Inflation to Structures: EFT all the way

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ABSTRACT: We investigate for an Effective Field Theory (EFT) framework that can consistently explain inflation to Large Scale Structures (LSS). With the development of the construction algorithm of EFT, we arrive at a properly truncated action for the entire scenario. Using this, we compute the two-point correlation function for quantum fluctuations from Goldstone modes and related inflationary observables in terms of coefficients of relevant EFT operators, which we constrain using Planck 2015 data. We then carry forward this primordial power spectrum with the same set of EFT parameters to explain the linear and non-linear regimes of LSS by loop-calculations of the matter overdensity two-point function. For comparative analysis, we make use of two widely accepted transfer functions, namely, BBKS and Eisenstein-Hu, thereby making the analysis robust. We finally corroborate our results with LSS data from SDSS-DR7 and WiggleZ. The analysis thus results in a consistent, model-independent EFT framework for inflation to structures.

Contents

1   Introduction

Physics of Large Scale Structure (LSS) provides us with a plethora of information about the initial condition and subsequent evolution of the universe. Studies of LSS are usually materialised by a linear perturbation theory (LPT) (see, for example, [1]) keeping in mind the observational fact that on the large scale the perturbations are linear. In this regime the Fourier modes of perturbations evolve independently of each other keeping the stochastic properties of the primordial fluctuations intact. However, at small scales the fluctuations become non-linear and the modes no longer remain independent of each other. Consequently, small scales affect the evolution of large scales [2, 3]. Studies of small scale physics thus become a bit tricky due to the overlapping of modes. Additionally, there are several ambiguities at small scales that leads to degeneracies among models, and a fairly clear understanding of non-linear regime from a non-linear cosmological perturbation theory (NLCPT) is yet to be achieved. Early attempts in this direction were made by calculating the next-to-leading order correction for Gaussian initial condition [4–6]. In this papers, for the linear regime, the usual Harrison-Zel'dovich power spectrum with power law behaviour $P(k) \propto k^n$ have been considered for $k_{IR} < k < k_{UV}$; where k_{IR} and k_{UV} are, respectively, infrared and ultra-violet cutoffs for the power spectrum; whereas for the non-linear regime,

the exponent n of the power spectrum was constrained by calculating the loop contributions and subjecting them to N-body simulation. Subsequently, in [7] the authors introduced renormalized perturbation to NLCPT and tried to cancel the cutoff dependence of the loops in small scales.

Given the difficulties in proposing a proper NLCPT, searches for alternative ideas of explaining the non-linear regime are in vogue. One such useful alternate technique is based on Effective Field Theory (EFT henceforth) approach. In [8] the authors describe how one can conveniently develop a perturbation theory based on EFT that can, to a considerably good extent, explain LSS perturbations. Here they integrate out small scales perturbations beyond a UV cutoff and study their effects on the large scales. Further developments of issues like renormalization, higher loop calculation, bispectrum calculation etc. can be found in [9–12]. The idea of [8] has also been extended to non-Gaussian initial condition in [13] for a nearly scale invariant power spectrum. [14–19] also discuss some very recent developments in the direction of LSS from EFT. In [20, 21] the authors pursue an alternative but related technique to [8] called Coarse Grained Perturbation Theory where the UV source terms themselves are measured using N-body simulations.

On the other hand, primordial cosmology based on inflationary paradigm provides meticulously the seeds of fluctuations that eventually grow due to gravitational instability and give rise to LSS. A consistent corroboration of any LSS theory with corresponding inflationary theory from the same platform is thus quite unavoidable. The usual method to study inflationary cosmology is to start with a scalar field characterized by a potential and study quantum fluctuations of the field. An extensive list of the potentials that are usually considered is given in [22] and an analysis of the models while comparing with CMB data can be found in[23]. In recent years an alternative approach to describe perturbations based on EFT has been introduced [24, 25]. Here, considering that the inflaton field ϕ is a scalar under all diffeomorphisms (diffs) but the fluctuation $\delta\phi$ is scalar only under spatial diffs and transforms under the time diffs., one can construct an effective action in unitary gauge where fluctuation in inflaton is zero, by allowing every possible gravitational fluctuation interaction terms that respect the symmetry. Using *Stückelberg mechanism* [26, 27] the gravitational perturbation can be written in terms of massless Goldstone boson and curvature perturbation can also be related to the Goldstone boson. One interesting feature of this scenario is that at high energies the Goldstone boson decouples from gravitational fluctuation and the corresponding Lagrangian becomes very simple to study perturbation and in different parameter space of EFT coefficients one can retrieve different inflationary models. In this regard this theory is a model independent description of inflation. Further developments of EFT of inflation from different prescriptions have also been done in [28–32]. In [33, 34] it has been shown how one can generate large non-Gaussianity from single field inflation with sound speed $c_s < 1$ using EFT. Some recent works on diverse aspects of EFT of inflation can also be found in [35–42].

However, it is very important to note that a systematic development of a consistent, model-independent LSS perturbations starting from an EFT for inflation is still left unexplored. This is due to the fact that EFT for LSS and EFT for inflation have till now been studied separately, from different EFT frameworks, without bothering much about the other. This raises serious concerns, such as whether or not the constraints on EFT parameters in one framework is consistent with the other. In this article our primary intention

is to build a model-independent description of LSS for Gaussian initial condition starting from EFT of inflation. In this theory, the primordial power spectrum is characterized only by the EFT parameters and cutoff energies of the theory. This also takes care of proper truncation as derived from the construction rules of EFT action, thereby leaving behind any ambiguity or inconsistency in the development of the theory. IN other words, the EFT action of our consideration is a consistent one developed from the first principles. Equipped with this, we would engage ourselves in analysing quantum fluctuations during inflation and finding out the primordial power spectrum and spectral tilt. We would also constrain the EFT parameters using Planck 2015 data [45, 46] thereby making the theory consistent with latest observations as well.

We would then try to explore the possibility of viability of the same EFT framework in LSS, in particular, to non-linear regime, by calculating loop-corrections to the matter overdensity two-point correlation function. The fundamental input in the LSS sector of EFT is that here we will not consider Harrison-Zel'dovich spectrum a priori. Rather, we will carry forward the same primordial power spectrum as obtained from our EFT of inflation. This will in turn take into account the effect of spectral tilt in the analysis of LSS, which is more accurate given present observational scenario. We will also be consistent with the constraints on the EFT parameters as obtained from CMB observations, thereby making the entire analysis self-consistent and coming from a single EFT with same cutoff throughout. From this primordial power spectrum we explore both the linear and non-linear regimes by calculating upto one-loop corrected power spectrum. In our analysis of LSS from EFT, we will make use of two widely accepted transfer functions, namely the BBKS [48] and Eisenstein-Hu [49], thereby making the analysis robust. This will also help us do a comparative analysis of the applicability of those fitting functions in the context of EFT for LSS. In the process we will confront our results with LSS data from SDSS-DR7 [50] and WiggleZ [51] data and search for consistent constraints on the EFT parameters, thereby bringing inflation and LSS under a common EFT framework.

2 Construction of consistent EFT for inflation

Let us begin with the construction of a consistent EFT which is applicable to both inflation and LSS. Although our construction is somewhat motivated by some earlier works in this field [24, 25, 28–32, 35–44], we develop the framework in our own language that will also help in ariving at a consistent description of inflationary framework as well as large scale structures.

2.1 Truncated action for EFT

The construction algorithm of the EFT action is as follows:

1. In this description we will deal with quasi de Sitter background with spatial slicing, which can be identified as a single physical clock using which one can describe the cosmological evolution smoothly. This can conveniently be described by time evolution of a single scalar field $\phi(t)$. In order to deal with the cosmological perturbations in this framework one needs to choose unitary gauge where all the slicing coincides with hypersurfaces of constant time i.e. $\delta\phi(t, \mathbf{x}) = 0$. This guarantees that cosmological

perturbations are fully described by metric fluctuation alone and no other explicit contribution from the perturbation of the scalar field appear in the theory.

2. The temporal diffeomorphisms are completely fixed by this specific gauge choice. Consequently, the metric fluctuation from graviton is described by three degrees of freedom: two projections of tensor helicities and one scalar mode. One can further interpret that the scalar perturbation is completely eaten by metric, which mimics the role of Goldstone boson as appearing in the context of any $SU(N)$ non abelian gauge theory.

3. In order to construct a general, model independent action of EFT in this framework, one needs to define a class of relevant operators $\delta\mathcal{O}_{\mathbf{EFT}}$ that are function of the background metric and invariant under time dependent spatial diffeomorphisms in linear regime as described by following coordinate transformation

$$x^i \rightarrow x^i + \xi^i(t, \mathbf{x}) \quad \forall \quad i = 1, 2, 3. \tag{2.1}$$

This implies that the spatial diffeomorphism is broken by an amount $\xi^i(t, \mathbf{x}) \ \forall \ i = 1, 2, 3$. For example, we consider temporal part of the background metric g^{00} and the extrinsic curvature of hypersurface with constant time extrinsic curvature $K_{\mu\nu}$, which are transforming like a scalar and tensor under time dependent spatial diffeomorphisms in linear regime respectively. It is important to mention here that, in the present context the preferred slicing of the spacetime is described by a function $\tilde{t}(x) = \tilde{t}(t, \mathbf{x})$, which has a timelike gradient, $\partial_\mu \tilde{t}(x) = \delta^0_\mu$. Such a function is necessary to implement time diffeomorphism in the linear regime. Covariant derivatives of $\partial_\mu \tilde{t}(x)$ (essentially the covariant derivatives of unit normal vector n_μ defined later) and its projection tensor $K_{\mu\nu}$ play significant role in the construction of the EFT action in unitary gauge. In the unitary gauge the temporal coordinate coincide with this specified function. This restricts all other additional degrees of freedom appearing in this function and the final form of the EFT action is free from any ambiguity arising from such contributions.

4. It is worthwhile to mention that in the construction of EFT, tensors are crucial. Let us give an example with two tensor operators $\hat{\mathcal{O}}_A$ and $\hat{\mathcal{O}}_B$ which can be defined as:

$$\hat{\mathcal{O}}_A = \left(\hat{\mathcal{O}}_A^{(0)} + \delta\hat{\mathcal{O}}_A\right), \quad \hat{\mathcal{O}}_B = \left(\hat{\mathcal{O}}_B^{(0)} + \delta\hat{\mathcal{O}}_B\right). \tag{2.2}$$

Here the superscript (0) stands for tensor operators in unperturbed FLRW background and the rest of the part signify fluctuations on these operators. The construction rule of these operators give rise to a composite operator

$$\hat{\mathcal{O}}_A\hat{\mathcal{O}}_B = \left[-\hat{\mathcal{O}}_A^{(0)}\hat{\mathcal{O}}_B^{(0)} + \delta\hat{\mathcal{O}}_A\delta\hat{\mathcal{O}}_B + \hat{\mathcal{O}}_A^{(0)}\hat{\mathcal{O}}_B + \hat{\mathcal{O}}_A\hat{\mathcal{O}}_B^{(0)}\right]. \tag{2.3}$$

where $\hat{\mathcal{O}}_A^{(0)} = \hat{\mathcal{O}}_A^{(0)}(g_{\mu\nu}, n_\mu, t)$ and $\hat{\mathcal{O}}_B^{(0)} = \hat{\mathcal{O}}_B^{(0)}(g_{\mu\nu}, n_\mu, t)$. In our construction of EFT, possible choices for the tensor operator $\hat{\mathcal{O}}_B$ are extrinsic curvature $K_{\mu\nu}$ and Riemann tensor $R_{\mu\nu\alpha\beta}$.

5. As mentioned earlier, covariant derivative of the unit normal vector n_μ also plays a significant role in the construction of EFT. For example:

$$\int d^4x \sqrt{-g} J(t) K_\mu^\mu = \int d^4x \sqrt{-g} J(t) h_\mu^\nu \nabla_\nu n^\mu = \int d^4x \sqrt{-g} \sqrt{-g^{00}} \dot{J}(t). \quad (2.4)$$

where $J(t)$ is a any arbitrary time dependent contribution appearing in the action.

