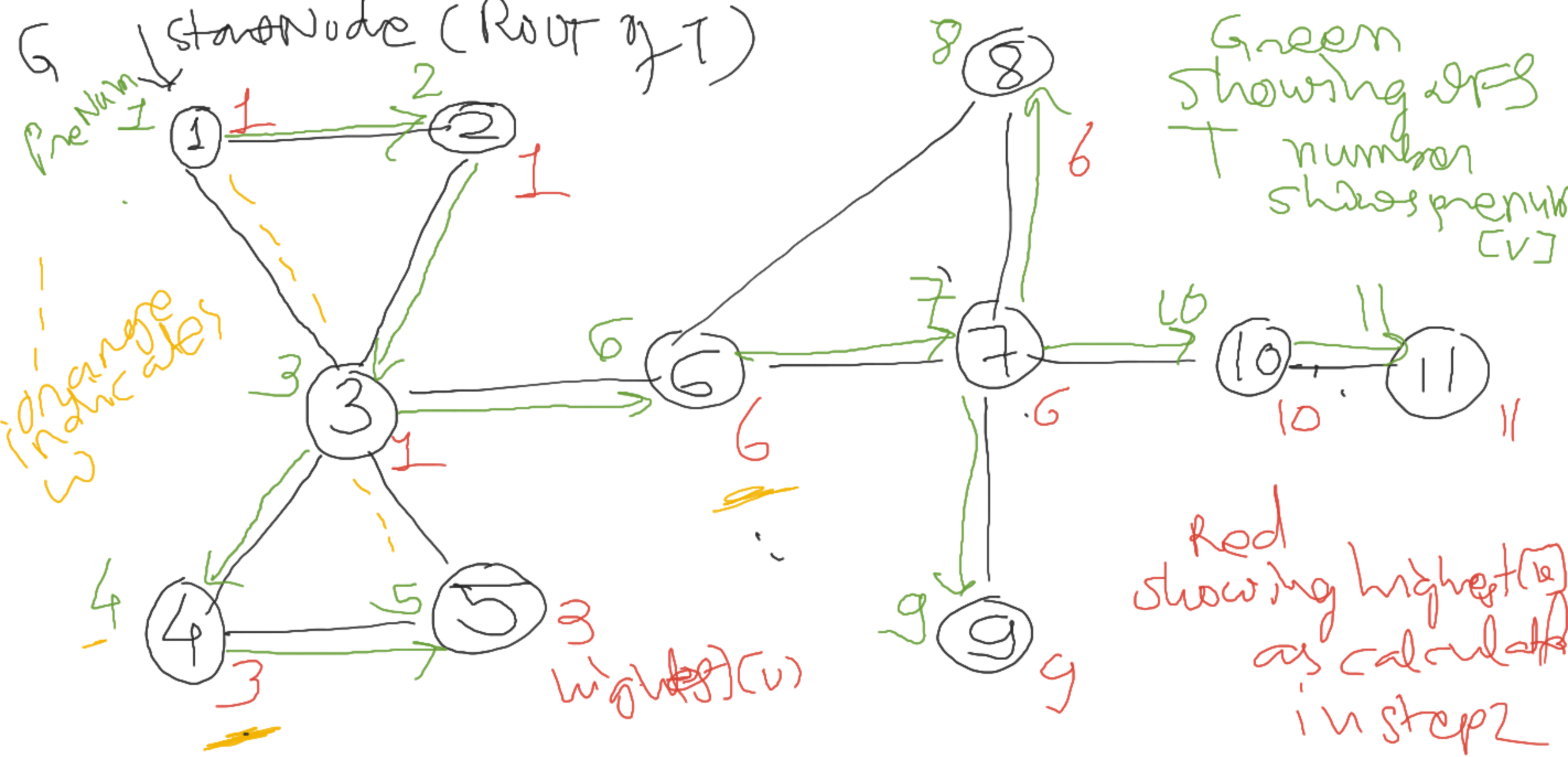
Articulation Point

A node v of a connected graph is an articulation point if the subgraph obtained by deleting v and all the edges incident on v is no longer connection.

Directed / Undirected ?

Algo:

1. Carry out a depth first search in G starting from any node. Let T be the tree generated by this search, and for each node v of G, let prenum[v] be the number assigned by the search.



v 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11

Step 2:

Traverse T in postorder.

For each node v visited calculate highest[v] as the minimum of

→ (a) Prenum[v]

→ (b) Prenum[w] for each node w such that there is an edge {v, w} in G with no corresponding edge in T;

→ (c) highest[x] for every child x of v

Post

5, 4, 8, 9, 11, 10, 7, 6, 3, 2, 1

highest[5] =

min{5, {3}, ____} ⇒ 3

highest[4] = 3

min{4, {_}, {3}} ⇒ 3

highest[8] =

min{8, {6}, {_}} = 6

highest[9]

= min{9, _, _} = 9

highest[11] = min{11, _, _} = 11

highest[10] = min{10, —, 11} = 10

highest[7] = min{7, —, 6, 9, 10} = 6

highest[6] = min{6, 8, 6} = 6

highest[3] = min{3, 1, 5, 3, 6} = 1

highest[2] = min{2, —, —1} = -1

highest[1] = min{1, 1, 1} = 1

Step3
Determine the articulation Points of G as follow:

a) the root of T is an articulation point if and only if it has more than one child (T).

b) Any other node v is an articulation point if it has a child x such that

lowest[x] ≥ prenum[v]

step3 for every nodes _ check

Node 1 ?

It is about having only one child.

So, it's not an articulation point

for every child of V

Node 2 ? highest[x] ≥ Prenum[v]

1 ≥ 2 ✗

Hence, node ② is Not an articulation point.

Node 3 ?

prev(3)

3 ✓

3

6

✓

3 is an articulation point.

Note that any single child satisfy condition then also select the node as ap.

Note 4?

3 $\geq$ 4 X'

So, N_4 is not an articulation

$\not\exists 0$

Nodes 9

As it is not having any child it can't be an articulation point of G. since if we delete it the remaining nodes are still connected by edges left in T.

Node 5 is not an articulation point.

Node 6 ?

$6 \geq 6$ ✓

Yes, node 6 is an articulation point.

Node 7 ?

$6 \geq 7$ ✗

$10 \geq 7$ ✓ $9 \geq 7$ ✓

Yes, node 7 is an articulation point

Node 8 ?
No child. So its Not an a.p.

Node 9 ?
No child. So its Not an a.p.

Node 10 ?
11 ≯ 10
- an a.p

Node 11 ? No child. So Not an a.p

Answer of the
Articulation Points are
$\{3, 6, 7, 10\}$