QFin Pairs Trading Report

Emerald Traders

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1 Important definitions

- **Strictly stationary process** $\rightarrow$ A stochastic process whose parameters are constant with respect to time. As a result, stationary processes have a constant mean and variance.
- **Order of integration** $\rightarrow$ Order of integration (denoted $I(d)$) reports the number of differences d required for a process to become stationary.
- **Cointegration** $\rightarrow$ Two processes x_t and y_t are cointegrated if x_t and y_t are $I(1)$ (integrated order 1) and $\exists\beta : z_t = x_t - \beta y_t$ where z_t is $I(0)$ (integrated order 0).
- **Correlation** $\rightarrow$ correlation, in a mathematical sense, is a measure of the linear relationship between two variables.

2 Background

Pairs trading is a strategy that works off of the assumption that the value of a pair of stocks is dependent on an underlying economic factor. Under this assumption, stock prices move together, however their movements aren't always in sync. Sometimes two stocks are further apart than they normally are, and sometimes they are closer together than they normally are. When this happens there is a statistical arbitrage opportunity.

When they are far apart, we can short the higher stock, and go long the higher stock. This way we can make money as the stocks return back to their baseline separation. This makes pairs trading a **market neutral strategy**; if the whole market collapses, we will lose money on our long position but, if the stocks are still moving together, make it back on the short position.

There are several details that we need to work out in order to construct a pairs trading algorithm:

- Choosing a pair
  - How do we determine whether a pair move together?
  - How can we reduce our exposure to the market?
- Developing a trading algorithm
  - How do we determine the baseline difference between two stocks?
  - How do we determine when we are far enough from the baseline to trade?
  - How do we determine when we have returned to the baseline and can close out our trade?
  - How do we determine when we are too far from the baseline and want to close out our trade to mitigate losses?

3 The nature of the data

Our data consists of 1 minute data for all stocks on the S&P500 from 03/02/2020 to 30/04/2020. In this time period, COVID-19 had a significant impact on the stock market, hence large fluctuations in stock prices are visible. In times of crisis, systemic risk amongst all stocks increase, which increases correlation amongst assets, as generally all stocks would experience a decrease in their price as people react to the crisis. This factor was considered in our determination of stocks by looking at 1.5 months worth of data instead of the full 3 months.
For some analyses, we aggregated the data to larger time intervals (mainly 5 minutes) in order to reduce the amount of noise in the data.

4 Picking a pair

For this project, we looked at trading equity pairs from the S&P500. There are many different ways (both qualitative and quantitative) that pairs can be selected. The approach that we went for was a mix of qualitative and quantitative.

First we grouped the stocks by industry with the goal of finding pairs within the same industry. Choosing pairs in the same industry allows the strategy to be sector neutral as well as market neutral. This way if there is an event that significantly effects the price of stocks in one sector, we won't be significantly impacted. Another consideration is which sector to choose. Stocks in the technology sector, although being in the same sector, may have weaker relationships between stocks. Intuitively the stock prices for major technology companies (eg. cell phone companies) will be dependent on their marketing, products they release, as well as the public's opinion of their products. These individual signals for each stock would make it unfeasible to implement a stocks trading algorithm with them. On the other hand, a sector such as energy or mining would have very similar products, and hence finding a pair of stocks here would be more viable.

Once we have selected the sectors of interest we needed to determine the way we wanted to quantitatively filter out stocks. There are multiple ways that we can go about choosing stocks. A couple options include **correlation** and **cointegration**. We ended up opting for cointegration as it seemed to be the more widely used method for pairs trading and the more robust method of the two, however there does seem to be a connection between cointegration and correlation which will be explored in a later section.
This test was performed over the most recent 30 days of our dataset. The cointegration test p-value can be interpreted as the probability of the two stock movements acting the way they are if they weren't cointegrated. Hence a low p-value would indicate that the stocks are likely cointegrated.

4.1 A bit more on cointegration

The mathematics behind cointegration (a topic in time series analysis) is quite dense. Despite this, an understanding of cointegration and its importance in the context of picking pairs can still be attained.