Using the above algorithm, a fairly general EFT action for inflationary cosmology can be written as

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_p^2 R - \Lambda(t) - c(t) g^{00} + \delta \mathcal{O}_{\mathbf{EFT}} \right] \quad (2.5)$$

where the first three terms in the parenthesis determine the background metric and the last term takes care of the perturbations. In terms of the EFT parameters for the background $\Lambda(t)$ and $c(t)$, the good old field equations for FLRW background can be expressed as [24]

$$H^2 = \left(\frac{\dot{a}}{a} \right)^2 = \frac{\Lambda(t) + c(t)}{3M_p^2}, \qquad H^2 + \dot{H} = \frac{\ddot{a}}{a} = \frac{\Lambda(t) - 2c(t)}{3M_p^2}. \quad (2.6)$$

where $H = \dot{a}/a$ is the usual Hubble parameter. For perfect fluid the components of the stress energy tensor can then be identified as

$$\rho = c(t) + \Lambda(t), \quad p = c(t) - \Lambda(t). \quad (2.7)$$

Recasting the above field equations one can get the following expressions for $c(t)$ and $\Lambda(t)$ [1]:

$$c(t) = -\dot{H} M_p^2, \quad \Lambda(t) = \left(\dot{H} + 3H^2 \right) M_p^2. \quad (2.10)$$

Plugging them back in the EFT action Eq(2.5) one can recast it in the following form

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_p^2 R - \left(\dot{H} + 3H^2 \right) M_p^2 + \dot{H} M_p^2 g^{00} + \delta \mathcal{O}_{\mathbf{EFT}} \right] \quad (2.11)$$

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[1]If we assume that the cosmological dynamics is governed by a single scalar field ϕ which has canonical kinetic term and minimally couples with Einstein's gravity, then in unperturbed FLRW background the matter action can be expressed as:

$$S = \int d^4x \sqrt{-g} \left[X - V(\phi) \right], \quad (2.8)$$

where $X = -g^{00}\dot{\phi}^2/2$, is the canonical kinetic term for the matter scalar field in FLRW background. Here we get following expressions for $c(t)$ and $\Lambda(t)$:

$$c(t) = \frac{\dot{\phi}^2}{2}, \quad \Lambda(t) = V(\phi). \quad (2.9)$$

Let us now elaborate the last term in the EFT action. The total contribution from the EFT operators from quantum fluctuation can be written as a sum of individual contributions from different types of operators

$$\delta\mathcal{O}_{\mathbf{EFT}} = Y(R_{\mu\nu\alpha\beta}, g^{00}, K_{\mu\nu}, \nabla_\mu, t) = \sum_{i=1}^{\infty} \hat{\mathcal{O}}_{(i)}, \tag{2.12}$$

where the individual operators $\hat{\mathcal{O}}_{(i)}$ are defined as:

$$\hat{\mathcal{O}}_{(1)} = \sum_{n=2}^{\infty} \frac{M_n^4(t)}{n!} (\delta g^{00})^n, \tag{2.13}$$

$$\hat{\mathcal{O}}_{(2)} = -\sum_{q=0}^{\infty} \frac{\bar{M}_1^{3-q}(t)}{(q+2)!} \delta g^{00} \left(\delta K_\mu^\mu\right)^{q+1}, \tag{2.14}$$

$$\hat{\mathcal{O}}_{(3)} = -\sum_{m=0}^{\infty} \frac{\bar{M}_2^{2-m}(t)}{(m+2)!} \left(\delta K_\mu^\mu\right)^{m+2}, \tag{2.15}$$

$$\hat{\mathcal{O}}_{(4)} = -\sum_{m=0}^{\infty} \frac{\bar{M}_3^{2-m}(t)}{(m+2)!} [\delta K]^{m+2}, \tag{2.16}$$

$$\cdots\cdots\cdots \quad \cdots\cdots\cdots\cdots \quad \cdots\cdots\cdots .$$

where $\cdots$ represent contributions from higher order fluctuations which involve higher derivatives of the metric. Here M_i, $\bar{M}_i$ etc are otherwise arbitrary EFT parameters which need to be constrained in the process of development of EFT theory of inflation using CMB data. This we will do later on in this article.

Further, we have introduced a symbol $[\delta K]$, which is defined by the following contraction rule:

$$[\delta K]^2 = \delta K_\nu^\mu \delta K_\mu^\nu, \tag{2.17}$$

$$[\delta K]^3 = \delta K_\nu^\mu \delta K_\delta^\nu \delta K_\mu^\delta, \tag{2.18}$$

$$\cdots\cdots\cdots \quad \cdots\cdots\cdots\cdots \quad \cdots\cdots\cdots$$

$$[\delta K]^{m+2} = \delta K_{\mu_2}^{\mu_1} \delta K_{\mu_3}^{\mu_2} \delta K_{\mu_4}^{\mu_3} \cdots \delta K_{\mu_{m+2}}^{\mu_{m+1}} \delta K_{\mu_1}^{\mu_{m+2}}. \tag{2.19}$$

In this context, we use the following definitions of extrinsic curvature $K_{\mu\nu}$, induced spatial

metric $h_{\mu\nu}$ and unit normal n_μ [2][24]:

$$h_\alpha^\mu h_\beta^\nu h_\gamma^\rho h_\delta^\sigma R_{\mu\nu\rho\sigma} = K_{\alpha\gamma}K_{\beta\delta} - K_{\beta\gamma}K_{\alpha\delta} + {}^{(3)}R_{\alpha\beta\gamma\delta}, \tag{2.21}$$

$$K_{\mu\nu} = h_\mu^\sigma \nabla_\sigma n_\nu; \tag{2.22}$$

$$h_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu; \tag{2.23}$$

$$n_\mu = \frac{\partial_\mu \tilde{t}}{\sqrt{-g^{\mu\nu}\partial_\mu \tilde{t}\partial_\nu \tilde{t}}} \underbrace{=}_{\text{unitary gauge}} \frac{\partial_\mu t}{\sqrt{-g^{\mu\nu}\partial_\mu t\partial_\nu t}} = \frac{\delta_\mu^0}{\sqrt{-g^{00}}}, \tag{2.24}$$

and also the fluctuations in the EFT action are quantified as [24]:

$$\delta K_{\mu\nu} = K_{\mu\nu} - K_{\mu\nu}^{(0)} = K_{\mu\nu} - a^2 H h_{\mu\nu}, \tag{2.25}$$

$$\delta g^{00} = g^{00} - (g^{00})^{(0)} = \left(g^{00} + 1\right). \tag{2.26}$$

We are now in a position to interpret each operator appearing in the EFT action (2.11). As said earlier, the first three terms represent the background part of the EFT action. Here the first operator is the Einstein-Hilbert gravitational term, second and third term mimic the role of kinetic term and the effective potential of the matter field in EFT action. Rest of the contributions are due to Wilsonian operators in the EFT action. All these operators are characterized by the coefficients $M_n \forall n \geq 2$ and $\bar{M}_n \forall n \geq 1$. In general the coefficients M_n and $\bar{M}_n$ have time dependence. Since, in the inflationary scenario, in which we are inserted in, H and its time derivatives slowly vary with time, the EFT operators and coefficients also expected to be slowly varying with time and approximately the time translational invariance holds good.

In principle, following symmetry requirements one can write down various operators in the EFT action which include higher derivative terms of the metric and also suppressed by the UV cut off scale of the EFT, that is fixed at the Planck scale M_p. For our analysis we only consider operators upto second order fluctuation in metric, which is sufficient to extract the information of two-point function from scalar and tensor modes. However, for the computation of three and four-point function one need to consider more terms to truncate the EFT action in a consistent way.

Since in this article we will be concentrating on two-point correlation functions only, both in the context of CMB and LSS, we will consider the following truncated EFT action derived using the EFT operators (2.12) [24]:

$$\mathcal{S} = \int d^4 x \sqrt{-g} \left[\frac{1}{2}M_p^2 R - \left(\dot{H} + 3H^2\right)M_p^2 + \dot{H}M_p^2 g^{00} + \frac{M_2^4(t)}{2}(\delta g^{00})^2 \right.$$
$$\left. - \frac{\bar{M}_1^3(t)}{2}\delta g^{00}\delta K_\mu^\mu - \frac{\bar{M}_2^2(t)}{2}\left(\delta K_\mu^\mu\right)^2 - \frac{\bar{M}_3^2(t)}{2}\delta K_\mu^\nu \delta K_\nu^\mu \right]. \tag{2.27}$$

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[2]In the present context the unit normal n_μ satisfy the following sets of constraints:

$$n^\alpha \nabla_\alpha n_\mu = -\frac{1}{2g_{00}}h_\mu^\alpha \partial_\alpha g^{00}, \quad n^\mu \nabla_\sigma n_\mu = 0. \tag{2.20}$$

Additionally, the determinant of the background metric can be expressed in terms of the induced metric as, $\sqrt{-g} = \sqrt{h}/\sqrt{-g^{00}}$.

Note that this is a proper truncation as derived from the construction rules of EFT action, thereby leaving behind any ambiguity or inconsistency in the development of the theory. Thus the above EFT action is a consistent one developed from the first principles.

2.2 The role of Goldstone Boson

In this section, our prime objective is to construct a theory of Goldstone boson starting from the EFT proposed in the previous section. We will use the fact that the EFT operators and the corresponding action break temporal diffeomorphism explicitly. This is commonly known as *Stückelberg mechanism* [26, 27] in gravity where broken temporal diffeomorphism of Goldstone realizes the symmetry in non-linear way. This is equivalent to the gauge transformation in $SU(N)$ non-abelian gauge theory where one studies the contribution from the longitudinal components of a massive gauge boson. In order to establish broken temporal diffeomorphism we use the following transformations

$$t \rightarrow \tilde{t} = t + \xi^0(t, \mathbf{x}); \qquad x^i \rightarrow \tilde{x}^i = x^i, \tag{2.28}$$

where $\xi^0(t, \mathbf{x})$ is a spacetime dependent parameter which is a measure of broken temporal diffeomorphism. See ref. [24, 54] for more details.

To proceed further, we note that under these set of transformation equations the metric and its inverse components transform as [3]:

$$g^{\mu\nu} \rightarrow \tilde{g}^{\mu\nu} = \left(\frac{\partial \tilde{x}^\mu}{\partial x^\alpha}\right) \left(\frac{\partial \tilde{x}^\nu}{\partial x^\beta}\right) g^{\alpha\beta} = \left(\delta^\mu_\alpha + \delta^\mu_0 \partial_\alpha \pi\right) \left(\delta^\nu_\beta + \delta^\nu_0 \partial_\beta \pi\right) g^{\alpha\beta}, \tag{2.31}$$

$$g_{\alpha\beta} \rightarrow \tilde{g}_{\alpha\beta} = \left(\frac{\partial x^\mu}{\partial \tilde{x}^\alpha}\right) \left(\frac{\partial x^\nu}{\partial \tilde{x}^\beta}\right) g_{\mu\nu} \approx \left(\delta^\mu_\alpha - \delta^\mu_0 \partial_\alpha \pi\right) \left(\delta^\nu_\beta - \delta^\nu_0 \partial_\beta \pi\right) g_{\mu\nu}, \tag{2.32}$$

which is perfectly consistent with the following constraint condition [4]:

$$g^{\alpha\beta} g_{\beta\gamma} \rightarrow \tilde{g}^{\alpha\beta} \tilde{g}_{\beta\gamma} = \delta^\alpha_\mu \delta^\sigma_\nu \delta^\rho_\gamma g^{\mu\nu} g_{\sigma\rho} = g^{\alpha\sigma} g_{\sigma\gamma} = \delta^\alpha_\gamma. \tag{2.35}$$

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[3]It is important to note that, in the present context any general rank m contravariant and covariant tensor transformation under broken temporal diffeomorphism as:

$$B^{\mu_1\mu_2\cdots\mu_m} \rightarrow \prod_{i=1}^{m} \left(\delta^{\mu_i}_{\alpha_i} + \delta^{\mu_i}_0 \partial_{\alpha_i} \pi\right) B^{\alpha_1\alpha_2\cdots\alpha_m}, \tag{2.29}$$

$$B_{\mu_1\mu_2\cdots\mu_m} \rightarrow \prod_{i=1}^{m} (\delta^{\alpha_i}_{\mu_i} + \delta^{\alpha_i}_0 \partial_{\mu_i} \pi)^{-1} B_{\alpha_1\alpha_2\cdots\alpha_m} \tag{2.30}$$

[4]Once we use the following truncated expression for the series expansion:

$$(\delta^\mu_\alpha + \delta^\mu_0 \partial_\alpha \pi)^{-1} \approx (\delta^\mu_\alpha - \delta^\mu_0 \partial_\alpha \pi), \tag{2.33}$$

we get one more additional constraint equation as given by:

$$\left(\delta^\alpha_\mu + \delta^\alpha_0 \partial_\mu \pi\right) \left(\delta^\beta_\nu + \delta^\beta_0 \partial_\nu \pi\right) \left(\delta^\sigma_\beta - \delta^\sigma_0 \partial_\beta \pi\right) \left(\delta^\rho_\gamma - \delta^\rho_0 \partial_\gamma \pi\right) = \delta^\alpha_\mu \delta^\sigma_\nu \delta^\rho_\gamma, \tag{2.34}$$

which is a necessary constraint to satisfy Eq (2.35).

