Discrete Maths

State in FSM

state represents a summary of the history of the machine.

Two states are equivalent if they represent summaries that are equivalent (identical) as far as the terminal behaviour of the machine is concerned.

Surprisingly,

equivalent states within a given machine can be combined without changing the terminal behaviour of the machine.

Moreover, less the number of states, less is going to be ~~implementation~~ & maintenance cost.

This also can be proved ~~mathematically~~.

How do we find states (2 or more)
are equivalent?

Can we say anything based
on their output?

Ideally, an exhaustive examination
of all possible input sequences
of arbitrary length is required.

What can we do to avoid exhaustive ?

Two finite state machines are said to be equivalent if, starting from their respective initial states, they will produce the same output sequence when they are given the same input sequence.

In other words,
equivalent machines have
identical terminal
(FINAL)
behaviors even though their
internal structures might
be different.

i.e Petrol or CNG or LPG
all drive car engine to
fuels motion.

State	Input 0	Input 1	
A	B	F	0
B	A	~~A~~ F	0
C	G	~~F~~ A	0
D	H	B	0
E	A	G	0
F	H	C	1
G	A	D	1
H	A	C	1

π_0

$\{\overline{ABCDE}, \overline{FGH}\}$

π_1

$\{\overline{ABE}\ \overline{CD}\ \overline{F}\ \overline{GH}\}$

$\pi_2 =$

$\{\overline{AB}\ \overline{CD}\ \overline{E}\ \overline{F}\ \overline{GH}\}$

Two states are in the same block in π_k if and only if (1) they are in the same block in π_{k-1} and (2) for (every) any input letter, ~~their successors~~ are in the same block in π_{k-1}

That is if there exist atleast one input letter, if its successor is NOT in same block then currently they can NOT be together.

Given [π_0 $\overline{ABCDE}$ $\overline{FGH}$]

I

π_1 formation

Successors

Input

	0		AND	1		
AB	B, A	✓		F, F	✓	T & T ⇒ T
(I) AC	B, G	X		F, A	X	~~F & F ⇒ F~~
AD	B, H	X		F, B	X	
AE	B, A	✓		F, G	✓	
B ~~C~~	A, G	X		F, A	X	
BD	A, H	X		F, B	X	
BE	A, A	✓		F, G	✓	
CD	G, H	✓		A, B	✓	
CE	G, A	X		A, G	X	
DE	H, A	X		B, G	X	

Refer given machine to populate

~~At least one~~ X then final X
NO   NO

F & T ⇒ F

T & F ⇒ F

FGH

π FG	HA ✗	CD ✓	NO
FH	HA ✗	CC ✓	NO
GH	AA ✓	DC ✓	YES

π_1

$\overline{ABE}$ $\overline{CD}$ $\overline{F}$ $\overline{GH}$

Given ~~Π_0~~, Π_1

Π_2 formation

In Π_2 Will ~~be together~~?

$\overline{ABE}$

	Input 0	Input 1	In Π_2
AB	B, A ✓	F, F ✓	Yes
AE	B, A ✓	F, G ×	NO
BE	A, A ✓	F, G ×	NO

~~Refer~~

Given machine to populate

$\overline{CD}$

	Input 0	Input 1	In Π_2
CD	G, H ✓	A, B ✓	Yes

$\overline{F}$

$\overline{GH}$

	Input 0	Input 1	In Π_2
GH	A A ✓	D C ✓	Yes

$\Pi_2 \quad \overline{AB} \quad \overline{E} \quad \overline{CD} \quad \overline{F} \quad \overline{GH}$

When should we stop ?

$\overline{A} \quad \overline{B} \quad \overline{E} \quad \overline{C} \quad \overline{D} \quad \overline{F} \quad \overline{G} \quad \overline{H}$

Exhaustive ?

when there is no difference from Π_k to Π_{k+1} stop

Very much like

Improved Bubble sort where two iterations do not have further exchange we don't go all iterations. As data is already sorted.

$\Rightarrow$

Given Π_2 calculate Π_3

$\overline{AB}$ $\overline{E}$ $\overline{CD}$ $\overline{F}$ $\overline{GH}$

	0	1	Will stay together?
AB	BA	EF	yes
E			yes
CD	GH	AB	
F			
GH	AA	DC	Yes

Π_2 ~~is same~~ as Π_3. Stop.

What is judgement?

Instead of 8 original states (A-H) the same machine can be developed using 5 states.

1) AB
2) E
3) CD
4) F
5) GH

Areas

(1) Given a single machine

try to reduce states

[machine with more or less ~~states~~

both are equivalent machines

(2) Given two different machines

find out they are same

or not ?

$m_A \Rightarrow m_A$ reduced

$m_B \Rightarrow m_B$ reduced

compare.

m_A and m_B are equivalent.

State	Input 0	Input 1	Output
AB (A')	AB (A')	FF (D')	0
CD (B')	GH (E')	AB (A')	0
E (C')	AB (A')	GH (E')	0
F (D')	GH (E')	CD (B')	1
GH (E')	AB (A')	CD (B')	1