Discrete Maths

22

~~Conditional~~ Probability

Rolling single die and getting '4'

$\frac{1}{6}$ $\because$ $|S| = 6$ $\{1, 2, 3, 4, 5, 6\}$

How about you were told that
number is even
now what is the probability of
getting '4'

~~Because~~,
it's even given already, it can be either
$S = \{2, 4, 6\}$ $|S| = 3$ $1/3$ (i.e. $1/3 > 1/6$)

A event of getting 4 $\quad 1/6$

B event of getting even number

$3/6 = 0.16667$

$P(A/B)$

Given B, what is probability of A

$= 1/3$

Note that $1/3 = 0.3334 > 3/6 = 0.16667$

The occurrence of B has effectively changed the probabilities associated with the samples in sample space.

that is

for even number like 4

probability has increased.

On the other side

given even number,

probability of 1, 3, 5 has

reduced to 0 from $1/6$ each

$$P(A/B) = \sum_{x_i \in A \cap B} P_B(x_i)$$

$$= \sum_{x_i \in A \cap B} \frac{P(x_i)}{P(B)}$$

$$= \frac{1}{P(B)} \cdot \sum_{x_i \in A \cap B} P(x_i)$$

$$= \frac{P(A \cap B)}{P(B)}$$

$P(A \cap B)$

Both ~~event A~~ and B occurs

A $\{4\}$ B $\{2,4,6\}$

$A \cap B \Rightarrow$ common sample points $\{4\}$

$S = \{1,2,3,4,5,6\}$

$|S| = 6$

$P(A \cap B) = 1/6$

$\therefore P(A/B) = \dfrac{1/6}{1/2} = \dfrac{P(A \cap B)}{P(B)} = \dfrac{2}{6} = 1/3$

Note that sum of all still ~~remains~~ 1

$$P\left(\frac{\{2\}}{\{2,4,6\}}\right) = \frac{1}{3}$$

$$P\left(\frac{\{4\}}{\{2,4,6\}}\right) = \frac{1}{3}$$

$$\frac{1}{3}+\frac{1}{3}+\frac{1}{3} = \frac{3}{3} = 1$$

$$P\left(\frac{\{6\}}{\{2,4,6\}}\right) = \frac{1}{3}$$

Example 2.40 +

Out of 100,000 people (~~Hundred Thousand~~ one lakh)

51,500 female $\rightarrow$ 9000 are bald

48,500 male $\rightarrow$ 30,200 are bald

suppose, talking about a person at random

P(female & Bald)

P(Female & with hair)

P(male & Bald)

P(male & with hair)

.... ex. 2.50

Let A denote the event that a bald person was chosen

B the event that a female was chosen

C the event that a male was chosen.

$P(A/B)$ Given female (B), what is probability of being bald (A) (no hair)

$$= \frac{P(A \cap B)}{P(B)} = \underline{\qquad}$$

$$P(B) = P(\text{female}) = \frac{51{,}500}{100{,}000} = 0.515$$

$P(A \cap B)$

that is ~~com~~

A bald person

$$\begin{array}{rl} 9000 & f \\ +\ 30{,}200 & m \\ \hline 39{,}200 & \text{total} \end{array}$$

B female 51,500

common points (samples) $\Rightarrow$ 9000

$$\frac{9000}{100{,}000} \Rightarrow 0.09$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

Given female, being bald

$$= \frac{0.09}{0.515}$$

$$= 0.175$$

$$P(B) = \frac{51,5\cancel{00}}{100,\cancel{000}} = 0.515$$

Observe that

$$P(A) = \frac{39,2\cancel{00}}{100,\cancel{000}}$$

$$0.175 < 0.392$$

$$= 0.392$$

$$P(A|B) < P(A)$$

On the other hand

$P(A/C)$ — C Given male, being bald

$$= \frac{P(A \cap C)}{P(C)}$$

$$= \frac{0.302}{0.485}$$

$$= 0.623$$

$P(C) = \frac{48,500}{100,000}$

male

$= 0.485$

$A \cap C$

bald — male

900 f — 48,500

30,200 m

$P(A \cap C)$

$= \frac{30,200}{100,000}$

$= 0.302$

Common sample

30,200

0.623 — 0.392

$P(A/C) > P(A)$

Further, $P(B/A) = \frac{0.090}{0.392} = 0.23$ $P(C/A) = \frac{0.302}{0.392} = 0.77$

~~Example:~~ 3 dice were rolled. $|S| = 6^3$

$\{1,2,3,4,5,6\} \times \{1,2,3,4,5,6\} \times \{1,2,3,4,5,6\}$

Given that no two faces were the same.

~~the dot is~~

it was like $\{1,2,3\}$ but not $\{1,2,2\}$

p.s. ace (one dot in dice face) (not six which is highest) in deck of cards

side note

sequence were ace, 2, 3, --, 9, 10, jack, que, king.

1

But ace has more weight.

one dot has less weight

Compare to six dots.

What is the probability that there was an ace (1 dot)?

A event: there was an ace (3 dice rolled)

$\frac{1}{6}$ one dice rolled

B event: no two faces ~~were same~~ (3 dice rolled)

B No two face were same.

That is no repetition allowed

3 dice' {1, 2, 3, 4, 5, 6}

6 outcomes

$$\underset{6}{\_} \times \underset{5}{\_} \times \underset{4}{\_} \qquad {}^6P_3 = 6 \times 5 \times 4 = \frac{6!}{(6-3)!} = \frac{6 \times 5 \times 4 \times \cancel{3!}}{\cancel{3!}}$$

$$\text{Probability}(B) = \frac{6\times5\times4}{6^3} = \frac{\text{Perm}(6,3)}{6^3}$$

$A\cap B$ that is common samples of no two faces same and ~~an ace is present~~

$\boxed{1}$	$\boxed{\ }$	$\boxed{\ }$	$\boxed{\ }$	$\boxed{1}$	$\boxed{\ }$	$\boxed{\ }$	$\boxed{\ }$	$\boxed{1}$
	5	4	5		4	5	4	

$5\,\text{Perm}\,2$

$$\frac{5!}{(5-2)!} = \frac{5\times4\times3!}{3!} = 5\times4$$

$$\text{Prob}(A\cap B) = \frac{3\times5\times4}{6^3}$$

$$3\,(5\times4)$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{\dfrac{3 \times 5 \times 4}{6^3}}{\dfrac{6 \times 5 \times 4}{6^3}}$$

$$= \frac{3 \times \cancel{5} \times \cancel{4}}{\overset{2}{\cancel{6}} \times \cancel{5} \times \cancel{4}}$$

$$= 1/2$$

Extra efforts

Ex

3 dice rolled

M exactly one dice shows 1. ~~It~~ can be any first, second or third.

N ~~atleast~~ one dice shows 1

(minimum one dice)

it can be two or more also.

M

exactly one

repetition of 2..6 allowed

$\boxed{1}$ _ _

2 ⋮ 6   2 ⋮ 6

$3 \times 5 \times 5$

N

minimum one time 1

~~repetition~~ of 1..6 allow

$\boxed{1}$ _ _

1 ⋮ 6   1 ⋮ 6

$3 \times 6 \times 6$

If we are not allowing any repetition at all

that is

$3 \times 5 \times 4$

$3 \times$ 5 Perm 2