From these set of equations we get the following transformation rule of the components of the contravariant metric:

$$g^{00} \to \tilde{g}^{00} = (1 + \dot{\pi})^2 g^{00} + 2(1 + \dot{\pi}) g^{0i} \partial_i \pi + g^{ij} \partial_i \pi \partial_j \pi, \tag{2.36}$$

$$g^{0i} \to \tilde{g}^{0i} = (1 + \dot{\pi}) g^{0i} + g^{ij} \partial_j \pi, \tag{2.37}$$

$$g^{ij} \to \tilde{g}^{ij} = g^{ij}, \tag{2.38}$$

and of the covariant metric:

$$g_{00} \to \tilde{g}_{00} = (1 + \dot{\pi})^2 g_{00}, \tag{2.39}$$

$$g_{0i} \to \tilde{g}_{0i} = (1 + \dot{\pi}) g_{0i} + g_{00} \dot{\pi} \partial_i \pi, \tag{2.40}$$

$$g_{ij} \to \tilde{g}_{ij} = g_{ij} + g_{0j} \partial_i \pi + g_{i0} \partial_j \pi. \tag{2.41}$$

Having done that, let us now introduce a local field $\pi(x) \equiv \pi(t, \mathbf{x})$ identified to be the Goldstone mode that transforms under the broken temporal diffeomorphism as [24]:

$$\pi(t, \mathbf{x}) \to \tilde{\pi}(t, \mathbf{x}) = \pi(t, \mathbf{x}) - \xi^0(t, \mathbf{x}). \tag{2.42}$$

Since we work in unitary gauge for the calculations of cosmological perturbations, the gauge fixing condition is given by $\pi(x) = \pi(t, \mathbf{x}) = 0$, that results in $\tilde{\pi}(t, \mathbf{x}) = -\xi^0(t, \mathbf{x})$. It is worthwhile to note that, any time-dependent dynamical function under broken time diffeomorphism transform as

$$f(t) \to f(t + \pi) = \left[\sum_{n=0}^{\infty} \frac{\pi^n}{n!} \frac{d^n}{dt^n} \right] f(t). \tag{2.43}$$

where the Taylor series expansion is terminated in appropriate order in Goldstone mode. Note that we can truncate this expansion for two-fold reasons:

1. During inflation Hubble parameter H and its time derivative $\dot{H}$ does not change significantly so that quasi de Sitter approximation holds good. In the present computation under broken time diffeomorphism the Hubble parameter transform as:

$$H(t + \pi) = \left[\sum_{n=0}^{\infty} \frac{\pi^n}{n!} \frac{d^n}{dt^n} \right] H(t), \tag{2.44}$$

which is the Taylor series expansion of the Hubble parameter by assuming the contribution from the Goldstone modes are small. Further we use, $\epsilon = -\dot{H}/H^2$ as the Hubble slow roll parameter in terms of which Eq (2.44) can be recast as

$$H(t + \pi) = \left[1 - \pi H(t) \epsilon - \frac{\pi^2 H(t)}{2} \left(\dot{\epsilon} - 2\epsilon^2 \right) + \cdots \right] H(t), \tag{2.45}$$

In the slow roll regime of inflation $\epsilon << 1$ and one can neglect all the higher order contributions in slow roll parameter ϵ and its time derivatives. Following similar logic one can also Taylor expand the term $c(t)$, which has been done explicitly in the next subsection.

2. Let us consider the EFT Wilson coefficient M_2 which transform under broken time diffeomorphism as:

$$M_2(t + \pi) = \left[\sum_{n=0}^{\infty} \frac{\pi^n}{n!} \frac{d^n}{dt^n} \right] M_2(t) \tag{2.46}$$

For further simplification we introduce here the canonically normalized Goldstone boson, which is defined as, $\pi_c = M_2^2 \pi$, using which Eq (2.46) can be recast as:

$$M_2(t + \pi) = \left[\sum_{n=0}^{\infty} \frac{\pi_c^n}{n! M_2^{2n}} \frac{d^n}{dt^n} \right] M_2(t) \approx M_2(t). \tag{2.47}$$

It clearly implies that, if we go higher order in the Taylor series expansion then we will get additional suppression from M_2. For the other three coefficients, $\bar{M}_1$, $\bar{M}_2$ and $\bar{M}_3$ the higher order contributions in the Taylor series expansion get also suppressed by M_2 as appearing in Eq (2.47).

Further, to construct the EFT action it is important to explicitly know the transformation rule of each an every contributions under broken time diffeomorphism, which are appended below:

1. The 3-hypersurface Ricci scalar and spatial component of the Ricci tensor transform as:

$$^{(3)}R \to {}^{(3)}\tilde{R} = {}^{(3)}R + \frac{4}{a^2} H(\partial^2 \pi), \tag{2.48}$$

$$^{(3)}R_{ij} \to {}^{(3)}\tilde{R}_{ij} = {}^{(3)}R_{ij} + H(\partial_i \partial_j \pi + \delta_{ij} \partial^2 \pi), \tag{2.49}$$

where the operator $\partial^2 = \partial_k^2$.

2. The trace and spatial component of the extrinsic curvature tensor transform as:

$$\delta K \to \delta \tilde{K} = \delta K - 3\pi \dot{H} - \frac{1}{a^2} (\partial^2 \pi), \tag{2.50}$$

$$\delta K_{ij} \to \delta \tilde{K}_{ij} = \delta K_{ij} - \pi \dot{H} h_{ij} - \partial_i \partial_j \pi. \tag{2.51}$$

3. The pure time component and mixed component of the extrinsic curvature tensor transform as:

$$\delta K_0^0 \to \delta \tilde{K}_0^0 = \delta K_0^0, \tag{2.52}$$

$$\delta K_i^0 \to \delta \tilde{K}_i^0 = \delta K_i^0, \tag{2.53}$$

$$\delta K_0^i \to \delta \tilde{K}_0^i = \delta K_0^i + 2H g^{ij} \partial_j \pi. \tag{2.54}$$

2.3 Pathology of weak coupling approximation

Let us now discuss the consequences of weak coupling approximation between gravity and Goldstone bosons on the EFT action Eq (2.5). We will begin by writing down the transfor-

mation of the function $c(t)g^{00}$ of the action under broken time diffeomorphism

$$c(t)g^{00} = -\dot{H}M_p^2 g^{00} \rightarrow c(t+\pi)\tilde{g}^{00} = -M_p^2\dot{H}(t+\pi)\left[(1+\dot{\pi})^2 g^{00} + 2(1+\dot{\pi})\partial_i\pi g^{0i}\right.$$
$$\left.+ g^{ij}\partial_i\pi\partial_j\pi\right],$$
$$= c(t)\left[1 + \frac{\pi}{\epsilon}\left(\dot{\epsilon} - 2H\epsilon^2\right) + \cdots\right]\left[(1+\dot{\pi})^2 g^{00}\right.$$
$$\left.+ 2(1+\dot{\pi})\partial_i\pi g^{0i} + g^{ij}\partial_i\pi\partial_j\pi\right]. \qquad (2.55)$$

For further simplification we decompose the temporal component of the metric g^{00} into background ($\bar{g}^{00} = -1$) and perturbations (δg^{00}) as

$$g^{00} = \bar{g}^{00} + \delta g^{00}, \qquad (2.56)$$

substitute Eq (2.56) in Eq (2.55) and consider only the first term appearing in the transformation rule. It contains a Kinetic term, $M_p^2\dot{H}\dot{\pi}^2\bar{g}^{00}$ and a mixing term, $M_p^2\dot{H}\dot{\pi}\delta g^{00}$. From the mixing term we define another normalized field $\delta g_c^{00} = M_p\delta g^{00}$, in terms of which the mixing term can be recast as

$$M_p^2\dot{H}\dot{\pi}\delta g^{00} \rightarrow \sqrt{\dot{H}}\dot{\pi}_c\delta g_c^{00}. \qquad (2.57)$$

It is important to note that, this mixing term has one less derivative compared to the kinetic term. Consequently, in the energy limit, $E > E_{mix} = \sqrt{\dot{H}}$, we can neglect this contribution from the EFT action. The mixing energy scale E_{mix} is the decoupling limit above which one can completely neglect the mixing between gravity and Goldstone boson fluctuation [24, 54].

It may be mentioned here that apart from the above mixing term, one can also have other mixing contributions in the EFT action, which can be shown to be negligibly small. Consider, for example, the term $M_p^2\dot{H}\dot{\pi}^2\delta g^{00}$. By recasting it after canonical normalization as

$$M_p^2\dot{H}\dot{\pi}^2\delta g^{00} \rightarrow \frac{\dot{\pi}_c^2\delta g_c^{00}}{M_p}. \qquad (2.58)$$

it becomes obvious that this term is Planck-suppressed. Similarly, just by doing dimensional analysis we can say that further higher order contributions in $\dot{\pi}$ will lead to additional Planck-suppression in the EFT action. Consequently, we can safely consider that $M_p^2\dot{H}\dot{\pi}\delta g^{00}$ is the leading order mixing term and hence neglect any contributions from the mixing terms as long as we are working in the energy regime $E > E_{mix}$.

In addition, from the expansion of $\dot{H}$, in the decoupling limit we get terms like, $\pi M_p^2\ddot{H}\dot{\pi}\bar{g}^{00}$, which can be recast as

$$\pi M_p^2\ddot{H}\dot{\pi}\bar{g}^{00} = -\pi\dot{\pi}M_p^2 H^2\left(\dot{\epsilon} - 2H\epsilon^2\right) \rightarrow \frac{\ddot{H}}{\dot{H}}\pi_c\dot{\pi}_c\bar{g}^{00} = \left(\frac{\dot{\epsilon}}{\epsilon} - 2H\epsilon\right)\pi_c\dot{\pi}_c\bar{g}^{00}. \qquad (2.59)$$

In the slow roll regime as, $\ddot{H}/\dot{H} << 1$, we can easily neglect this contribution from the EFT action. This argument essentially implies that if we go higher in powers of π, then contributions are more suppressed by Planck scale after canonical normalization.

So, at the end of the day, considering weak coupling approximation leads to the following simplified expression for the transformation rule of the function $c(t)g^{00}$ under broken time diffeomorphism in the decoupling limit

$$c(t)g^{00} = -\dot{H}M_p^2 g^{00} \rightarrow c(t+\pi)\tilde{g}^{00} \approx c(t)g^{00}\left[\dot{\pi}^2 - \frac{1}{a^2}(\partial_i\pi)^2\right]. \qquad (2.60)$$

In the above mentioned transformation rule we use the fact that liner π terms are absent as the contributions from the tadpole diagrams are irrelevant in the EFT action [54].

However, one needs to keep in mind that if one goes slightly below the decoupling scale, the weak coupling approximation still holds good and one can then consider the effect of the leading order mixing term $M_p^2\dot{H}\dot{\pi}\delta g^{00}$ neglecting the subleading corrections. This will lead to relatively more accurate description of perturbations without violating any physical principle as such. We will elaborate on this in the next section.

This completes the formal development of the properly truncated EFT action for inflationary cosmology. Equipped with this, in the rest of the article we will engage ourselves in analysing quantum fluctuations during inflation and large scale structures therefrom. In the process we will confront our results with observations and search for consistent constraints on the EFT parameters that lead to correct power spectrum both in CMB and LSS.

3 Quantum fluctuation from Goldstone modes

3.1 Two-point correlation function

To construct the two-point correlation function from the quantum fluctuation of the Goldstone modes we have to be consistent with derivatives of Goldstone. For our computation, we will consider the contribution from the back reaction to be very small and will also drop any contributions that contain quadratic derivatives of the Goldstone mode involving space and time. Consequently, one can drop all the other EFT parameters barring $\bar{M}_1$ and M_2 from the truncated action (2.27). As a result, the second order perturbed EFT action for the decoupling regime turns out to be

$$\mathcal{S}^{(2)} = \int dt\, d^3x\, a^3 \left(\frac{\frac{\bar{M}_1^3}{2}H - M_p^2\dot{H}}{c_S^2}\right)\left[\dot{\pi}^2 - c_S^2\frac{(\partial_i\pi)^2}{a^2}\right], \qquad (3.1)$$

where the effective sound speed squared parameter is defined as

$$c_S^2 = \frac{\frac{\bar{M}_1^3}{2}H - M_p^2\dot{H}}{2M_2^4 - M_p^2\dot{H}}. \qquad (3.2)$$

See ref. [24, 30, 52, 54] for the discussion on similar issues. For further simplification one can express the spatial component of the metric fluctuation as

$$g_{ij} = a^2(t)\left[(1 + 2\zeta(t,\mathbf{x}))\,\delta_{ij} + \gamma_{ij}\right], \qquad (3.3)$$

where $\zeta(t,\mathbf{x})$ is the curvature perturbation that takes into account scalar fluctuations and the spin-2, transverse and traceless tensor γ_{ij} represent tensor fluctuations. In order to

exploit the correspondence between the Goldstone fluctuation and scalar fluctuation, we start with the transformation rule of the scale factor under broken time diffeomorphism, which can be expressed considering terms upto linear order in Goldstone mode as

$$a(t) \rightarrow \tilde{a}(t - \pi) \approx a(t)\left[1 - H\pi\right]. \tag{3.4}$$

Comparing Eq (3.3) and Eq (3.4) one readily obtains $\zeta(t, \mathbf{x}) = -H\pi(t, \mathbf{x})$ [24, 54] which shows explicitly the correspondence between curvature perturbation and Goldstone fluctuations.