We can view a stock as being composed as being composed as having a signal component and a noise component. The signal is a time varying function that determines the movement of the stock, and the noise component is stochastic noise (with constant variance) causing the stock price to fluctuate about the signal. Different stocks may be affected by a signal to different extents.

Ideally, we would like two stocks that move together (ie. they have the same scaled signal). If we have two stock prices x_t and y_t, and we can find an optimal constant β, by evaluating

$$z_t = x_t - \beta y_t$$

we would end up eliminating the signal just leaving the noise (z_t would fluctuate stochastically about $z_t = 0$). If, at anytime, z_t is sufficiently above 0, then x_t would be over-valued, and y_t

would be under-valued. By shorting x_t and going long on y_t, gains can be made while the stocks re-establish equilibrium. Likewise, if z_t is sufficiently below 0, we would go long on x_t and short y_t.

4.2 Implementing cointegration tests in the Energy sector

We performed pairwise cointegration tests on the 23 stocks from the S&P500 in the Energy sector. We performed cointegration twice, once over the data belonging to the whole time period, and another time only using the data from the month of April. As a lot of stocks were impacted by COVID-19 before in February and March, we wanted to see whether we received different cointegration results using the two timeframes.

Figure 1: Cointegration for the energy sector from 03/02/2020 to 30/04/2020

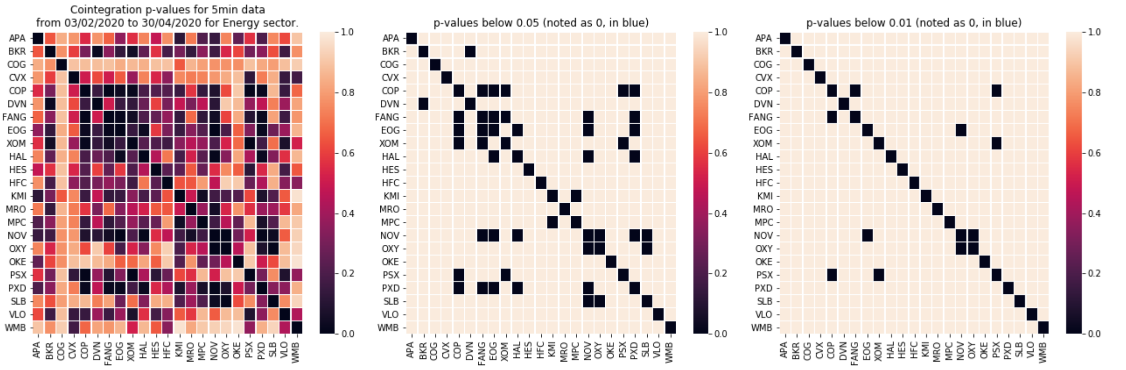
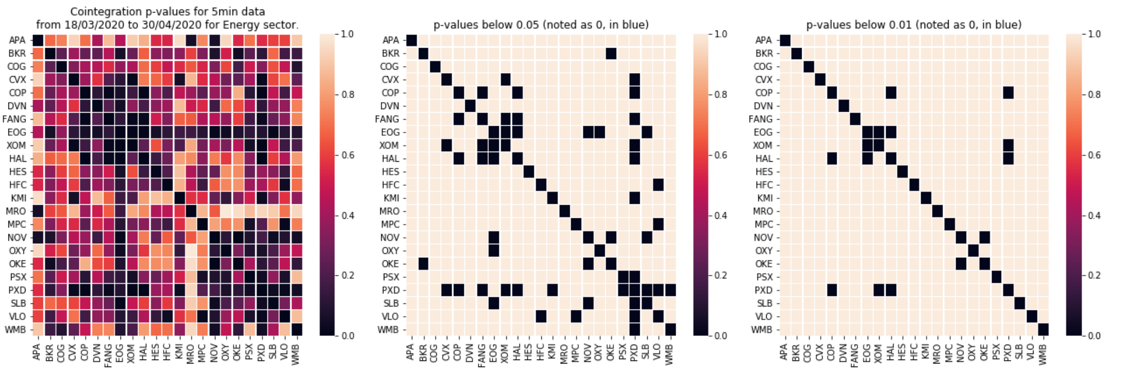


Figure 2: Cointegration for the energy sector from 18/03/2020 to 30/04/2020



From these heatmaps, its evident that the two timeframes have roughly the same number of pairs, however the pairs themselves very different. A lot of the pairs with p-values below 0.05 in the full timeframe do not appear to be cointegrated on one month timeframe.

