A) = \frac{0.302}{0.392} = 0.77$$

Given male, losing bald

$$\left| \begin{array}{l} P(C) = \frac{48,500}{100,000} \\ \text{male} \\ = 0.485 \end{array} \right.$$

$$\left| \begin{array}{ccc} A \cap C & & P(A \cap C) \\ \nearrow \text{bald} & \searrow \text{male} & \\ 9000 f & & 48,500 \\ 30,200 m & & = \frac{30,200}{100,000} \\ & & = 0.302 \end{array} \right.$$

$$\text{Common sample} = 30,200$$

Example: 3 dice were rolled. (6^3)

$$\{1, 2, 3, 4, 5, 6\} \times \{1, 2, 3, 4, 5, 6\} \times \{1, 2, 3, 4, 5, 6\}$$

Given that no two faces were the same.

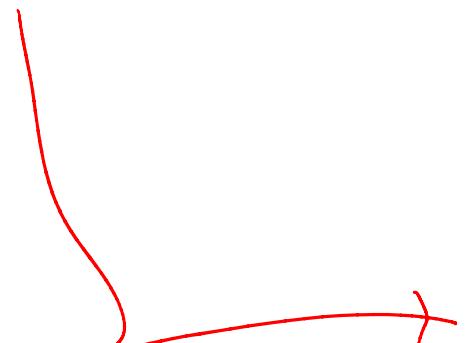
Method 1)

it was like $\{1, 2, 3\}$ but not $\{1, 2, 2\}$

q.s. Ace (one dot in dice face) (not six which is highest)
in deck of cards

sequence will be ace, 2, 3, ..., 9, 10
↓
jack, queen, king.

Side note



But ace has more weight.
one dot has less weight
(Compare to 8R dots.)

What is the probability that there was
an ace (1 dot)?

A event there was an ace
(3 dice rolled)

$\frac{1}{6}$ one dice
rolled

B event no two faces were same
(3 dice rolled)

(B) No two face were same.

That is no repetition allowed

3 dice's $\{1, 2, 3, 4, 5, 6\}$

$\xleftarrow{\text{6 outcomes}}$

$$\frac{6 \times 5 \times 4}{6 \times 5 \times 4} = 6 \times 5 \times 4 = \frac{6!}{(6-3)!} = \frac{6 \times 5 \times 4 \times 3!}{3!}$$

$$P_{\text{subseq}}(B) = \frac{6 \times 5 \times 4}{6^3} = \frac{P_{\text{Perm}}(6, 3)}{6^3}$$

$A \cap B$ that is common samples
of no two faces same and
an ace is present

$\boxed{1}$ \square \square	\square $\boxed{1}$ \square	\square $\boxed{1}$ \square
5 4	5 4	5 4

$5 P_{\text{Perm}} 2$

$$\frac{5!}{(5-2)!} = \frac{5 \times 4 \times 3!}{3!} = 5 \times 4$$

$$3 (5 \times 4)$$

$$P_{\text{subseq}}(A \cap B) = \frac{3 \times 5 \times 4}{6^3}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{3 \times 5 \times 4}{6^3}$$

$\overbrace{}^{6 \times 5 \times 4}$

$$= \frac{3 \times 5 \times 4}{2 \cancel{4} \times 5 \times 4}$$
$$= \frac{1}{2}$$

□

Extra efforts

Ex

3 dice rolled

M exactly one dice shows

From be any
1. first, second or third.

N atleast one dice shows 1

(minimum one dice)

It can be two or more
also.

M exactly one repetition of 2-6 allowed

1 D D
2 2
6 ;
6 ;

$$3 \times 5 \times 5$$

N minimum onetime 1 repetition of 1-6 all

1 1
1 6

$$3 \times 6 \times 6$$

If we are not allowing any repetition
at all
that is

$$3 \times 5 \times 4$$

$$3 \times 5 \text{ Pqsm 2}$$