Subsequently, the Mukhanov-Sasaki variable for this EFT setup can be defined as

$$v(\tau, \mathbf{x}) = -z\ \pi(\tau, \mathbf{x})HM_p, \tag{3.5}$$

where z is defined as

$$z = \left(\frac{\sqrt{2}\sqrt{\frac{\bar{M}_1^{\ 3}}{2}H - M_p^2 \dot{H}}}{c_S H M_p}\right)a = \frac{a}{c_S}\sqrt{\frac{\bar{M}_1^{\ 3}}{HM_p^2} + 2\epsilon} = 2M_2^4 - M_p^2 \dot{H}. \tag{3.6}$$

The scale factor for quasi de Sitter case can be expressed in terms of conformal time τ as $a(\tau) = -\frac{1}{H\tau}(1 + \epsilon)$. In terms of the Mukhanov-Sasaki variable $v(\tau, \mathbf{x})$ the second order perturbed EFT action (3.1) can be recast as

$$\mathcal{S}^{(2)} = \frac{1}{2}\int d\tau\ d^3x\ \left[v'^2 + v^2\left(\frac{z'}{z}\right)^2 - 2vv'\left(\frac{z'}{z}\right) - c_S^2(\partial_i v)^2\right], \tag{3.7}$$

where $(\cdots)' \equiv d/d\tau$. Following usual technique with the boundary conditions

$$\left[\frac{z'}{z}\right]_{\delta\Omega} = 0, \qquad [v(\tau, \mathbf{x})]_{\delta\Omega} = 0, \tag{3.8}$$

we arrive at the following simplified version of the EFT action for scalar perturbation

$$\mathcal{S}^{(2)} = \frac{1}{2}\int d\tau\ d^3x\ \left[v'^2 + v^2\left(\frac{z''}{z}\right) - c_S^2(\partial_i v)^2\right], \tag{3.9}$$

Further decomposition into Fourier modes of the Mukhanov-Sasaki variable $v(\tau, \mathbf{x})$

$$v(\tau, \mathbf{x}) = \int \frac{d^3k}{(2\pi)^3}v_{\mathbf{k}}(\tau)\ e^{i\mathbf{k}.\mathbf{x}} \tag{3.10}$$

leads to the expression for the perturbed EFT action for the scalar perturbation in Fourier space

$$\mathcal{S}^{(2)} = \frac{1}{2}\int d\tau\ \frac{d^3k}{(2\pi)^3}\ \left[v_{\mathbf{k}}'^2(\tau) + \left(k^2 c_S^2 - \frac{z''}{z}\right)v_{\mathbf{k}}^2(\tau)\right], \tag{3.11}$$

where $k = |\mathbf{k}| = \sqrt{\mathbf{k} \cdot \mathbf{k}}$. Hence, as in the case of standard curvature perturbations, Eq (3.11) represents an exactly parametric harmonic oscillator characterized by a time dependent frequency

$$\omega^2(k, \tau) = \left(k^2 c_S^2 - \frac{z''}{z} \right). \tag{3.12}$$

Let us now pause for a moment and take a careful look on the above EFT action (3.1) for scalar perturbations. The above action is strictly valid in the decoupling regime, i.e., at scales higher than the one at which gravity and Goldstone boson completely decouple. As discussed in Section 2.3, in this regime one can completely drop all the terms arising from mixing between gravity and Goldstone Boson. So, strictly speaking, this action will lead to results accurate upto order $\frac{H^2}{M_p^2}$ and ϵ [53, 54]. However, as discussed in [53, 54], the mass term of order $3\epsilon H^2$ coming from the leading order mixing term $M_p^2 \dot{H} \dot{\pi} \delta g^{00}$ results in a more accurate expression for spectral tilt. As observational precision improve, such as a 5-σ detection of spectral index by Planck 2015 [45, 46], our intention would be to obtain results which are accurate at least upto next order leading to a better fit with observational results. This can be materialised by modifying the action (3.1) by going slightly below the decoupling scale, where the weak coupling approximation still holds good. One can then consider the effect of the leading order mixing term neglecting the subleading corrections. As mentioned in the previous section 2.3, this will lead to relatively more accurate description of perturbations without violating any physical principle as such.

Including the leading order mixing term leads to the following expression for the second order perturbed EFT action

$$\mathcal{S}^{(2)} = \int dt \, d^3x \, a^3 \left(\frac{\frac{\bar{M}_1^3}{2} H - M_p^2 \dot{H}}{c_S^2} \right) \left[\dot{\pi}^2 - c_S^2 \frac{(\partial_i \pi)^2}{a^2} + 3\epsilon H^2 \pi^2 \right], \tag{3.13}$$

A straightforward exercise as before leads to a parametric oscillator akin to Eq. (3.11) with modified time dependent frequency

$$\omega_{eff}^2(k, \tau) = \left(k^2 c_S^2 - \frac{z''}{z} + 3(aH)^2 \epsilon \right). \tag{3.14}$$

that contains explicit dependence on the slow roll parameter ϵ, and is hence more accurate. Consequently, the equation of motion in Fourier space can be written as

$$v_{\mathbf{k}}''(\tau) + \omega_{eff}^2(k, \tau) v_{\mathbf{k}}(\tau) = 0 \tag{3.15}$$

In what follows we will restrict our calculation upto first order in slow roll and first order in $\frac{2M_2^4}{Mpl^2 H^2 \epsilon}$ for which

$$\frac{z''}{z} - 3(aH)^2 \epsilon \equiv \frac{1}{\tau^2} \left(\nu^2 - \frac{1}{4} \right) \approx \frac{1}{\tau^2} \left[2 + 3\epsilon + \frac{3}{2}\eta + \frac{2M_2^4}{M_p^2 H^2} \left(-\frac{3\eta}{2\epsilon} - \frac{11\eta}{4} + \frac{3}{2} - \frac{\eta\kappa}{4\epsilon} + \frac{\eta^2}{8\epsilon} + 3\epsilon \right) \right]. \tag{3.16}$$

For quasi de Sitter evolution of the background and considering the terms in Eq (3.16), the general solution to the equation of motion (3.15) can be readily obtained

$$v_{\mathbf{k}}(\tau) = \sqrt{-\tau}\left[\alpha H_{\nu}^{(1)}(-c_S k\tau) + \beta H_{\nu}^{(2)}(-c_S k\tau)\right],\qquad(3.17)$$

where α and β are the arbitrary integration constants and the numerical values of them are fixed by the choice of initial vacuum. For Bunch-Davies vacuum, $\alpha = \sqrt{\pi/2}$, $\beta = 0$ and the solution for the mode function finally boils down to

$$v_{\mathbf{k}}(\tau) = \sqrt{-\frac{\pi\tau}{2}}H_{\nu}^{(1)}(-c_S k\tau).\qquad(3.18)$$

where the argument for the Hankel function ν is

$$\nu = \frac{3}{2} + \frac{1}{2}\left(2\epsilon + \eta + \frac{2M_2^4}{M_p^2 H^2}\left[1 - \frac{\eta}{\epsilon} - \frac{11\eta}{6} - \frac{\eta\kappa}{6\epsilon} + \frac{\eta^2}{12\epsilon} + 2\epsilon\right]\right)\qquad(3.19)$$

with $\epsilon = -\frac{H'}{aH^2}$; $\eta = \frac{\epsilon'}{aH\epsilon}$; $\kappa = \frac{\eta'}{aH\eta}$. It shows explicit role of the EFT parameters and weak coupling pathology on the mode function that will further reflect in two point function.

To understand the behaviour of the solution let us consider two limiting cases $kc_S\tau \to -\infty$ and $kc_S\tau \to 0$, where the behaviour of the Hankel functions of the first kind are given by:

$$\lim_{kc_S\tau\to-\infty} H_{\nu}^{(1)}(-kc_S\tau) = \sqrt{\frac{2}{\pi}}\frac{1}{\sqrt{-kc_S\tau}}\ e^{-ikc_S\tau}\ e^{-\frac{i\pi}{2}\left(\nu+\frac{1}{2}\right)},\qquad(3.20)$$

$$\lim_{kc_S\tau\to0} H_{\nu}^{(1)}(-kc_S\tau) = \frac{i}{\pi}\ \Gamma(\nu)\ \left(-\frac{kc_S\tau}{2}\right)^{-\nu}.\qquad(3.21)$$

One can, in principle, analyse solutions for both super-horizon and sub-horizon modes. However, as is well-known, it is the super-horizon modes that actually freeze out and appear as scalar perturbations at a later epoch. Thus, they have the major contribution to the two-point function. So, for all practical purpose, one can simply take into account the two-point function of the super-horizon modes, which is given by

$$\langle\zeta(\tau,\mathbf{k})\zeta(\tau,\mathbf{q})\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{q})P_{\zeta}(k,\tau),\qquad(3.22)$$

where $P_{\zeta}(k,\tau)$ is the primordial power spectrum for scalar fluctuation at time τ. Written explicitly in the context of EFT, it reads

$$P_{\zeta}(k,\tau) = \frac{|v_{\mathbf{k}}(\tau)|^2}{z^2 M_p^2} = \frac{2^{2\nu-3}H^2(-kc_S\tau)^{3-2\nu}}{4c_S(1+\epsilon)^2 M_p^2\cdot\left(\frac{\bar{M}_1^3}{HM_p^2}+2\epsilon\right)}\frac{1}{k^3}\left|\frac{\Gamma(\nu)}{\Gamma\left(\frac{3}{2}\right)}\right|^2(1+k^2c_S^2\tau^2).\qquad(3.23)$$

As a result, for the relevant modes for CMB observables, the primordial power spectrum at the horizon crossing $k_* c_S\tau = -1$ takes the following form

$$P_{\zeta}(k_*) = \frac{(-c_s\tau)^3 2^{2\nu-3}H^2}{4c_S(1+\epsilon)^2 M_p^2\cdot\left(\frac{\bar{M}_1^3}{HM_p^2}+2\epsilon\right)}\left|\frac{\Gamma(\nu)}{\Gamma\left(\frac{3}{2}\right)}\right|^2 = 2^{2\nu}\frac{(-\tau)}{4\pi}(\Gamma(\nu))^2\frac{H^2}{a^2(-M_p^2\dot{H}+2M_2^4)}.\qquad(3.24)$$

As usual, one can also define a dimensionless power spectrum therefrom $\Delta_\zeta(k) = \Delta_\zeta(k_*) = \frac{1}{2\pi^2} P_\zeta(k_*)$, where k_* is the pivot scale on which $P_\zeta(k_*)$ and $\Delta_\zeta(k_*)$ are evaluated.

Consequently, the spectral tilt turns out to be

$$
\begin{aligned}
n_\zeta(k_*) - 1 &= \left[\frac{d\ln \Delta_\zeta(k)}{d\ln k}\right]_{k_* c_S \tau = -1} \\
&= (3 - 2\nu) \\
&= -\left(2\epsilon + \eta + \frac{2M_2^4}{M_p^2 H^2}\left[1 - \frac{\eta}{\epsilon} - \frac{11\eta}{6} - \frac{\eta\kappa}{6\epsilon} + \frac{\eta^2}{12\epsilon} + 2\epsilon\right]\right)
\end{aligned} \tag{3.25}
$$

One can further introduce a new set of parameters, $\epsilon_v = -\frac{\dot{H}}{H^2} \equiv \epsilon$, $\eta_v = 2\epsilon_v - \frac{\dot{\epsilon}}{H\epsilon}$ and $\kappa_v \equiv \kappa$ in terms of which the spectral tilt takes the form

$$
n_\zeta(k_*) - 1 = -\left[6\epsilon_v - 2\eta_v + \frac{M_2^4}{M_p^2 H^2}\left(-3 + \frac{8\eta_v}{\epsilon_v} - 4\epsilon_v + \frac{7\eta_v}{3} + \frac{2\kappa_v}{3} - \frac{\eta_v\kappa_v}{3\epsilon_v} + \frac{\eta_v^2}{3\epsilon_v}\right)\right], \tag{3.26}
$$

which is more akin to the form available in the literature for standard inflationary framework.

3.2 Constraining EFT parameters from CMB data

We are now in a position to put possible constraints on the EFT parameters of our consideration using inflationary parameters from CMB observations. For this let us first summarize the values of those parameters from Planck 2015 [45, 46] as follows

Planck 2015 + high L (TT) + low P :

$$
\begin{aligned}
\ln(10^{10} P_\zeta) &= 3.089 \pm 0.036 \quad (2\sigma \text{ CL}), \\
n_\zeta &= 0.9655 \pm 0.0062 \quad (3\sigma \text{ CL}), \\
0.23 &< c_S \leq 1 \quad (2\sigma \text{ CL}), \\
\epsilon &< 0.0066 \quad (2\sigma \text{ CL}), \\
\eta &= 0.030^{+0.007}_{-0.006} \quad (1\sigma \text{ CL}).
\end{aligned}
$$

As mentioned earlier, in this article our primary intention is to give a consistent theoretical description for inflation (and LSS) starting from EFT and demonstrate that it works perfectly with latest observations. We leave a detailed numerical calculations for constraining EFT by a joint analysis using CMB and LSS data for a follow-up work. In what follows we shall make use of the best fit values only. Using the best fit values of the inflationary parameters from Planck 2015 given above, we arrive at the following best fit value for the EFT parameter M_2 :

$$
M_2 = 1.35 \times 10^{16} GeV. \tag{3.27}
$$

In order to constrain the other EFT parameter $\bar{M}_1$, one notices that it has got a degeneracy with sound speed c_S via Eq (3.2). Since Planck 2015 [45] gives only a bound on c_S as $0.23 < c_S \leq 1$ (2σ CL), the best fit value of this EFT parameter $\bar{M}_1$ cannot be precisely determined using present data. One can, however, find out an allowed region for $\bar{M}_1$ using
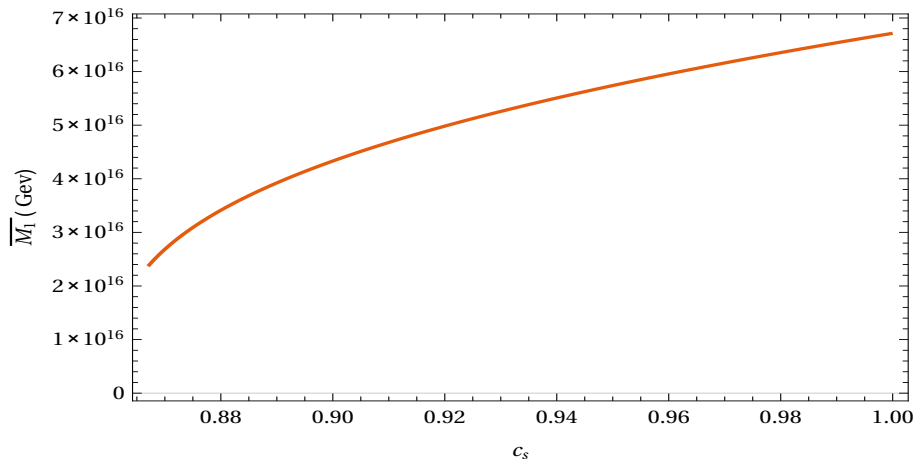
Figure 1. Variation of the EFT parameter $\bar{M}_1$ with c_S determined from Planck 2015 data [45] at the first order of slow roll.

this bound for c_S. This is what we have done here. Fig.1 shows the variation of $\bar{M}_1$ with c_S. Further theoretical constraint i.e., a real value of $\bar{M}_1$ restricts the lower bound of c_S to be 0.86. This can possibly be interpreted in two different ways. The defensive interpretation is that the present EFT analysis cannot take into account inflationary models for which $c_S < 0.86$. However, a more interesting interpretation can be that though inflationary scenario for very small c_S can be derived from EFT, in this article we are terminating the series in second order of gravitational fluctuation which only has upto second order derivative. So we are not deviating much from slow roll but adding the leading order corrections characterized by the two EFT parameters M_2 and $\bar{M}_1$. So, in this framework, one should not expect that sound speed will deviate much from 1 and it is coming naturally from the analysis.