4.3 Filtering Stocks Pair Candidates

The most challenging element of pairs trading is determining the actual pairs to trade. Through mathematical tests, cointegrated pairs can be found however there still lies the risk of false positive data, that if you took the results at face value and traded them, could lead to significant loss. Thus a more helpful approach instead of purely quantitative models, is to combine these models with fundamental analysis.

A further quantitative approach was considered to help select pairs called a Bonferonni test[1].

[1] https://www.investopedia.com/terms/b/bonferroni-test.asp

This was applied, at first, to reduce false positives but only three pairs held a significant p-value for the cointegration test. It is a well known problem that Bonferroni for larger datasets often over-corrects and many results become false negatives. Thus, we decided that it would be better to work with false positives and filter them out with other diagnostics at run time instead of using the limited possibilities that the Bonferonni correction left us.

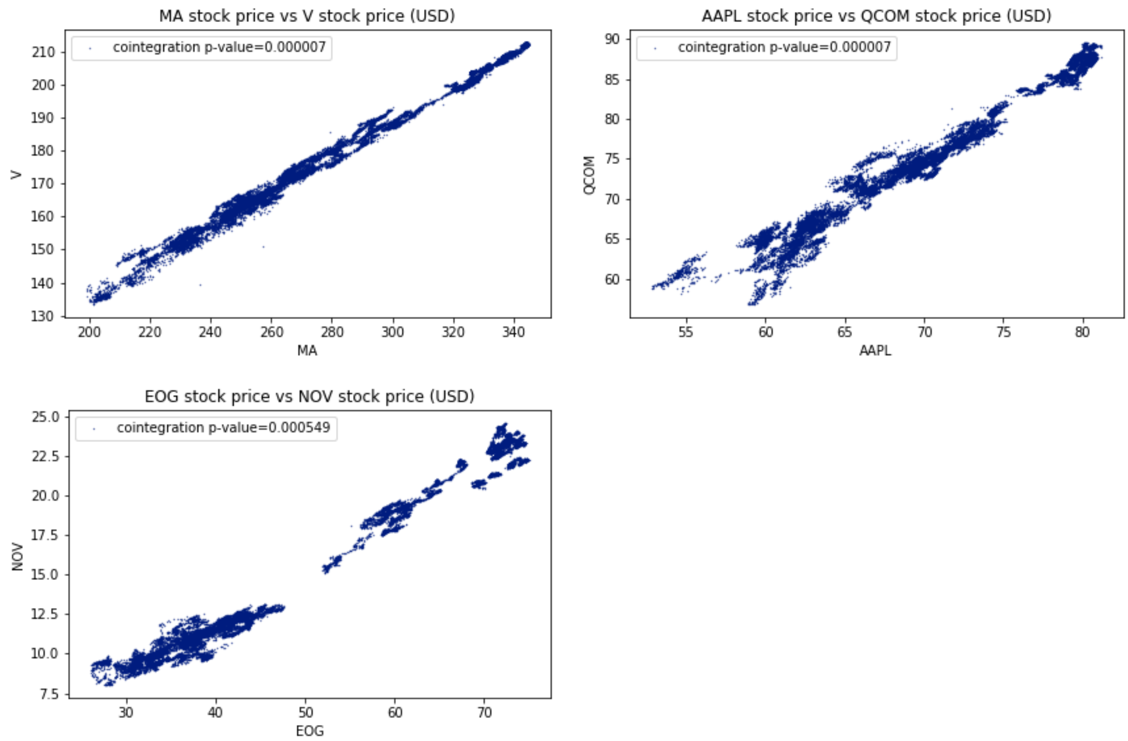
Some fundamental analysis factors[2] considered were:

- **The sectors:** Applying the cointegration test across all 500 stocks would have likely resulted in multiple comparison biases occurring. This was reduced by looking within a specific industry, that are subject to similar risk. So the relationship between the two stocks would be less likely to be arbitrary. Hence why we decided to only look into the Energy industry.
- **Volatility/ Market Beta:** Two stocks with dissimilar volatility, are unlikely to have their stock prices to move together, thus two stocks of similar volatility would be ideal. This factor is taken into consideration in the algorithm.
- **Liquidity:** Statistical arbitrage operates under the assumption that there it will be easy to close our position cause there is a high volume of demand. But keeping in mind times of crisis, where it is harder to liquidate stocks. Stocks also needed to be chosen based on how liquid they are. This was also taken into consideration in the implementation of the algorithm

4.4 A connection between cointegration and correlation

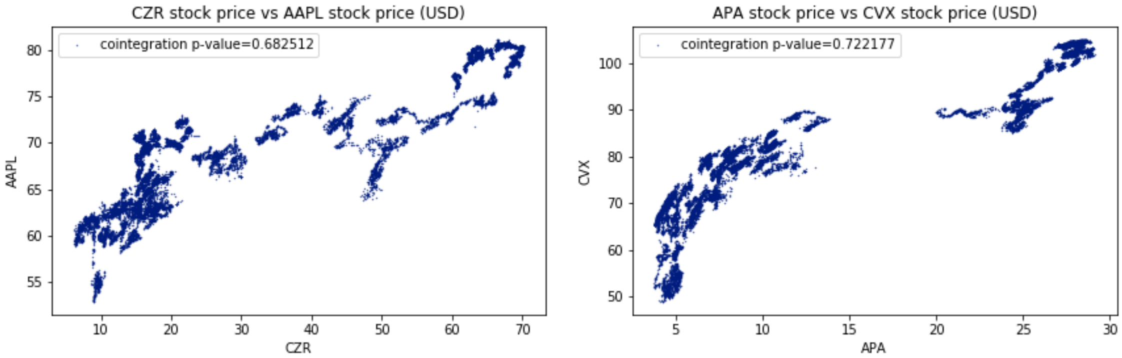
Interestingly, by examining plots of the data is seems that stocks that are cointegrated also have significant correlation, and vice versa.
Examining the key differences between correlation and cointegration, and the implications of using correlation to build an algorithm could be an interesting path of enquiry in future.

Figure 3: Cointegrated stocks with varying p-values



[2]https://www.maths.ox.ac.uk/system/files/attachments/593233.pdf

Figure 4: Non-cointegrated stocks



5 The Algorithm

5.1 General Overview

The strategy employed was built by starting off with the code in a Quantconnect implementation[3].

The strategy used by this algorithm was to start off with some predetermined stocks and only trade those which deviated least from the mean of their difference. Such stocks are the ones which tend to revert back to the mean more strongly and so are the most consistent to pairs trade with. The algorithm reran these calculations every six months in order to update the pairs that were being traded.

It would take a market position once the difference of two stocks was more than two standard deviations away from the mean (if it had not already) and liquidate their assets if the difference was less than two standard deviations away from their mean otherwise.

We have made the decision to run the algorithm on daily data (like the Quantconnect implementation) as any smaller timescale is too small for it to make any profit, and even when it makes a small return the Quantconnect fees diminish them.

5.2 Changes

We changed the inputs to turn our cointegrated pairs into a list of stocks that could potentially be cointegrated (under the assumptions that if two stocks are cointegrated with a third then they are likely cointegrated with each other). In order to mitigate the risk of this procedure we only took stocks from the energy sector, as this would reduce the potential for false positives.

We then added some basic blanket error handling in the case that Quantconnect API could not provide specific data. We ignored the stock in such cases that this happened. We also made the addition of liquidating all stocks that were deemed too unstable to trade instead of holding them (like the Quantconnect implementation did). This made our algorithm more stable to periods of market volatility and less prone to risk. This change also made it possible to update the stocks we were trading more often (every four months instead of six) without risk of losing them in the portfolio.

Another change that we made is that we changed the threshold to liquidate assets to one standard deviation instead of two. This reduced the cost of fees as a bigger shift in price difference was needed to liquidate assets while keeping profits as when sold the stocks would have a chance to converge to the mean.

