(1) $\Rightarrow$ main problem

n problem solve

Recursive calls

a

(2) $\Rightarrow$ n/b $\quad$ n/b $\quad$ n/b $\quad$ n/b $\quad$ n/b

Size of subproblem it solves

$$T(n) \Rightarrow aT(n/b) + f(n)$$

$n^{\log_b a}$?

$\Rightarrow a^{\log_b n}$ $\because$ log formulas

How many number of leaves will be there

a^H

what is height with respect to n

i.e. 1024

2^{10}

$\log_b n$.

$a^H \Rightarrow a^{\log_b n}$

$n^{\log_b a} \Leftrightarrow a^{\log_b n}$

work done root level $f(n)$

comparison between $n^{\log_b a}$ (leaf level (work done

Case 1 Theory $f(n) = O\left(n^{\log_b a - \varepsilon}\right)$

Soln 1 $\Theta\left(n^{\log_b a}\right)$ means RHS is greater

Case 2 $f(n) = \Theta\left(n^{\log_b a}\right)$

Sol 3 $\Theta\left(f(n) \log n\right)$ or $\Theta\left(n^{\log_b a} \log n\right)$

Case 3 Theorem $f(n) = \Omega\left(n^{\log_b a + \varepsilon}\right)$

Soln 3 $\Theta\left(f(n)\right)$ means LHS is greater

$n \quad 1024 \rightarrow 2^{10}$

$\log n \rightarrow \log_2 2^{10} \rightarrow 10$

height

Ex: 4

$$T(n) = 1 \cdot T(n/2) + 1$$

Binary Search

$$T(n) = \underline{a}\, T(n/b) + f(n)$$

a 1	b 2	1

$f(n)$	$n^{\log_b a}$
1	$n^{\log_2 1} \Rightarrow n^0 \Rightarrow 1$

Conclusion is

Solution to given recurrence is $\Theta\left(\dfrac{f(n)\log n}{n^{\log_b a}\log n}\right) \Rightarrow \Theta(\log n)$

Limitations / Prerequisites of master theorem :

Divide

combine / Conquer

(1) The formula

$$T(n) = a\underline{T(n/b)} + \underline{f(n)}$$

If the recurrence is not of above form we can not apply MT

(2) Note that the three cases do not cover all the possibilities for f(n).

) There is a gap between cases 1 and 2 when $f(n)$ is smaller than $n^{\log_b a}$ but not polynomially smaller.

$(-\varepsilon$

$\varepsilon > 0$

) There is a gap between cases 2 and 3 when $f(n)$ is larger than $n^{\log_b a}$ but not polynomially larger.

$(+\varepsilon$

$\varepsilon > 0$

Both above gap based recurrences can not be handled. We cannot solve using MT.

- Regularity Condition

Combine

$f(n)$ is 3rd order higher

if $a f(n/b) \leq c f(n)$ for some constants $c < 1$

0.92 0.83

If the function $f(n)$ falls into one of these gaps or if the regularity condition in case 3 fails to hold, we cannot use master theorem to solve the recurrence.

<u>Ex. 5</u> $T(n) = 2T(n/2) + n \lg n$

$T(n) = aT(n/b) + f(n)$

a 2	b 2	$f(n)$ $n\log n$

(combine)

LHS $f(n)$

Root
Divide
level $n \log n$

$n^{\log_b a}$ leaf level work

$n^{\log_2 2} \Rightarrow \underline{n}$

Tough
$n \log n$ is asymptotically greater than n

Case 3 looks promising but let's check

It is not polynomially larger.
The ratio $\frac{f(n)}{n^{\log_b a}} \Rightarrow \frac{n\log n}{n} \Rightarrow \log n$ is
asymptotically less than n^{ϵ} for $\epsilon > 0$.
Consequently, the recurrence falls into the gap between case 2 and case 3.
Hence, we can not solve this recurrence using Master Theorem.