The above values/bounds of the EFT parameters $\bar{M}_1$ and M_2 set the UV cutoff of EFT theory close to M_p, which is consistent with the bound of c_S under consideration. This has been consistently used throughout in the analysis of LSS in the next section, thereby bringing both inflation and LSS under a common EFT platform.

4 Structures from EFT

As mentioned in Introduction, a consistent EFT of inflation and LSS is still eluding us. Some earlier works of LSS from EFT considers Harrison-Zel'dovich spectrum a priori [4–6]. and analyses the scenario by simulated results, without considering any direct connection with inflation. In their approach initial power spectrum was power law type, the power n of the power spectrum was constrained by calculating the loop contribution and matching them with N-body simulation. Further developments on diverse aspects in this directions have also been reported in the Introduction. However, as is well-known, Planck 2015 [45, 46] gives $n_\zeta \neq 1$ at 5-σ CL, thereby ruling out Harrison-Zel'dovich spectrum at 5-σ CL. So, one needs to revisit the LSS scenario using those parameters. It is very important to note that a systematic study of higher order LSS perturbation starting from primordial cosmology is still left unexplored.

In addition, physics of the non-linear regime is not yet well-understood. There are couple of ambiguities that leads to degeneracies among models, in particular, at very small scales, and a fairly clear understanding of non-linear regime is yet to be achieved. Keeping in mind these ambiguities, we would try to explore the possibility of applicability of the present EFT framework to LSS, in particular, to non-linear regime. In other words, we would like to investigate how the loop-corrected results from EFT fare with the uncertainties of non-linear regime.

Our fundamental inputs in the entire analysis of LSS from EFT are as follows:

- As stated, we will not consider Harrison-Zel'dovich spectrum a priori. Rather, we will carry forward the same power spectrum as obtained from EFT of inflation in the previous section. This will in turn take into account the effect of spectral tilt in the analysis of LSS, which is more accurate given present observational scenario.

- We will also be consistent with the constraints on the EFT parameters $\bar{M}_1$ and M_2 as given by CMB data, thereby making the entire analysis self-consistent and coming from a single EFT with same cutoff throughout.

4.1 Linear and non-linear regimes: transfer function

For completeness, let us briefly describe our choice of transfer functions used in the analysis of LSS from EFT. At large scales, i.e., in the linear regime, the power spectrum is given by the primordial power spectrum with slight spectral tilt as derived earlier. At relatively smaller scales, where the non-linear regime sets in, the behaviour of perturbations can be manifestly written in the language of a momentum dependent fitting function, called transfer function $T(k)$, that basically transfers the primordial power to smaller scales. This can be expressed in terms of the gravitational potential as

$$\Phi(a, \mathbf{k}) = \frac{3}{2} \frac{H_0^2}{ak^2} \Omega_m \delta_g(a, \mathbf{k}) = \frac{9}{10} \Phi_{\text{prim}}(\mathbf{k}) T(k) \frac{D_g(a)}{a}, \tag{4.1}$$

where $\Phi_{\text{prim}}(\mathbf{k})$ is the primordial potential and $D_g(a)$ is the growth function. Written explicitly,

$$D_g(a) = \frac{5}{2} \Omega_m \frac{H(a)}{H_0} \int_0^a \left(\frac{H_0}{\tilde{a} H(\tilde{a})} \right)^3 d\tilde{a}. \tag{4.2}$$

For flat, matter dominated universe it simply boils down to a.

The matter overdensities can then be conveniently written as

$$\delta_g(a, \mathbf{k}) = \frac{2}{3} \Phi(a, \mathbf{k}) \frac{ak^2}{\Omega_m H_0^2} = -\frac{2}{3} \zeta(\tau, \mathbf{k}) \frac{ak^2}{\Omega_m H_0^2} = \frac{3}{5} \Phi_{\text{prim}}(\mathbf{k}) T(k) D_g(a) \frac{k^2}{\Omega_m H_0^2}. \tag{4.3}$$

An age-old transfer function is given by [47]

$$T(k) = \begin{cases} 1 & \text{for } k_{IR} < k < k_{eq} \\ 12 \left(\frac{k_{eq}}{k} \right)^2 \ln \left(\frac{k}{8k_{eq}} \right), & \text{for } k_{eq} < k < k_{UV}. \end{cases} \tag{4.4}$$

where k_{eq} is the corresponding momentum scale for matter-radiation equality which can be expressed in terms of matter abundance $(\Omega_m h^2)$ as:

$$k_{eq} = a_{eq} H(a_{eq}) = 0.073 \text{ Mpc}^{-1} \; \Omega_m h^2. \tag{4.5}$$

However, the above transfer function is not too useful as it has a discontinuity near k_{eq}. Further, given latest data, one needs to have a more accurate fitting function for the same that can describe the observable universe more accurately. Couple of good fitting functions are available in the literature that can more or less successfully serve the purpose.

As it will be revealed in the next section, an essential feature of our analysis based on EFT compared to the analysis based on simulated results is that in our analysis the two-point correlation function is computed by a loop-by-loop calculations, the effects of which may play a non-trivial role in changing the shape of power spectrum (compared to the one from standard perturbation theory). So, one cannot say a priori which fitting function would be able to describe the observable universe more accurately. Rather, one needs to figure out the role of each transfer function separately, or may even need to propose new fitting function for it.

In our analysis of LSS, we will make use of two most widely accepted transfer functions, namely the BBKS [48] and Eisenstein-Hu [49] transfer function, thereby making the analysis robust. This will help us do a comparative analysis of the viability of those fitting functions in the context of EFT for LSS.

The BBKS transfer function [48] dates back to 1986. It was proposed to resolve the discontinuity issue of the previous fitting function and to give a better fit than the previous one. The form of BBKS transfer function is given by

$$T_{\text{BBKS}} \left(x \equiv \frac{k}{k_{eq}} \right) = \frac{\ln[1 + 0.171x]}{0.171x} \left[1 + 0.284x + (1.18x)^2 + (0.399x)^3 + (0.490x)^4 \right]^{-0.25} \tag{4.6}$$

It can be readily checked that this transfer function gives rise to an approximately scale-independent power spectrum for linear regime and the deviations arise at relatively large value of x and k. So, it takes into account a smooth transition from linear to non-linear regimes without facing any discontinuity in between.

The second transfer function of our consideration was proposed by Eisenstein-Hu (EH hereafter) in 1998 by taking into account perturbation of Baryonic part as well. Written in a compact form, it looks [49]

$$T_{\text{EH}}(k) = \frac{\Omega_b}{\Omega_c} T_b(k) + \frac{\Omega_c}{\Omega_0} T_c(k) \tag{4.7}$$

where, Ω_b and Ω_c are baryon and cold dark matter (CDM) density and $T_b(k)$ and $T_c(k)$ are baryon and CDM transfer functions respectively. The individual components have rather complicated forms. Elaborating further, the CDM transfer function with different components can be written as

$$T_c(k) = f\tilde{T}_0(k, 1, \beta_c) + (1 - f)\tilde{T}_0(k, \alpha_c, \beta_c) \tag{4.8}$$

where

$$\tilde{T}_0(k, \alpha_c, \beta_c) = \frac{\ln(e + 1.8\beta_c q)}{\ln(e + 1.8\beta_c q + Cq^2)} \tag{4.9}$$

and the rest of the components are given by :

$$f = \frac{1}{1 + (\frac{ks}{5.4})^4}, \quad C = \frac{14.2}{\alpha_c} + \frac{386}{1 + 69.9q^{1.08}}, \quad q = \frac{k}{13.41k_{eq}},$$

$$\alpha_c = a_1^{\Omega_b/\Omega_0} a_2^{(\Omega_b/\Omega_0)^3}, \quad \beta_c^{-1} = 1 + b_1[(\frac{\Omega_c}{\Omega_0})^{b_2} - 1],$$

$$a_1 = (46.9\Omega_0 h^2)^0.67[1 + (32.1\Omega_0 h^2)^{-0.532}], \quad a_2 = (12.0\Omega_0 h^2)^{0.424}[1 + (45.0\Omega_0 h^2)^{-0.582}],$$

$$b_1 = 0.944[1 + (458\Omega_0 h^2)^{-0.708}]^{-1}, \quad b_2 = (0.395\Omega_0 h^2)^{-0.0266}.$$

Similarly, the baryonic counterpart of this transfer function can be obtained by collecting all the contributions

$$T_b = \left(\frac{\tilde{T}_0(k,1,1)}{1 + (\frac{ks}{2})^2} + \frac{\alpha_b}{1 + (\frac{\beta_b}{ks})^3} \exp[-(\frac{k}{k_{silk}})^{1.4}] \right) j_0(k\tilde{s}) \tag{4.10}$$

Here j_0 is Bessel function, s is sound horizon at the drag epoch and

$$\alpha_b = 2.07k_{eq}s(1 + R_d)^{(-3/4)}G(1 + z_{eq}/(1 + z_d)),$$

$$G(y) = y[-6\sqrt{1+y} + (2 + 3y)\ln(\frac{\sqrt{1+y}+1}{\sqrt{1+y}-1})],$$

$$\beta_b = 0.5 + \frac{\Omega_b}{\Omega_0} + (3 - 2\frac{\Omega_b}{\Omega_0})\sqrt{(17.2\Omega_0 h^2)^2 + 1},$$

$$\tilde{s}(k) = \frac{s}{[1 + (\frac{\beta_{node}}{ks})^3]^{1/3}}, \quad \beta_{node} = 8.41(\Omega_0 h^2)^{0.435}.$$

Though like BBKS, this transfer function too takes into account both linear and non-linear regimes, it provides relatively better fit due to the inclusion of Baryonic physics.

4.2   Loop calculation technique for matter overdensity two-point function

We are now in a position to discuss the loop calculation rules for computing the two-point correlation function for matter overdensities. Using Eq (4.3) one can express the two-point function for the matter overdensities as

$$\langle \delta_g(\tau, \mathbf{k})\delta_g(\tau, \mathbf{q}) \rangle = \frac{4}{9}\langle \Phi(\tau, \mathbf{k})\Phi(\tau, \mathbf{q}) \rangle \frac{a^2 k^4 q^2}{\Omega_m^2 H_0^4}$$

$$= \frac{9}{25}\langle \Phi_{\text{prim}}(\mathbf{k})\Phi_{\text{prim}}(\mathbf{q}) \rangle T(k)T(q)D_g^2(a)\frac{k^4 q^2}{\Omega_m^2 H_0^4}$$

$$= (2\pi)^3 \delta^{(3)}(\mathbf{k} + \mathbf{q})P_{\delta_g}(k, \tau), \tag{4.11}$$

This cumulative two-point function for overdensities can be expressed as a sum of all possible loop contribution as:

$$\langle \delta_g(\tau, \mathbf{k})\delta_g(\tau, \mathbf{q}) \rangle = \sum_{n=0}^{\infty} \langle \delta_g(\tau, \mathbf{k})\delta_g(\tau, \mathbf{q}) \rangle^{(n)}. \tag{4.12}$$

Here the superscript n stands for n-loop contribution to the matter power spectrum. This loop corrected two-point function can be re-expressed in a more physical manner as a sum of all possible contributions coming from auto correlations and cross correlations for different order of expansions as:

$$\langle \delta_g(\tau, \mathbf{k}) \delta_g(\tau, \mathbf{q}) \rangle = \sum_{i,j=1, i+j=\mathbf{even}}^{\infty} \langle \delta_g^{(i)}(\tau, \mathbf{k}) \delta_g^{(j)}(\tau, \mathbf{q}) \rangle \tag{4.13}$$

where the matter overdensity field has been defined as

$$\delta_g(\tau, \mathbf{k}) = \sum_{j=1}^{\infty} \delta_g^{(j)}(\tau, \mathbf{k}). \tag{4.14}$$

The superscript j stands for the contribution from the j-th order term in the expansion of overdensity field. The physical restriction $i + j = even$ leads to

$$i + j = 2m = \mathbf{even}, \quad \forall m = 1(\mathbf{Tree}), 2(\mathbf{1-loop}), 3(\mathbf{2-loop}), \cdots \tag{4.15}$$

and other contributions identically vanishes if we consider Gaussian initial conditions. However, if one allows the contribution from the non-Gaussianity in the initial condition then one needs to consider all possible contributions from auto correlations and cross correlations.