[3] https://www.Quantconnect.com/tutorials/strategy-library/pairs-trading-with-stocks

5.3 Rebalance

The Rebalance algorithm is set to run every three months. This determines which stocks to trade for the next three months.

The top four stocks are picked every three months and the metric by which they are ranked is the sum of their square residuals (from the difference). Stocks which tend to deviate very far from the mean are more likely to never revert and so are less likely to be considered. This is the diagnostic used to attempt to filter out suspected false positives at runtime.

Stocks no longer being traded are liquidated.

5.4 Taking a market position

OnData calculates the differences between two stocks and normalizes it. This gives us a random variable that is continuous, and being standardized, it has a mean of zero.

We determine that we want to take a market position if the current value deviates more than two standard deviations from the mean if we do not already have a market position. If the current value is positive, this indicates that either the first stock has increased in value, or the second stock has decreased in value. In both situations we take a short position on the first stock and a long position on the second stock. If we have a negative value, this indicates the opposite movements, and we take a long position on the first stock and short the second.

If the current value is less than one standard deviation from the mean and we already have a market position then we will liquidate all of our stocks as the data has reverted to the mean.

These two behaviours when coupled together will take a market position when the two stocks diverge from the mean and then we liquidate our assets when they converge to the mean.

It is important to know that when taking a long position on one stock and a short position on a cointegrated stock that we need to invest the same monetary amount in both. We divide the cost of the first stock by the second to determine the ratio of the prices, and then multiply the quantity of the first stock traded by the ratio to calculate the quantity of the second stock that we need to trade. Trading stocks in the ratio of their prices ensures that if the market crashes our algorithm does not turn a loss, and when the stocks revert to their mean our algorithm will turn a profit.

5.5 Results

Averaging some backtesting data the algorithm seems to return a profit of 8%. This is not a random or fair sample but merely an estimate of the algorithms potential performance.

Over a period of 7 years there are only about 150 trades on average. This is approximately two monthly trades on average. This hints that the frequency at which pairs diverge from and then revert to the mean is on average twice a month. This indicates that the timescale that which our algorithm must consider is at minimum one month (two periods). And it is sensible to consider more than two periods of history so that the moving average is not only more accurate but the algorithm does not react too sporadically to jumps in price.

It is noteworthy that for a lot of pairs there is a great spike upwards or downwards in our profit during the time COVID-19 hit (the first few months of 2020). This spike often dictates what the overall return of the algorithm is and might be due to stocks no longer being cointegrated during the pandemic. Furthermore behaviour after the pandemic tended to be very sporadic as the historical data being used was bias.

