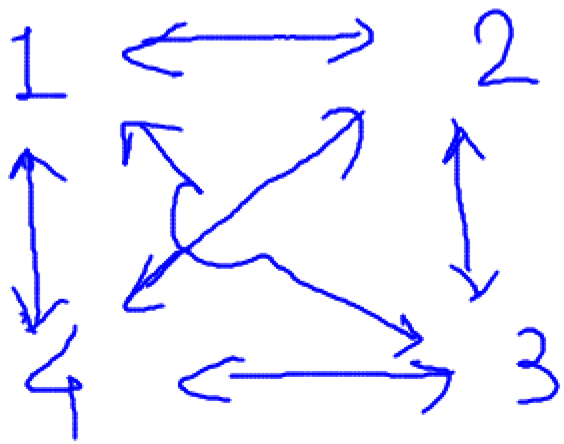


	1	2	3	4
1	0	10	15	20
2	5	0	9	10
3	6	13	0	12
4	8	8	9	0

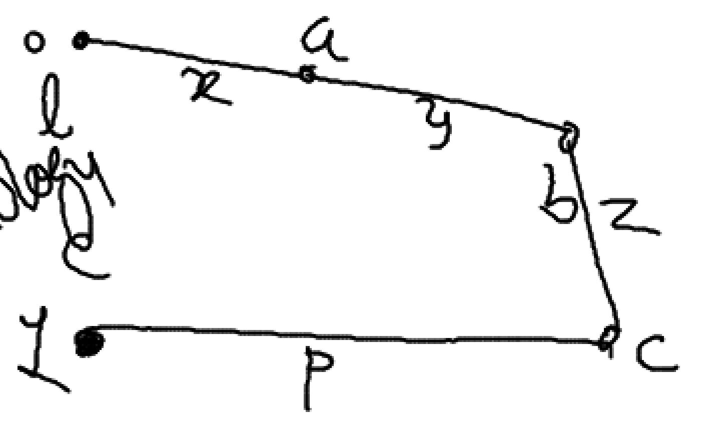
Travelling Salesperson Problem

Using Dynamic Programming

Let $g(i, S)$ be the length of a shortest path starting vertex i going through all vertices in S and ending at 1 (Assuming that route starts from vertex 1 for simplicity)

$S\ \{a, b, c\}$

Helping the terminology



$g(i, S)$

$\Rightarrow x + y + z + p$

So, $g(1, V - \{1\})$ || V is set of vertices of graph $G(V, E)$

It is starting from vertex 1. ✓

It is going to end at vertex 1 ✓

All vertices are covered exactly once except the source. ✓

$g(1, V - \{1\})$ is going to be TSP answer.

Minimum cost of a tour starting 1.

$g(1, V - \{1\})$

$$
\min \{
\begin{array}{ll}
A & C_{12} + g(2, V - \{1, 2\}), \\
B & C_{13} + g(3, V - \{1, 3\}), \\
C & C_{14} + g(4, V - \{1, 4\}), \\
D & C_{15} + g(5, V - \{1, 5\}), \\
\vdots & \vdots \\
N & C_{1n} + g(n, V - \{1, n\})
\end{array}
\}
$$

$$g(1, V-\{1\}) = \min_{2 \le k \le n} \{ c_{1k} + g(k, V-\{1,k\}) \} \quad \text{— (1)}$$

Generally,

$$g(i, S) = \min_{j \in S} \{ c_{ij} + g(j, S-\{j\}) \} \quad \text{— (2)}$$

To find,

$g(1, V-\{1\}) \rightarrow g(k, V-\{1,k\})$

For all choices of k

$g(i, \phi)$ Empty Set	c_{i1}	$\lvert S \rvert = 0$ $g()$	
$g(1,\phi)$	c_{11}	0	$j1$
$g(2,\phi)$	c_{21}	5	$j1$
$g(3,\phi)$	c_{31}	6	$j1$
$g(4,\phi)$	c_{41}	8	$j1$

$\lvert S \rvert = 1$

$g(2,\{3\}) = c_{23} + g(3,\phi) = 9 + 6 = 15$ $j3$

$g(2,\{4\}) = c_{24} + g(4,\phi) = 10 + 8 = 18$ $j4$

$g(3,\{2\}) = c_{32} + g(2,\phi) = 13 + 5 = 18$ $j2$

$g(3,\{4\}) = c_{34} + g(4,\phi) = 12 + 8 = 20$ $j4$

$g(4,\{2\}) = c_{42} + g(2,\phi) = 8 + 5 = 13$ $j2$

$g(4,\{3\}) = c_{43} + g(3,\phi) = 9 + 6 = 15$ $j3$

$|S| = 2$

Now, we compute $g(i, S)$ with $|S|=2$

$i \neq 1$, $1 \notin S$ and $i \notin S$

1/1

$g(2, \{3,4\})$

min of

$c_{23} + g(3,\{4\})$

$9+20=29$

$2 \to 3 \to 4 \to 1$

$c_{24} + g(4,\{3\})$

$10+15=25$

$2 \to 4 \to 3 \to 1$

$g(3, \{2,4\})$

min of

$c_{32}+g(2,\{4\})$

$13+18 = 31$

$3 \to 2 \to 4 \to 1$

$c_{34}+g(4,\{2\})$

$12+13=25$

$3 \to 4 \to 2 \to 1$

$g(4, \{2,3\})$

j2 min of j3

$c_{42}+g(2,\{3\})$

$8+15=23$

$c_{43}+g(3,\{2\})$

$9+18=27$

$4 \to 3 \to 2 \to 1$

$|S|=3$

$g(1, \{2,3,4\})$

min of

$c_{12}+g(2,\{3,4\})$

$10+25=35$ j2

$1 \to 2 \to 4 \to 3 \to 1$

$c_{13}+g(3,\{2,4\})$

$15+25=40$ j3

$c_{14}+g(4,\{2,3\})$

$20+23=43$ j=4

1	2	3	4	1	?
1	2	4	3	1	35
1	3	2	4	1	?
1	3	4	2	1	40
1	4	2	3	1	43
1	4	3	2	1	?

Increasing more cost