Since the tree level is the dominant contribution, we would restrict our analysis upto one-loop only. So, considering the contributions upto one-loop in the non-linear perturbation we get the following simplified expansion for the total two-point function :

$$\langle \delta_g(\tau, \mathbf{k}) \delta_g(\tau, \mathbf{q}) \rangle = \sum_{n=0}^{1} \langle \delta_g(\tau, \mathbf{k}) \delta_g(\tau, \mathbf{q}) \rangle^{(n)}$$
$$= \left[\langle \delta_g(\tau, \mathbf{k}) \delta_g(\tau, \mathbf{q}) \rangle^{(0)} + \langle \delta_g(\tau, \mathbf{k}) \delta_g(\tau, \mathbf{q}) \rangle^{(1)} + \cdots \right], \tag{4.16}$$

where the individual contributions are given by

$$\langle \delta_g(\tau, \mathbf{k}) \delta_g(\tau, \mathbf{q}) \rangle^{(0)} = \langle \delta_g(\tau, \mathbf{k}) \delta_g(\tau, \mathbf{q}) \rangle_{\mathbf{Tree}}$$
$$= \langle \delta_g^{(1)}(\tau, \mathbf{k}) \delta_g^{(1)}(\tau, \mathbf{q}) \rangle, \tag{4.17}$$
$$\langle \delta_g(\tau, \mathbf{k}) \delta_g(\tau, \mathbf{q}) \rangle^{(1)} = \langle \delta_g(\tau, \mathbf{k}) \delta_g(\tau, \mathbf{q}) \rangle_{\mathbf{1-loop}}$$
$$= \langle \delta_g^{(2)}(\tau, \mathbf{k}) \delta_g^{(2)}(\tau, \mathbf{q}) \rangle + 2 \langle \delta_g^{(1)}(\tau, \mathbf{k}) \delta_g^{(3)}(\tau, \mathbf{q}) \rangle, \tag{4.18}$$
$$\cdots\cdots\cdots \quad \cdots\cdots\cdots\cdots\cdots .$$

Here in Eq (4.18) a factor of 2 is appearing due to the symmetry in the cross correlations

$$\langle \delta_g^{(i)}(\tau, \mathbf{k}) \delta_g^{(j)}(\tau, \mathbf{q}) \rangle = \langle \delta_g^{(j)}(\tau, \mathbf{k}) \delta_g^{(i)}(\tau, \mathbf{q}) \rangle \quad \forall i, j = 1, 2, \cdots \text{with } i \neq j. \tag{4.19}$$

Consequently, the total matter power spectrum for the overdensity field

$$P_{\delta_g}(k, \tau) = \sum_{n=0}^{\infty} P_{\delta_g}^{(n)}(k, \tau) = \sum_{i,j=1, i+j=\mathbf{even}}^{\infty} P_{\delta_g}^{(ij)}(k, \tau). \tag{4.20}$$

reduces to the following truncated power spectrum upto one-loop correction:

$$P_{\delta_g}(k,\tau) = \sum_{n=0}^{1} P_{\delta_g}^{(n)}(k,\tau) = \left[P_{\delta_g}^{(0)}(k,\tau) + P_{\delta_g}^{(1)}(k,\tau) \right]. \tag{4.21}$$

Finally, comparing Eq (4.20) and Eq (4.21), and collecting the contributions from tree level and one loop separately, one arrives at

$$P_{\delta_g}^{(0)}(k,\tau) = P_{\delta_g}^{\textbf{Tree}}(k,\tau) = P_{\delta_g}^{(11)}(k,\tau), \tag{4.22}$$

$$P_{\delta_g}^{(1)}(k,\tau) = P_{\delta_g}^{\textbf{1-loop}}(k,\tau) = P_{\delta_g}^{(22)}(k,\tau) + 2P_{\delta_g}^{(13)}(k,\tau) \tag{4.23}$$

Note that, likewise one can also derive the weight functions for smoothing in the context of EFT of LSS. Since our intention in this article is not to deal with errors explicitly, we refrain from doing so.

The above two are the master equations in computing, loop by loop, two-point correlation function for matter overdensities from EFT using the primordial power spectrum derived in Sections 3.1 and 3.2 for the transfer functions of our choice from equations (4.6) and (4.7). In the rest of the article we will engage ourselves in this.

4.3 Tree level results for two-point function

In this subsection we look into only $n = 0$ contribution in the loop expansion for the matter power spectrum for the overdensity field. By fixing $n = 0$ in the two-point function for the matter overdensity, we get

$$\langle \delta_g(\tau, \mathbf{k}) \delta_g(\tau, \mathbf{q}) \rangle^{(0)} = \langle \delta_g(\tau, \mathbf{k}) \delta_g(\tau, \mathbf{q}) \rangle_{\textbf{Tree}} = (2\pi)^3 \delta^{(3)}(\mathbf{k} + \mathbf{q}) P_{\delta_g}^{(0)}(k,\tau) \tag{4.24}$$

or equivalently one can write

$$\langle \delta_g^{(1)}(\tau, \mathbf{k}) \delta_g^{(1)}(\tau, \mathbf{q}) \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k} + \mathbf{q}) P_{\delta_g}^{(11)}(k,\tau). \tag{4.25}$$

Plugging the primordial power spectrum into Eq (4.3), and subsequently, collecting the tree level contribution as above, one arrives at the matter power spectrum for the overdensity field. Separating the time dependent and momentum dependent parts of the resulting power spectrum, one can express it in a convenient way as below:

$$P_{\delta_g}^{(0)}(k,\tau) = P_{11} = \mathcal{G}_{\text{EFT}} \; k^{4-2\nu} T(k)^2 = \mathcal{G}_{\text{EFT}}(\tau, c_S) \; k^{1-2u} T(k)^2 \tag{4.26}$$

Written explicitly, the term $\mathcal{G}_{\text{EFT}}(\tau, c_S)$ reads

$$\mathcal{G}_{\text{EFT}}(\tau, c_S) = \frac{4}{25} \frac{1}{\Omega_{m0}^2 H_0^4} \left(\frac{c_s}{2} \right)^{(-2\nu)} \frac{(-\tau)^{1-2\nu}}{4\pi} (\Gamma(\nu))^2 \frac{H^2}{a^2(-M_p^2 \dot{H} + 2M_2^4)}. \tag{4.27}$$

It shows explicitly the effect of EFT parameters $\bar{M}_1$ and M_2 on LSS. We remind the reader that we take here the same values/bounds for those parameters as derived from EFT of inflation using CMB. Here the momentum dependent function $T(k)$ is/are the transfer

function(s) of our consideration, thereby bringing the entire scenario of inflation and LSS under a common EFT theory.

In the above expression of power spectrum, we have also introduced a new variable

$$2u = (3 - 2\nu) = \left(2\epsilon + \eta + \frac{2M_2^4}{M_p^2 H^2}\left[1 - \frac{\eta}{\epsilon} - \frac{11\eta}{6} - \frac{\eta\kappa}{6\epsilon} + \frac{\eta^2}{12\epsilon} + 2\epsilon\right]\right) \tag{4.28}$$

As has been pointed out in this article, our intention is to carry the primordial information forward to structure formation. So, the best fit value of u should have to be obtained from the best fit value of the primordial spectral tilt. Using the result of Planck 2015 [45, 46] as given in Section 3.2, the best fit value for u turns out to be 0.016 which we will use in numerical computations in the subsequent sections.

As mentioned earlier, we will take into account two separate transfer functions, namely, BBKS and Eisenstein-Hu. Hence, in order to get the tree level results, all we are left with is to plug them in separately from equations (4.6) and (4.7) and calculate the tree level power spectrum in Eq (4.26). This has been done subsequently and the behaviour for tree level power spectra has been explicitly shown in plots in Figs. 2 and 3 respectively by the green lines. We will discuss more on this in Section 4.5.

4.4   One loop corrections from EFT

In this subsection we look into only $n = 1$ contribution in the loop expansion for the matter power spectrum for the overdensity field for which the power spectrum looks

$$\langle\delta_g(\tau, \mathbf{k})\delta_g(\tau, \mathbf{q})\rangle^{(1)} = \langle\delta_g(\tau, \mathbf{k})\delta_g(\tau, \mathbf{q})\rangle_{\mathbf{1-loop}} = (2\pi)^3\delta^{(3)}(\mathbf{k} + \mathbf{q})P_{\delta_g}^{(1)}(k, \tau) \tag{4.29}$$

By decomposing the above equation using (4.18), we obtain

$$\begin{aligned}
\langle\delta_g(\tau, \mathbf{k})\delta_g(\tau, \mathbf{q})\rangle^{(1)} &= \langle\delta_g^{(2)}(\tau, \mathbf{k})\delta_g^{(2)}(\tau, \mathbf{q})\rangle + 2\langle\delta_g^{(1)}(\tau, \mathbf{k})\delta_g^{(3)}(\tau, \mathbf{q})\rangle \\
&= (2\pi)^3\delta^{(3)}(\mathbf{k} + \mathbf{q})\left[P_{\delta_g}^{(22)}(k, \tau) + 2P_{\delta_g}^{(13)}(k, \tau)\right],
\end{aligned} \tag{4.30}$$

where we use the following results:

$$\langle\delta_g^{(2)}(\tau, \mathbf{k})\delta_g^{(2)}(\tau, \mathbf{q})\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k} + \mathbf{q})P_{\delta_g}^{(22)}(k, \tau), \tag{4.31}$$

$$\langle\delta_g^{(1)}(\tau, \mathbf{k})\delta_g^{(3)}(\tau, \mathbf{q})\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k} + \mathbf{q})P_{\delta_g}^{(13)}(k, \tau). \tag{4.32}$$

In terms of momentum kernels, the above expressions for the one-loop contributions can be written as [5]

$$P_{\delta_g}^{(13)}(k, \tau) = 6\int d^3q \, F_3^s(k, q, -q) \, P_{\delta_g}^{(11)}(k, \tau)P_{\delta_g}^{(11)}(q, \tau), \tag{4.33}$$

$$P_{\delta_g}^{(22)}(k, \tau) = 2\int d^3q \, [F_2^s(\mathbf{k} - \mathbf{q}, \mathbf{q})]^2 \, P_{\delta_g}^{(11)}(|\mathbf{k} - \mathbf{q}|, \tau)P_{\delta_g}^{(11)}(q, \tau). \tag{4.34}$$

Here $F_n^{(s)}$ are symmetrized kernels of order n which satisfy the following properties:

1. Using the momentum conservation in COM coordinate one can write

$$\mathbf{k} = \sum_{i=1}^{n} \mathbf{q}_i. \tag{4.35}$$

When $\mathbf{k} \to 0$ the individual momentum satisfy additional constraint condition $\mathbf{q}_i \neq 0 \ \forall i = 1, \cdots, n$. Consequently, the momentum dependence of the symmetrized kernels of order n can be quantified as

$$\lim_{\mathbf{k} \to 0, \ \mathbf{q}_i \neq 0 \ \forall i=1,\cdots,n} F_n^{(s)} \propto k^2. \tag{4.36}$$

2. When the total momentum $\mathbf{k}$ is fixed to a certain value, it is possible that some of the individual momentum arguments are large and for $|\mathbf{p}| >> |\mathbf{q}_i| \ \forall i = 1, \cdots, n$ limiting case the momentum dependence of the symmetrized kernels of order n can be quantified as

$$\lim_{|\mathbf{p}|>>|\mathbf{q}_i| \ \forall i=1,\cdots,n} F_n^{(s)} \propto k^2/p^2. \tag{4.37}$$

3. If one of the individual momentum goes to zero i.e. $\mathbf{q}_i \to 0$, then in this limiting case the momentum dependence of the symmetrized kernels of order n can be quantified as

$$\lim_{\mathbf{q}_i \to 0} F_n^{(s)} \propto \lim_{\mathbf{q}_i \to 0} \mathbf{q}_i/|\mathbf{q}_i|^2 \to \text{IR divergence.} \tag{4.38}$$