Figure 5: Spike in profits during the COVID-19 pandemic

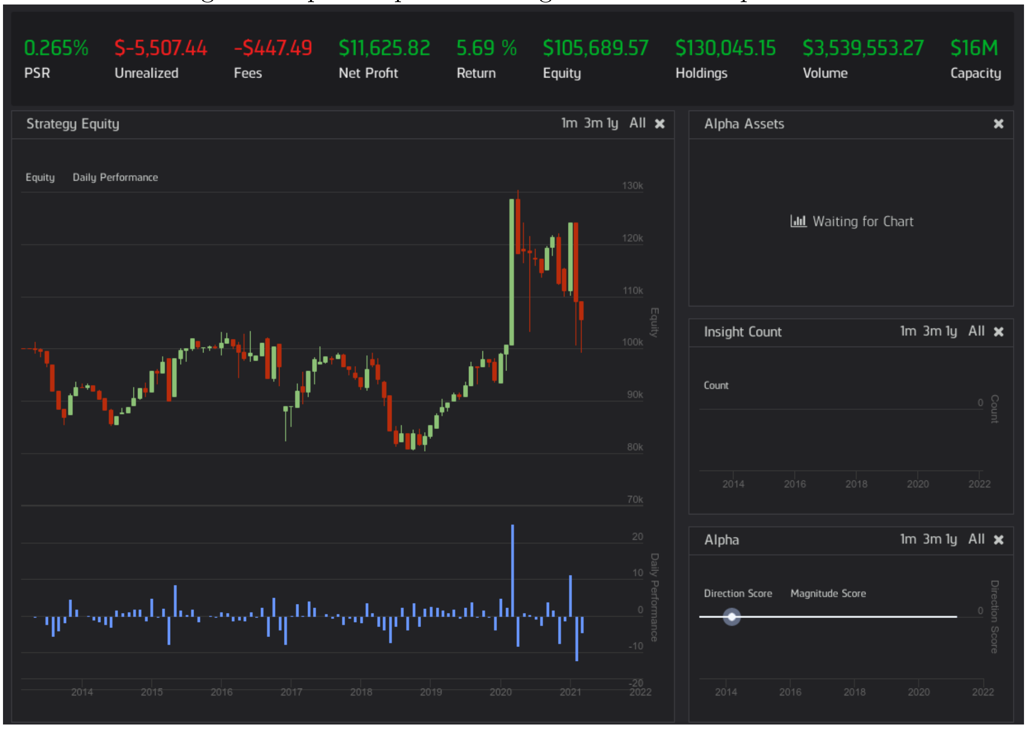
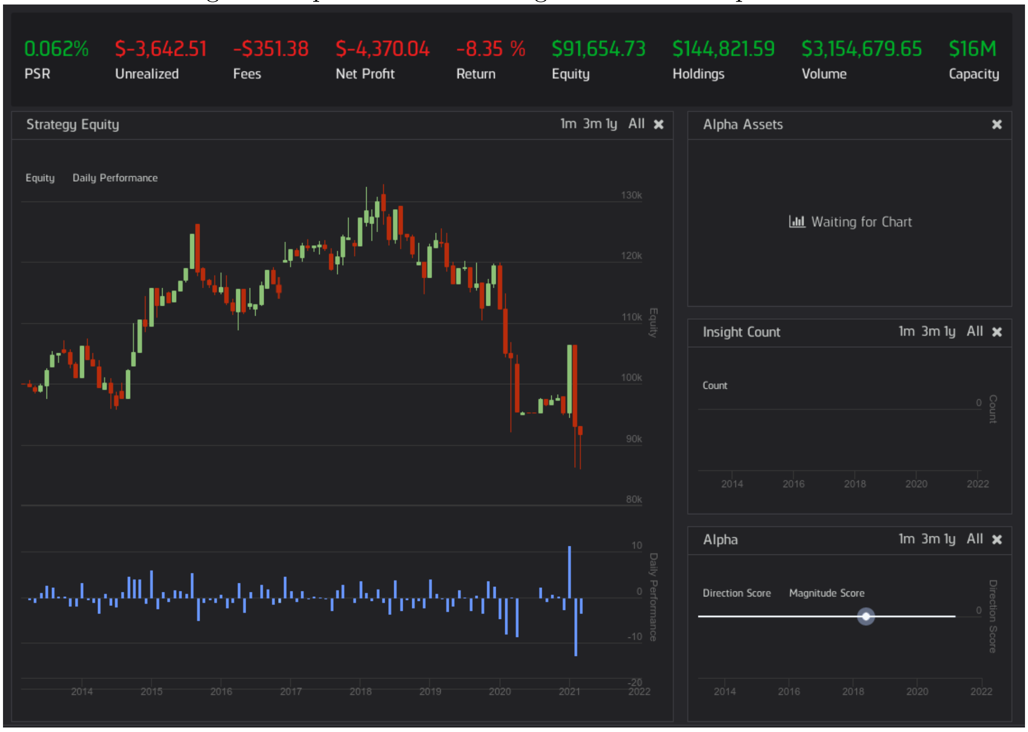


Figure 6: Spike in losses during the COVID-19 pandemic



While the algorithm on average earns a profit it is not stable and cannot be depended on to perform consistently. While sometimes it makes a relatively steady profit, at other times it fails. The figures below show how the algorithm can output sporadic results in both directions:

Figure 7: Long term profit (over 10 years)