For $n = 2$ and $n = 3$ the structures of the kernels can be expressed as [2, 10]

$$F_2^s(\mathbf{k} - \mathbf{q}, \mathbf{q}) = \frac{5}{7} + \frac{1}{2} \frac{(\mathbf{k} - \mathbf{q}).\mathbf{q}}{|\mathbf{k} - \mathbf{q}||\mathbf{q}|} \left(\frac{|\mathbf{k} - \mathbf{q}|}{|\mathbf{q}|} + \frac{|\mathbf{q}|}{|\mathbf{k} - \mathbf{q}|} \right) + \frac{2}{7} \frac{((\mathbf{k} - \mathbf{q}).\mathbf{q})^2}{|\mathbf{k} - \mathbf{q}|^2 |\mathbf{q}|^2}, \tag{4.39}$$

$$\begin{aligned}
F_3^s(k, q, -q) = &\frac{1}{|\mathbf{k} - \mathbf{q}|^2} \left[\frac{5k^2}{126} - \frac{11\mathbf{k}.\mathbf{q}}{108} + \frac{7(\mathbf{k}.\mathbf{q})^2}{108k^2} - \frac{k^2(\mathbf{k}.\mathbf{q})^2}{54q^4} + \frac{4(\mathbf{k}.\mathbf{q})^3}{189q^4} \right. \\
&\left. - \frac{23k^2\mathbf{k}.\mathbf{q}}{756q^2} + \frac{25(\mathbf{k}.\mathbf{q})^2}{252q^2} - \frac{2(\mathbf{k}.\mathbf{q})^3}{27k^2q^2} \right] \\
&+ \frac{1}{|\mathbf{k} + \mathbf{q}|^2} \left[\frac{5k^2}{126} + \frac{11\mathbf{k}.\mathbf{q}}{108} - \frac{7(\mathbf{k}.\mathbf{q})^2}{108k^2} - \frac{4k^2(\mathbf{k}.\mathbf{q})^2}{27q^4} - \frac{53(\mathbf{k}.\mathbf{q})^3}{189q^4} \right. \\
&\left. + \frac{23k^2\mathbf{k}.\mathbf{q}}{756q^2} - \frac{121(\mathbf{k}.\mathbf{q})^2}{756q^2} - \frac{5(\mathbf{k}.\mathbf{q})^3}{27k^2q^2} \right].
\end{aligned} \tag{4.40}$$

In order to evaluate those integrals, one needs to express all the terms in terms of ultraviolet cutoff k_{uv} of the EFT. Rescaling things in terms of the parameter $x = k/k_{uv}$ the one loop contribution can be written in the following way

$$P_{\delta_g}^{(ij)}(k, \tau) = \mathcal{G}_{\text{EFT}} \ k_{uv}^{5-2u} p_{\delta_g,1}^{(ij)}(x, \Lambda) T(x, k_{uv})^2, \tag{4.41}$$

where $\mathcal{G}_{\text{EFT}}$ is defined as in the previous subsection. We also want to introduce some other dimensionless quantities, $t = q/k_{eq}$ for internal momentum, $\Lambda = k_{uv}/k_{IR}$ for ultraviolet cutoff and $\lambda = \frac{\mathbf{k}.\mathbf{q}}{|\mathbf{k}||\mathbf{q}|}$ for the angles between two momentum vectors $\mathbf{k}$ and $\mathbf{q}$. Here the range $k_{IR} < k < k_{UV}$ can be translated in terms of x and Λ as, $1/\Lambda < x < 1$.

In the above, the two different transfer functions of our consideration should also have to be recast consistently. In terms of these new set of dimensionless variables $\{x, \ t\}$, the BBKS transfer function (4.6) $T_{\mathrm{BBKS}}(x, k_{uv})$ reads

$$T_{\mathrm{BBKS}}(x, k_{uv}) = \frac{k_{eq} \log\left[\frac{0.171 k_{uv}}{k_{eq}} x + 1\right]}{0.171 k_{uv} x \left[\left(\frac{0.49 k_{uv}}{k_{eq}} x\right)^4 + \left(\frac{0.399 k_{uv}}{k_{eq}} x\right)^3 + \left(\frac{1.18 k_{uv}}{k_{eq}} x\right)^2 + \frac{0.284 k_{uv}}{k_{eq}} x + 1\right]^{0.25}} \quad (4.42)$$

Similarly, the Eisenstein-Hu transfer function (4.7) can also be recast as

$$T(x, k_{uv}) = \frac{\Omega_b}{\Omega_c} T_b(x, k_{uv}) + \frac{\Omega_c}{\Omega_0} T_c(x, k_{uv}) \quad (4.43)$$

where,

$$T_c(x, k_{uv}) = f \tilde{T}_0(x k_{uv}, 1, \beta_c) + (1 - f) \tilde{T}_0(x k_{uv}, \alpha_c, \beta_c) \quad (4.44)$$

and

$$T_b(x, k_{uv}) = \left(\frac{\tilde{T}_0(x k_{uv}, 1, 1)}{1 + (\frac{x k_{uv} s}{2})^2} + \frac{\alpha_b}{1 + (\frac{\beta_b}{x k_{uv} s})^3} \exp[-(\frac{x k_{uv}}{k_{silk}})^{1.4}]\right) j_0(x k_{uv} \tilde{s}). \quad (4.45)$$

Consequently, in terms of of dimensionless variables $\{x, \ t\}$, the one loop contributions to the matter power spectrum read

$$p_{\delta_g}^{(13)}(x, \Lambda) = \int_{1/\Lambda}^1 dt \ G(x, t, \Lambda) \qquad p_{\delta_g}^{(22)}(x, \Lambda) = \int_{1/\Lambda}^1 dt \ K(x, t, \lambda), \quad (4.46)$$

where, for general momentum dependence within the interval $1/\Lambda < x < 1$, we introduce here two integral kernels $G(x, t, \Lambda)$ and $K(x, t, \lambda)$, which have got rather clumsy expressions. Still, for the sake of completeness, let us write down the explicit expressions for those kernels:

$$G(x, t, \Lambda) = t^{3-2u} x^{1-2u} T(x, k_{uv})^2 T(t, k_{uv})^2$$
$$\times \left(\frac{\left((t^2 - x^2)^3 (7t^2 + 2x^2)\right) \ln\left(\frac{|t+x|}{|x-t|}\right)}{42 t^5 x^3} + \frac{-21t^6 + 50t^4 x^2 - 79t^2 x^4 + 6x^6}{63 t^4 x^2}\right) (4.47)$$

$$K(x, t, \lambda) = \int_{\lambda_{min}}^{\lambda_{max}} d\lambda \frac{1}{49} x^4 t^{1-2u} T(t, k_{uv})^2 T(\sqrt{x^2 + t^2 - 2xt\lambda}, k_{uv})^2$$
$$\times \left(-10k^2 t + 7\lambda x + 3t\right)^2 \left(-2\lambda t x + t^2 + x^2\right)^{-u-\frac{3}{2}}. \quad (4.48)$$

The cutoff dependence of the power spectrum results in the constraints on λ_{min} and λ_{max} which can be given as

$$\lambda_{\min} = \mathrm{Max}\left\{-1, \frac{x^2 + t^2 - 1}{2xt}\right\}, \qquad \lambda_{\max} = \mathrm{Min}\left\{-1, \frac{x^2 + t^2 - \Lambda^{-2}}{2xt}\right\}. \quad (4.49)$$

We are finally in a position to evaluate the integrals and subsequently, find out the cumulative contribution for one-loop correction to the power spectrum. Precisely, our job

is to plug the above two rescaled transfer functions (4.42) and (4.43) in the expressions of the individual loops and compute the kernel integrals using (4.47) and (4.48) to find out the one-loop contribution to the power spectrum for the two different transfer functions. Once again, we have analysed that numerically. The behaviour of the one-loop corrected power spectra have been shown in Figs. 2 and 3 respectively by the red lines. We will elaborate on this in the next subsection.

4.5 Corroboration with LSS data and discussions

Let us now engage ourselves in investigating how the loop corrected results from EFT fare with LSS data. The tree level computations for Section 4.2 are more or less easy to handle. However, evaluating the integrals for one-loop correction are a bit tricky to the complicated structures of the kernels.

We have succeeded in evaluating the integrals numerically. For numerical integrations, we have set the values of the relevant parameters as: $\Omega_{m0} = 0.315$ and $h = 0.6731$. Let us recall that while computing the consistent EFT parameters in Section 3.2, we have restricted to the region of sound speed which does not deviate much from 1. Therefore, the-ultra violet cutoff coming from effective field theory of inflation is of the order of M_p. However, since we will be working on mildly non-linear regime, we can set our cutoff at a relatively lower scale $k_{uv} \sim 10 hMpc^{-1}$. Further, the infrared cutoff has been set at $k_{IR} = 0.001 hMpc^{-1}$ which is also consistent with our previous computations.

Figures 2 and 3 depict the behaviour of one-loop corrected power spectra with respect to the corresponding tree level power spectra for two different transfer functions of our consideration. Fig. 2 deals with BBKS transfer function while Fig. 3 represents the one by Eisenstein-Hu. In both the figures, green lines represent the tree level power spectrum as derived from Section 4.2 whereas the red lines represent the one-loop corrected power spectrum. Obviously, by 'one-loop corrected', we mean the cumulative effect of tree level (Section 4.2) and one-loop calculations (Section 4.4).
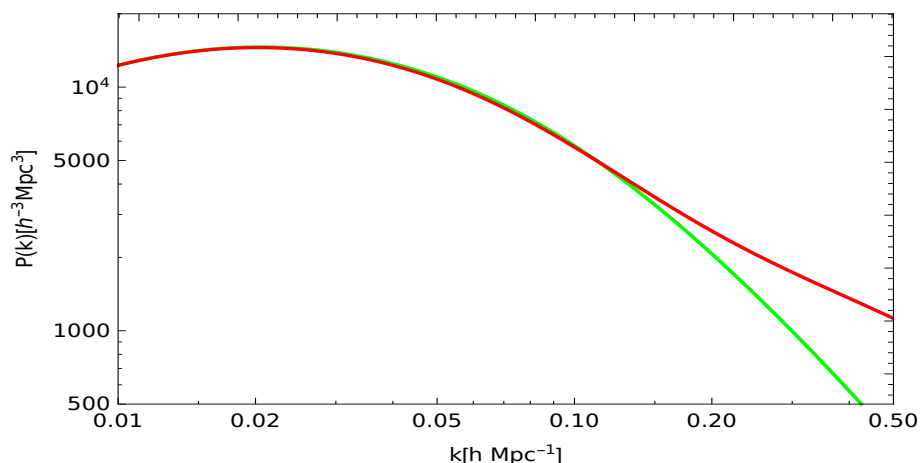

Figure 2. Power spectrum as computed with BBKS transfer function. Green line denotes linear power spectrum while red line denotes one loop corrected power spectrum. We have used $k_{eq} = 0.073 \Omega_{m0} h^2 Mpc^{-1}$, $k_{uv} = 10 h^{-1} Mpc$ and $k_{IR} = 0.001 h^{-1} Mpc$ as a result $\Lambda = 10000$
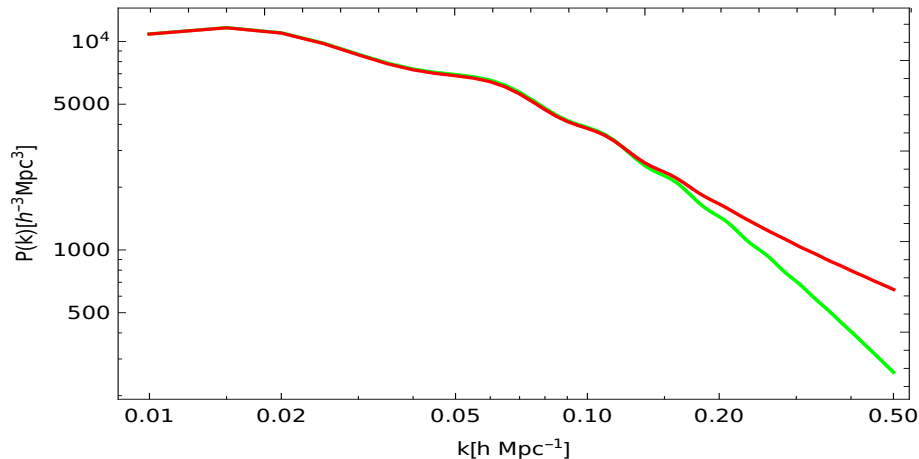
Figure 3. Power spectrum as computed with Eisenstein-Hu transfer function. Blue line denotes linear power spectrum while green line denotes one loop corrected power spectrum. We used $k_{uv} = 10h^{-1}Mpc$ and $k_{IR} = 0.001h^{-1}Mpc$ as a result Λ=10000

As it appears from the plots, for the linear regime, there is no significant contribution from one-loop corrections and tree level results are the dominant contribution for both the transfer functions. This is not much surprising, as the transfer functions were proposed keeping in mind that they behave pretty close to $T(k) \sim k$ in the linear regime so as to fit the observational data and we use a primordial power spectrum with the EFT parameters which are consistent with CMB data. This in turn validates our EFT both for inflation and LSS at linear regime. Nevertheless, as in the case of standard perturbations, in EFT too the EH transfer function explains more accurately the wiggles at moderately non-linear regime compared to BBKS, as the former include Baryonic perturbations as well.

However, for non-linear regimes, the contributions from one-loop corrections for both the transfer functions are found to be non-trivial. Precisely, in both the cases, the contributions are enhancing the power at small scales (roughly at $k \geq 0.15$) but at different level. Thus, at these scales, they are expected to show interesting features.