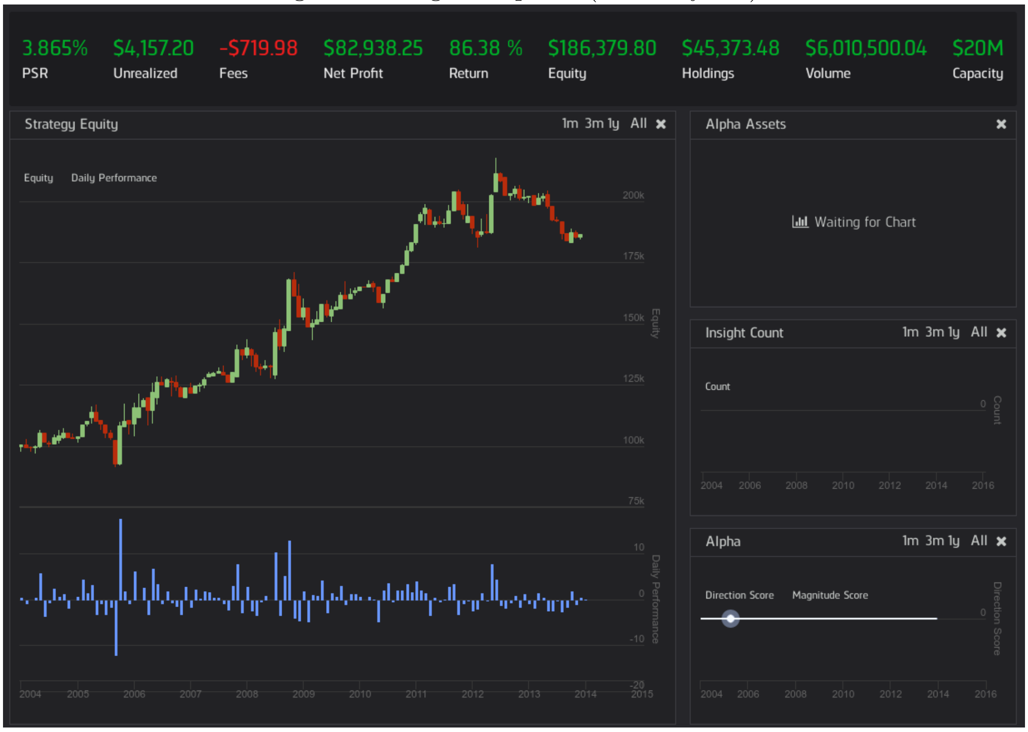
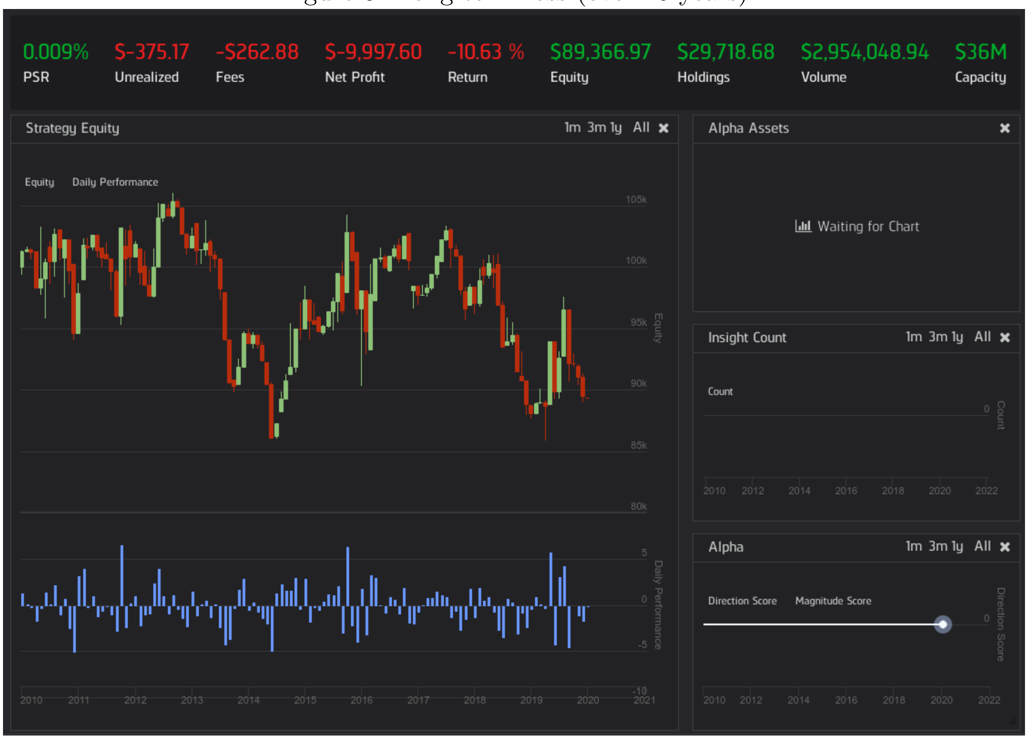


Figure 8: Long term loss (over 10 years)



Cointegration analysis on stocks in the months surrounding March 2020 showed that very few stocks remained cointegrated across February, April and March of 2020, meaning trading relying on cointegration during the beginning of the pandemic would be unreliable. Since our algorithm was updating the pairs of stocks it was trading every three months and needed at minimum one month of history such a quick shift in the market would be difficult to detect in time to take any action.

Figure 9: Cointegration for the energy sector from 01/02/2020 to 1/03/2020

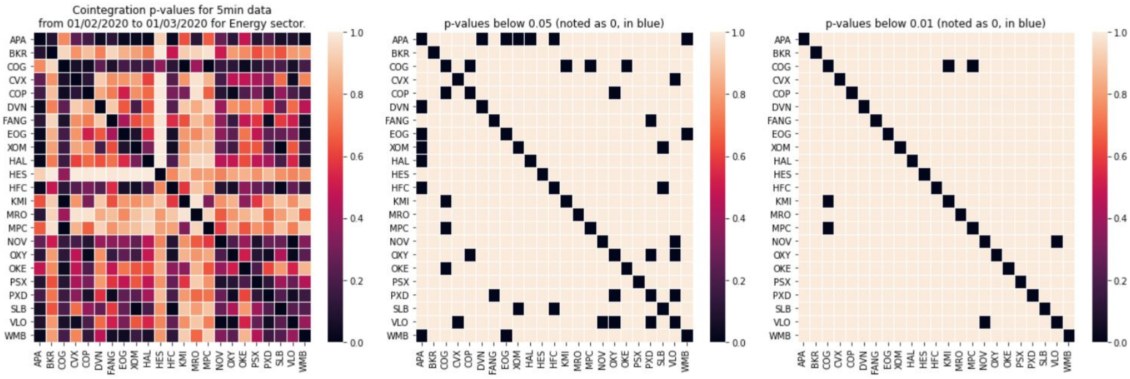


Figure 10: Cointegration for the energy sector from 01/03/2020 to 1/04/2020

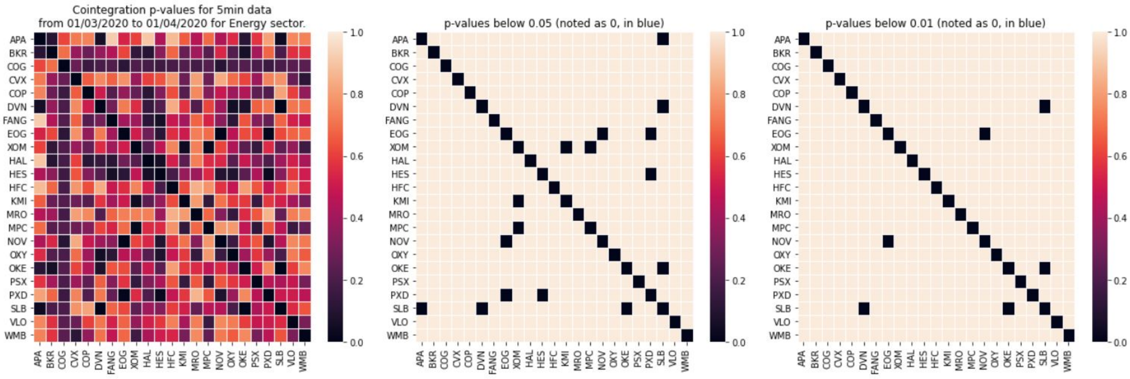


Figure 11: Cointegration for the energy sector from 01/04/2020 to 1/05/2020