In Fig. 4 we do a more realistic analysis by comparing our results with two different datasets, namely, SDSS DR7 [50] and WiggleZ [51] data. They are represented by blue and magenta colours respectively. As there is no significant deviation in the linear regime, we concentrate here only on the non-linear regime. As is well-known, there is a slight difference between the amplitudes of SDSS and Wiggle Z data because of different galaxies they probe and different window functions they use. In our analysis, in order to derive LSS from a consistent EFT of inflation, we are using CMB normalization. As a result, the amplitude of the plots for two different transfer functions (BBKS : red, EH: black) are slightly apart keeping the shapes intact. Hence, even though the loop-corrected results for BBKS transfer function show proximity with SDSS-DR7 whereas for EH transfer function are closer with WiggleZ data, it might be just an artifact. To be honest, one cannot comment from this result alone whether or not this is a real improvement. For this one needs to do error analysis for the entire EFT scenario of our consideration using both CMB and LSS data. From the present analysis, all one can say is that, like the standard perturbation technique, both the transfer functions fare equally well in the context of EFT too.
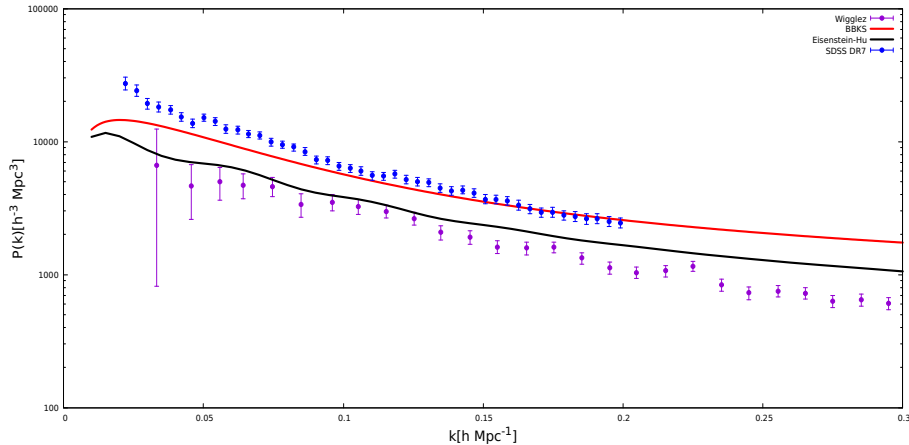
Figure 4. Comparison the one-loop corrected power spectra for LSS with Wiggle Z and SDSS-DR7 LRG data. The red and black lines represent the power spectra at non-linear regime for BBKS and Eisenstein-Hu transfer functions respectively. The blue and magenta plots represent actual data points with error bars from SDSS-DR7 and WiggleZ respectively.

What one can readily conclude from the Fig. 4 is that the theoretical power spectrum plots as calculated from EFT for both the transfer functions resemble the nature of observational plots. This, at the first place, proves the validity of our EFT for LSS as well, thereby providing a consistent EFT theory of inflation and LSS under a common umbrella. This was our major point of interest in this article and we hope that we have succeeded in convincing the reader of our proximate goal. We leave a detailed numerical calculations for constraining EFT by joint analysis using CMB and LSS data for a follow-up work.

5   Summary and outlook

In this article we have proposed a consistent Effective Field Theory that can successfully explain inflation to Large Scale Structures. The EFT we derive is consistently developed with proper truncation. With this EFT, we have studied quantum fluctuation for Goldstone bosons and computed the two-point correlation function resulting in the primordial power spectrum and spectral tilt. These observable parameters are found to be completely described by couple of parameters of the theory. We have then constrained these EFT parameters using latest CMB data from Planck 2015.

Next, we have carried the same EFT forward in formulating a theory for LSS by calculating the loop corrections in the two-point correlation function for matter overdensities, taking into account linear as well as mildly non-linear regimes. Analysis of LSS has two fundamental inputs, one, the primordial power spectrum as obtained from the EFT of inflation, and two, a proper transfer function that takes care of the momentum transfer. Due to the fact that in this scenario the two-point correlation function is derived by a loop-by-loop calculations, the effects of which may play a non-trivial role in changing the shape of power spectrum, one cannot say a priori which transfer function would be able to describe the observable universe more accurately. So, we have made use of two widely accepted fitting functions, namely the BBKS and Eisenstein-Hu transfer functions, thereby making the

analysis robust. Using them separately, we have computed the one-loop corrected matter power spectrum numerically and subsequently confronted the results with LSS data from SDSS-DR7 and WiggleZ. The theoretical power spectrum plots for both the transfer functions resemble the nature of observational plots. This, at the first place, proves the validity of our EFT framework in the context of LSS as well, thereby providing a consistent EFT theory for inflation to LSS. This was the major success of this article.

In this article our primary intention was to give a consistent theoretical description for inflation and LSS starting from EFT and to demonstrate that it corroborates perfectly with latest observations. Having convinced ourselves of a consistent theory of inflation all the way to LSS, one can now plan to look beyond the theoretical sectors and do a rigorous analysis to search for the possible improvements, if any, of the EFT framework compared to the standard perturbation theory. This can be achieved by probing deep inside non-linear regime where the role of loop corrections become more severe. One can also try to investigate which one between the two transfer functions used in this article fit better the observational data in the EFT framework. Also, in the literature there are couple of other transfer functions that are relatively more useful that BBKS and Eisenstein-Hu in explaining non-linear features. It will be interesting to see how loop calculations starting from EFT improve the results for those transfer functions. A joint analysis of the EFT using both CMB and LSS data to find out the combined constraints on the model parameters is already in progress. We are also planning to explore the non-linear regime using other transfer functions. We hope to report on some of these aspects in future.

Acknowledgments

References

[1] F. Bernardeau, S. Colombi, E. Gaztanaga, R. Scoccimarro, Phys. Rept. 367 (2002) 1-248 [arXiv:astro-ph/0112551]

[2] M. H. Goroff, B. Grinstein, S. J. Rey, M. B. Wise, Astrophys.J. 311 (1986) 6-14

[3] R. Scoccimarro, J. Frieman, Astrophys.J.Suppl. 105 (1996) 37 [arXiv:astro-ph/9509047]

[4] B. Jain, E. Bertschinger, Astrophys.J. 431 (1994) 495 [arXiv:astro-ph/9311070]

[5] R. Scoccimarro, J. Frieman, Astrophys.J. 473 (1996) 620 [arXiv:astro-ph/9602070]

[6] N. Makino, M. Sasaki, Y. Suto, Phys. Rev. D 46 (1992) 585

[7] M. Crocce, R. Scoccimarro, Phys.Rev.D73 (2006) 063519 [arXiv:astro-ph/0509418]

[8] J. Joseph, M. Carrasco, M. P. Hertzberg, L. Senatore, JHEP 9 (2012) 82. [arXiv:1206.2926 [astro-ph.CO]]

[9] E. Pajer, M. Zaldarriaga, JCAP 08 (2013) 037 [arXiv:1301.7182 [astro-ph.CO]]

[10] J. Joseph, M. Carrasco, S. Foreman, D. Green, L. Senatore, JCAP 07 (2014) 056 [arXiv:1304.4946 [astro-ph.CO]]

[11] T. Baldauf, L. Mercolli, M. Mirbabayi, E. Pajer, JCAP 05 (2015) 007 [arXiv:1406.4135 [astro-ph.CO]]

[12] L. Senatore, JCAP 11 (2015) 007 [arXiv:1406.7843 [astro-ph.CO]]

[13] V. Assassi, D. Baumann, E. Pajer, Y. Welling, D.v.d. Woude, JCAP 11 (2015) 024 [arXiv:1505.06668 [astro-ph.CO]]

[14] M. Lewandowski, A. Maleknejad, L. Senatore, JCAP 05 (2017) 038 [arXiv:1611.07966 [astro-ph.CO]]

[15] A. Perko, L. Senatore, E. Jennings, R. H. Wechsler [arXiv:1610.09321 [astro-ph.CO]]

[16] M. Lewandowski, L. Senatore [arXiv:1701.07012 [astro-ph.CO]]

[17] L. F. Bella, D. Regan, D. Seery, S. Hotchkiss [arXiv:1704.05309 [astro-ph.CO]]

[18] D. Munshi, D. Regan [arXiv:1705.07866 [astro-ph.CO]]

[19] M. P. Hertzberg, Phys. Rev. D **89** (2014) no.4, 043521 [arXiv:1208.0839 [astro-ph.CO]].

[20] M. Pietroni, G. Mangano, N. Saviano, M. Viel, JCAP 01 (2012) 019 [arXiv:1108.5203 [astro-ph.CO]]

[21] A. Manzotti, M. Peloso, M. Pietroni, M. Viel and F. Villaescusa-Navarro, JCAP 09 (2014) 047 [arXiv:1407.1342 [astro-ph.CO]]

[22] J. Martin, C. Ringeval, V. Vennin, [astro-ph/1303.3787]

[23] J. Martin et. al., JCAP 03 (2014) 039 [astro-ph/1312.3529]

[24] C. Cheung, P. Creminelli, A.L. Fitzpatrick, J. Kaplan, L. Senatore, JHEP 03 (2008) 014 [arXiv:0709.0293 [hep-th]].

[25] S. Weinberg, Phys. Rev. D 77 (2008) 123541 [arXiv:0804.4291 [hep-th]].

[26] E. C. G. Stueckelberg, Helv. Phys, Acta 11 (1938) 225-244.

[27] E. C. G. Stueckelberg. Helv. Phys, Acta 11 (1938) 299-312.

[28] L. Senatore, M.Zaldarriaga, JHEP 04 (2012) 024. [arXiv:1009.2093]

[29] T. Noumi, M. Yamaguchi, D. Yokoyama, JHEP 06 (2013) 051. [arXiv:1211.1624 [hep-th]]

[30] L. Senatore, K. M. Smith, M. Zaldarriaga, JCAP 01 (2010) 028. [arXiv:0905.3746 [astro-ph.CO]]

[31] R. Gwyn, G. A. Palma, M. Sakellariadou, S. Sypsas JCAP 04 (2013) 004, [arXiv:1210.3020 [hep-th]].

[32] J. Gong, M. Seo, S. Sypsas, JCAP 03 (2015) 009 [arXiv:1407.8268 [hep-th]].

[33] L. Senatore, M. Zaldarriaga, JCAP 01 (2011) 003 [arXiv:1004.1201 [hep-th]].

[34] P. Creminelli, G. D'Amico, M. Musso, J. Norena, E. Trincherini, JCAP 02 (2011) 006 [arXiv:1011.3004 [hep-th]].

[35] Y. Cai, Y. Wan, H. Li, T. Qiu, Y Piao, JHEP 01 (2017) 090 [arXiv:1610.03400 [gr-qc]].

[36] L. V. Delacretaz, V. Gorbenko, L. Senatore, JHEP 03 (2017) 063 [arXiv:1610.04227 [hep-th]].

[37] Y. Cai, H. Li, T. Qiu, Y Piao, Eur.Phys.J. C 77 (2017) 369 [arXiv:1701.04330 [gr-qc]].

[38] J. Gong, M. Yamaguchi, Phys. Rev. D 95, 083510 (2017) [arXiv:1701.05875 [astro-ph.CO]].

[39] O. Ozsoy, J. T. Giblin Jr., E. Nesbit, G. Sengor, S. Watson [arXiv:1701.01455 [hep-th]].

[40] T. Rostami, A. Karami, H. Firouzjahi [arXiv:1702.03744 [astro-ph.CO]].

[41] H. Motohashi, W. Hu [arXiv:1704.01128 [hep-th]].

[42] L. Bordin, G. Cabass, P. Creminelli, F. Vernizzi [arXiv:1706.03758 [astro-ph.CO]].

[43] F. Piazza, F. Vernizzi, 2013 Class. Quantum Grav. 30 (2013) 214007 [arXiv:1307.4350 [hep-th]].

[44] G. Gubitosi, F. Piazza, F. Vernizzi, JCAP 02 (2013) 032 [arXiv:1210.0201 [hep-th]].

[45] P. A. R. Ade et. al. [Planck collaboration], A&A 594, A20 (2016) [arXiv:1502.02114].

[46] P. A. R. Ade et. al. [Planck collaboration], A&A 594, A13 (2016) [arXiv:1502.01589].

[47] S. Weinberg, Cosmology, Oxford Univ. Press (2008).

[48] J. M. Bardeen, J. R. Bond, N. Kaiser, A. S. Szalay, Astrophys.J. 304 (1986). 15B

[49] D. Eisenstein, W. Hu, Astrophys.J. 496 (1998) 605E.

[50] A. B. Reid et. al., Mon. Not. R. Astron. Soc. 404 (2010) 60-85. [arXiv:0907.1659[astro-ph.CO]]

[51] D. Parkinson et. al., Phys.Rev.D 86 (2012) 103518 [1210.2130[astro-ph.CO]].

[52] Quantum Field Theory in de Sitter space, Lecture by D. Baumann, http://www.damtp.cam.ac.uk/user/db275/TEACHING/ICTP/QFTinDS.pdf

[53] D. Baumann, D. Green, JCAP 09 (2011) 014 [arXiv:1102.5343 [hep-th]].

[54] C. Cheung, A. L. Fitzpatrick, J. Kaplan, L. Senatore, JCAP 02 (2008) 021 [arXiv:0709.0295 [hep-th]].